IB Math AI HL — Topic 1 [Set 2]

Number Toolkit

Standard Form · Approximation · Upper & Lower Bounds · Percentage Error · Accuracy & Estimation · GDC Equations

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Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Marks each	3	4	5
Total Marks	30	40	50

Name:____________________________ Date:______________

Score:_________/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Calculators are permitted where indicated.

Key Formulae

Standard form: $a \times 10^k, \quad 1 \le a < 10,\ k \in \mathbb{Z}$

Percentage error: $\%\,\text{error} = \dfrac{|v_A - v_E|}{|v_E|} \times 100$ (v_A = approximate, v_E = exact/true)

Bounds (rounded to nearest u): Lower $= x - \frac{u}{2}$, Upper $= x + \frac{u}{2}$

Bounds (to n s.f.): half-unit in the last significant figure

Upper bound of product/sum: use upper bounds of each factor

Upper bound of quotient $\dfrac{A}{B}$: use upper bound of A **and lower** bound of B

Estimation: round each value to 1 s.f., compute, then check via % error

GDC Tips

1. **Polynomial equations:** Graph $y = f(x)$, use Analyse $\rightarrow$ Zero (Casio) or 2nd CALC $\rightarrow$ Zero (TI) to find all x-intercepts. A cubic can have up to 3 real roots — check the full graph.
2. **Simultaneous equations:** Use the Equation/Linear mode (Casio) or `rref` on the augmented matrix (TI) for systems of 2 or more equations.
3. **Exponential equations** like $a \cdot b^t = c$: either graph both sides and find intersection, or solve directly via the Equation solver.
4. **Standard form on GDC:** The display shows `8.2E5`; always rewrite as 8.2×10^5 in your answer — calculator notation scores A0.

Common Mistakes to Avoid

1. **Percentage error:** divide by the *exact* value v_E, not the approximate. Using v_A in the denominator is the most common algebraic error.
2. **Bounds of a quotient:** to maximise A/B, use the *upper* bound of A and the *lower* bound of B. Swapping these is a trap on every bounds question.
3. **Standard form coefficient:** 14.7×10^3 is *not* standard form; rewrite as 1.47×10^4.
4. **Rounding vs truncating:** 3.749 rounded to 1 d.p. is 3.7, *not* 3.8. Truncation (dropping digits) is not rounding.
5. **Bounds from significant figures:** 8.5 g to 2 s.f. has bounds 8.45 and 8.55, not 8.4 and 8.6.
6. **GDC — missing roots:** a cubic may have three real roots. Always zoom out to see the full graph before reading off intercepts.

Section A — Easy EASY (10 questions $\times$ 3 marks = 30 marks)

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1. EASY No Calculator **[3 marks]**

A cloud storage server holds 2.46×10^{12} bytes of data. A single photograph file takes 3.0×10^{6} bytes.

(a) Find the number of photographs the server can store. Give your answer as an integer. **[2]**

(b) Write your answer to part (a) in the form $a \times 10^{k}$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$. **[1]**

2. EASY No Calculator **[3 marks]**

The length of a garden path is 162.8 cm.

(a) Round 162.8 to the nearest ten. **[1]**

(b) Round 162.8 to 3 significant figures. **[1]**

(c) Write down the difference between your answers to (a) and (b). **[1]**

3. EASY Calculator **[3 marks]**

A plank of wood is measured as 3.8 m, to the nearest 0.1 m.

(a) Write down the lower bound of the length of the plank. **[1]**

(b) Write down the upper bound of the length of the plank. **[1]**

(c) A carpenter cuts 5 equal planks of this stated length. Write down the upper bound of the total length. **[1]**

4. EASY Calculator **[3 marks]**

A student estimates the height of a tree as 12 m. The actual height is 10.5 m.

(a) Calculate the percentage error in the student's estimate. **[2]**

(b) State whether the student overestimated or underestimated, and by what absolute amount. **[1]**

5. EASY Calculator **[3 marks]**

Use a GDC to solve the equation

$$x^2 + 5x - 14 = 0.$$

Give both solutions.

6. EASY No Calculator **[3 marks]**

The wavelength of green light is 5.5×10^{-7} m. A radio wave used in a local broadcast has a wavelength of 2.75 m.

