IB Math AI HL — Topic 3

Geometry Toolkit, Coordinate Geometry, Perpendicular Bisectors & Arcs and Sectors

Coordinate Geometry · Perpendicular Bisectors · Arcs & Sectors (Degrees) · Radian Measure · Arcs & Sectors (Radians)

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Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Marks each	3	4	5
Total Marks	30	40	50

Grand Total: 120 marks

Name:_____ Date:_____

Score:_____/120

Instructions: Answer all 30 questions. Show all working clearly. Give answers correct to 3 significant figures unless stated otherwise. A GDC is permitted throughout. Diagrams are not to scale unless stated.

Key Formulae

Distance: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **Midpoint:** $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ **Gradient:** $m = \frac{y_2 - y_1}{x_2 - x_1}$

Perp. gradient: $m_\perp = -\frac{1}{m}$ **Line:** $y - y_1 = m(x - x_1)$

Arc length (degrees): $l = \frac{\theta}{360} \times 2\pi r$ **Sector area (degrees):** $A = \frac{\theta}{360} \times \pi r^2$

Radian conversion: $\theta\,(\text{rad}) = \theta\,(\text{deg}) \times \frac{\pi}{180}$ 2π rad $= 360°$

Arc length (radians): $l = r\theta$ **Sector area (radians):** $A = \frac{1}{2}r^2\theta$

GDC Tips

- **Angle mode:** Always check your GDC is in the correct mode (DEG or RAD) before computing trig functions or arc/sector values. A wrong mode gives completely incorrect answers.
- **Distance & midpoint:** Store coordinates as variables if reusing them to avoid rounding errors in multi-step questions.
- **Simultaneous equations:** Use the Equation Solver or graph intersection to find where two perpendicular bisectors meet (circumcentre).

- **Sector fractions:** For degrees, always enter $\theta/360$; for radians, use $\theta/(2\pi)$ if using the fraction approach, or simply $A = \frac{1}{2}r^2\theta$ directly.
- π **button:** Use the GDC's π key for full precision; never approximate $\pi \approx 3.14$ mid-calculation.

Common Mistakes to Avoid

1. **Perpendicular gradient:** The perpendicular gradient is $-1/m$, not $-m$. If the original gradient is 2, the perpendicular is $-\frac{1}{2}$, not -2.
2. **Midpoint before bisector:** The perpendicular bisector *must* pass through the midpoint of the segment. Always find the midpoint first.
3. **Degrees vs radians:** Ensure your GDC mode matches the question. Radians give arc length $l = r\theta$ directly; degrees require the $\theta/360$ fraction.
4. **Sector area vs triangle area:** Area of a sector is $\frac{1}{2}r^2\theta$; do not confuse with the triangle area $\frac{1}{2}r^2 \sin\theta$.
5. **Perimeter of sector:** The perimeter includes *two radii plus the arc*: $P = 2r + r\theta = r(2+\theta)$. Students often forget the radii.
6. **Distance formula:** Square both differences before adding — never write $\sqrt{(x_2 - x_1) + (y_2 - y_1)}$.

Section A — Easy EASY (Questions 1–10, 3 marks each = 30 marks)

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1. EASY Calculator [3 marks]

Point A has coordinates $(1,\ 4)$ and point B has coordinates $(7,\ 12)$.

(a) Find the distance AB. [1]

(b) Find the midpoint M of AB. [2]

2. EASY Calculator [3 marks]

A straight path passes through the points $P(2,\ 1)$ and $Q(6,\ 9)$.

(a) Find the gradient of PQ. [1]

(b) Write down the equation of the line through P and Q. [2]

3. EASY Calculator [3 marks]

Points $C(0,\ 2)$ and $D(8,\ 6)$ are the endpoints of a line segment. Find the equation of the perpendicular bisector of CD.

4. EASY Calculator [3 marks]

A circular garden feature has radius 5 m. A sprinkler at the centre irrigates a sector of angle $72°$.

(a) Find the arc length of the irrigated sector. [2]

(b) Write down the fraction of the full circle covered by the sprinkler. [1]

5. EASY Calculator [3 marks]

A sector of a circle has radius 9 cm and angle $120°$.

(a) Find the area of the sector. [2]

(b) Find the arc length of the sector. [1]

6. EASY Calculator [3 marks]

(a) Convert $210°$ to radians. Give your answer as an exact multiple of π. [1]

(b) Convert $\frac{5\pi}{4}$ radians to degrees. [1]

(c) An arc of a circle with radius 6 cm subtends an angle of $\frac{5\pi}{4}$ radians at the centre. Find the arc length. [1]

7. EASY Calculator [3 marks]

A sector of a circle has radius 4 cm and central angle $\dfrac{3\pi}{4}$ radians.

