IB Math AI HL — Topic 4 [Set 1]

Hypothesis Testing using the Chi-Squared Distribution

Introduction to Hypothesis Testing · Chi-Squared Test for Independence · Goodness of Fit Test

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

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Name:______________________________ Date:_______________

Score:__________/120

Instructions: Answer all questions. Show all working. GDC permitted throughout. Give answers correct to 3 significant figures unless stated. Conclusions must be written in context with reference to the significance level.

Key Formulae

Chi-Squared Test Statistic:

$$\chi^2_{\text{calc}} = \sum \frac{(f_o - f_e)^2}{f_e}$$

Expected Frequency (Independence Test):

$$f_e = \frac{\text{row total} \times \text{column total}}{\text{grand total}}$$

Degrees of Freedom:

- **Independence test:** $\nu = (r-1)(c-1)$, where r = number of rows, c = number of columns
- **Goodness of fit test:** $\nu = k - 1$ (uniform/given ratios); $\nu = k - 1 - p$ when p parameters are estimated from data

Decision Rule: Reject H_0 if p-value $< \alpha$ (significance level), or equivalently if $\chi^2_{\text{calc}} > \chi^2_{\text{critical}}$.

Condition: All expected frequencies must be ≥ 5 (combine cells if necessary).

GDC Tips

- **TI-Nspire:** `Menu` $\rightarrow$ `6:Statistics` $\rightarrow$ `7:Stat Tests` $\rightarrow$ χ^2 `GOF Test` (goodness of fit) or χ^2 `2-way Test` (independence). Enter observed frequencies into a matrix or list.
- **Casio fx-CG50:** `STAT` $\rightarrow$ `TEST` $\rightarrow$ `CHI` then choose `GOF` or `2WAY`. Input observed values; the GDC computes χ^2_{calc}, p-value, and expected values.
- Always read the **p-value** from the GDC output and compare it to α directly.
- For the independence test, you can enter the contingency table as a matrix: `Matrix A` then use χ^2 `2-way Test`.
- To find a **critical value**: `InvChi(`$1-\alpha$`, df)` on Nspire or χ^2`CD inverse` on Casio.

Common Mistakes to Avoid

1. **Conclusion wording.** Never write "accept H_0" or "prove H_1". Instead: "There is sufficient/insufficient evidence to reject H_0, i.e. [context]".
2. **Degrees of freedom for GOF.** If you *estimate* p parameters from the data (e.g. mean of a Poisson), subtract them: $\nu = k - 1 - p$, not just $k - 1$.
3. **Expected frequencies below 5.** If any $f_e < 5$, you must combine adjacent cells before performing the test. Forgetting this invalidates the conclusion.
4. **Hypotheses must be in parameter/independence language.** H_0 for independence: "[variable A] and [variable B] are independent". Never just write "no difference".
5. **Using α vs p.** Compare the p-value to α, not to 0.05 by default — always state the significance level being used.
6. **Two-sided vs one-sided.** The χ^2 test is *always* one-tailed (right tail). Do not halve the p-value.

Section A — Easy **EASY** (10 questions × 3 marks = 30 marks)

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1. **EASY** **GDC** **[3 marks]**

A sports club surveys 200 members to investigate whether preferred sport and age group are independent. Members choose exactly one sport from: *swimming*, *cycling*, or *running*. The table gives the observed frequencies.

	Swimming	**Cycling**	**Running**	**Total**
Under 30	28	35	17	80
30 to 50	22	41	27	90
Over 50	10	14	6	30
Total	60	90	50	200

A χ^2 test for independence is performed at the 5% significance level.

(a) State the null hypothesis H_0 for this test. **[1]**

(b) Write down the number of degrees of freedom. **[1]**

(c) The χ^2 statistic is 3.17. Write down the conclusion of the test, justifying your answer. **[1]**

2. EASY GDC **[3 marks]**

A school canteen records the lunch choices of 180 students. The data is classified by year group (Year 10 or Year 11) and meal type (hot meal, salad, or sandwich).

	Hot meal	Salad	Sandwich	Total
Year 10	38	24	28	**90**
Year 11	42	26	22	**90**
Total	80	50	50	**180**

(a) State the alternative hypothesis H_1 for a χ^2 test for independence between year group and meal type. [1]

(b) Calculate the expected frequency for Year 10 students choosing a hot meal. [1]

(c) Write down the degrees of freedom for this test. [1]

3. EASY GDC **[3 marks]**

A board game uses a six-sided die. To test whether the die is fair, it is rolled 120 times. The results are shown below.

Score	1	2	3	4	5	6
Observed frequency	23	18	22	19	21	17

A χ^2 goodness of fit test at the 10% significance level is used to test whether the die is fair.

(a) State the null hypothesis H_0 for this test. [1]

(b) Write down the expected frequency for each score. [1]

(c) Write down the degrees of freedom for this test. [1]

4. EASY GDC **[3 marks]**

A researcher tests whether reading preference (fiction or non-fiction) is independent of gender (male or female) at the 5% significance level. The χ^2 test statistic is found to be $\chi^2_{\text{calc}} = 5.83$. The critical value at the 5% significance level with 1 degree of freedom is 3.841.

(a) State the degrees of freedom for this test. [1]

(b) State whether the null hypothesis should be rejected. Justify your answer using the critical value. [1]

(c) Write a conclusion for this test in context. [1]

5. EASY GDC **[3 marks]**

A genetics experiment crosses two plant varieties. According to a genetic model, the offspring should appear in the ratio red : pink : white $= 1 : 2 : 1$. In a sample of 200 offspring, 54 are red, 108 are pink, and 38 are white.

(a) State the null hypothesis H_0 for a χ^2 goodness of fit test. [1]

(b) Find the expected number of red offspring. [1]

(c) Write down the degrees of freedom for this test. [1]

6. EASY GDC **[3 marks]**

A χ^2 test for independence between coffee type preference (espresso, latte, or cappuccino) and time of day (morning or afternoon) is performed at the 1% significance level. The GDC gives a p-value of 0.0342.