(a) Write 2.75 in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$. **[1]**

(b) Find how many times longer the radio wavelength is than the green-light wavelength. Give your answer in standard form. **[2]**

7. EASY Calculator **[3 marks]**

A rectangular field is measured as 120 m long and 85 m wide, both to the nearest metre.

(a) Write down the lower bound of the length. **[1]**

(b) Write down the lower bound of the width. **[1]**

(c) Hence find the lower bound of the area of the field. **[1]**

8. EASY Calculator **[3 marks]**

A cinema has 18 rows of 24 seats.

(a) Estimate the total number of seats by rounding each number to 1 significant figure. **[2]**

(b) Calculate the percentage error of your estimate compared with the exact total. **[1]**

9. EASY No Calculator **[3 marks]**

The table shows the masses of five parcels.

Parcel	A	B	C	D	E
Mass (kg)	1.2	3.8	5.1	2.4	4.6

(a) Round each mass to the nearest kilogram. **[2]**

(b) Find the sum of the rounded masses and state how it compares to the exact total of 17.1 kg. **[1]**

10. EASY Calculator **[3 marks]**

Use a GDC to solve the simultaneous equations:

$$2x - 3y = 1 \qquad x + 4y = 10.$$

Give your answers correct to 3 significant figures.

Section B — Medium MEDIUM (10 questions $\times$ 4 marks = 40 marks)

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11. MEDIUM Calculator **[4 marks]**

A cyclist records a distance of 42.0 km (to 3 s.f.) in a time of 1.5 hours (to 2 s.f.).

(a) Write down the upper and lower bounds of the distance. **[1]**

(b) Write down the upper and lower bounds of the time. **[1]**

(c) Find the upper bound of the cyclist's average speed, in km h^{-1}. Give your answer correct to 3 significant figures. **[2]**

12. MEDIUM Calculator **[4 marks]**

An approximation gives a result of 360. The percentage error is 4.0%.

(a) Show that the true value v_E satisfies $|360 - v_E| = 0.04\, v_E$. **[1]**

(b) Hence find the two possible values of v_E. **[3]**

13. MEDIUM Calculator **[4 marks]**

The diameter of a virus is $d = 2.4 \times 10^{-7}$ m, measured to 2 significant figures.

(a) Write down the upper and lower bounds of d. **[1]**

(b) A biologist places viruses end to end across a 1 mm $(= 1 \times 10^{-3}$ m) slide. Using the upper bound of d, find the minimum number of viruses needed. **[2]**

(c) Using the lower bound of d, find the maximum number of viruses that could fit. Give your answer in standard form. **[1]**

14. MEDIUM Calculator **[4 marks]**

Use a GDC to find all real solutions of

$$2x^3 - 3x^2 - 11x + 6 = 0.$$

Give each answer as an exact value.

15. MEDIUM Calculator **[4 marks]**

A pendulum's period is given by $T = 2\pi\sqrt{\dfrac{L}{g}}$, where $L = 0.650$ m (to 3 s.f.) and $g = 9.81$ m s^{-2} (exact).

(a) Calculate T using $L = 0.650$. Give your answer to 4 significant figures. **[2]**

(b) Find the upper bound of T, using the upper bound of L. **[1]**

(c) Find the percentage error between the upper bound of T and the exact value. Give your answer to 3 significant figures. **[1]**

16. MEDIUM Calculator **[4 marks]**

The table shows the masses of packages delivered by a courier service on one day.

Mass (kg)	**Frequency**
$0 < m \leq 2$	15
$2 < m \leq 5$	28
$5 < m \leq 10$	12

(a) Using the midpoint of each class, estimate the total mass of all packages. **[3]**

(b) Explain why this is an estimate. **[1]**

17. MEDIUM No Calculator **[4 marks]**

The speed of sound in air is approximately 343 m s^{-1}.

(a) Convert this speed to km h^{-1}. Give your answer in standard form. **[2]**

(b) An aircraft travels at Mach 2.3 (i.e. 2.3 times the speed of sound). Find the distance it travels in 45 minutes. Give your answer in km correct to 3 significant figures. **[2]**

18. MEDIUM Calculator **[4 marks]**

The mass of a metal block is $m = 6.40$ kg, measured to 3 significant figures. Its volume is $V = 820$ cm^3, measured to the nearest 10 cm^3.