(a) Find the area of the sector. [2]

(b) Find the arc length of the sector. [1]

8. EASY Calculator [3 marks]

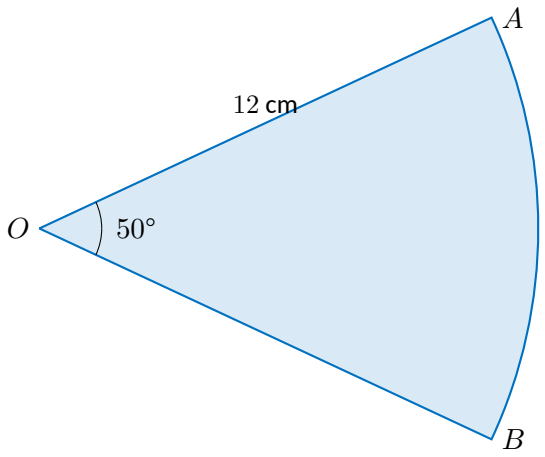
A sector of a circle has radius 10 m and central angle 0.8 radians. Find the perimeter of the sector.

9. EASY Calculator [3 marks]

A windscreen wiper sweeps through an angle of 1.2 radians. The blade has length 0.35 m. Find the area swept by the wiper blade.

10. EASY Calculator [3 marks]

The diagram shows a sector OAB of a circle with centre O, radius 12 cm, and angle $AOB = 50°$.



(a) Find the area of the sector OAB. [2]

(b) Find the arc length AB. [1]

Section B — Medium MEDIUM (Questions 11–20, 4 marks each = 40 marks)

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11. MEDIUM Calculator **[4 marks]**

Points $A(-3,\ 1)$, $B(5,\ 5)$ and $C(3,\ -3)$ form a triangle.

(a) Find the length of side AB. [1]

(b) Find the midpoint M of BC. [1]

(c) Find the gradient of the median AM. [2]

12. MEDIUM Calculator **[4 marks]**

A line ℓ_1 passes through $R(1,\ 7)$ and $S(5,\ 3)$.

(a) Find the gradient of ℓ_1. [1]

(b) A second line ℓ_2 is perpendicular to ℓ_1 and passes through $S(5,\ 3)$. Find the equation of ℓ_2. [3]

13. MEDIUM Calculator **[4 marks]**

Points $E(2,\ 9)$ and $F(10,\ 3)$ are marked on a coordinate grid.

(a) Find the gradient of EF. [1]

(b) Find the midpoint of EF. [1]

(c) Find the equation of the perpendicular bisector of EF, giving your answer in the form $y = mx + c$. [2]

14. MEDIUM Calculator **[4 marks]**

Triangle PQR has vertices $P(0,\ 6)$, $Q(8,\ 6)$ and $R(4,\ 0)$.

(a) Write down the equation of the perpendicular bisector of PQ. [1]

(b) Find the equation of the perpendicular bisector of QR. [2]

(c) Find the circumcentre of triangle PQR. [1]

15. MEDIUM Calculator **[4 marks]**

A decorative window is in the shape of a sector of angle $135°$ and radius 0.8 m.

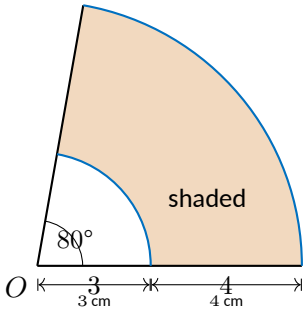
(a) Find the arc length of the window's curved edge. [2]

(b) Find the total perimeter of the window. [1]

(c) Find the area of glass needed for the window. [1]

16. MEDIUM Calculator **[4 marks]**

The diagram shows two sectors, both centred at O. The inner sector has radius 3 cm and the outer sector has radius 7 cm. Both sectors have angle $\theta = 80°$.



Find the area of the shaded region (between the two sectors). **[4 marks]**

17. MEDIUM Calculator **[4 marks]**

A sector has arc length 15 cm and radius 6 cm.

(a) Find the central angle, in radians. [1]

(b) Convert your answer to degrees, giving your answer to 3 significant figures. [1]

(c) Find the area of the sector. [2]

18. MEDIUM Calculator **[4 marks]**

A sector of radius r cm and angle 1.4 radians has an area of 44.1 cm^2.

