IB Math AI HL — Topic 3 [Set 1]

Geometry of 3D Shapes, 3D Coordinate Geometry & Volume/Surface Area

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

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Name:______________________ Date:____________ Score:________/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 sf unless stated otherwise. A GDC is permitted.

Key Formulae — Geometry of 3D Shapes

Distance in 3D:
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Midpoint in 3D:
$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$

Volume — Sphere: $V = \frac{4}{3}\pi r^3$

Surface Area — Sphere: $A = 4\pi r^2$

Volume — Cone: $V = \frac{1}{3}\pi r^2 h$

Slant height — Cone: $l = \sqrt{r^2 + h^2}$

Curved SA — Cone: $A = \pi r l$

Volume — Cylinder: $V = \pi r^2 h$

Curved SA — Cylinder: $A = 2\pi r h$

Volume — Prism: $V = A_{\text{base}} \times h$

Volume — Pyramid: $V = \frac{1}{3} A_{\text{base}} \times h$

Volume — Hemisphere: $V = \frac{2}{3}\pi r^3$

Curved SA — Hemisphere: $A = 2\pi r^2$

Volume — Cuboid: $V = l \times w \times h$

Space diagonal — Cuboid: $d = \sqrt{l^2 + w^2 + h^2}$

GDC Tips

- Use **MATH** $\to$ **NUM** $\to$ **nDeriv / fnInt** to check volume-by-revolution or surface integrals numerically.
- For 3D distance: store coordinates in lists and use the distance formula directly in the Home screen.
- Store the exact symbolic expression for volume, then **solve(V = k, r)** to find unknown radii.
- When working with composite solids, sketch each component separately and label shared faces before combining.
- Use π from the GDC keypad (not 3.14159) to avoid premature rounding in multi-step problems.

Common Mistakes to Avoid

1. **Slant height vs vertical height.** The cone formula $V = \frac{1}{3}\pi r^2 h$ uses *perpendicular* height h, not slant height l. Always draw a right-angle triangle inside the cone.
2. **Composite solid surface areas.** When two solids share a face, *remove* that shared face from both before adding surface areas together.
3. **3D distance formula.** Remember there are *three* squared terms: forgetting the z-component is the most common error.
4. **Midpoint in 3D.** Average *all three* coordinates separately — do not confuse with 2D midpoint.
5. **Units consistency.** Convert all lengths to the same unit *before* computing volumes/areas, not after.
6. **Hemisphere surface area.** The total SA of a hemisphere includes both the curved surface $2\pi r^2$ *and* the flat circular base πr^2, giving $3\pi r^2$ in total.

Section A — Easy EASY (10 questions $\times$ 3 marks = 30 marks)

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1. EASY GDC **[3 marks]**

A storage tank is in the shape of a cylinder with internal radius 4.5 m and internal height 12 m.

(a) Write down the formula for the volume of a cylinder. **[1]**

(b) Find the volume of the tank. Give your answer in m^3, correct to 3 significant figures. **[2]**

2. EASY GDC **[3 marks]**

A decorative ball has a diameter of 18 cm.

(a) Write down the radius of the ball. **[1]**

(b) Find the total surface area of the ball. Give your answer in cm^2, correct to 3 significant figures. **[2]**

3. EASY GDC **[3 marks]**

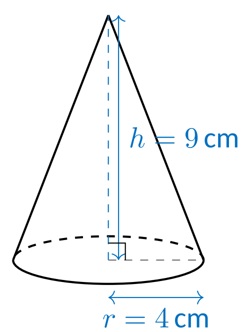
Point A has coordinates $(1,\ 3,\ -2)$ and point B has coordinates $(5,\ 7,\ 4)$ in a 3D coordinate system where distances are in metres.

(a) Write down the 3D distance formula. **[1]**

(b) Find the distance AB. Give your answer correct to 3 significant figures. **[2]**

4. EASY GDC **[3 marks]**

The diagram shows a cone with perpendicular height $h = 9$ cm and base radius $r = 4$ cm.