(a) Find the upper and lower bounds of m and V. **[2]**

(b) Find the lower bound of the density $\rho = m/V$. Give your answer in g cm^{-3} to 3 significant figures. **[2]**

19. MEDIUM Calculator **[4 marks]**

A colony of mould grows according to $N = N_0 \times 1.08^t$, where N is the number of spores and t is time in hours. Initially $N_0 = 2.5 \times 10^3$ spores.

(a) Find the number of spores after 24 hours. Give your answer in standard form to 3 significant figures. **[2]**

(b) Use a GDC to find the value of t when the colony first exceeds 10^5 spores. Give your answer correct to 3 significant figures. **[2]**

20. MEDIUM Calculator **[4 marks]**

A measuring instrument has a maximum percentage error of 2.5%. A student measures a voltage and obtains $V_{\text{approx}} = 48.0$ V.

(a) Find the maximum and minimum possible true values of the voltage. Give your answers to 3 significant figures. **[3]**

(b) A circuit requires the voltage to be strictly above 46.5 V. Determine whether the measurement guarantees this. Justify your answer. **[1]**

Section C — Hard HARD (10 questions $\times$ 5 marks = 50 marks)

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21. HARD Calculator **[5 marks]**

A measurement x is recorded as n cm to the nearest centimetre. The upper bound of x is 17.5 cm.

(a) Write down the value of n. **[1]**

(b) A second measurement y is recorded as m cm to the nearest 0.5 cm. The lower bound of y is 12.25 cm. Find m. **[2]**

(c) Find the upper bound of the ratio x/y. Give your answer to 3 significant figures. **[2]**

22. HARD Calculator **[5 marks]**

A formula gives an approximation $A_{\text{approx}} = kx^2$ where k is a constant and $x = 4.0$ (exact). The true value is $A_{\text{true}} = 52.0$.

(a) Given that the percentage error is exactly 4.00%, find the two possible values of k. **[4]**

(b) Given that $A_{\text{approx}} < A_{\text{true}}$, state which value of k applies. **[1]**

23. HARD Calculator **[5 marks]**

In a dataset, $Q_1 = p$ and $Q_3 = p + 16$ for some value $p > 0$.

(a) Write down the IQR in terms of p. **[1]**

(b) The value $v = 82$ lies exactly on the upper outlier fence. Find the value of p. **[2]**

(c) Determine whether the value $w = 1$ is an outlier. Justify your answer. **[2]**

24. HARD Calculator **[5 marks]**

A pharmacist uses the formula $D = \dfrac{W \times d}{70}$ to calculate a drug dose D mg, where W kg is the patient's weight and $d = 250$ mg (exact). The patient's weight is recorded as $W = 84$ kg to the nearest kg.

(a) Find the lower and upper bounds of W. **[1]**

(b) Find the lower and upper bounds of D. **[1]**

(c) The maximum safe dose is 305 mg. Find the percentage error between the upper bound of D and the maximum safe dose. Give your answer to 3 significant figures. **[2]**

(d) Determine whether the pharmacist can be certain the dose is safe. Justify your answer. **[1]**

25. HARD Calculator **[5 marks]**

A forest fire spreads in a circular pattern. The radius of the burnt area, measured from aerial photographs, is $r = 3.2$ km correct to 2 significant figures.

(a) Find the upper and lower bounds of r. **[1]**

(b) Calculate the upper bound of the burnt area $A = \pi r^2$. Give your answer in standard form to 3 significant figures. **[2]**