(a) Show that $r^2 = 63$. [2]

(b) Hence find r, giving your answer to 3 significant figures. [1]

(c) Find the perimeter of the sector. [1]

19. MEDIUM Calculator **[4 marks]**

A playground roundabout has an inner safety rail of radius 1.2 m and an outer edge of radius 2.0 m. Both radii subtend the same central angle of $\frac{4\pi}{3}$ radians at the centre of the roundabout.

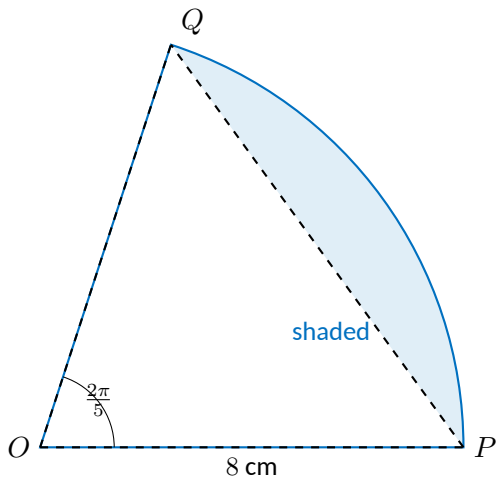
(a) Find the length of the inner rail arc. [1]

(b) Find the length of the outer edge arc. [1]

(c) Find the area of the annular sector (the region between the two arcs). [2]

20. MEDIUM Calculator **[4 marks]**

The diagram shows a sector OPQ of a circle with centre O, radius 8 cm and central angle $\frac{2\pi}{5}$ radians. Triangle OPQ is also drawn.



(a) Find the area of sector OPQ. [1]

(b) Find the area of triangle OPQ. [2]

(c) Find the shaded area (sector minus triangle). [1]

Section C — Hard HARD (Questions 21–30, 5 marks each = 50 marks)

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21. HARD Calculator **[5 marks]**

Triangle ABC has vertices $A(1,\ 1)$, $B(7,\ 1)$ and $C(7,\ 9)$.

(a) Write down the equation of the perpendicular bisector of AB. [1]

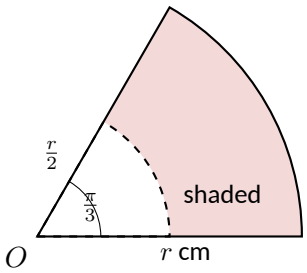
(b) Write down the equation of the perpendicular bisector of BC. [1]

(c) Hence find the circumcentre of triangle ABC. [1]

(d) Show that the circumradius equals 5 units. [2]

22. HARD Calculator **[5 marks]**

A decorative tile consists of a large sector of radius r cm and angle $\frac{\pi}{3}$ radians, with a smaller sector of radius $\frac{r}{2}$ cm and the same angle cut from its corner (as shown). The shaded area of the tile is 75π cm^2.



(a) Show that the shaded area equals $\frac{\pi r^2}{8}$. [3]

(b) Hence find the value of r. [2]

23. HARD Calculator [5 marks]

A quadrilateral field has vertices $J(0,\,0)$, $K(6,\,4)$, $L(10,\,0)$ and $M(6,\,-4)$ on a coordinate grid where 1 unit $= 10$ m.

(a) Show that $JK = JM$. [2]

(b) Verify that JL and KM are perpendicular to each other. [2]

(c) Find the area of the field in m^2. [1]

24. HARD Calculator [5 marks]

Point G has coordinates $(2,\,k)$ and point H has coordinates $(8,\,2)$. The gradient of GH is $-\frac{1}{2}$.

(a) Find the value of k. [2]

(b) Find the equation of the line through G and H. [1]

(c) Find the distance GH. [2]

25. HARD Calculator **[5 marks]**

Points $S(1,\ 2)$ and $T(9,\ 6)$ are given.

(a) Find the equation of the perpendicular bisector ℓ of ST. [3]

(b) The line $y = 2x - 4$ intersects ℓ at point V. Find the coordinates of V. [2]

26. HARD Calculator **[5 marks]**

Points $X(p,\ 3)$ and $Y(7,\ p)$ are such that the perpendicular bisector of XY has gradient 2.

(a) Find the value of p. [3]

(b) Find the equation of the perpendicular bisector of XY. [2]

27. HARD Calculator [5 marks]

A sector of a circle has radius r cm and central angle θ radians. The arc length equals the chord length PQ.

