IB Math AI HL — Topic 4

Poisson Distribution

Poisson Model · Calculating Probabilities · Mean Rescaling · Combined Distributions · Binomial–Poisson Fusion · Hypothesis Testing

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Marks each	3	4	5
Section total	30	40	50

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Name:________________________ Date:______________

Score:__________/120

Instructions: Answer all questions. Show all working clearly. Give probabilities correct to 3 significant figures unless stated. A graphical display calculator (GDC) is permitted throughout.

Key Formulae — Poisson Distribution

Poisson Distribution: $X \sim \text{Po}(\lambda)$, where $\lambda > 0$ is the mean rate.

$$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}, \quad k = 0, 1, 2, \ldots$$

Mean and Variance: $E(X) = \lambda, \quad \text{Var}(X) = \lambda.$

Mean Rescaling: If events occur at rate λ per unit time, then in an interval of length t, the count $\sim \text{Po}(\lambda t)$.

Sum of independent Poisson RVs: If $X \sim \text{Po}(\lambda_1)$ and $Y \sim \text{Po}(\lambda_2)$ independently, then $X + Y \sim \text{Po}(\lambda_1 + \lambda_2)$.

Complementary events: $P(X \geq k) = 1 - P(X \leq k - 1)$.

Conditions for a Poisson model: (1) events occur independently; (2) mean rate λ is constant; (3) two events cannot occur simultaneously; (4) only the number of events in a fixed interval matters.

GDC Tips

Casio fx-CG50: `MENU` $\to$ `STAT` $\to$ `DIST` $\to$ `POISN` $\to$ `Ppd` (exact) or `Pcd` (cumulative $\leq$).
TI-84/TI-Nspire: `2nd DISTR` $\to$ `poissonpdf(`λ, k`)` or `poissoncdf(`λ, k`)`.
Always scale λ to the interval asked *before* entering the GDC. For $P(X \geq k)$: compute $1 -$ `poissoncdf`$(\lambda, k-1)$. Check: if your probability exceeds 1, you have the wrong interval.

Common Mistakes to Avoid

- **Forgetting to rescale λ:** if the rate is per hour but the question asks for a 30-minute window, use $\lambda/2$, not λ.
- **Wrong tail:** $P(X > k) = 1 - P(X \leq k)$, NOT $1 - P(X \leq k - 1)$.
- **Using Poisson for small n, large p:** Poisson approximates Binomial only when n is large and p is small (so $np = \lambda$ is moderate).
- **Assuming the sum of two Poisson RVs is Poisson in all cases:** this only holds when the two streams are *independent*.
- **Ignoring independence in repeated-interval questions:** "probability of no events in each of n intervals" requires raising to the nth power, not multiplying raw probabilities differently.

Section A **EASY** (Questions 1–10, 3 marks each = 30 marks)

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1. **EASY** **Calculator** **[3 marks]**

A wildlife ranger records the number of elephants crossing a river ford each hour. The number of crossings per hour can be modelled by a Poisson distribution with mean 2.5.

Let X be the number of crossings in a randomly chosen hour.

(a) Write down the value of $E(X)$. [1]

(b) Find $P(X = 3)$. Give your answer correct to 3 significant figures. [2]

2. **EASY** **Calculator** **[3 marks]**

A printing machine produces faults at a mean rate of 1.8 faults per page. The number of faults per page follows a Poisson distribution.

Let F be the number of faults on a randomly chosen page.

(a) Find $P(F = 0)$. Give your answer correct to 3 significant figures. [1]

(b) Find $P(F \leq 2)$. Give your answer correct to 3 significant figures. [2]

3. EASY Calculator **[3 marks]**

A call centre receives telephone enquiries at a mean rate of 4.2 calls per minute, modelled by a Poisson distribution.

(a) Find the probability that exactly 5 calls arrive in a randomly chosen minute. Give your answer correct to 3 significant figures. [2]

(b) Write down the variance of the number of calls per minute. [1]

4. EASY Calculator **[3 marks]**

The number of customers entering a bookshop follows a Poisson distribution with mean 3.6 customers per 10-minute period.

The table gives the observed frequencies of customers entering in 60 ten-minute periods.

Number of customers	0	1	2	3	4	≥ 5
Observed frequency	2	8	14	13	12	11

(a) Write down the mean and variance of the Poisson distribution used to model the number of customers. [1]

(b) Find the probability that exactly 4 customers enter in a randomly chosen 10-minute period. Give your answer correct to 3 significant figures. [2]

5. EASY Calculator **[3 marks]**

Shooting stars are observed at a remote observatory at a mean rate of 0.8 per minute, following a Poisson distribution.

(a) Find the probability that **no** shooting stars are observed in a randomly chosen minute. Give your answer correct to 3 significant figures. [1]

(b) Find the probability that **at most 2** shooting stars are observed in a randomly chosen minute. Give your answer correct to 3 significant figures. [2]

6. EASY Calculator **[3 marks]**

Defective bolts occur on an assembly line at a mean rate of 1.2 defective bolts per metre of output. The number of defective bolts per metre follows a Poisson distribution.