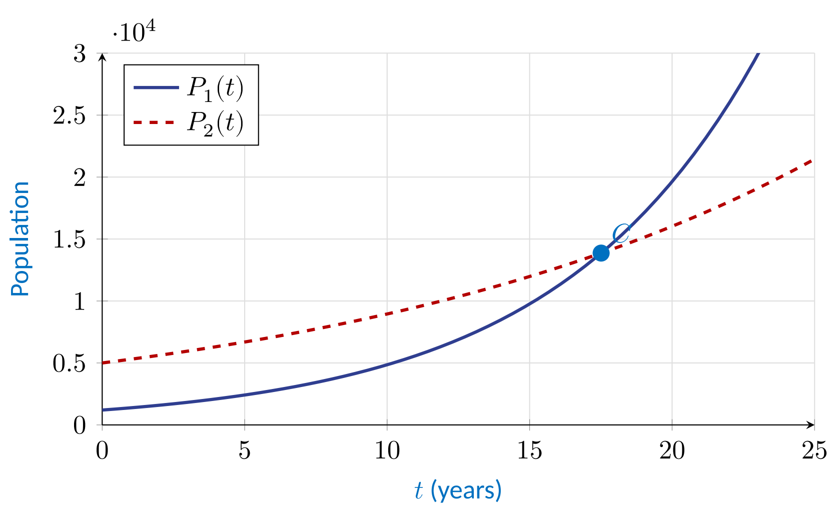
(c) An emergency team estimates the area as 32 km^2 (using $\pi \approx 3$ and $r = 3.2$ exactly). Find the percentage error of this estimate compared with the exact area (using $r = 3.2$ and π from your GDC). Give your answer to 3 significant figures. **[2]**

26. HARD Calculator **[5 marks]**

Two animal species have populations modelled by

$$P_1(t) = 1.2 \times 10^3 \times 1.15^{\,t} \qquad P_2(t) = 5.0 \times 10^3 \times 1.06^{\,t}$$

where t is the number of years from now. The graph below shows both populations for $0 \leq t \leq 25$.



(a) Write down the initial population of each species. **[1]**

(b) Use a GDC to find the value of t at which $P_1 = P_2$. Give your answer to 3 significant figures. **[2]**

(c) Find the combined population at the time found in (b). Give your answer in standard form to 3 significant figures. **[2]**

27. HARD Calculator **[5 marks]**

A rocket engine burns fuel at a rate of $r = 4.50 \times 10^3$ kg s^{-1}, measured to 3 significant figures. The total burn time is $T = 145$ s, measured to the nearest 5 s.

(a) Write down the bounds of r and T. **[2]**

(b) Find the upper bound of the total fuel mass $M = rT$. Give your answer in standard form to 3 significant figures. **[1]**

(c) Using the values as given ($r = 4.50 \times 10^3$, $T = 145$), calculate M and find the percentage error between this and the upper bound found in (b). Give your answer to 3 significant figures. **[2]**

28. **HARD** **Calculator** **[5 marks]**

A city council estimates daily water usage. The table gives the council's estimates for a city of population $N = 3.2 \times 10^5$ residents.

Source	**Usage per person per day (litres)**
Drinking & cooking	5
Bathing & hygiene	65
Laundry	40
Garden & outdoor	30

(a) Find the estimated total daily water usage for the city. Give your answer in standard form. **[2]**

(b) The water reservoir holds 7.0×10^7 litres. Estimate the number of days the reservoir could supply the city without refilling. Give your answer to 3 significant figures. **[2]**

(c) A water audit finds that actual usage is 12% higher than the council's estimate. Find the percentage error in the council's estimate. Give your answer to 3 significant figures. **[1]**

29. HARD Calculator **[5 marks]**

A signal's intensity I (W m^{-2}) at distance d m from a transmitter of power P W is given by

$$I = \frac{P}{4\pi d^2}.$$

(a) A receiver at distance $d = 1.50 \times 10^3$ m records $I = 2.40 \times 10^{-6}$ W m^{-2}. Show that $P = 6.786 \times 10^4$ W correct to 4 significant figures. **[2]**

(b) A second receiver records $I = 8.50 \times 10^{-7}$ W m^{-2}. Use a GDC to find the distance d to this receiver. Give your answer in standard form to 3 significant figures. **[2]**

(c) A technician uses $P = 6.80 \times 10^4$ W (rounded to 3 s.f.) to predict I at the distance found in (b). Find the percentage error of this predicted I compared with the true $I = 8.50 \times 10^{-7}$ W m^{-2}. Give your answer to 3 significant figures. **[1]**

30. HARD Calculator **[5 marks]**

A formula approximates the area of an ellipse as $A \approx \pi ab$ where a and b are the semi-axes. For a particular ellipse, $a = k$ and $b = 2k - 1$ for some constant $k > 1$.