The chord length formula is $PQ = 2r\sin\left(\frac{\theta}{2}\right)$.

(a) Show that $r\theta = 2r\sin\left(\frac{\theta}{2}\right)$ and hence that $\theta = 2\sin\left(\frac{\theta}{2}\right)$. [2]

(b) Use your GDC to solve $\theta = 2\sin\left(\frac{\theta}{2}\right)$ for $0 < \theta < \pi$, giving your answer correct to 3 significant figures. [2]

(c) Hence find the area of the sector if $r = 8$ cm. [1]

28. HARD Calculator [5 marks]

In a sector of radius r cm and angle θ radians, the perimeter of the sector is 30 cm and the area of the sector is 50 cm^2.

(a) Write down two equations in r and θ. [2]

(b) Show that $r^2 - 15r + 50 = 0$. [2]

(c) Find the two possible values of r and the corresponding values of θ. [1]

29. HARD Calculator [5 marks]

Points $A(0,\ 0)$, $B(12,\ 0)$ and $C(6,\ 6\sqrt{3})$ form an equilateral triangle.

(a) Find the side length AB. [1]

(b) The circumcentre of the triangle is at W. Find the coordinates of W. [2]

(c) A circle centred at W passes through A, B and C (the circumcircle). Find the area of one of the three equal sectors formed by the radii WA, WB, WC, giving your answer in exact form. [2]

30. HARD Calculator [5 marks]

Two lighthouses $L_1(2,\ 10)$ and $L_2(14,\ 4)$ are positioned on a coordinate map, where units are in kilometres. A coastguard patrol boat is always equidistant from both lighthouses and moves along their perpendicular bisector.

(a) Find the equation of the path of the patrol boat. [3]

(b) A radar with range 7.5 km is placed at the point on the patrol boat's path where $x = 5$. Find the area of the sea covered by the radar (the full circle). [2]

Worked Solutions

Official Mark-Scheme Style (M/A/R notation)

Section A — Easy

Q1 — Distance and Midpoint [EASY]

(a) $AB = \sqrt{(7-1)^2 + (12-4)^2} = \sqrt{36+64} = \sqrt{100}$ **M1**

$AB = 10$ **A1**

(b) $M = \left(\frac{1+7}{2}, \frac{4+12}{2}\right) = (4,\ 8)$ **A1**

Note: Award A1 for both coordinates correct. Award A0 if parentheses omitted.

Q2 — Equation of Line [EASY]

(a) $m = \frac{9-1}{6-2} = \frac{8}{4} = 2$ **A1**

(b) $y - 1 = 2(x-2)$ **M1**

$y = 2x - 3$ **A1**

Note: Accept equivalent forms, e.g. $2x - y - 3 = 0$. FT from their gradient in (a).

Q3 — Perpendicular Bisector [EASY]

Midpoint of CD: $\left(\frac{0+8}{2}, \frac{2+6}{2}\right) = (4,\ 4)$ **A1**

Gradient of CD: $m = \frac{6-2}{8-0} = \frac{4}{8} = \frac{1}{2}$ **(A1)**

Perpendicular gradient: $m_{\perp} = -2$ **M1**

Equation: $y - 4 = -2(x-4) \implies y = -2x + 12$ **A1**

Note: Award final A1 for any correct equivalent form.

Q4 — Arc Length (Degrees) [EASY]

(a) $l = \frac{72}{360} \times 2\pi \times 5 = \frac{1}{5} \times 10\pi$ **M1**

$l = 2\pi \approx 6.28\,\text{m}$ **A1**

(b) $\dfrac{72}{360} = \dfrac{1}{5}$ **A1**

Q5 — Sector Area and Arc Length (Degrees) [EASY

] **(a)** $A = \dfrac{120}{360} \times \pi \times 9^2 = \dfrac{1}{3} \times 81\pi$ **M1**

$A = 27\pi \approx 84.8\,\text{cm}^2$ **A1**

(b) $l = \dfrac{120}{360} \times 2\pi \times 9 = \dfrac{1}{3} \times 18\pi = 6\pi \approx 18.8\,\text{cm}$ **A1**

Q6 — Radian Conversion and Arc Length [EASY

] **(a)** $210° \times \dfrac{\pi}{180} = \dfrac{7\pi}{6}$ radians **A1**

(b) $\dfrac{5\pi}{4} \times \dfrac{180}{\pi} = 225°$ **A1**

(c) $l = r\theta = 6 \times \dfrac{5\pi}{4} = \dfrac{15\pi}{2} \approx 23.6\,\text{cm}$ **A1**