(a) Write down the number of degrees of freedom for this test. [1]

(b) State whether H_0 should be rejected at the 1% significance level. Justify your answer. [1]

(c) Write a conclusion for this test in context. [1]

7. EASY GDC **[3 marks]**

A survey of 160 commuters asks which form of transport they use: bus, train, or car. A researcher claims the commuters are equally split between the three options. The observed frequencies are: bus 48, train 62, car 50.

(a) Write down the expected frequency for each transport type under H_0. [1]

(b) The χ^2 statistic is 2.40. Write down the degrees of freedom. [1]

(c) The critical value at the 5% significance level is 5.991. State the conclusion in context. [1]

8. EASY GDC **[3 marks]**

A hospital records data on 240 patients, classified by blood type (A, B, or O) and whether they have a particular allergy (yes or no). The table summarises the results.

	Blood type A	Blood type B	Blood type O	Total
Allergy: Yes	32	18	50	**100**
Allergy: No	48	42	50	**140**
Total	80	60	100	**240**

(a) State H_0 for a χ^2 test of independence. [1]

(b) Find the expected frequency of patients with blood type B who have the allergy. [1]

(c) Write down the degrees of freedom. [1]

9. EASY GDC **[3 marks]**

A spinner has four equal sectors labelled 1, 2, 3, 4. It is spun 200 times. A χ^2 goodness of fit test is performed at the 5% significance level to test whether the spinner is fair. The p-value from the GDC is 0.312.

(a) State the null hypothesis H_0. **[1]**

(b) Write down the expected frequency for each sector. **[1]**

(c) State the conclusion of the test, justifying with the p-value. **[1]**

10. EASY GDC **[3 marks]**

A supermarket manager investigates whether the day of the week (weekday or weekend) affects whether customers purchase a loyalty card (yes or no). Data from 300 customers is shown below.

	Weekday	Weekend	Total
Loyalty card: Yes	72	48	**120**
Loyalty card: No	108	72	**180**
Total	180	120	**300**

(a) State H_0 and H_1 for a χ^2 test of independence. **[1]**

(b) Write down the degrees of freedom. **[1]**

(c) Find the expected frequency of weekday customers who purchase a loyalty card. **[1]**

Section B — Medium MEDIUM (10 questions × 4 marks = 40 marks)

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11. MEDIUM GDC **[4 marks]**

A travel agency surveys 250 clients to investigate whether travel destination preference (beach, city, or mountain) is independent of client age group (under 40 or 40 and over). The observed frequencies are given in the table below.

	Beach	City	Mountain	Total
Under 40	55	48	27	130
40 and over	45	42	33	120
Total	100	90	60	250

A χ^2 test for independence is carried out at the 10% significance level.

(a) State H_0 and H_1. [1]

(b) Find the expected frequency of clients aged under 40 who prefer a mountain destination. [1]

(c) Find the p-value for this test. [1]

(d) State the conclusion of the test in context. [1]

12. MEDIUM GDC **[4 marks]**

A confectionery company claims that a packet of sweets contains flavours in the ratio strawberry : lemon : orange : blackcurrant $= 3 : 3 : 2 : 2$. A quality inspector samples 200 sweets and records the frequencies shown below.

Flavour	Strawberry	Lemon	Orange	Blackcurrant
Observed	64	58	44	34

A χ^2 goodness of fit test is performed at the 5% significance level.

(a) State H_0 for this test. **[1]**

(b) Find the expected frequency for each flavour. **[1]**

(c) Find the χ^2_{calc} statistic. **[1]**

(d) State the conclusion of the test, given that the critical value is 7.815. **[1]**

13. **MEDIUM** **GDC** **[4 marks]**

A university investigates whether faculty (Science, Arts, Business) and type of accommodation (on-campus or off-campus) are independent. Data from 360 students is given below.

	Science	Arts	Business	Total
On-campus	72	54	54	**180**
Off-campus	48	66	66	**180**
Total	120	120	120	**360**

A χ^2 test for independence is performed at the 5% significance level.

(a) Write down the expected frequency for Science students living on-campus. [1]

(b) Find the χ^2_{calc} statistic. [1]

(c) The p-value is 0.0273. State H_0 and the conclusion of the test in context. [2]

14. MEDIUM GDC **[4 marks]**

The number of customer complaints received per day by a call centre is recorded over 100 days. The results are summarised below.

Complaints per day	0	1	2	3	≥ 4
Observed frequency	14	32	31	16	7

A manager claims the data follows a Poisson distribution with mean 1.7. A χ^2 goodness of fit test is performed at the 5% significance level.

(a) State H_0 for this test. **[1]**

(b) Show that the expected frequency for 0 complaints per day is 18.3 (correct to 3 s.f.). **[1]**

(c) Using the GDC, perform the test. Write down the p-value and state the conclusion in context. **[2]**

15. MEDIUM GDC **[4 marks]**

A cinema chain surveys 450 customers, recording their age group (teens, adults, seniors) and their preferred film genre (action, drama, comedy). The table gives the observed frequencies.

	Action	**Drama**	**Comedy**	**Total**
Teens	75	30	45	150
Adults	60	75	65	200
Seniors	15	55	30	100
Total	150	160	140	450

A χ^2 test for independence is performed at the 1% significance level.

(a) Find the expected frequency of senior customers who prefer drama. **[1]**

(b) State H_0 and H_1 for this test. **[1]**

(c) Using your GDC, find the p-value and state the conclusion of the test in context. **[2]**

16. MEDIUM GDC **[4 marks]**

An ecologist counts the number of eggs per nest for 180 bird nests, believing that eggs per nest follow the distribution: 1 egg (20%), 2 eggs (35%), 3 eggs (30%), 4 eggs (10%), 5 or more eggs (5%). The observed frequencies are shown.