(a) Find the volume of the cone. Give your answer correct to 3 significant figures. **[2]**

(b) Find the slant height of the cone. Give your answer correct to 3 significant figures. **[1]**

5. EASY GDC **[3 marks]**

Point P has coordinates $(2, -1, 6)$ and point Q has coordinates $(8, 5, 0)$.

(a) Find the coordinates of the midpoint M of PQ. **[2]**

(b) Write down the z-coordinate of M. **[1]**

6. EASY GDC **[3 marks]**

A rectangular fish tank has internal dimensions: length 80 cm, width 35 cm, and height 40 cm.

(a) Find the volume of the tank in cm^3. **[1]**

(b) Convert your answer to litres (1 litre $= 1000$ cm^3). **[1]**

(c) The tank is filled to 85% of its capacity. Find the volume of water in litres. **[1]**

7. EASY GDC **[3 marks]**

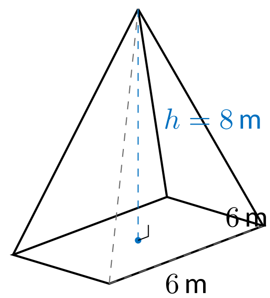
A rectangular box has dimensions 6 cm $\times$ 8 cm $\times$ 10 cm.

(a) Write down the formula for the space diagonal of a cuboid. **[1]**

(b) Find the length of the space diagonal of the box. Give your answer correct to 3 significant figures. **[2]**

8. EASY GDC **[3 marks]**

The diagram shows a square-based pyramid with base side length 6 m and perpendicular height 8 m.



Find the volume of the pyramid. **[3 marks]**

9. EASY GDC **[3 marks]**

A tin can is in the shape of a closed cylinder with radius 3.5 cm and height 11 cm.

(a) Find the total surface area of the tin can. Give your answer correct to 3 significant figures. **[2]**

(b) The can is made from sheet metal costing \$0.04 per cm^2. Find the cost of manufacturing one can, correct to the nearest cent. **[1]**

10. EASY GDC **[3 marks]**

The table gives the coordinates of three vertices of a rectangular box (cuboid) in a 3D coordinate system where distances are in centimetres.

Vertex	Coordinates	Description
A	$(0,\ 0,\ 0)$	Origin corner
B	$(5,\ 0,\ 0)$	Along x-axis
C	$(0,\ 4,\ 0)$	Along y-axis
D	$(0,\ 0,\ 3)$	Along z-axis

(a) Write down the coordinates of the vertex E diagonally opposite A. **[1]**

(b) Find the length of the space diagonal AE. **[1]**

(c) Find the distance from B to D. **[1]**

Section B — Medium **MEDIUM** (10 questions × 4 marks = 40 marks)

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11. MEDIUM GDC **[4 marks]**

A garden water feature consists of a solid concrete cylinder with a hemispherical hollow carved into its top face. The cylinder has radius $r = 0.6$ m and total height $H = 1.4$ m. The hemisphere has the same radius as the cylinder.

(a) Find the volume of the hemisphere. **[1]**

(b) Find the volume of the solid concrete feature (cylinder volume minus hemisphere). Give your answer correct to 3 significant figures. **[2]**

(c) Concrete costs $85 per m^3. Find the cost of the concrete used, correct to the nearest dollar. **[1]**

12. MEDIUM GDC **[4 marks]**

Two drone waypoints are located at $P(4, -2, 7)$ and $Q(-2, 6, 3)$ in a coordinate system where each unit represents 10 m.

(a) Find the coordinates of the midpoint M of PQ. **[2]**

(b) Find the distance PQ in metres, correct to 3 significant figures. **[2]**

13. MEDIUM GDC **[4 marks]**

A conical paper cup has base radius 3 cm and perpendicular height 7 cm.