Q7 — Sector Area and Arc Length (Radians) [EASY

] **(a)** $A = \dfrac{1}{2}r^2\theta = \dfrac{1}{2} \times 4^2 \times \dfrac{3\pi}{4} = \dfrac{1}{2} \times 16 \times \dfrac{3\pi}{4}$ **M1**

$A = 6\pi \approx 18.8\,\text{cm}^2$ **A1**

(b) $l = r\theta = 4 \times \dfrac{3\pi}{4} = 3\pi \approx 9.42\,\text{cm}$ **A1**

Q8 — Perimeter of Sector (Radians) [EASY

] Arc length $= r\theta = 10 \times 0.8 = 8\,\text{m}$ **M1**

Perimeter $= 2r + \text{arc} = 2 \times 10 + 8 = 28\,\text{m}$ **A1**

Note: Award M1 for correct arc formula; A1 for adding two radii.

Q9 — Area Swept by Wiper (Radians) [EASY

] $A = \dfrac{1}{2}r^2\theta = \dfrac{1}{2} \times (0.35)^2 \times 1.2$ **M1**

$= \dfrac{1}{2} \times 0.1225 \times 1.2 = 0.0735\,\text{m}^2$ **A1**

Note: No FT for using diameter instead of radius.

Q10 — Sector Area and Arc (Degrees) [EASY

] **(a)** $A = \frac{50}{360} \times \pi \times 12^2 = \frac{50}{360} \times 144\pi$ **M1**

$= 20\pi \approx 62.8\,\text{cm}^2$ **A1**

(b) $l = \frac{50}{360} \times 2\pi \times 12 = \frac{50}{360} \times 24\pi = \frac{10\pi}{3} \approx 10.5\,\text{cm}$ **A1**

Section B — Medium

Q11 — Distance, Midpoint and Median Gradient [MED]

(a) $AB = \sqrt{(5-(-3))^2 + (5-1)^2} = \sqrt{64+16} = \sqrt{80} = 4\sqrt{5} \approx 8.94$ **A1**

(b) $M = \left(\frac{5+3}{2}, \frac{5+(-3)}{2}\right) = (4,\ 1)$ **A1**

(c) Gradient of $AM = \frac{1-1}{4-(-3)} = \frac{0}{7} = 0$ **M1 A1**

Note: The median AM is horizontal. Award M1 for substituting their M and $A(-3, 1)$.

Q12 — Perpendicular Line [MED

] **(a)** $m_1 = \frac{3-7}{5-1} = \frac{-4}{4} = -1$ **A1**

(b) Perpendicular gradient $m_\perp = 1$ **M1**

Through $S(5, 3)$: $y - 3 = 1(x - 5)$ **M1**

$y = x - 2$ **A1**

Note: FT from their m_1 for the perpendicular gradient.

Q13 — Perpendicular Bisector Equation [MED

] **(a)** $m_{EF} = \frac{3-9}{10-2} = \frac{-6}{8} = -\frac{3}{4}$ **A1**

(b) Midpoint $= \left(\frac{2+10}{2}, \frac{9+3}{2}\right) = (6,\ 6)$ **A1**

(c) Perpendicular gradient $= \frac{4}{3}$ **(M1)**

$y - 6 = \frac{4}{3}(x - 6) \implies y = \frac{4}{3}x - 2$ **A1**

Note: Accept equivalent forms. FT from their midpoint and gradient.

Q14 — Circumcentre [MED

] **(a)** PQ is horizontal (both have $y = 6$); perpendicular bisector is $x = 4$ **A1**

(b) Midpoint of QR: $\left(\frac{8+4}{2}, \frac{6+0}{2}\right) = (6,\ 3)$ **(A1)**

Gradient of QR: $m = \frac{0-6}{4-8} = \frac{-6}{-4} = \frac{3}{2}$; perp. gradient $= -\frac{2}{3}$ **(A1)**

Bisector of QR: $y-3=-\frac{2}{3}(x-6) \implies y=-\frac{2}{3}x+7$ **A1**

(c) Solve $x=4$ and $y=-\frac{2}{3}(4)+7=-\frac{8}{3}+7=\frac{13}{3}$ **A1**

Circumcentre $=\left(4,\ \frac{13}{3}\right)$ (or $(4,\ 4.33)$)