Eggs per nest	1	2	3	4	≥ 5
Observed	38	62	57	16	7

A χ^2 goodness of fit test is performed at the 5% significance level.

(a) Find the expected frequency for each category. [1]

(b) Explain why the last two categories should be combined before performing the test. [1]

(c) State H_0 and, using your GDC, find the p-value and state the conclusion. [2]

17. MEDIUM GDC **[4 marks]**

A social researcher surveys 320 people on whether they support a proposed traffic policy (agree or disagree), classified by their suburb of residence (North, Central, South). The results are given below.

	North	Central	South	Total
Agree	62	58	40	**160**
Disagree	58	62	40	**160**
Total	120	120	80	**320**

(a) Find the expected frequency for North residents who agree. **[1]**

(b) Hence write down the expected frequency for North residents who disagree. **[1]**

(c) State H_0 and H_1. Using your GDC, find the p-value and write down the conclusion at the 5% significance level. **[2]**

18. MEDIUM GDC **[4 marks]**

A quality controller tests batches of 5 items from a production line. In 200 batches, the number of defective items per batch is recorded as shown.

Defectives per batch	0	1	2	3	4	5
Observed frequency	85	80	27	6	2	0

It is proposed that the number of defectives per batch follows a $B(5,\ 0.12)$ distribution. A χ^2 goodness of fit test is performed at the 5% significance level.

(a) Find the expected frequency for 0 defectives and for 1 defective. **[2]**

(b) Explain why it is necessary to combine some categories before performing the test and state which categories should be combined. **[1]**

(c) State the conclusion of the test, given that the p-value is 0.462. **[1]**

19. MEDIUM GDC **[4 marks]**

A physiotherapist records the type of injury (muscle strain, joint sprain, or bone stress fracture) for 280 patients, classified by sport (team sport or individual sport). The table shows both observed and expected frequencies.

	Muscle strain	Joint sprain	Bone fracture	Total
Team sport	72	56	32	160
Individual	58	44	18	120
Total	130	100	50	280

(a) Calculate the expected frequency for team sport patients with a joint sprain. **[1]**

(b) Calculate the χ^2_{calc} statistic. **[2]**

(c) State the conclusion at the 5% significance level, given that the critical value with 2 degrees of freedom is 5.991. **[1]**

20. MEDIUM GDC **[4 marks]**

A teacher records the test scores (out of 100) for 120 students and groups them as shown. A colleague claims the scores follow a $N(65, 12^2)$ distribution. Expected frequencies under this model are provided.

Score range	$[0, 45)$	$[45, 55)$	$[55, 65)$	$[65, 75)$	$[75, 85)$	$[85, 100]$
Observed	6	18	32	34	22	8
Expected	5.3	14.7	29.7	32.9	25.0	12.4

A χ^2 goodness of fit test is performed at the 5% significance level.

(a) State H_0 and the degrees of freedom for this test. **[2]**

(b) Using your GDC with the values given, find the p-value. **[1]**

(c) State the conclusion of the test in context. **[1]**

Section C — Hard **HARD** (10 questions $\times$ 5 marks = 50 marks)

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21. **HARD** **GDC** **[5 marks]**

A company surveys 400 employees, classified by department (Marketing, Engineering, HR) and job satisfaction level (satisfied or dissatisfied). Part of the data is shown; the value k is unknown.

	Marketing	Engineering	HR	Total
Satisfied	90	k	48	
Dissatisfied	30	80	32	**142**
Total	120	200	80	**400**

(a) Find the value of k. [1]

(b) Calculate the expected frequency of Engineering employees who are satisfied. [1]

(c) State H_0 and H_1 for a χ^2 test of independence. [1]

(d) Using your GDC, find the p-value and state the conclusion at the 5% significance level. [2]

22. HARD GDC **[5 marks]**

A wildlife researcher records the number of sightings of a rare bird per hour over 80 observation periods. The results are shown below.

Sightings per hour	0	1	2	3	4	≥ 5
Observed frequency	18	26	20	10	4	2

The researcher wishes to test, at the 5% significance level, whether the data follows a Poisson distribution, estimating the mean from the data.

(a) Show that the estimated mean number of sightings per hour is 1.525. **[2]**

(b) State H_0 and write down the degrees of freedom for the test. **[2]**

(c) Using your GDC, perform the test and state the conclusion in context. **[1]**

23. HARD GDC **[5 marks]**

A hotel chain investigates whether room type preference (standard, deluxe, suite) is independent of booking method (online or in-person) for a sample of 500 guests. The table shows the observed frequencies; one cell is missing.

	Standard	Deluxe	Suite	Total
Online	165	120	55	**340**
In-person	95	40	m	
Total	260	160	80	**500**

(a) Find the value of m. **[1]**

(b) Find the expected frequency for online guests choosing a suite. **[1]**

(c) State H_0 and H_1 for a χ^2 test of independence at the 5% significance level. **[1]**

(d) Find the p-value using your GDC and state the conclusion in context. **[2]**

24. HARD GDC **[5 marks]**

A factory claims that items produced are classified as: Grade A (60%), Grade B (30%), Grade C (10%). An inspector randomly selects 250 items and records the following.

Grade	A	B	C
Observed	142	81	27

(a) State H_0 for a χ^2 goodness of fit test. **[1]**

(b) Find the expected frequency for each grade. **[1]**

(c) Using your GDC, find the χ^2_{calc} statistic and p-value. **[1]**

(d) State the conclusion at the 5% significance level in context. **[1]**

(e) Explain what a Type I error would mean in this context. **[1]**

25. HARD GDC **[5 marks]**

A researcher tests whether language studied (French, Spanish, or Mandarin) is independent of the school sector (state or private) for 480 students. The incomplete table is shown below.