(a) Write down an expression for A in terms of k. **[1]**

(b) It is given that the percentage error between the approximate area A and the true area $A_{\text{true}} = 28.0$ is exactly 3.50%. Form an equation in k and use a GDC to find all values of $k > 1$. Give your answers to 3 significant figures. **[3]**

(c) For the value of k found in (b) such that $A < A_{\text{true}}$, find the dimensions a and b. Give your answers to 3 significant figures. **[1]**

Worked Solutions

Q1 — Standard Form EASY

(a)

$$\frac{2.46 \times 10^{12}}{3.0 \times 10^{6}} \qquad (\textbf{M1})$$

$= 820\,000$ photographs **A1**

(b) 8.2×10^{5} **A1**

Note: FT from (a). Do not accept `8.2E5` *or any other calculator notation.*

Q2 — Approximation EASY

(a) 160 **A1**

(b) 163 **A1**

(c) $|163 - 160| = 3$ **A1**

Q3 — Upper & Lower Bounds EASY

(a) Lower bound $= 3.75$ m **A1**

(b) Upper bound $= 3.85$ m **A1**

(c) Upper bound of total $= 5 \times 3.85 = 19.25$ m **A1 FT**

Note: FT from their upper bound in (b).

Q4 — Percentage Error EASY

(a)

$$\%\ \text{error} = \frac{|10.5 - 12|}{10.5} \times 100 \qquad (\textbf{M1})$$

$= \dfrac{1.5}{10.5} \times 100 = 14.285\ldots \approx 14.3\%$ **A1**

(b) Overestimated by 1.5 m **A1**

Note: Must state overestimated and give 1.5 m for A1.

Q5 — Solving Equations using a GDC EASY

Using GDC to solve $x^2 + 5x - 14 = 0$ **(M1)**

$x = -7$ or $x = 2$ **A1 A1**

Note: (M1) implied by either correct root. Accept $x = -7$ and $x = 2$ in any order. METHOD 2: factoring $(x + 7)(x - 2) = 0$ earns M1 and both A1.

Q6 — Standard Form EASY

(a) 2.75×10^0 **A1**

(b) $\dfrac{2.75}{5.5 \times 10^{-7}}$ **(M1)**

$= 5.0 \times 10^6$ times longer **A1**

Note: Award A0 for answer not in standard form.

Q7 — Upper & Lower Bounds EASY

(a) Lower bound of length $= 119.5$ m **A1**

(b) Lower bound of width $= 84.5$ m **A1**

(c) Lower bound of area $= 119.5 \times 84.5 = 10097.75 \approx 10\,098$ m^2 **A1 FT**

Q8 — Accuracy & Estimation EASY

(a) $18 \approx 20$; $24 \approx 20$; estimate $= 20 \times 20 = 400$ seats **(M1) A1**

Exact $= 18 \times 24 = 432$

(b) $\%$ error $= \dfrac{|432 - 400|}{432} \times 100 = \dfrac{32}{432} \times 100 = 7.407\ldots \approx 7.41\%$ **A1**

Q9 — Approximation EASY

(a) Rounded masses: $1,\ 4,\ 5,\ 2,\ 5$ kg **A1 A1**

Note: Award A1 for 3 or more correct; A1 for all five correct.

(b) Sum $= 1 + 4 + 5 + 2 + 5 = 17$ kg; this is 0.1 kg *less than* the exact total of 17.1 kg **A1**

Q10 — Solving Equations using a GDC EASY

Setting up the system in GDC **(M1)**

$x = \dfrac{34}{11} = 3.09\ldots \approx 3.09$ **A1**

$y = \dfrac{19}{11} = 1.72\ldots \approx 1.73$ **A1**

Note: (M1) implied by at least one correct value. Accept exact fractions $x = 34/11$, $y = 19/11$.