Q15 — Arc, Perimeter and Area of Window (Degrees) [MED]

(a) $l=\frac{135}{360}\times 2\pi\times 0.8=\frac{3}{8}\times 1.6\pi=0.6\pi\approx 1.88\,\text{m}$ **M1 A1**

(b) Perimeter $=2r+l=2(0.8)+0.6\pi=1.6+0.6\pi\approx 3.48\,\text{m}$ **A1**

(c) $A=\frac{135}{360}\times\pi\times(0.8)^2=\frac{3}{8}\times 0.64\pi=0.24\pi\approx 0.754\,\text{m}^2$ **A1**

Q16 — Annular Sector Area (Degrees) [MED

] Area of outer sector $=\frac{80}{360}\times\pi\times 7^2=\frac{2}{9}\times 49\pi=\frac{98\pi}{9}$ **M1**

Area of inner sector $=\frac{80}{360}\times\pi\times 3^2=\frac{2}{9}\times 9\pi=2\pi$ **M1**

Shaded area $=\frac{98\pi}{9}-2\pi=\frac{98\pi-18\pi}{9}=\frac{80\pi}{9}\approx 27.9\,\text{cm}^2$ **A1 A1**

Note: First A1 for correct setup of difference; second A1 for numerical answer.

Q17 — Finding Angle from Arc Length (Radians) [MED

] **(a)** $l=r\theta \implies \theta=\frac{l}{r}=\frac{15}{6}=2.5$ radians **A1**

(b) $2.5\times\frac{180}{\pi}=\frac{450}{\pi}\approx 143°$ **A1**

(c) $A=\frac{1}{2}r^2\theta=\frac{1}{2}\times 36\times 2.5=45\,\text{cm}^2$ **M1 A1**

Note: FT from their θ in (a) for parts (b) and (c).

Q18 — Unknown Radius from Area (Radians) [MED

] **(a)** $A=\frac{1}{2}r^2\theta$; substituting: $44.1=\frac{1}{2}\times r^2\times 1.4$ **M1**

$r^2 = \dfrac{44.1 \times 2}{1.4} = \dfrac{88.2}{1.4} = 63$ **AG** **A1**

(b) $r = \sqrt{63} = 3\sqrt{7} \approx 7.94$ cm **A1**

(c) Arc $= r\theta = 7.937\ldots \times 1.4 = 11.11\ldots$ cm
Perimeter $= 2r + l = 2(7.937\ldots) + 11.11\ldots = 26.98\ldots \approx 27.0$ cm **A1**

Note: FT from their r for the perimeter.

Q19 — Annular Sector (Radians) [MED

] **(a)** Inner arc $= r\theta = 1.2 \times \dfrac{4\pi}{3} = 1.6\pi \approx 5.03$ m **A1**

(b) Outer arc $= 2.0 \times \dfrac{4\pi}{3} = \dfrac{8\pi}{3} \approx 8.38$ m **A1**

(c) Area of annular sector $= \dfrac{1}{2}(R^2 - r^2)\theta = \dfrac{1}{2}(2.0^2 - 1.2^2) \times \dfrac{4\pi}{3}$ **M1**

$= \dfrac{1}{2}(4 - 1.44) \times \dfrac{4\pi}{3} = \dfrac{1}{2} \times 2.56 \times \dfrac{4\pi}{3} = \dfrac{5.12\pi}{3} \approx 5.36\,\text{m}^2$ **A1**

Q20 — Sector Minus Triangle (Radians) [MED

] **(a)** Area of sector $= \dfrac{1}{2}r^2\theta = \dfrac{1}{2} \times 64 \times \dfrac{2\pi}{5} = \dfrac{64\pi}{5} \approx 40.2\,\text{cm}^2$ **A1**

(b) Area of triangle $OPQ = \dfrac{1}{2}r^2 \sin\theta = \dfrac{1}{2} \times 64 \times \sin\left(\dfrac{2\pi}{5}\right)$ **M1**

$= 32\sin(72°) = 32 \times 0.9511\ldots = 30.43\ldots \approx 30.4\,\text{cm}^2$ **A1**

(c) Shaded area $= \dfrac{64\pi}{5} - 30.43\ldots = 40.21\ldots - 30.43\ldots \approx 9.78\,\text{cm}^2$ **A1**

Note: FT from their sector and triangle areas. Accept $40.2 - 30.4 = 9.8$.