(a) Show that the slant height of the cup is $\sqrt{58}$ cm. **[2]**

(b) Find the total external surface area of the closed conical cup (base circle + curved surface). Give your answer correct to 3 significant figures. **[2]**

14. MEDIUM GDC **[4 marks]**

A gift box is in the shape of a closed cuboid with length $l = 24$ cm, width $w = 15$ cm, and height $h = 10$ cm.

(a) Find the total surface area of the gift box. **[2]**

(b) Wrapping paper costs $0.006 per cm^2. Find the minimum cost to wrap the box, correct to the nearest cent. **[1]**

(c) Find the length of the space diagonal of the gift box, correct to 3 significant figures. **[1]**

15. MEDIUM GDC **[4 marks]**

A telecommunications tower is supported by three cables anchored at ground level. In a coordinate system (distances in metres), the top of the tower is at $T(0,\ 0,\ 45)$ and the three anchor points are at $A_1(12,\ 0,\ 0)$, $A_2(-6,\ 9,\ 0)$, and $A_3(-6,\ -9,\ 0)$.

The table gives information about the cables.

Cable	From	To	Length (m)
1	$T(0,0,45)$	$A_1(12,0,0)$	
2	$T(0,0,45)$	$A_2(-6,9,0)$	
3	$T(0,0,45)$	$A_3(-6,-9,0)$	

(a) Find the length of Cable 1, correct to 3 significant figures. **[2]**

(b) Find the lengths of Cables 2 and 3, and complete the table. **[1]**

(c) Find the total length of cable needed for all three cables, correct to 3 significant figures. **[1]**

16. MEDIUM GDC **[4 marks]**

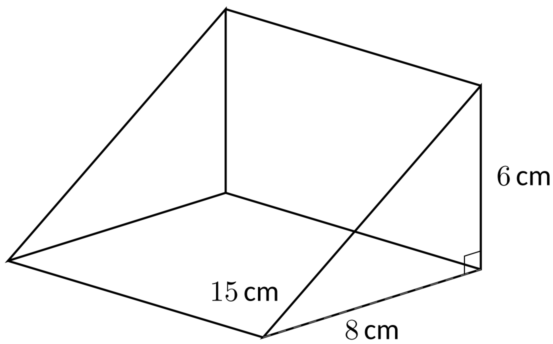
A meteorological balloon is spherical. When fully inflated, it has a volume of $1700\,\text{m}^3$.

(a) Find the radius of the inflated balloon, correct to 3 significant figures. **[2]**

(b) Find the surface area of the inflated balloon, correct to 3 significant figures. **[2]**

17. MEDIUM GDC **[4 marks]**

The diagram shows a wedge (triangular prism). The triangular face is a right-angled triangle with legs 6 cm and 8 cm. The length of the prism is 15 cm.



(a) Find the area of the triangular cross-section. **[1]**

(b) Find the volume of the wedge. **[1]**

(c) Find the total surface area of the wedge. **[2]**

18. MEDIUM GDC **[4 marks]**

The midpoint of segment RS is $M(3,\, 1,\, -4)$. Point R has coordinates $(7,\, -3,\, 2)$.

(a) Find the coordinates of point S. **[2]**

(b) Find the length RS, correct to 3 significant figures. **[2]**

19. MEDIUM GDC **[4 marks]**

A decorative planter is in the shape of a frustum (a cone with the top cut off). The frustum has a large circular base of radius $R = 18$ cm, a small circular top of radius $r = 10$ cm, and perpendicular height $h = 22$ cm.

The volume of a frustum is given by $V = \dfrac{\pi h}{3}(R^2 + Rr + r^2)$.

(a) Find the volume of the planter in cm^3, correct to 3 significant figures. **[2]**

(b) Convert your answer to litres, correct to 3 significant figures. **[1]**

(c) The planter is filled with soil to 75% of its capacity. Find the volume of soil used, in litres, correct to 3 significant figures. **[1]**

20. MEDIUM GDC **[4 marks]**

A novelty lamp shade is formed by joining a hemisphere on top of a hollow open cone. The hemisphere and the cone share the same circular opening of radius 12 cm. The cone has perpendicular height 16 cm. There is no material along the base of the cone or along the flat face of the hemisphere.