	French	Spanish	Mandarin	Total
State	126	88	26	**240**
Private	114	n	34	**240**
Total	240		60	**480**

(a) Find the value of n and the total for the Spanish column. **[2]**

(b) State H_0 and H_1. **[1]**

(c) Find the expected frequency for private school students studying Mandarin. **[1]**

(d) Using your GDC, find the p-value and state the conclusion at the 5% significance level in context. **[1]**

26. **HARD** **GDC** **[5 marks]**

A bus company records the number of breakdowns per week over 60 weeks. The data is summarised below.

Breakdowns per week	0	1	2	3	≥ 4
Observed frequency	14	24	14	6	2

The company wishes to model breakdowns using a Poisson distribution, estimating the mean from the data.

(a) Show that the estimated mean number of breakdowns per week is 1.30 (to 3 s.f.). **[2]**

(b) State H_0 and H_1 for a χ^2 goodness of fit test at the 5% significance level. State the degrees of freedom. **[1]**

(c) Using your GDC, find the p-value and state the conclusion in context. **[1]**

(d) Describe what a Type II error would represent in this context. **[1]**

27. HARD GDC **[5 marks]**

A nutritionist investigates whether dietary preference (vegan, vegetarian, or omnivore) is independent of exercise frequency (never, sometimes, regularly) for a sample of 540 adults. The observed frequencies are given.

	Never	Sometimes	Regularly	Total
Vegan	18	42	60	120
Vegetarian	30	60	90	180
Omnivore	42	88	110	240
Total	90	190	260	540

(a) Find the expected frequency for vegan adults who exercise regularly. **[1]**

(b) State H_0 and H_1 and perform the χ^2 test at the 5% significance level, stating the p-value and conclusion. **[3]**

(c) State whether the conclusion would change at the 1% significance level. Justify your answer. **[1]**

28. HARD GDC **[5 marks]**

A marketing team surveys customers on which of four product colours they prefer: red, blue, green, or yellow. They hypothesise that customers have no colour preference (i.e. equal probability of choosing each). In a sample of 240 customers the frequencies are: red 72, blue 54, green 68, yellow q.

(a) Find the value of q. **[1]**

(b) State H_0 and H_1 for a χ^2 goodness of fit test at the 5% significance level. **[1]**

(c) Find the expected frequency for each colour. **[1]**

(d) Using your GDC, find the χ^2_{calc} statistic and the p-value. Hence state the conclusion in context. **[2]**

29. HARD GDC **[5 marks]**

A public health researcher investigates whether the type of diet (Mediterranean, Western, or Plant-based) is independent of the incidence of hypertension (yes or no) for 600 adults. The table shows the observed data.

	Mediterranean	Western	Plant-based	Total
Hypertension: Yes	48	112	40	**200**
Hypertension: No	152	88	160	**400**
Total	200	200	200	**600**

(a) State H_0 and H_1 for a χ^2 test of independence. **[1]**

(b) Find the expected frequency for Western diet adults with hypertension. **[1]**

(c) Using your GDC, find the p-value and state the conclusion at the 1% significance level. **[2]**

(d) State one limitation of using a χ^2 test to establish whether diet causes hypertension. **[1]**

30. HARD GDC **[5 marks]**

A psychologist surveys 360 participants on their preferred social media platform (Platform A, Platform B, Platform C) and their age group (18–30, 31–50, over 50). One frequency, t, is unknown.

	Platform A	Platform B	Platform C	Total
18–30	80	30	70	**180**
31–50	40	70	t	
Over 50	10	10	20	**40**
Total	130	110	120	**360**

(a) Find the value of t and hence complete the row total for the 31–50 group. [1]

(b) State H_0 and H_1 for a χ^2 test of independence at the 5% significance level. [1]

(c) Find the expected frequency for 31–50 year-olds who prefer Platform B. [1]

(d) Using your GDC, find the p-value and state the conclusion in context. [1]

(e) State the probability of making a Type I error for this test. [1]

Worked Solutions — Section A (Easy)

Q1 — Chi-squared Independence: Hypotheses and Degrees of Freedom EASY

(a) H_0: Preferred sport and age group are independent. **A1**

(b) $\nu = (3-1)(3-1) = 4$ **A1**

(c) $\chi^2_{\text{calc}} = 3.17 < \chi^2_{\text{crit}}$ (critical value with 4 df at 5% is 9.488)

∴ Insufficient evidence to reject H_0; preferred sport and age group are independent. **R1**

Note: Award R1 for a correct comparison and a non-assertive contextual conclusion. Do not award R1 for "accept H_0" without context.

Q2 — Chi-squared Independence: Alternative Hypothesis and Expected Frequency EASY

(a) H_1: Year group and meal type are not independent. **A1**

(b) $f_e(\text{Y10, Hot}) = \dfrac{90 \times 80}{180} = 40$ **A1**

(c) $\nu = (2-1)(3-1) = 2$ **A1**

Q3 — Chi-squared GOF: Fair Die Hypotheses and Expected Frequency EASY

(a) H_0: The die is fair (each score is equally likely, i.e. $P(\text{score}) = \frac{1}{6}$). **A1**

(b) Expected frequency for each score $= \dfrac{120}{6} = 20$ **A1**

(c) $\nu = 6 - 1 = 5$ **A1**

Q4 — Chi-squared Independence: Conclusion from Critical Value EASY

(a) $\nu = 1$ **A1**

(b) $\chi^2_{\text{calc}} = 5.83 > 3.841 = \chi^2_{\text{crit}} \Rightarrow$ Reject H_0. **R1**

(c) There is sufficient evidence at the 5% significance level to conclude that reading preference and gender are not independent (i.e. there is a relationship between them). **A1**

Q5 — Chi-squared GOF: Genetic Ratios EASY

(a) H_0: The offspring follow a red : pink : white $= 1:2:1$ ratio. **A1**

(b) Expected red $= \dfrac{1}{4} \times 200 = 50$ **A1**

(c) $\nu = 3 - 1 = 2$ **A1**

Q6 — Chi-squared Independence: p-value Conclusion EASY

(a) $\nu = (3-1)(2-1) = 2$ **A1**

(b) $p = 0.0342 > 0.01 = \alpha \Rightarrow$ Do **not** reject H_0 at the 1% significance level. **R1**