Q11 — Upper & Lower Bounds MEDIUM

(a) $41.95 \le d < 42.05$ km **A1**

(b) $1.45 \le t < 1.55$ hours **A1**

(c) Upper speed $= \dfrac{\text{upper } d}{\text{lower } t} = \dfrac{42.05}{1.45}$ **(M1)**

$= 29.0\overline{0}\ldots \approx 29.0$ km h^{-1} **A1**

Q12 — Percentage Error MEDIUM

(a) % error $= \dfrac{|360 - v_E|}{v_E} \times 100 = 4.0 \implies |360 - v_E| = 0.04\,v_E$ **A1 (AG)**

(b) *Case 1:* $360 - v_E = 0.04\,v_E \implies 360 = 1.04\,v_E$ **M1**

$v_E = \dfrac{360}{1.04} = 346.15\ldots \approx 346$ **A1**

Case 2: $v_E - 360 = 0.04\,v_E \implies 0.96\,v_E = 360$

$v_E = \dfrac{360}{0.96} = 375$ **A1**

Q13 — Upper & Lower Bounds MEDIUM

(a) $2.35 \times 10^{-7} \le d < 2.45 \times 10^{-7}$ m **A1**

(b) Minimum number $= \dfrac{1 \times 10^{-3}}{2.45 \times 10^{-7}} = 4081.6\ldots$ **(M1)**

$\implies 4082$ viruses (round up) **A1**

(c) Maximum $= \dfrac{1 \times 10^{-3}}{2.35 \times 10^{-7}} = 4255.3\ldots$; in standard form: 4.26×10^3 **A1**

Note: (b) must round up to the next integer; truncation scores A0.

Q14 — Solving Equations using a GDC MEDIUM

Using GDC to graph $y = 2x^3 - 3x^2 - 11x + 6$ and find zeros **(M1)**

$x = -2$ **A1**

$x = \frac{1}{2}$ **A1**

$x = 3$ **A1**

Note: Verify: $2(-8) - 3(4) - 11(-2) + 6 = -16 - 12 + 22 + 6 = 0$; $2(\frac{1}{8}) - 3(\frac{1}{4}) - \frac{11}{2} + 6 = 0$; $2(27) - 3(9) - 33 + 6 = 0$. *Exact fractional answer* $\frac{1}{2}$ *required; decimal* 0.5 *also accepted.*

Q15 — Upper & Lower Bounds (Pendulum) MEDIUM

(a) $T = 2\pi\sqrt{\dfrac{0.650}{9.81}}$ **(M1)**

$= 2\pi \times 0.25742\ldots = 1.617\ldots \approx 1.617$ s (4 s.f.) **A1**

(b) Upper bound of $L = 0.6505$; $\quad T_{\text{upper}} = 2\pi\sqrt{\dfrac{0.6505}{9.81}} = 1.6180\ldots$ s **A1**

(c) $\%$ error $= \dfrac{|1.6180 - 1.617|}{1.617} \times 100 = 0.0618\ldots \approx 0.0618\%$ **A1**

Note: Accept answers in range $[0.061\%, 0.063\%]$ *due to rounding in (a).*

Q16 — Accuracy & Estimation MEDIUM

(a) Midpoints: $1,\ 3.5,\ 7.5$

$\sum fx = 1(15) + 3.5(28) + 7.5(12)$ **M1**

$= 15 + 98 + 90 = 203$ kg **(M1) A1**

(b) The exact mass of each individual package is unknown; only the class interval is recorded. **A1**

Q17 — Standard Form MEDIUM

(a) $343 \times 3600 \div 1000 = 1234.8$ km h^{-1} **(M1)**

$= 1.2348 \times 10^3 \approx 1.23 \times 10^3$ km h^{-1} **A1**

(b) Speed at Mach 2.3 $= 2.3 \times 1234.8 = 2840.04$ km h^{-1} **(M1)**

Distance in 45 min $= 2840.04 \times \dfrac{45}{60} = 2130.03\ldots \approx 2130$ km **A1**

Note: FT from their (a). Accept 2130 *km.*

Q18 — Upper & Lower Bounds (Density) MEDIUM

(a) $6395 \le m < 6405$ g; $815 \le V < 825$ cm^3 **A1 A1**

Note: Convert kg to g: $6.395 \le m < 6.405$ *kg or equivalently* $6395 \le m < 6405$ *g. Both acceptable.*

(b) Lower density $= \dfrac{\text{lower } m}{\text{upper } V} = \dfrac{6395}{825}$ **(M1)**