(a) Find the probability of **more than 2** defective bolts in a randomly chosen metre. Give your answer correct to 3 significant figures. [2]

(b) Write down the standard deviation of the number of defective bolts per metre. Give your answer correct to 3 significant figures. [1]

7. EASY Calculator **[3 marks]**

A hospital records emergency admissions at a rate of 6 per hour, modelled by a Poisson distribution. Let Y be the number of emergency admissions in a 30-minute period.

(a) Write down the distribution of Y, including its parameter. [1]

(b) Find $P(Y = 4)$. Give your answer correct to 3 significant figures. [2]

8. EASY Calculator **[3 marks]**

A botanist counts the number of rare orchid flowers in randomly chosen 1 m^2 plots of a meadow. The count can be modelled by a Poisson distribution with mean 2.1.

(a) Find the probability that a randomly chosen plot contains **exactly 2** orchids. Give your answer correct to 3 significant figures. [2]

(b) Write down $E(X^2)$ for $X \sim \text{Po}(2.1)$. [*Hint: use* $\text{Var}(X) = E(X^2) - [E(X)]^2$] [1]

9. EASY Calculator **[3 marks]**

A sports commentator claims that, on average, 1.5 goals are scored per match in a regional football league. Assume a Poisson model applies to each match.

(a) Find the probability that a randomly chosen match has **at least 1** goal. Give your answer correct to 3 significant figures. [2]

(b) State one assumption required for the Poisson model to be valid in this context. [1]

10. **EASY** **Calculator** **[3 marks]**

Flaws in a roll of fabric occur at a mean rate of 0.6 flaws per metre, modelled by a Poisson distribution.

(a) Find the probability that a randomly chosen 2-metre length contains **exactly 1** flaw. Give your answer correct to 3 significant figures. [2]

(b) Write down the mean number of flaws in a 5-metre length. [1]

Section B MEDIUM (Questions 11–20, 4 marks each = 40 marks)

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11. MEDIUM Calculator **[4 marks]**

A hydroponics farm records the number of equipment malfunctions per week. The mean number of malfunctions is 2.4 per week, following a Poisson distribution.

Let M be the number of malfunctions in a randomly chosen week.

(a) Find $P(M \geq 3)$. Give your answer correct to 3 significant figures. [2]

(b) Find $P(1 \leq M \leq 4)$. Give your answer correct to 3 significant figures. [2]

12. MEDIUM Calculator **[4 marks]**

Vehicles pass a speed-monitoring point at a mean rate of 3.6 per minute, following a Poisson distribution.

(a) Find the probability that **more than 5** vehicles pass in a randomly chosen minute. Give your answer correct to 3 significant figures. [2]

(b) Find the probability that **at most 2** vehicles pass in a randomly chosen 30-second interval. Give your answer correct to 3 significant figures. [2]

13. MEDIUM Calculator **[4 marks]**

A marine biologist records the number of dolphin sightings per hour from a research vessel. The hourly count follows a Poisson distribution.

The table gives the number of sightings recorded each hour over 50 hours of observation.

Sightings per hour (x)	0	1	2	3	4	5	≥ 6
Frequency (f)	3	9	14	11	8	4	1

(a) Calculate the mean number of sightings per hour from the table above. Give your answer correct to 3 significant figures. [2]

(b) Hence find the probability that, in a randomly chosen hour, there are **exactly 3** dolphin sightings. Use your mean from part (a). Give your answer correct to 3 significant figures. [2]

14. MEDIUM Calculator **[4 marks]**

A pastry chef records the number of imperfections (e.g. air bubbles, uneven glaze) on each tart produced. The number of imperfections per tart follows a Poisson distribution with mean m.

It is known that $P(\text{no imperfections}) = 0.202$.

(a) Show that $m = 1.60$ correct to 3 significant figures. [2]

(b) Hence find the probability that a randomly chosen tart has **more than 2** imperfections. Give your answer correct to 3 significant figures. [2]

15. MEDIUM Calculator **[4 marks]**

Two independent streams of internet packets arrive at a router. Stream A delivers packets at a mean rate of 2.3 per millisecond and Stream B at 1.7 per millisecond. Both streams follow Poisson distributions.

Let $T = A + B$ be the total number of packets per millisecond.

(a) Write down the distribution of T, including its parameter. [1]

(b) Find $P(T \leq 3)$. Give your answer correct to 3 significant figures. [1]

(c) Find $P(T > 5)$. Give your answer correct to 3 significant figures. [2]

16. MEDIUM Calculator **[4 marks]**

A city bus arrives at a particular stop following a Poisson process with an average of 0.9 buses per 5-minute interval.

(a) Find the probability that **no** buses arrive in a randomly chosen 5-minute interval. Give your answer correct to 3 significant figures. [1]

(b) Find the probability that **no** buses arrive in **each** of three consecutive independent 5-minute intervals. Give your answer correct to 3 significant figures. [2]

(c) Find the mean number of buses that arrive in a 20-minute period. [1]

17. MEDIUM Calculator **[4 marks]**

The number of typos in a student's essay follows a Poisson distribution with mean 3.2 typos per page.

(a) Find the probability that a randomly chosen page has **at least 3** typos. Give your answer correct to 3 significant figures. [2]

(b) Eight pages are chosen at random. Find the probability that **exactly 3** of these pages each contain at least 3 typos. Give your answer correct to 3 significant figures. [2]

18. MEDIUM Calculator **[4 marks]**

A geologist counts the number of microfractures in core samples. Each 10 cm core sample contains microfractures modelled by a Poisson distribution with mean 1.5.