(c) There is insufficient evidence at the 1% significance level to conclude that coffee type preference and time of day are not independent. **A1**

Q7 — Chi-squared GOF: Uniform Transport EASY

(a) Expected frequency for each type $= \dfrac{160}{3} = 53.\overline{3}$ **A1**

(b) $\nu = 3 - 1 = 2$ **A1**

(c) $\chi^2_{\text{calc}} = 2.40 < 5.991 = \chi^2_{\text{crit}} \Rightarrow$ Do not reject H_0.
There is insufficient evidence that commuters are not equally distributed among the three transport types.
R1

Q8 — Chi-squared Independence: Blood Type and Allergy EASY

(a) H_0: Blood type and allergy are independent. **A1**

(b) $f_e(\text{B, Yes}) = \dfrac{60 \times 100}{240} = 25$ **A1**

(c) $\nu = (2-1)(3-1) = 2$ **A1**

Q9 — Chi-squared GOF: Fair Spinner EASY

(a) H_0: Each sector is equally likely (i.e. $P = \frac{1}{4}$ for each). **A1**

(b) Expected frequency for each sector $= \dfrac{200}{4} = 50$ **A1**

(c) $p = 0.312 > 0.05 = \alpha \Rightarrow$ Do not reject H_0.
There is insufficient evidence at the 5% level that the spinner is unfair. **R1**

Q10 — Chi-squared Independence: Loyalty Card EASY

(a) H_0: Day of week and loyalty card purchase are independent.
H_1: Day of week and loyalty card purchase are not independent. **A1**

(b) $\nu = (2-1)(2-1) = 1$ **A1**

(c) $f_e(\text{Weekday, Yes}) = \dfrac{180 \times 120}{300} = 72$ **A1**

Worked Solutions — Section B (Medium)

Q11 — Full Independence Test: Travel Preferences MEDIUM

(a) H_0: Travel destination preference and age group are independent.
H_1: Travel destination preference and age group are not independent. **A1**

(b) $f_e(\text{Under 40, Mountain}) = \dfrac{130 \times 60}{250} = 31.2$ **A1**

(c) Using GDC (2-way χ^2 test with the 2×3 table): **(M1)**
p-value $= 0.449\ldots \approx 0.449$ **A1**

(d) $p = 0.449 > 0.10 = \alpha \Rightarrow$ Do not reject H_0.
There is insufficient evidence at the 10% level that travel destination preference and age group are not independent. **R1**

Note: FT on p-value from GDC. Accept conclusion consistent with their p-value.

Q12 — Full GOF Test: Sweet Flavour Ratios MEDIUM

(a) H_0: Flavours follow the ratio strawberry : lemon : orange : blackcurrant $= 3 : 3 : 2 : 2$. **A1**

(b) Total ratio parts $= 10$.
Expected: Strawberry $= 60$, Lemon $= 60$, Orange $= 40$, Blackcurrant $= 40$. **A1**

(c) $\chi^2_{\text{calc}} = \dfrac{(64-60)^2}{60} + \dfrac{(58-60)^2}{60} + \dfrac{(44-40)^2}{40} + \dfrac{(34-40)^2}{40}$ **M1**

$= \dfrac{16}{60} + \dfrac{4}{60} + \dfrac{16}{40} + \dfrac{36}{40} = 0.2\overline{6} + 0.0\overline{6} + 0.4 + 0.9 = 1.633\ldots \approx 1.63$ **A1**

(d) $\chi^2_{\text{calc}} = 1.63 < 7.815 = \chi^2_{\text{crit}}\ (\nu = 3) \Rightarrow$ Do not reject H_0.
There is insufficient evidence at the 5% level that the flavour distribution differs from the claimed $3 : 3 : 2 : 2$ ratio. **R1**

Q13 — Independence Test: Faculty and Accommodation MEDIUM

(a) Since all column totals are equal (120 each) and the row totals are equal (180 each):
$f_e(\text{Science, On-campus}) = \dfrac{120 \times 180}{360} = 60$ **A1**

(b) By symmetry of the table, all expected values in each row are 60 (on-campus) and 60 (off-campus).
$\chi^2_{\text{calc}} = \dfrac{(72-60)^2}{60} + \dfrac{(54-60)^2}{60} + \dfrac{(54-60)^2}{60} + \dfrac{(48-60)^2}{60} + \dfrac{(66-60)^2}{60} + \dfrac{(66-60)^2}{60}$ **M1**

$= \dfrac{144 + 36 + 36 + 144 + 36 + 36}{60} = \dfrac{432}{60} = 7.2$ **A1**

(c) H_0: Faculty and accommodation type are independent. **A1**
$p = 0.0273 < 0.05 = \alpha \Rightarrow$ Reject H_0.
There is sufficient evidence at the 5% level that faculty and accommodation type are not independent. **R1**

Q14 — GOF Test: Poisson with Given Mean (Complaints) MEDIUM

(a) H_0: The number of complaints per day follows a Poisson distribution with mean 1.7. **A1**

(b) $P(X = 0) = e^{-1.7} \times \dfrac{1.7^0}{0!} = e^{-1.7} = 0.18268\ldots$ **M1**

Expected frequency $= 100 \times 0.18268\ldots = 18.268\ldots \approx 18.3$ **AG**

(c) Using GDC for GOF test with $\lambda = 1.7$, $n = 100$, 5 categories (combining ≥ 4 into last cell):
Expected frequencies: 0: 18.3, 1: 31.1, 2: 26.4, 3: 14.9, ≥ 4: 9.3 **(M1)**

p-value $= 0.637\ldots \approx 0.637$ **A1**

$p = 0.637 > 0.05 \Rightarrow$ Do not reject H_0.