(a) Find the slant height of the cone, correct to 3 significant figures. **[1]**

(b) Find the total external surface area of the lamp shade (curved cone surface + curved hemisphere surface). Give your answer correct to 3 significant figures. **[3]**

Section C — Hard **HARD** (10 questions $\times$ 5 marks = 50 marks)

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21. **HARD** **GDC** **[5 marks]**

A silo consists of a cylinder topped by a cone. The cylinder has radius $r = 3$ m and height h_c m. The cone has the same base radius and perpendicular height 4 m. The total volume of the silo is 120 m^3.

(a) Write an expression for the total volume of the silo in terms of h_c. **[2]**

(b) Hence find the value of h_c, correct to 3 significant figures. **[2]**

(c) Find the total external surface area of the silo (lateral cylinder surface + lateral cone surface only; open at base and at the join). Give your answer correct to 3 significant figures. **[1]**

22. HARD GDC **[5 marks]**

A spherical water tank has radius measured as 2.8 m, correct to the nearest 0.1 m.

(a) Write down the upper and lower bounds for the radius of the tank. **[1]**

(b) Find the upper bound for the volume of the tank, correct to 4 significant figures. **[2]**

(c) The volume calculated using the measured radius is V_{calc}. Calculate the percentage error between the upper-bound volume and V_{calc}, correct to 3 significant figures. **[2]**

23. HARD GDC **[5 marks]**

Points $A(6,\,2,\,-1)$, $B(-2,\,6,\,3)$, and $C(k,\,0,\,5)$ are in a 3D coordinate system where distances are in metres.

(a) Find the midpoint M of AB. **[2]**

(b) Given that $MC = 7$ m, find the two possible values of k. Give your answers correct to 3 significant figures. **[3]**

24. HARD GDC **[5 marks]**

A sphere is inscribed in a closed cylinder so that the sphere touches the curved surface and both flat ends. The cylinder has radius r cm.

(a) Show that the height of the cylinder is $2r$ cm. **[1]**

(b) Show that the ratio of the volume of the sphere to the volume of the cylinder is $\frac{2}{3}$. **[2]**

(c) Given that the volume of the sphere is $\frac{500\pi}{3}$ cm^3, find the total surface area of the cylinder. Give your answer in terms of π. **[2]**

25. HARD GDC **[5 marks]**

A closed cone (with base) has a fixed total surface area of 300π cm^2. Let the base radius be r cm and the slant height be l cm.

(a) Show that $l = \frac{300\pi - \pi r^2}{\pi r} = \frac{300 - r^2}{r}$. **[2]**

(b) Hence write the perpendicular height h in terms of r only. **[1]**

(c) Use your GDC to find the value of r that maximises the volume, and find the maximum volume. Give your answers correct to 3 significant figures. **[2]**

26. HARD GDC **[5 marks]**

A closed cylindrical canister has total surface area $628\,\text{cm}^2$ and height $h = 10\,\text{cm}$.

(a) Show that $\pi r^2 + 10\pi r - 314 = 0$, where r cm is the radius of the canister. **[2]**

(b) Hence find the radius of the canister, correct to 3 significant figures. **[2]**

(c) Find the volume of the canister, correct to 3 significant figures. **[1]**

27. HARD GDC **[5 marks]**

A triangular-based pyramid (tetrahedron) has vertices at $A(0,\,0,\,0)$, $B(6,\,0,\,0)$, $C(3,\,6,\,0)$, and $D(3,\,2,\,8)$.

(a) Find the coordinates of the centroid G of the base triangle ABC. **[1]**

(b) Find the perpendicular height of the tetrahedron (the distance from D to the base plane $z = 0$). **[1]**

(c) Find the area of the base triangle ABC. **[1]**

(d) Hence find the volume of the tetrahedron. **[2]**

28. HARD GDC **[5 marks]**

A grain storage building consists of a hemisphere on top of a cylinder, both with radius r m. The total height of the building (cylinder + hemisphere) is 9 m.