(a) Find the probability that a randomly chosen 10 cm sample contains **fewer than 2** microfractures. Give your answer correct to 3 significant figures. [2]

(b) Find the probability that a randomly chosen 30 cm sample contains **exactly 5** microfractures. Give your answer correct to 3 significant figures. [2]

19. **MEDIUM** **Calculator** **[4 marks]**

A quality inspector samples packets of seed. The number of contaminated seeds per packet follows a Poisson distribution with mean 2.7.

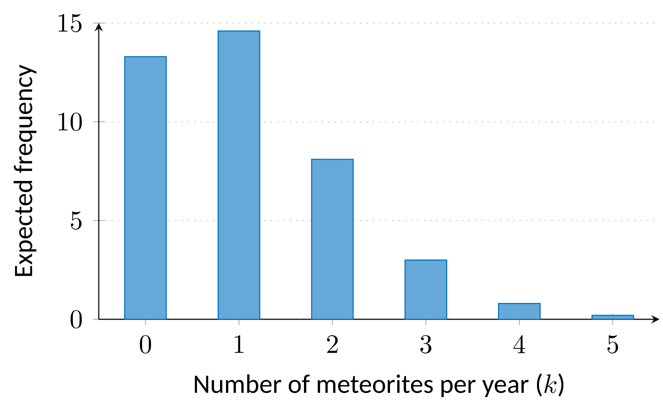
(a) Find $P(X = 0)$ and $P(X = 1)$, where X is the number of contaminated seeds in a randomly chosen packet. Give your answers correct to 3 significant figures. [2]

(b) Hence, or otherwise, find $P(X \geq 2)$. Give your answer correct to 3 significant figures. [2]

20. **MEDIUM** **Calculator** **[4 marks]**

Meteorites of detectable size fall in a particular desert region at a mean rate of 1.1 per year, modelled by a Poisson distribution.

The bar chart below shows the expected frequency distribution for the number of meteorites per year over 40 years, based on this model.



(a) Using the bar chart, write down the expected number of years (out of 40) in which **no** meteorites fall. [1]

(b) Find the probability that, in a randomly chosen year, **at most 2** meteorites fall. Give your answer correct to 3 significant figures. [2]

(c) Find the probability that, in a randomly chosen year, **more than 2** meteorites fall. Give your answer correct to 3 significant figures. [1]

Section C HARD (Questions 21–30, 5 marks each = 50 marks)

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21. HARD Calculator **[5 marks]**

Power outages in a city district are recorded at a mean rate of λ per week, following a Poisson distribution. It is known that the probability of **no outages** in a given week is 0.0821.

(a) Show that $\lambda = 2.50$ correct to 3 significant figures. [2]

(b) Hence find the probability of **at most 3** outages in a randomly chosen week. Give your answer correct to 3 significant figures. [2]

(c) State one reason why a Poisson distribution may not be a suitable model for the number of power outages per week in this district. [1]

22. HARD Calculator **[5 marks]**

Cosmic ray events are detected by a physics experiment at a mean rate of 3.8 per hour, following a Poisson distribution.

(a) Find the probability of **more than 4** events in a randomly chosen hour. Give your answer correct to 3 significant figures. [2]

(b) Six one-hour observation periods are chosen at random. Find the probability that **exactly 2** of them each record more than 4 events. Give your answer correct to 3 significant figures. [3]

23. HARD Calculator **[5 marks]**

Two independent teams monitor bird activity at separate sites. Team P records arrivals of migratory birds at a mean rate of 2.6 per day. Team Q records arrivals at a mean rate of 1.9 per day. Both counts follow Poisson distributions.

(a) Find the probability that the **combined** total of arrivals across both sites is exactly 4 in a randomly chosen day. Give your answer correct to 3 significant figures. [2]

(b) Find the probability that Team P records **more arrivals** than Team Q on a randomly chosen day, given that the combined total is exactly 4. Give your answer correct to 3 significant figures. [3]

24. HARD Calculator **[5 marks]**

A factory inspector monitors a production line for defective items. Defects occur at a mean rate of μ per batch, modelled by a Poisson distribution.

The inspector will flag a batch for review if it contains **3 or more** defects. It is required that the probability of a batch being flagged is **less than** 0.20.

(a) Find the largest value of μ (correct to 3 significant figures) such that the probability of a batch being flagged is less than 0.20. [3]

(b) Using this value of μ, find the probability that a batch contains **exactly 2** defects. Give your answer correct to 3 significant figures. [2]

25. HARD Calculator **[5 marks]**

Vehicles passing through a border checkpoint are flagged for secondary inspection at a mean rate of λ per hour. Border officials will activate an additional inspection booth if the probability of **at least 5** flaggings in an hour exceeds 0.35.

(a) Show that if $\lambda = 3.0$, the probability of at least 5 flaggings in an hour is 0.185 correct to 3 significant figures. [2]

(b) Find the smallest integer value of λ for which the additional booth must be activated. [3]

26. HARD Calculator **[5 marks]**

A climate station records lightning strikes in a $50\,\text{km}^2$ study area at a mean rate of 0.4 strikes per hour during storm events. The number of strikes per hour follows a Poisson distribution.