There is insufficient evidence that the number of daily complaints does not follow a Poisson distribution with mean 1.7. **R1**

Note: df = 4 (since 5 cells, no parameters estimated from data).

Q15 — Independence Test: Genre and Age Group MEDIUM

(a) $f_e(\text{Seniors, Drama}) = \dfrac{100 \times 160}{450} = 35.5\overline{5} \approx 35.6$ **A1**

(b) H_0: Age group and film genre preference are independent.
H_1: Age group and film genre preference are not independent. **A1**

(c) Using GDC with the 3×3 observed table: **(M1)**

p-value $= 0.000138\ldots \approx 1.38 \times 10^{-4}$ **A1**

$p < 0.01 = \alpha \Rightarrow$ Reject H_0.

There is sufficient evidence at the 1% level that age group and film genre preference are not independent. **R1**

Q16 — GOF Test: Eggs per Nest (Combining Cells) MEDIUM

(a) Expected frequencies (out of 180):
1 egg: $0.20 \times 180 = 36$; 2 eggs: $0.35 \times 180 = 63$; 3 eggs: $0.30 \times 180 = 54$
4 eggs: $0.10 \times 180 = 18$; ≥ 5 eggs: $0.05 \times 180 = 9$ **A1**

(b) The expected frequency for "≥ 5 eggs" is 9, which is above 5; however the expected frequency for "4 eggs" is 18 (above 5) so in this case both are above 5. The *original* last category has $f_e = 9 \geq 5$, so no combination is strictly required here. However, to be safe (as some textbooks require $f_e \geq 5$ strictly), combining categories 4 and ≥ 5 gives $f_e = 27$. **R1**

Note: Award R1 if student states the expected frequency for ≥ 5 eggs is 9 (≥ 5) and explains that combining is done when any cell falls below 5. Award if student correctly notes all expected values are ≥ 5 and that combining is not strictly necessary here; both answers are acceptable.

(c) H_0: The number of eggs per nest follows the given distribution (20%, 35%, 30%, 10%, 5%). **A1**
Without combining: $\nu = 5 - 1 = 4$; with combining last two: $\nu = 4 - 1 = 3$.

Using GDC: p-value $= 0.907\ldots$ (4 categories) or $0.873\ldots$ (5 categories, no combining) **A1**

$p > 0.05 \Rightarrow$ Do not reject H_0.

There is insufficient evidence that the distribution of eggs per nest differs from the stated model. **R1**

Q17 — Independence Test: Traffic Policy Support MEDIUM

(a) $f_e(\text{North, Agree}) = \dfrac{120 \times 160}{320} = 60$ **A1**

(b) Since the row total for North is 120 and expected for Agree is 60:

$f_e(\text{North, Disagree}) = 120 - 60 = 60$ **A1**

(c) H_0: Support for the traffic policy and suburb of residence are independent.

H_1: Support for the traffic policy and suburb of residence are not independent. **A1**

Using GDC with the 2×3 table: p-value $= 0.875\ldots$ **(M1)**

$p = 0.875 > 0.05 \Rightarrow$ Do not reject H_0.

Insufficient evidence that policy support and suburb are not independent. **R1**

Note: FT on p-value from GDC.

Q18 — GOF Test: Binomial B(5, 0.12) Defectives MEDIUM

(a) $P(X = 0) = \binom{5}{0}(0.12)^0(0.88)^5 = 0.5277\ldots$ **M1**

Expected frequency for 0 defectives $= 200 \times 0.5277\ldots = 105.5\ldots \approx 105.6$ **A1**

$P(X = 1) = \binom{5}{1}(0.12)^1(0.88)^4 = 5 \times 0.12 \times 0.5997\ldots = 0.3598\ldots$

Expected frequency for 1 defective $= 200 \times 0.3598\ldots = 71.97\ldots \approx 72.0$ **A1**

Note: Award A1A1 for both correct (accept values in range [105,106] and [71.9, 72.1]).

(b) For 2 defectives: $E = 200 \times P(X = 2) \approx 200 \times 0.0981 = 19.6$; for 3: ≈ 6.0; for 4: ≈ 1.04; for 5: ≈ 0.085.

Categories 3, 4, and 5 each have expected frequency < 5; these must be combined into a single "≥ 3" category. **R1**

(c) p-value $= 0.462 > 0.05 \Rightarrow$ Do not reject H_0.

There is insufficient evidence that the number of defectives per batch does not follow a $B(5, 0.12)$ distribution. **R1**

Q19 — Computing χ^2_{calc} from a 2×3 Table: Injuries MEDIUM

(a) $f_e(\text{Team, Joint sprain}) = \dfrac{160 \times 100}{280} = 57.\overline{14} \approx 57.1$ **A1**

(b) Full expected table:

$f_e(\text{Team, Muscle}) = \dfrac{160 \times 130}{280} = 74.28\overline{5}$; $f_e(\text{Team, Fracture}) = \dfrac{160 \times 50}{280} = 28.57\ldots$ **M1**

$f_e(\text{Ind., Muscle}) = 55.71\ldots$; $f_e(\text{Ind., Sprain}) = 42.85\ldots$; $f_e(\text{Ind., Fracture}) = 21.42\ldots$

$$\chi^2 = \frac{(72-74.29)^2}{74.29} + \frac{(56-57.14)^2}{57.14} + \frac{(32-28.57)^2}{28.57} + \frac{(58-55.71)^2}{55.71} + \frac{(44-42.86)^2}{42.86} + \frac{(18-21.43)^2}{21.43}$$

$= 0.0706 + 0.0228 + 0.4117 + 0.0941 + 0.0304 + 0.5490 = 1.179\ldots \approx 1.18$ **A1**

(c) $\chi^2_{\text{calc}} = 1.18 < 5.991 = \chi^2_{\text{crit}}$ ($\nu = 2$); equivalently p-value $\approx 0.555 > 0.05$.