(a) Write an expression for the total volume V of the building in terms of r. **[2]**

(b) Find the value of r that makes $V = 250\,\text{m}^3$, correct to 3 significant figures. **[2]**

(c) For the value of r found in part (b), find the total external surface area of the building (curved cylinder surface + curved hemisphere surface + circular base only). Give your answer correct to 3 significant figures. **[1]**

29. HARD GDC **[5 marks]**

A tent is in the shape of a square-based pyramid with base side length 4 m. The apex of the tent is at point $T(2,\,2,\,3)$ in a coordinate system where $A(0,\,0,\,0)$, $B(4,\,0,\,0)$, $C(4,\,4,\,0)$, $D(0,\,4,\,0)$ are the base vertices.

(a) Find the length of the slant edge AT. **[2]**

(b) Find the perpendicular slant height from the apex to the midpoint of edge AB, correct to 3 significant figures. **[2]**

(c) Using your answer to part (b), find the total surface area of the four triangular faces of the tent. Give your answer correct to 3 significant figures. **[1]**

30. HARD GDC **[5 marks]**

A cylindrical water trough of radius 0.5 m and length L m lies on its side. In a coordinate system, the centre of one circular end is at $E_1(0,\,0,\,0)$ and the centre of the other circular end is at $E_2(0,\,0,\,L)$.
The midpoint of the axis of the trough is at $M(0,\,0,\,2)$.

(a) Write down the value of L. **[1]**

(b) Find the total external surface area of the trough (two circular ends + curved surface), in terms of π. **[2]**

(c) A paint can covers $8\,\text{m}^2$ per litre. Find the minimum number of full litres of paint needed to coat the entire outside of the trough. **[2]**

Worked Solutions

(M = Method mark; A = Accuracy mark; R = Reasoning mark; AG = Answer Given)

Section A — Easy

Q1 — Volume of Cylinder EASY

(a) $V = \pi r^2 h$ **A1**

(b) $V = \pi \times (4.5)^2 \times 12 = \pi \times 20.25 \times 12$ **M1**

$= 763.407\ldots \approx 763\,\text{m}^3$ **A1**

Q2 — Surface Area of Sphere EASY

(a) Radius $= 18 \div 2 = 9\,\text{cm}$ **A1**

(b) $A = 4\pi r^2 = 4\pi \times (9)^2 = 4\pi \times 81$ **M1**

$= 1017.87\ldots \approx 1020\,\text{cm}^2$ **A1**

Q3 — 3D Distance EASY

(a) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ **A1**

(b) $AB = \sqrt{(5-1)^2 + (7-3)^2 + (4-(-2))^2} = \sqrt{16+16+36}$ **M1**

$= \sqrt{68} = 8.246\ldots \approx 8.25\,\text{m}$ **A1**

Q4 — Volume and Slant Height of Cone EASY

(a) $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times (4)^2 \times 9$ **M1**

$= 48\pi = 150.796\ldots \approx 151\,\text{cm}^3$ **A1**

(b) $l = \sqrt{r^2 + h^2} = \sqrt{16+81} = \sqrt{97} = 9.849\ldots \approx 9.85\,\text{cm}$ **A1**

Q5 — Midpoint in 3D EASY

(a) $M = \left(\dfrac{2+8}{2}, \dfrac{-1+5}{2}, \dfrac{6+0}{2}\right) = (5, 2, 3)$ **M1 A1**