(a) Find the probability of **at least 2** strikes in a randomly chosen hour. Give your answer correct to 3 significant figures. [2]

(b) During a 3-hour storm, find the probability that there are **fewer than 3** strikes in total. Give your answer correct to 3 significant figures. [2]

(c) State one assumption you have made about the strikes across the 3-hour period. [1]

27. HARD Calculator **[5 marks]**

A railway signal system records faults at a mean rate of m per month, following a Poisson distribution.

An engineer proposes a hypothesis test. Over a 3-month trial period, the engineer will observe the total number of faults T and will conclude that the fault rate has **decreased** if $T \leq c$.

The historical mean is $m_0 = 2.8$ faults per month.

(a) State suitable null and alternative hypotheses in terms of the monthly mean m. [1]

(b) Find the smallest value of c such that the probability of a Type I error is less than 0.05. [3]

(c) State the conclusion of the test if 5 faults are observed over the 3-month period. [1]

28. **HARD** **Calculator** **[5 marks]**

A hospital records the number of rare adverse drug reactions (ADRs) each month. ADRs historically occur at a mean rate of 1.6 per month, modelled by a Poisson distribution.

Following a change in prescribing policy, the hospital observes ADRs over the next 6 months and records the following monthly counts.

Month	1	2	3	4	5	6
ADR count	0	2	1	0	1	2

A test is conducted at the 5% significance level to determine whether the mean rate has **decreased**.

(a) State the null and alternative hypotheses. [1]

(b) State the distribution of S, the total number of ADRs in 6 months, under H_0. [1]

(c) Find the p-value for the observed data and state the conclusion of the test in context. [3]

29. HARD Calculator **[5 marks]**

A school librarian models the number of book returns per day using a Poisson distribution with mean μ. The librarian claims that $\mu = 8.5$.

A colleague believes the actual mean is **higher** and records the number of book returns on 5 randomly chosen days.

Day	1	2	3	4	5
Returns	11	9	13	10	12

A hypothesis test is conducted at the 5% significance level using the total number of returns R over the 5 days.

(a) State the null and alternative hypotheses in terms of μ. [1]

(b) State the distribution of R under H_0, including its parameter. [1]

(c) Find the p-value for the observed data and state the conclusion of the test in context. [3]

30. **HARD** **Calculator** **[5 marks]**

Calls to an emergency dispatch centre follow a Poisson distribution with mean λ calls per hour. The centre has capacity to handle at most 8 calls per hour without delay. The manager requires that the probability of the centre being overwhelmed (i.e. receiving **more than 8** calls in an hour) is **less than** 0.10.

(a) Show that if $\lambda = 5.0$, the probability of more than 8 calls in an hour is 0.0681 correct to 3 significant figures. [2]

(b) Find the largest value of λ (correct to 1 decimal place) such that the manager's requirement is satisfied. [3]

Worked Solutions

Section A — Easy (Q1–10)

Q1 EASY

(a) $E(X) = 2.5$ **A1**

(b) $X \sim \text{Po}(2.5)$

$P(X = 3) = \dfrac{e^{-2.5} \times 2.5^3}{3!}$ **(M1)**

$= 0.213\,846\ldots \approx 0.214$ **A1**

Note: Accept answers obtained directly from GDC. Award (M1) for correct Poisson pdf formula or GDC setup with $\lambda = 2.5$, $k = 3$.

Q2 EASY

$F \sim \text{Po}(1.8)$

(a) $P(F = 0) = e^{-1.8} = 0.165\,299\ldots \approx 0.165$ **A1**

(b) $P(F \leq 2) = P(F = 0) + P(F = 1) + P(F = 2)$ **(M1)**

$= e^{-1.8}(1 + 1.8 + 1.8^2/2) = 0.730\,974\ldots \approx 0.731$ **A1**

Note: GDC: poissoncdf(1.8, 2). Award (M1) for correct setup.

Q3 EASY

$X \sim \text{Po}(4.2)$

(a) $P(X = 5) = \dfrac{e^{-4.2} \times 4.2^5}{5!}$ **(M1)**

$= 0.149\,7\ldots \approx 0.150$ **A1**

(b) $\text{Var}(X) = \lambda = 4.2$ **A1**

Q4 EASY

$X \sim \text{Po}(3.6)$

(a) Mean $= 3.6$; Variance $= 3.6$ **A1**

Note: Both values required for the mark.

(b) $P(X = 4) = \dfrac{e^{-3.6} \times 3.6^4}{4!}$ **(M1)**

$= 0.191\,2\ldots \approx 0.191$ **A1**

Q5 EASY

$X \sim \text{Po}(0.8)$

(a) $P(X = 0) = e^{-0.8} = 0.449\,328\ldots \approx 0.449$ **A1**

(b) $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$ **(M1)**
$= e^{-0.8}(1 + 0.8 + 0.32) = 0.952\,577\ldots \approx 0.953$ **A1**

Q6 EASY

$X \sim \text{Po}(1.2)$
(a) $P(X > 2) = 1 - P(X \leq 2)$ **(M1)**
$P(X \leq 2) = e^{-1.2}(1 + 1.2 + 0.72) = 0.878\,095\ldots$
$P(X > 2) = 1 - 0.878\,095\ldots = 0.121\,904\ldots \approx 0.122$ **A1**
(b) $\text{SD}(X) = \sqrt{1.2} = 1.09544\ldots \approx 1.10$ **A1**