$\Rightarrow$ Do not reject H_0. Insufficient evidence that injury type and sport type are not independent. **R1**

Note: FT on χ^2_{calc} from part (b). Accept any value in the range $[1.17, 1.19]$.

Q20 — GOF: Normal Distribution Fitted to Score Data MEDIUM

(a) H_0: Test scores follow a $N(65, 12^2)$ distribution. **A1**

Degrees of freedom: $\nu = 6 - 1 = 5$

Note: No parameters were estimated from the data (mean and SD were given), so $\nu = k - 1 = 5$. **A1**

(b) Using GDC with observed $= [6, 18, 32, 34, 22, 8]$ and expected $= [5.3, 14.7, 29.7, 32.9, 25.0, 12.4]$: **(M1)**

p-value $= 0.396\ldots \approx 0.396$ **A1**

(c) $p = 0.396 > 0.05 \Rightarrow$ Do not reject H_0.

There is insufficient evidence at the 5% level that test scores do not follow a $N(65, 144)$ distribution. **R1**

Worked Solutions — Section C (Hard)

Q21 — Independence Test with Unknown Cell: Satisfaction HARD

(a) Engineering total $= 200$; Dissatisfied in Engineering $= 80$.

$k = 200 - 80 = 120$ **A1**

(b) Satisfied total $= 90 + 120 + 48 = 258$.

$f_e(\text{Eng., Satisfied}) = \dfrac{200 \times 258}{400} = 129$ **A1**

(c) H_0: Job satisfaction and department are independent.

H_1: Job satisfaction and department are not independent. **A1**

(d) Using GDC with observed table (Satisfied row: 90, 120, 48; Dissatisfied row: 30, 80, 32): **M1**

p-value $= 0.0161\ldots \approx 0.0161$ **A1**

$p = 0.0161 < 0.05 \Rightarrow$ Reject H_0.

There is sufficient evidence at the 5% level that job satisfaction and department are not independent. **R1**

Q22 — GOF Poisson: Estimated Mean and Adjusted df (Bird Sightings) HARD

(a) Total sightings $= 0(18) + 1(26) + 2(20) + 3(10) + 4(4) + 5(2)$ **M1**

$= 0 + 26 + 40 + 30 + 16 + 10 = 122$

$\hat{\lambda} = \dfrac{122}{80} = 1.525$ **AG**

(b) H_0: The number of sightings per hour follows a Poisson distribution with mean estimated from data.

H_1: The number of sightings per hour does not follow such a Poisson distribution. **A1**

Categories after combining ($f_e(\geq 4) = 3.92 < 5$ and $f_e(\geq 5) = 1.58 < 5$, so combine ≥ 4): cells are 0, 1, 2, 3, ≥ 4 giving 5 cells; $\nu = 5 - 1 - 1 = 3$ **A1**

(c) Expected: $f_e(0) = 17.4$, $f_e(1) = 26.6$, $f_e(2) = 20.2$, $f_e(3) = 10.3$, $f_e(\geq 4) = 5.51$ **(M1)**

Observed: 18, 26, 20, 10, 6.

p-value $= 0.993\ldots \approx 0.993$ **A1**

$p = 0.993 > 0.05 \Rightarrow$ Do not reject H_0.

There is insufficient evidence that the sightings per hour do not follow a Poisson distribution. **R1**

Q23 — Independence Test with Missing Cell: Hotel Bookings HARD

(a) Suite column total $= 80$; Online suites $= 55$; In-person suites: $m = 80 - 55 = 25$ **A1**

(b) $f_e(\text{Online, Suite}) = \dfrac{340 \times 80}{500} = 54.4$ **A1**

(c) H_0: Room type preference and booking method are independent.

H_1: Room type preference and booking method are not independent. **A1**

(d) In-person total $= 500 - 340 = 160$. Observed in-person row: Standard 95, Deluxe 40, Suite 25.

Using GDC with the complete 2×3 table: **M1**
p-value $= 0.000201\ldots \approx 2.01 \times 10^{-4}$ **A1**
$p < 0.05 \Rightarrow$ Reject H_0.
There is sufficient evidence at the 5% level that room type preference and booking method are not independent. **R1**

Q24 — GOF Test with Type I Error: Production Grades HARD

(a) H_0: Items are produced in the ratio Grade A : B : C $= 6 : 3 : 1$ (60% : 30% : 10%). **A1**

(b) Expected: A $= 0.60 \times 250 = 150$; B $= 0.30 \times 250 = 75$; C $= 0.10 \times 250 = 25$ **A1**

(c) $\chi^2_{\text{calc}} = \dfrac{(142-150)^2}{150} + \dfrac{(81-75)^2}{75} + \dfrac{(27-25)^2}{25}$ **M1**

$= \dfrac{64}{150} + \dfrac{36}{75} + \dfrac{4}{25} = 0.4267 + 0.48 + 0.16 = 1.0667\ldots \approx 1.07$ **A1**

p-value $= 0.586\ldots \approx 0.586$ **A1**

(d) $p = 0.586 > 0.05 \Rightarrow$ Do not reject H_0.
There is insufficient evidence at the 5% level that the production grade distribution differs from the claimed 60% : 30% : 10% split. **R1**

(e) A Type I error would mean concluding that the grade distribution is not 60% : 30% : 10% when in reality it is (i.e. incorrectly rejecting a true H_0). **R1**

Q25 — Independence Test with Unknown Column Total: Languages HARD

(a) Private school Spanish: $n = 240 - 114 - 34 = 92$
Spanish column total $= 88 + 92 = 180$ **A1 A1**

(b) H_0: Language studied and school sector are independent.
H_1: Language studied and school sector are not independent. **A1**

(c) $f_e(\text{Private, Mandarin}) = \dfrac{240 \times 60}{480} = 30$ **A1**

(d) Using GDC with the complete 2×3 observed table: **(M1)**
State school: French 126, Spanish 88, Mandarin 26; Private: French 114, Spanish 92, Mandarin 34.
p-value $= 0.416\ldots \approx 0.416$ **A1**
$p = 0.416 > 0.05 \Rightarrow$ Do not reject H_0.
There is insufficient evidence at the 5% level that language preference and school sector are not independent. **R1**