Section C — Hard

Q21 — Circumcentre and Circumradius [HARD

] **(a)** $A(1,1)$ and $B(7,1)$ both have $y=1$, so AB is horizontal. Perpendicular bisector is vertical through midpoint $(4,1)$: $x=4$ **A1**

(b) $B(7,1)$ and $C(7,9)$ both have $x=7$, so BC is vertical. Perpendicular bisector is horizontal through midpoint $(7,5)$: $y=5$ **A1**

(c) Circumcentre = intersection of $x=4$ and $y=5$: $(4,\ 5)$ **A1**

(d) Distance from $(4,5)$ to $A(1,1)$: $\sqrt{(4-1)^2+(5-1)^2}=\sqrt{9+16}=\sqrt{25}=5$ **M1**

Hence circumradius $=5$ units. **AG** **A1**

Note: Candidates may check with B or C: distance $(4,5)$ to $B(7,1)=\sqrt{9+16}=5$. ✓

Q22 — Composite Sector with Unknown Radius [HARD

] **(a)** Large sector area $=\frac{1}{2}r^2\cdot\frac{\pi}{3}=\frac{\pi r^2}{6}$ **M1**

Small sector area $=\frac{1}{2}\left(\frac{r}{2}\right)^2\cdot\frac{\pi}{3}=\frac{1}{2}\cdot\frac{r^2}{4}\cdot\frac{\pi}{3}=\frac{\pi r^2}{24}$ **M1**

Shaded area $=\frac{\pi r^2}{6}-\frac{\pi r^2}{24}=\frac{4\pi r^2-\pi r^2}{24}=\frac{3\pi r^2}{24}=\frac{\pi r^2}{8}$ **AG** **A1**

(b) $\frac{\pi r^2}{8}=75\pi \implies r^2=600$ **M1**

$r=\sqrt{600}=10\sqrt{6}\approx 24.5$ cm **A1**

Q23 — Coordinate Geometry: Kite [HARD

] **(a)** $JK=\sqrt{(6-0)^2+(4-0)^2}=\sqrt{36+16}=\sqrt{52}$ **M1**

$JM=\sqrt{(6-0)^2+(-4-0)^2}=\sqrt{36+16}=\sqrt{52}$

Hence $JK=JM=\sqrt{52}$ **A1**

(b) Gradient of JL: $m_{JL}=\frac{0-0}{10-0}=0$ (horizontal) **M1**

KM connects $K(6,4)$ to $M(6,-4)$ — a vertical segment ($x=6$ throughout)

Horizontal $\perp$ vertical, so $JL\perp KM$. **A1**

(c) Area of kite $=\frac{1}{2}d_1d_2$ where $d_1=JL=10$ units, $d_2=KM=8$ units

Area $=\frac{1}{2}\times 10\times 8=40$ square units $=40\times 100=4000\,\text{m}^2$ **A1**

Q24 — Unknown Vertex from Gradient [HARD

] **(a)** $m_{GH} = \dfrac{2-k}{8-2} = \dfrac{2-k}{6} = -\dfrac{1}{2}$ **M1**

$2-k = -3 \implies k = 5$ **A1**

(b) $G = (2,5)$, gradient $= -\dfrac{1}{2}$: $y - 2 = -\dfrac{1}{2}(x-8) \implies y = -\dfrac{x}{2} + 6$ **A1**

(c) $GH = \sqrt{(8-2)^2 + (2-5)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5} \approx 6.71$ **M1 A1**

Q25 — Perpendicular Bisector Meets a Line [HARD

] **(a)** Midpoint of $ST = \left(\dfrac{1+9}{2}, \dfrac{2+6}{2}\right) = (5,\ 4)$ **A1**

Gradient of $ST = \dfrac{6-2}{9-1} = \dfrac{4}{8} = \dfrac{1}{2}$; perp. gradient $= -2$ **M1**

$\ell:\ y - 4 = -2(x-5) \implies y = -2x + 14$ **A1**

(b) Set $2x - 4 = -2x + 14$: $4x = 18$, $x = 4.5$; $y = 2(4.5) - 4 = 5$ **M1**

$V = (4.5,\ 5)$ **A1**

Q26 — Perpendicular Bisector with Unknown Parameter [HARD

] **(a)** Gradient of $XY = \dfrac{p-3}{7-p}$. For the perpendicular bisector to have gradient 2, we need:

$\dfrac{p-3}{7-p} \times 2 = -1 \implies \dfrac{p-3}{7-p} = -\dfrac{1}{2}$ **M1**