Q7 EASY

(a) In a 30-minute period, $\lambda = 6 \times 0.5 = 3$.
$Y \sim \text{Po}(3)$ **A1**
Note: Award A0 if the rescaled parameter is not stated.
(b) $P(Y = 4) = \dfrac{e^{-3} \times 3^4}{4!}$ **(M1)**
$= 0.168\,031\ldots \approx 0.168$ **A1**

Q8 EASY

$X \sim \text{Po}(2.1)$
(a) $P(X = 2) = \dfrac{e^{-2.1} \times 2.1^2}{2!}$ **(M1)**
$= 0.270\,0\ldots \approx 0.270$ **A1**
(b) $\text{Var}(X) = E(X^2) - [E(X)]^2 \Rightarrow E(X^2) = \text{Var}(X) + [E(X)]^2$
$= 2.1 + 2.1^2 = 2.1 + 4.41 = 6.51$ **A1**

Q9 EASY

$X \sim \text{Po}(1.5)$
(a) $P(X \geq 1) = 1 - P(X = 0) = 1 - e^{-1.5}$ **(M1)**
$= 1 - 0.223\,130\ldots = 0.776\,869\ldots \approx 0.777$ **A1**
(b) Any one valid assumption, e.g.:
"Goals are scored independently of one another", or
"The mean rate of goals is constant throughout each match." **R1**
Note: Accept equivalent valid statements about independence, constant rate, or events not occurring simultaneously.

Q10 EASY

(a) In a 2-metre length, $\lambda = 0.6 \times 2 = 1.2$.

$P(X = 1) = \dfrac{e^{-1.2} \times 1.2^1}{1!} = 1.2\,e^{-1.2}$ **(M1)**

$= 0.361\,432\ldots \approx 0.361$ **A1**

(b) Mean in 5 metres $= 0.6 \times 5 = 3.0$ **A1**

Section B — Medium (Q11–20)

Q11 MEDIUM

$M \sim$ Po(2.4)

(a) $P(M \geq 3) = 1 - P(M \leq 2)$ **(M1)**

$P(M \leq 2) = e^{-2.4}(1 + 2.4 + 2.88) = 0.569\,591\ldots$

$P(M \geq 3) = 1 - 0.569\,591\ldots = 0.430\,408\ldots \approx 0.430$ **A1**

(b) $P(1 \leq M \leq 4) = P(M \leq 4) - P(M = 0)$ **(M1)**

$P(M \leq 4) = 0.903\,7\ldots \quad P(M = 0) = e^{-2.4} = 0.090\,71\ldots$

$P(1 \leq M \leq 4) = 0.903\,7\ldots - 0.090\,71\ldots = 0.813\,0\ldots \approx 0.813$ **A1**

Note: GDC: poissoncdf(2.4,4) — poissonpdf(2.4,0), or poissoncdf(2.4,4)—poissoncdf(2.4,0).

Q12 MEDIUM

(a) $X \sim$ Po(3.6)

$P(X > 5) = 1 - P(X \leq 5)$ **(M1)**

$P(X \leq 5) = 0.844\,0\ldots$

$P(X > 5) = 0.156\,0\ldots \approx 0.156$ **A1**

(b) In 30 seconds, $\lambda = 3.6 \times 0.5 = 1.8$.

$Y \sim$ Po(1.8) **(M1)**

$P(Y \leq 2) = e^{-1.8}(1 + 1.8 + 1.62) = 0.730\,974\ldots \approx 0.731$ **A1**

Note: Award (M1) for correctly rescaling λ to 1.8. FT from their rescaled λ.

Q13 MEDIUM

(a) $\bar{x} = \dfrac{0(3) + 1(9) + 2(14) + 3(11) + 4(8) + 5(4) + 6(1)}{50}$

$= \dfrac{0 + 9 + 28 + 33 + 32 + 20 + 6}{50} = \dfrac{128}{50}$ **(M1)**

$= 2.56$ **A1**

(b) $X \sim$ Po(2.56)

$P(X = 3) = \dfrac{e^{-2.56} \times 2.56^3}{3!}$ **(M1)**

$= 0.217\,4\ldots \approx 0.217$ **A1**

Note: FT from their mean in (a) provided it is positive. Award M1 for correct Poisson pdf formula or GDC setup using their mean.

Q14 MEDIUM

(a) $P(X=0)=e^{-m}=0.202$ **(M1)**
$\Rightarrow m=-\ln(0.202)=1.5995\ldots=1.60$ (3 s.f.) **A1 AG**
Note: The unrounded value 1.5995...must be seen before the rounded answer 1.60. No mark for AG line.

(b) $X \sim \text{Po}(1.60)$
$P(X>2)=1-P(X\le 2)$ **(M1)**
$P(X\le 2)=e^{-1.6}(1+1.6+1.28)=0.7827\ldots$
$P(X>2)=1-0.7827\ldots=0.2172\ldots\approx 0.217$ **A1**
Note: Accept $m=1.5995\ldots$ in part (b); if $m=1.60$ (3 s.f.) used, answer is 0.217 (same to 3 s.f.).