Q26 — GOF Poisson (Estimated Mean) + Type II Error: Bus Breakdowns HARD

(a) Total breakdowns $= 0(14) + 1(24) + 2(14) + 3(6) + 4(2) = 0 + 24 + 28 + 18 + 8 = 78$ **M1**

$\hat{\lambda} = \frac{78}{60} = 1.30$ **AG**

(b) H_0: Number of breakdowns per week follows a Poisson distribution with mean estimated from data.
H_1: Number of breakdowns does not follow such a Poisson distribution. **A1**
With $\hat{\lambda} = 1.30$, $n = 60$: $f_e(\geq 4) = 60 \times P(X \geq 4) \approx 2.59 < 5$; combine 3 and ≥ 4:
Combined $f_e(\geq 3) = 60 \times P(X \geq 3) \approx 8.57 \geq 5$. Cells: 0, 1, 2, ≥ 3 (4 cells).
$\nu = 4 - 1 - 1 = 2$ **A1**

(c) Expected: $f_e(0) = 16.4$, $f_e(1) = 21.3$, $f_e(2) = 13.8$, $f_e(\geq 3) = 8.57$
Observed: 14, 24, 14, 8. **(M1)**
p-value $= 0.693\ldots \approx 0.693$ **A1**
$p = 0.693 > 0.05 \Rightarrow$ Do not reject H_0.
There is insufficient evidence that the weekly breakdowns do not follow a Poisson distribution. **R1**

(d) A Type II error would mean failing to reject H_0 when in fact the number of breakdowns per week does *not* follow a Poisson distribution (i.e. failing to detect that the model is wrong). **R1**

Q27 — Independence Test + Significance Level Comparison: Diet and Exercise HARD

(a) $f_e(\text{Vegan, Regularly}) = \frac{120 \times 260}{540} = 57.\overline{7} \approx 57.8$ **A1**

(b) H_0: Dietary preference and exercise frequency are independent.
H_1: Dietary preference and exercise frequency are not independent. **A1**
Using GDC with the 3×3 table: **(M1)**
p-value $= 0.893\ldots \approx 0.893$ **A1**
$p = 0.893 > 0.05 \Rightarrow$ Do not reject H_0.
There is insufficient evidence at the 5% level that dietary preference and exercise frequency are not independent. **R1**

(c) Since $p = 0.893 > 0.01$, the conclusion remains the same at the 1% level: H_0 is not rejected. **R1**

Q28 — GOF Test with Unknown Frequency: Colour Preference HARD

(a) $q = 240 - 72 - 54 - 68 = 46$ **A1**

(b) H_0: Customers have no colour preference (each colour is equally likely, $P = \frac{1}{4}$).
H_1: Customers do have a colour preference. **A1**

(c) Expected frequency for each colour $= \frac{240}{4} = 60$ **A1**

(d) $\chi^2_{\text{calc}} = \dfrac{(72-60)^2}{60} + \dfrac{(54-60)^2}{60} + \dfrac{(68-60)^2}{60} + \dfrac{(46-60)^2}{60}$ **M1**

$= \dfrac{144+36+64+196}{60} = \dfrac{440}{60} = 7.3\overline{3} \approx 7.33$ **A1**

p-value $= 0.0620\ldots \approx 0.0620$ **A1**

$p = 0.0620 > 0.05 \Rightarrow$ Do not reject H_0 at the 5% level.

There is insufficient evidence that customers have a colour preference. **R1**

Note: $\nu = 3$; critical value at 5% is 7.815. Since $\chi^2_{calc} = 7.33 < 7.815$, do not reject H_0 (consistent with p-value).

Q29 — Independence Test + Limitation: Diet and Hypertension HARD

(a) H_0: Diet type and hypertension are independent.

H_1: Diet type and hypertension are not independent. **A1**

(b) $f_e(\text{Western, Yes}) = \dfrac{200 \times 200}{600} = 66.\overline{6} \approx 66.7$ **A1**

(c) Using GDC with the 2×3 observed table: **M1**

p-value $= 6.06 \times 10^{-16} \approx 6.06 \times 10^{-16}$ **A1**

$p \ll 0.01 \Rightarrow$ Reject H_0.

There is very strong evidence at the 1% level that diet type and hypertension are not independent. **R1**

(d) A χ^2 test establishes association (non-independence) but cannot establish causation; other confounding factors (e.g. age, exercise level, genetics) may explain the relationship between diet and hypertension. **R1**

Note: Accept any reasonable limitation, e.g. the test only shows association, not that diet causes hypertension; the sample may not be representative; other variables not controlled for.

Q30 — Independence Test with Unknown Cell + Type I Error: Social Media HARD

(a) Platform C total $= 120$; C values: 70 (18–30) + t (31–50) + 20 (Over 50) $= 120$

$t = 120 - 70 - 20 = 30$

Row total for 31–50: $40 + 70 + 30 = 140$ **A1**

(b) H_0: Social media platform preference and age group are independent.

H_1: Social media platform preference and age group are not independent. **A1**

(c) $f_e(\text{31–50, Platform B}) = \dfrac{140 \times 110}{360} = 42.\overline{7} \approx 42.8$ **A1**

(d) Using GDC with the complete 3×3 observed table

(Row 1: 80, 30, 70; Row 2: 40, 70, 30; Row 3: 10, 10, 20): **(M1)**

p-value $= 1.47 \times 10^{-9}$ **A1**

$p \ll 0.05 \Rightarrow$ Reject H_0.

There is sufficient evidence at the 5% level that social media platform preference and age group are not independent. **R1**

(e) The probability of a Type I error equals the significance level: $P(\text{Type I error}) = 0.05$ **A1**

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