(b) z-coordinate of $M = 3$ **A1**

Q6 — Volume of Cuboid and Conversion EASY

(a) $V = 80 \times 35 \times 40 = 112\,000\,\text{cm}^3$ **A1**

(b) $V = 112\,000 \div 1000 = 112$ litres **A1**

(c) $V_{\text{water}} = 0.85 \times 112 = 95.2$ litres **A1**

Q7 — Space Diagonal of Cuboid EASY

(a) $d = \sqrt{l^2 + w^2 + h^2}$ **A1**

(b) $d = \sqrt{6^2 + 8^2 + 10^2} = \sqrt{36 + 64 + 100} = \sqrt{200}$ **M1**

$= 14.142\ldots \approx 14.1\,\text{cm}$ **A1**

Q8 — Volume of Square-based Pyramid EASY

Base area $= 6^2 = 36\,\text{m}^2$ **(A1)**

$V = \frac{1}{3} \times 36 \times 8$ **M1**

$= 96\,\text{m}^3$ **A1**

Q9 — Total Surface Area of Cylinder EASY

(a) $A = 2\pi r^2 + 2\pi rh = 2\pi(3.5)^2 + 2\pi(3.5)(11)$ **M1**

$= 2\pi(12.25) + 2\pi(38.5) = 2\pi(50.75) = 319.022\ldots \approx 319\,\text{cm}^2$ **A1**

(b) Cost $= 319.022\ldots \times 0.04 = 12.760\ldots \approx \$12.76$ **A1**

Q10 — 3D Coordinates of Cuboid Vertices EASY

(a) $E = (5,\ 4,\ 3)$ **A1**

(b) $AE = \sqrt{5^2 + 4^2 + 3^2} = \sqrt{25 + 16 + 9} = \sqrt{50} = 7.071\ldots \approx 7.07\,\text{cm}$ **A1**

(c) $BD = \sqrt{(0-5)^2 + (0-0)^2 + (3-0)^2} = \sqrt{25 + 0 + 9} = \sqrt{34} = 5.831\ldots \approx 5.83\,\text{cm}$ **A1**

Section B — Medium

Q11 — Composite Solid: Cylinder with Hemispherical Hollow MEDIUM

(a) $V_{\text{hemi}} = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(0.6)^3 = \frac{2}{3}\pi(0.216) = 0.1440\pi = 0.45239\ldots\ \text{m}^3 \approx 0.452\,\text{m}^3$ **A1**

(b) $V_{\text{cylinder}} = \pi(0.6)^2(1.4) = 0.504\pi = 1.58340\ldots\ \text{m}^3$ **M1**

$V_{\text{concrete}} = V_{\text{cylinder}} - V_{\text{hemi}} = 1.58340\ldots - 0.45239\ldots = 1.13100\ldots \approx 1.13\,\text{m}^3$ **A1**

(c) Cost $= 1.13100\ldots \times 85 = 96.135\ldots \approx \$96$ **A1**

Note: FT from their answer to (b).

Q12 — 3D Midpoint and Distance: Drone Waypoints MEDIUM

(a) $M = \left(\dfrac{4+(-2)}{2}, \dfrac{-2+6}{2}, \dfrac{7+3}{2}\right) = (1,\ 2,\ 5)$ **M1 A1**

(b) $PQ = \sqrt{(-2-4)^2 + (6-(-2))^2 + (3-7)^2} = \sqrt{36+64+16}$ **M1**

$= \sqrt{116} = 10.770\ldots$ units $\times 10 = 107.70\ldots \approx 108$ m **A1**

Q13 — Cone Slant Height and Total Surface Area MEDIUM

(a) $l = \sqrt{r^2+h^2} = \sqrt{3^2+7^2} = \sqrt{9+49} = \sqrt{58}$ cm **M1 AG**

Note: Both $\sqrt{9+49}$ and $\sqrt{58}$ must be seen for AG to be awarded.