$= 7.7515\ldots \approx 7.75$ g cm^{-3} **A1**

Q19 — Standard Form & GDC MEDIUM

(a) $N = 2500 \times 1.08^{24}$ **(M1)**

$= 2500 \times 6.3412\ldots = 15\,853\ldots \approx 1.59 \times 10^4$ **A1**

(b) Solve $2500 \times 1.08^{\,t} = 100\,000 \implies 1.08^{\,t} = 40$ **(M1)**

$t = \dfrac{\ln 40}{\ln 1.08} = 46.2\ldots \approx 46.2$ hours **A1**

Q20 — Percentage Error MEDIUM

(a) $\dfrac{|48.0 - v_E|}{v_E} \times 100 = 2.5 \implies |48.0 - v_E| = 0.025\,v_E$ **M1**

Case 1 (over-estimate): $1.025\,v_E = 48.0 \implies v_E = 46.829\ldots \approx 46.8$ V **A1**

Case 2 (under-estimate): $0.975\,v_E = 48.0 \implies v_E = 49.230\ldots \approx 49.2$ V **A1**

(b) Minimum true value $= 46.8$ V > 46.5 V, so the measurement **does** guarantee the circuit requirement. **R1**

Q21 — Upper & Lower Bounds (Parameter) HARD

(a) Upper bound $= n + 0.5 = 17.5 \implies n = 17$ **A1**

(b) Lower bound $= m - 0.25 = 12.25 \implies m = 12.5$ cm **M1 A1**

(c) Upper bound of $\dfrac{x}{y} = \dfrac{17.5}{12.25}$ **(M1)**

$= 1.4285\ldots \approx 1.43$ **A1**

Q22 — Percentage Error (Unknown Parameter) HARD

(a) $A_{\text{approx}} = k(4.0)^2 = 16k$

$\dfrac{|16k - 52|}{52} \times 100 = 4.00 \implies |16k - 52| = 2.08$ **M1**

Case 1: $16k - 52 = 2.08 \implies 16k = 54.08 \implies k = 3.38$ **M1 A1**

Case 2: $16k - 52 = -2.08 \implies 16k = 49.92 \implies k = 3.12$ **A1**

(b) $A_{\text{approx}} < A_{\text{true}} \implies 16k < 52 \implies k < 3.25$, so $k = 3.12$ **A1**

Q23 — Outlier Fence (Parameter) HARD

(a) IQR $= (p + 16) - p = 16$ **A1**

(b) Upper fence $= Q_3 + 1.5 \times$ IQR $= (p + 16) + 1.5(16) = p + 40$ **M1**

$p + 40 = 82 \implies p = 42$ **A1**

(c) Lower fence $= Q_1 - 1.5 \times 16 = 42 - 24 = 18$ **M1**

Since $w = 1 < 18$, w is an outlier. **A1**

Q24 — Bounds + % Error + Decision HARD

(a) $83.5 \leq W < 84.5$ kg **A1**

(b) Lower $D = \dfrac{83.5 \times 250}{70} = 298.214\ldots \approx 298$ mg; Upper $D = \dfrac{84.5 \times 250}{70} = 301.785\ldots \approx 302$ mg
(M1) A1

(c) $\%$ error $= \dfrac{|301.785 - 305|}{305} \times 100$ **M1**

$= \dfrac{3.214}{305} \times 100 = 1.053\ldots \approx 1.05\%$ **A1**

(d) Upper bound of $D = 301.79$ mg < 305 mg, so the dose is **always safe**. **R1**

Q25 — Bounds & % Error (Circular Area) HARD

(a) $3.15 \leq r < 3.25$ km **A1**

(b) Upper $A = \pi(3.25)^2 = \pi \times 10.5625 = 33.183\ldots$ **M1**

$\approx 3.32 \times 10^1$ km^2 **A1**

(c) Exact $A = \pi(3.2)^2 = 32.169\ldots$ km^2

$\%$ error $= \dfrac{|32.169 - 32|}{32.169} \times 100$ **M1**

$= 0.525\ldots \approx 0.525\%$ **A1**

Q26 — Standard Form & GDC (Population Crossing) HARD

(a) $P_1(0) = 1200; \quad P_2(0) = 5000$ **A1**

(b) Solve $1200 \times 1.15^t = 5000 \times 1.06^t$ on GDC **(M1)**

$t = \dfrac{\ln(25/6)}{\ln(1.15/1.06)} = 17.507\ldots \approx 17.5$ years **A1**

(c) $P_1(17.5) = 1200 \times 1.15^{17.507} \approx 13\,863$

Combined $= 2 \times 13\,863 = 27\,726\ldots \approx 2.77 \times 10^4$ **(M1) A1**

Note: FT from their t. Accept 2.76×10^4 to 2.78×10^4.