Q15 MEDIUM

(a) $T=A+B\sim \text{Po}(2.3+1.7)=\text{Po}(4.0)$ **A1**
(b) $P(T\le 3)=e^{-4}(1+4+8+\frac{32}{3})=0.4334\ldots\approx 0.433$ **A1**
(c) $P(T>5)=1-P(T\le 5)$ **(M1)**
$P(T\le 5)=0.7851\ldots$
$P(T>5)=1-0.7851\ldots=0.2148\ldots\approx 0.215$ **A1**
Note: In (a), independence must be assumed (implied by the word "independent" in the stem). FT (b)(c) from their parameter in (a).

Q16 MEDIUM

$X\sim \text{Po}(0.9)$
(a) $P(X=0)=e^{-0.9}=0.406569\ldots\approx 0.407$ **A1**
(b) $P(\text{no buses in each of 3 intervals})=[P(X=0)]^3$ **(M1)**
$=(e^{-0.9})^3=e^{-2.7}=0.067206\ldots\approx 0.0672$ **A1**
Note: Award (M1) for cubing their answer from (a). FT provided $0<p<1$.
(c) In 20 minutes (four 5-minute intervals): mean $=0.9\times 4=3.6$ **A1**

Q17 MEDIUM

$X\sim \text{Po}(3.2)$
(a) $P(X\ge 3)=1-P(X\le 2)$ **(M1)**
$P(X\le 2)=e^{-3.2}(1+3.2+5.12)=0.3803\ldots$
$P(X\ge 3)=1-0.3803\ldots=0.6197\ldots\approx 0.620$ **A1**
(b) Let $p=P(X\ge 3)=0.6197\ldots$
$Y\sim \text{B}(8,\ p)$ **(M1)**
$P(Y=3)=\binom{8}{3}p^3(1-p)^5$ **(M1)**

$= 56 \times (0.6197)^3 \times (0.3803)^5 = 0.162\,5\ldots \approx 0.163$ **A1**

Note: Award M1M1A1 for correct binomial setup with their p from (a), and correct evaluation. FT from part (a) provided $0 < p < 1$.

Q18 MEDIUM

(a) $X \sim$ Po(1.5)

$P(X < 2) = P(X = 0) + P(X = 1) = e^{-1.5}(1 + 1.5)$ **(M1)**

$= 2.5\,e^{-1.5} = 0.557\,825\ldots \approx 0.558$ **A1**

(b) In a 30 cm sample, $\lambda = 1.5 \times 3 = 4.5$.

$Y \sim$ Po(4.5) **(M1)**

$P(Y = 5) = \dfrac{e^{-4.5} \times 4.5^5}{5!} = 0.170\,6\ldots \approx 0.171$ **A1**

Note: Award M1 for correctly rescaling to $\lambda = 4.5$. FT from their rescaled parameter.

Q19 MEDIUM

$X \sim$ Po(2.7)

(a) $P(X = 0) = e^{-2.7} = 0.067\,206\ldots \approx 0.0672$ **A1**

$P(X = 1) = 2.7\,e^{-2.7} = 0.181\,457\ldots \approx 0.181$ **A1**

(b) $P(X \geq 2) = 1 - P(X = 0) - P(X = 1)$ **(M1)**

$= 1 - 0.067\,206\ldots - 0.181\,457\ldots$

$= 0.751\,336\ldots \approx 0.751$ **A1**

Note: FT from (a): $P(X \geq 2) = 1 -$ their $P(X = 0) -$ their $P(X = 1)$, provided both values were correctly computed.

Q20 MEDIUM

$X \sim$ Po(1.1)

(a) From the bar chart, expected frequency for $k = 0$ is $13.3 \approx 13.3$ years. **A1**

Note: Accept 13 or 13.3; the bar is clearly read from the chart.

(b) $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$ **(M1)**

$= e^{-1.1}(1 + 1.1 + 0.605) = 0.332\,87\ldots + 0.366\,16\ldots + 0.201\,39\ldots$

$= 0.900\,4\ldots \approx 0.900$ **A1**

(c) $P(X > 2) = 1 - P(X \leq 2) = 1 - 0.900\,4\ldots = 0.099\,6\ldots \approx 0.0996$ **A1**

Note: FT from (b): $P(X > 2) = 1 -$ their $P(X \leq 2)$.

Section C — Hard (Q21–30)

Q21 HARD

$X \sim \text{Po}(\lambda)$

(a) $P(X=0) = e^{-\lambda} = 0.0821$ **(M1)**

$\lambda = -\ln(0.0821) = 2.4996\ldots = 2.50$ (3 s.f.) **A1 AG**

Note: The full precision value 2.4996...must appear before the rounded 2.50. AG carries no mark.

(b) $X \sim \text{Po}(2.50)$

$P(X \leq 3) = e^{-2.5}(1 + 2.5 + 3.125 + 2.604\overline{1}\ldots)$ **(M1)**

$= 0.757\,576\ldots \approx 0.758$ **A1**

(c) Any one valid reason, e.g.:

"Power outages are unlikely to be independent — one outage may trigger others due to grid cascade failure", or

"The mean rate may not be constant throughout the year (seasonal variation)." **R1**

Note: Accept any contextually justified statement about non-independence or non-constant rate.