(b) Curved SA $= \pi r l = \pi(3)(\sqrt{58}) = 3\pi\sqrt{58}$ **M1**

Base circle $= \pi r^2 = 9\pi$

Total $= \pi(9+3\sqrt{58}) = 9\pi + 3\pi\sqrt{58} = 28.274\ldots + 71.710\ldots = 99.985\ldots \approx 100\,\text{cm}^2$ **A1**

Q14 — Cuboid Surface Area, Cost and Space Diagonal MEDIUM

(a) $A = 2(lw+lh+wh) = 2(24\times15 + 24\times10 + 15\times10)$ **M1**

$= 2(360+240+150) = 2(750) = 1500\,\text{cm}^2$ **A1**

(b) Cost $= 1500 \times 0.006 = \$9.00$ **A1**

(c) $d = \sqrt{24^2+15^2+10^2} = \sqrt{576+225+100} = \sqrt{901} = 30.016\ldots \approx 30.0$ cm **A1**

Q15 — 3D Distances: Telecommunications Cables MEDIUM

(a) Cable 1 $= \sqrt{(12-0)^2+(0-0)^2+(0-45)^2} = \sqrt{144+0+2025} = \sqrt{2169}$ **M1**

$= 46.572\ldots \approx 46.6$ m **A1**

(b) Cable 2 $= \sqrt{(-6)^2+9^2+(-45)^2} = \sqrt{36+81+2025} = \sqrt{2142} = 46.283\ldots \approx 46.3$ m **A1**

Cable 3: same as Cable 2 by symmetry $= \sqrt{2142} \approx 46.3$ m

(c) Total $= 46.572 + 46.283 + 46.283 = 139.138\ldots \approx 139$ m **A1**

Note: FT from (a) and (b). Award A1 for correct addition of their three cable lengths.

Q16 — Sphere: Volume to Radius to Surface Area MEDIUM

(a) $V = \frac{4}{3}\pi r^3 \Rightarrow r^3 = \dfrac{3V}{4\pi} = \dfrac{3 \times 1700}{4\pi} = \dfrac{5100}{4\pi}$ **M1**

$r = \sqrt[3]{\dfrac{5100}{4\pi}} = \sqrt[3]{405.845\ldots} = 7.40337\ldots \approx 7.40$ m **A1**

(b) $A = 4\pi r^2 = 4\pi(7.40337\ldots)^2 = 4\pi(54.809\ldots) = 688.89\ldots \approx 689$ m^2 **M1 A1**

Note: FT from their radius in (a).

Q17 — Triangular Prism (Wedge): Area, Volume and SA MEDIUM

(a) $A_\triangle = \frac{1}{2} \times 8 \times 6 = 24$ cm^2 **A1**

(b) $V = 24 \times 15 = 360$ cm^3 **A1**

(c) Hypotenuse of triangle $= \sqrt{6^2 + 8^2} = \sqrt{100} = 10$ cm **(M1)**

Total SA $= 2 \times 24 + (6 \times 15) + (8 \times 15) + (10 \times 15)$

$= 48 + 90 + 120 + 150 = 408$ cm^2 **A1**

Q18 — 3D Coordinates: Finding a Vertex from Midpoint MEDIUM

(a) Midpoint formula: $\dfrac{x_R + x_S}{2} = 3 \Rightarrow x_S = 6 - 7 = -1$; similarly $y_S = 2 - (-3) = 5$; $z_S = -8 - 2 = -10$ **M1**

$S = (-1,\ 5,\ -10)$ **A1**

(b) $RS = \sqrt{(7 - (-1))^2 + (-3 - 5)^2 + (2 - (-10))^2} = \sqrt{64 + 64 + 144}$ **M1**

$= \sqrt{272} = 16.492\ldots \approx 16.5$ (units) **A1**

Note: Also accept RS computed as $2 \times |RM| = 2\sqrt{(7-3)^2 + (-3-1)^2 + (2-(-4))^2} = 2\sqrt{68} = \sqrt{272}$.

Q19 — Volume of Frustum and Unit Conversion MEDIUM

(a) $V = \dfrac{\pi \times 22}{3}(18^2 + 18 \times 10 + 10^2) = \dfrac{22\pi}{3}(324 + 180 + 100)$ **M1**

$= \dfrac{22\pi}{3}(604) = \dfrac{13\,288\pi}{3} = 13\,927.2\ldots \approx 13\,900$ cm^3 **A1**

(b) $V = 13\,927.2\ldots \div 1000 = 13.927\ldots \approx 13.9$ litres **A1**

(c) $V_{\text{soil}} = 0.75 \times 13.927\ldots = 10.445\ldots \approx 10.4$ litres **A1**

Note: FT from (b) for (c).