Q27 — Bounds & % Error (Rocket Fuel) HARD

(a) $4.495 \times 10^3 \le r < 4.505 \times 10^3$ kg s^{-1}; $\quad 142.5 \le T < 147.5$ s **A1 A1**

(b) Upper $M = 4.505 \times 10^3 \times 147.5 = 664\,487.5$ kg $\approx 6.64 \times 10^5$ kg **A1**

(c) $M_{\text{given}} = 4.50 \times 10^3 \times 145 = 652\,500 = 6.525 \times 10^5$ kg **(M1)**

$\%$ error $= \dfrac{|6.525 - 6.645|}{6.645} \times 100 = 1.806\ldots \approx 1.81\%$ **A1**

Note: Use exact upper bound $6.64487\ldots$ in denominator. FT from (b).

Q28 — Estimation & Standard Form (Water Usage) HARD

(a) Total per person $= 5 + 65 + 40 + 30 = 140$ litres/day **M1**

City total $= 140 \times 3.2 \times 10^5 = 4.48 \times 10^7$ litres/day **A1**

(b) Days $= \dfrac{7.0 \times 10^7}{4.48 \times 10^7}$ **(M1)**

$= 1.5625\ldots \approx 1.56$ days **A1**

(c) True usage $= 1.12 \times v_A$; $\quad \%$ error $= \dfrac{0.12\,v_A}{1.12\,v_A} \times 100 = \dfrac{0.12}{1.12} \times 100 = 10.714\ldots \approx 10.7\%$ **A1**

Q29 — Standard Form & GDC (Signal Intensity) HARD

(a) $P = I \times 4\pi d^2 = 2.40 \times 10^{-6} \times 4\pi \times (1.50 \times 10^3)^2$ **M1**

$= 2.40 \times 10^{-6} \times 4\pi \times 2.25 \times 10^6 = 2.40 \times 4\pi \times 2.25 = 67\,857\ldots \approx 6.786 \times 10^4$ W **A1 (AG)**

(b) $8.50 \times 10^{-7} = \dfrac{6.786 \times 10^4}{4\pi d^2} \implies d^2 = \dfrac{6.786 \times 10^4}{4\pi \times 8.50 \times 10^{-7}}$ **M1**

$d = \sqrt{6.353 \times 10^9} = 7.971 \times 10^4\ldots \approx 7.97 \times 10^4$ m **A1**

(c) Using $P = 6.80 \times 10^4$: $I_{\text{pred}} = \dfrac{6.80 \times 10^4}{4\pi(7.971 \times 10^4)^2} = 8.519 \times 10^{-7}$

$\%$ error $= \dfrac{|8.519 - 8.50|}{8.50} \times 10^{-7}/10^{-7} \times 100 = 0.224\ldots \approx 0.224\%$ **A1**

Q30 — GDC + % Error (Ellipse Parameter) **HARD**

(a) $A = \pi k(2k-1)$ **A1**

(b) Percentage error condition: $\dfrac{|\pi k(2k-1) - 28.0|}{28.0} \times 100 = 3.50$ **M1**

$\implies |\pi k(2k-1) - 28| = 0.98$

Case 1: $\pi k(2k-1) = 28.98 \implies 2\pi k^2 - \pi k - 28.98 = 0$; GDC: $k = 2.384\ldots \approx 2.38$ $(k > 1)$ **A1**

Case 2: $\pi k(2k-1) = 27.02 \implies 2\pi k^2 - \pi k - 27.02 = 0$; GDC: $k = 2.298\ldots \approx 2.30$ $(k > 1)$ **A1**

(c) $A < A_{\text{true}}$ means $A < 28$, so use Case 2: $k = 2.30$

$a = k = 2.30$ cm; $\quad b = 2(2.30) - 1 = 3.60$ cm **A1**

Note: (b) GDC solutions for $k > 1$ only; negative roots discarded. FT from their k in (c).