Q22 HARD

$X \sim \text{Po}(3.8)$

(a) $P(X > 4) = 1 - P(X \leq 4)$ **(M1)**

$P(X \leq 4) = e^{-3.8}(1 + 3.8 + 7.22 + 9.1786\overline{6} + 8.7197\ldots) \times e^{-3.8}$

GDC: $P(X \leq 4) = 0.668\,4\ldots$

$P(X > 4) = 1 - 0.668\,4\ldots = 0.331\,6\ldots \approx 0.332$ **A1**

(b) Let $p = P(X > 4) = 0.3316\ldots$ Let $N \sim \text{B}(6,\ p)$ **(M1)**

$P(N = 2) = \binom{6}{2}p^2(1-p)^4$ **(M1)**

$= 15 \times (0.3316)^2 \times (0.6684)^4 = 15 \times 0.11000\ldots \times 0.19952\ldots$

$= 0.329\,7\ldots \approx 0.330$ **A1**

Note: Award M1 for recognising a Binomial model with $n = 6$ and their p from (a); M1 for correct $P(N = 2)$ expression; A1 for correct answer. FT from (a) provided $0 < p < 1$.

Q23 HARD

$P \sim \text{Po}(2.6)$, $Q \sim \text{Po}(1.9)$, independent; $T = P + Q \sim \text{Po}(4.5)$

(a) $P(T = 4) = \dfrac{e^{-4.5} \times 4.5^4}{4!}$ **(M1)**

$= 0.189\,7\ldots \approx 0.190$ **A1**

(b) We need $P(P > Q \mid T = 4)$.

Given $T = P + Q = 4$, the value of P is $\text{B}(4,\ \frac{2.6}{4.5})$ **(M1)**

$p_P = \dfrac{2.6}{4.5} = \dfrac{26}{45}$

Events where $P > Q$: $P = 3, Q = 1$ or $P = 4, Q = 0$ **(M1)**

$P(P > Q \mid T = 4) = \binom{4}{3}\left(\frac{26}{45}\right)^3\left(\frac{19}{45}\right)^1 + \binom{4}{4}\left(\frac{26}{45}\right)^4$

$= 4 \times (0.5778)^3 \times 0.4222 + (0.5778)^4$

$= 4 \times 0.19285\ldots \times 0.4222 + 0.11148\ldots$

$= 0.32573\ldots + 0.11148\ldots = 0.437\,2\ldots \approx 0.437$ **A1**

Note: Award M1 for identifying the conditional binomial structure; M1 for listing the correct cases ($P = 3$ or $P = 4$); A1 for correct final answer.

Q24 HARD

$X \sim$ Po(μ). Require $P(X \geq 3) < 0.20$, i.e. $1 - P(X \leq 2) < 0.20$, so $P(X \leq 2) > 0.80$.

(a) $P(X \leq 2) = e^{-\mu}(1 + \mu + \frac{\mu^2}{2}) > 0.80$ **(M1)**

Using GDC or systematic search: **(M1)**

μ	$P(X \leq 2)$	Satisfies > 0.80?
1.5	0.8088	Yes
1.6	0.7834	No
1.53	0.8020	Yes (borderline)
1.54	0.7997	No

Largest μ such that $P(X \leq 2) > 0.80$: $\mu = 1.53$ (3 s.f.) **A1**

(b) $P(X = 2) = \dfrac{e^{-1.53} \times 1.53^2}{2}$ **(M1)**

$= \dfrac{0.216\,5\ldots \times 2.3409}{2} = 0.253\,5\ldots \approx 0.254$ **A1**

Note: Award M1A1A1 in (a) for setting up the inequality, systematic or GDC search, and correct critical value. Award M1A1 in (b) for using their μ in the Poisson pdf formula.

Q25 HARD

$X \sim$ Po(λ). Require $P(X \geq 5) > 0.35$.

(a) When $\lambda = 3.0$:

$P(X \geq 5) = 1 - P(X \leq 4)$ **(M1)**

$P(X \leq 4) = e^{-3}(1 + 3 + 4.5 + 4.5 + 3.375) = e^{-3} \times 16.375$

$= 0.049787\ldots \times 16.375 = 0.815\,263\ldots$

$P(X \geq 5) = 1 - 0.815\,263\ldots = 0.184\,737\ldots \approx 0.185$ **A1 AG**

Note: The full-precision value 0.18473...must be visible. AG carries no mark.

(b) Search integer values: **(M1)**

λ (integer)	$P(X \geq 5)$
3	0.185
4	0.371
5	0.560

$\lambda = 4$: $P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.628\,8\ldots = 0.371\,2\ldots > 0.35$ **(A1)**

$\lambda = 3$: $P(X \geq 5) = 0.185 < 0.35$ (does not trigger)

Smallest integer λ for which additional booth must be activated: $\lambda = 4$ **A1**

Note: Award M1 for systematic evaluation of integer λ values; A1 for correct identification that $\lambda = 4$ first exceeds 0.35; A1 for conclusion.