$2(p-3) = -(7-p) \implies 2p - 6 = -7 + p \implies p = -1$ **M1 A1**

(b) $X(-1,\ 3), Y(7,\ -1)$; midpoint $= \left(\dfrac{-1+7}{2}, \dfrac{3+(-1)}{2}\right) = (3,\ 1)$ **A1**

Perpendicular bisector: $y - 1 = 2(x-3) \implies y = 2x - 5$ **A1**

Q27 — Arc Equals Chord: Transcendental Equation [HARD

] **(a)** Arc length $= r\theta$; chord $PQ = 2r\sin\left(\dfrac{\theta}{2}\right)$

Setting equal: $r\theta = 2r\sin\left(\dfrac{\theta}{2}\right)$ **M1**

Dividing by $r > 0$: $\theta = 2\sin\left(\dfrac{\theta}{2}\right)$ **AG** **A1**

(b) Using GDC (in radian mode): solve $\theta = 2\sin\left(\dfrac{\theta}{2}\right)$ for $0 < \theta < \pi$ **(M1)**

$\theta \approx 1.90$ rad (full value: $1.8955\ldots$) **A1**

(c) Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 64 \times 1.8955\ldots = 60.7\text{ cm}^2$ **A1**

Q28 — Sector from Perimeter and Area Conditions [HARD

] **(a)** Perimeter: $2r + r\theta = 30 \implies r(2+\theta) = 30$ ...(1) **A1**

Area: $\frac{1}{2}r^2\theta = 50 \implies r^2\theta = 100$ ...(2) **A1**

(b) From (1): $\theta = \frac{30}{r} - 2$. Substitute into (2): **M1**

$r^2\left(\frac{30}{r} - 2\right) = 100 \implies 30r - 2r^2 = 100$

$2r^2 - 30r + 100 = 0 \implies r^2 - 15r + 50 = 0$ **AG** **A1**

(c) $(r-5)(r-10) = 0 \implies r = 5$ or $r = 10$ **A1**

When $r = 5$: $\theta = \frac{30}{5} - 2 = 4$ rad; when $r = 10$: $\theta = \frac{30}{10} - 2 = 1$ rad

Q29 — Equilateral Triangle: Circumcentre and Sector [HARD

] **(a)** $AB = \sqrt{(12-0)^2 + 0^2} = 12$ units **A1**

(b) By symmetry, circumcentre lies on $x = 6$. Midpoint of $AB = (6, 0)$; perp bisector is $x = 6$.

Midpoint of $AC = (3, 3\sqrt{3})$; gradient of $AC = \frac{6\sqrt{3} - 0}{6 - 0} = \sqrt{3}$; perp gradient $= -\frac{1}{\sqrt{3}}$ **M1**

Bisector of AC: $y - 3\sqrt{3} = -\frac{1}{\sqrt{3}}(x - 3)$. At $x = 6$: $y = 3\sqrt{3} - \frac{3}{\sqrt{3}} = 3\sqrt{3} - \sqrt{3} = 2\sqrt{3}$

Circumcentre $W = (6,\ 2\sqrt{3})$ **A1**

(c) Circumradius $R = WA = \sqrt{(6-0)^2 + (2\sqrt{3})^2} = \sqrt{36 + 12} = \sqrt{48} = 4\sqrt{3}$

Three equal sectors: each has angle $\frac{2\pi}{3}$ (equilateral triangle, so $120°$ between circumradii) **M1**

Area of one sector $= \frac{1}{2}R^2 \cdot \frac{2\pi}{3} = \frac{1}{2} \times 48 \times \frac{2\pi}{3} = 16\pi\text{ cm}^2$ **A1**

Q30 — Perpendicular Bisector and Radar Coverage [HARD

] **(a)** Midpoint of $L_1L_2 = \left(\frac{2+14}{2},\ \frac{10+4}{2}\right) = (8,\ 7)$ **A1**

Gradient of $L_1L_2 = \dfrac{4-10}{14-2} = \dfrac{-6}{12} = -\dfrac{1}{2}$; perp. gradient $= 2$ **M1**

Patrol path: $y - 7 = 2(x-8) \implies y = 2x - 9$ **A1**

(b) At $x = 5$: $y = 2(5) - 9 = 1$. Radar centred at $(5,\ 1)$, range 7.5 km. **M1**

Area $= \pi r^2 = \pi \times (7.5)^2 = 56.25\pi \approx 176.7$ km^2 **A1**

End of Worksheet

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