Q26 HARD

$X \sim$ Po(0.4)

(a) $P(X \geq 2) = 1 - P(X \leq 1) = 1 - e^{-0.4}(1 + 0.4)$ **(M1)**

$= 1 - 0.670\,320\ldots = 0.329\,679\ldots - 1 + 1$

$= 1 - 1.4\,e^{-0.4} = 1 - 0.938\,448\ldots = 0.061\,551\ldots \approx 0.0616$ **A1**

(b) Over 3 hours, $Y \sim$ Po$(0.4 \times 3) =$ Po(1.2) **(M1)**

$P(Y < 3) = P(Y \leq 2) = e^{-1.2}(1 + 1.2 + 0.72)$ **(M1)**

$= 2.92\,e^{-1.2} = 0.878\,095\ldots \approx 0.878$ **A1**

(c) *"The Poisson rate of strikes is constant throughout the 3-hour storm period"*, or *"lightning strikes occur independently within the 3-hour period."* **R1**

Q27 HARD

(a) H_0: $m = 2.8$ (mean fault rate is 2.8 per month)

H_1: $m < 2.8$ (mean fault rate has decreased) **A1**

Note: Accept hypotheses stated in terms of the 3-month mean (i.e. H_0: $\mu = 8.4$).

(b) Under H_0, total over 3 months: $T \sim$ Po$(3 \times 2.8) =$ Po(8.4)

Type I error: reject H_0 when H_0 is true, i.e. $P(T \leq c) < 0.05$ **(M1)**

Check values: **(M1)**

c	$P(T \leq c)$
3	0.0134
4	0.0395
5	0.0959

$c = 4$: $P(T \leq 4) = 0.0395 < 0.05$ ✓

$c = 5$: $P(T \leq 5) = 0.0959 > 0.05$ ×

Smallest c such that $P(T \leq c) < 0.05$: $c = 4$ **A1**

(c) Observed $T = 5$. Since $5 > 4 = c$, we **do not reject** H_0.

There is insufficient evidence at the 5% level to conclude that the fault rate has decreased. **A1**

Note: Award A1 for a non-assertive contextual conclusion referencing the comparison $T = 5 > c = 4$.

Q28 HARD

(a) H_0: $\mu = 1.6$ (mean ADR rate has not changed)

H_1: $\mu < 1.6$ (mean ADR rate has decreased) **A1**

(b) Under H_0, total over 6 months: $S \sim$ Po$(6 \times 1.6) =$ Po(9.6) **A1**

(c) Observed total: $S = 0 + 2 + 1 + 0 + 1 + 2 = 6$ **(M1)**
p-value $= P(S \leq 6) =$ poissoncdf$(9.6,\ 6)$ **(M1)**
$= 0.127\,6\ldots \approx 0.128$
Since $0.128 > 0.05$, we **do not reject** H_0. **A1**
There is insufficient evidence at the 5% significance level to conclude that the mean number of ADRs has decreased.
Note: Award (M1) for computing the observed total $S = 6$; (M1) for finding $P(S \leq 6)$ under Po(9.6); A1 for correct p-value, comparison, and non-assertive contextual conclusion. Conclusion must reference the context (ADRs / policy).

Q29 **HARD**

(a) H_0: $\mu = 8.5$ (mean daily book returns is 8.5)
H_1: $\mu > 8.5$ (mean daily book returns is greater than 8.5) **A1**

(b) Under H_0, total over 5 days: $R \sim$ Po$(5 \times 8.5) =$ Po(42.5) **A1**

(c) Observed total: $R = 11 + 9 + 13 + 10 + 12 = 55$ **(M1)**
p-value $= P(R \geq 55) = 1 - P(R \leq 54)$ **(M1)**
GDC: $P(R \leq 54) =$ poissoncdf$(42.5,\ 54) = 0.955\,0\ldots$
p-value $= 1 - 0.955\,0\ldots = 0.0450\ldots \approx 0.0450$
Since $0.0450 < 0.05$, we **reject** H_0. **A1**
There is sufficient evidence at the 5% significance level to suggest that the mean number of daily book returns is greater than 8.5.
Note: Award (M1) for finding the total $R = 55$; (M1) for evaluating $P(R \geq 55)$ under Po(42.5); A1 for correct p-value, comparison to 0.05, and non-assertive contextual conclusion. Conclusion must not state "proves" or "the mean is greater".

Q30 **HARD**

$X \sim$ Po(λ)
(a) When $\lambda = 5.0$:
$P(X > 8) = 1 - P(X \leq 8)$ **(M1)**
GDC: $P(X \leq 8) =$ poissoncdf$(5.0,\ 8) = 0.931\,906\ldots$
$P(X > 8) = 1 - 0.931\,906\ldots = 0.068\,093\ldots \approx 0.0681$ **A1 AG**
Note: Full-precision value 0.06809...must appear. AG carries no mark.

(b) Require $P(X > 8) < 0.10$, i.e. $1 - P(X \leq 8) < 0.10$,
so $P(X \leq 8) > 0.90$.
Systematic search (to 1 d.p.): **(M1)**

λ	$P(X \leq 8)$
5.0	0.9319
5.5	0.9077
6.0	0.8472
5.6	0.9003
5.7	0.8894

$\lambda = 5.6$: $P(X \leq 8) = 0.9003 > 0.90$ ✓
$\lambda = 5.7$: $P(X \leq 8) = 0.8894 < 0.90$ × **(A1)**
Largest λ to 1 d.p. satisfying the requirement: $\lambda = 5.6$ **A1**
Note: Award M1 for a systematic search at 1 d.p. precision; A1 for correctly identifying the boundary pair; A1 for the correct conclusion $\lambda = 5.6$.

End of Worked Solutions

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