Q20 — Composite Surface Area: Hemisphere + Cone MEDIUM

(a) $l = \sqrt{r^2 + h^2} = \sqrt{12^2 + 16^2} = \sqrt{144 + 256} = \sqrt{400} = 20$ cm **A1**

(b) Curved cone SA $= \pi r l = \pi(12)(20) = 240\pi$ **M1**

Curved hemisphere SA $= 2\pi r^2 = 2\pi(12)^2 = 288\pi$ **M1**

Total $= 240\pi + 288\pi = 528\pi = 1658.76\ldots \approx 1660$ cm^2 **A1**

Section C — Hard

Q21 — Silo: Cylinder + Cone with Unknown Height HARD

(a) $V_{\text{cone}} = \frac{1}{3}\pi(3)^2(4) = 12\pi$ **(A1)**

$V_{\text{total}} = \pi(3)^2 h_c + 12\pi = 9\pi h_c + 12\pi$ **A1**

(b) $9\pi h_c + 12\pi = 120$ **M1**

$$h_c = \frac{120 - 12\pi}{9\pi} = \frac{120 - 37.699\ldots}{28.274\ldots} = \frac{82.300\ldots}{28.274\ldots} = 2.9109\ldots \approx 2.91 \text{ m}$$ **A1**

(c) $l_{\text{cone}} = \sqrt{3^2 + 4^2} = 5$ m

Lateral cylinder SA $= 2\pi(3)(2.9109\ldots) = 54.912\ldots$ m^2

Lateral cone SA $= \pi(3)(5) = 15\pi = 47.123\ldots$ m^2

Total SA $= 54.912\ldots + 47.123\ldots = 102.036\ldots \approx 102$ m^2 **A1**

Note: FT from their h_c.

Q22 — Spherical Tank: Bounds and Percentage Error HARD

(a) Lower bound: 2.75 m; Upper bound: 2.85 m **A1**

(b) $V_{\text{upper}} = \frac{4}{3}\pi(2.85)^3 = \frac{4}{3}\pi(23.149\ldots) = 97.225\ldots \approx 97.23$ m^3 **M1 A1**

(c) $V_{\text{calc}} = \frac{4}{3}\pi(2.8)^3 = \frac{4}{3}\pi(21.952) = 92.153\ldots$ m^3 **M1**

$$\%\text{ error} = \frac{|97.225\ldots - 92.153\ldots|}{92.153\ldots} \times 100 = \frac{5.071\ldots}{92.153\ldots} \times 100 = 5.504\ldots \approx 5.50\%$$ **A1**

Q23 — 3D Coordinates: Unknown Parameter from Distance Condition HARD

(a) $M = \left(\dfrac{6+(-2)}{2}, \dfrac{2+6}{2}, \dfrac{-1+3}{2}\right) = (2,\ 4,\ 1)$ **M1 A1**

(b) $MC = \sqrt{(k-2)^2 + (0-4)^2 + (5-1)^2} = \sqrt{(k-2)^2 + 16 + 16}$ **M1**

Setting $MC = 7$: $(k-2)^2 + 32 = 49$

$(k-2)^2 = 17 \Rightarrow k - 2 = \pm\sqrt{17}$ **M1**

$k = 2 + \sqrt{17} = 6.123\ldots \approx 6.12$ or $k = 2 - \sqrt{17} = -2.123\ldots \approx -2.12$ **A1**

Q24 — Sphere Inscribed in Cylinder: Volume Ratio and SA HARD

(a) The sphere touches both ends of the cylinder, so the diameter of the sphere equals the height of the cylinder: $h = 2r$. **R1 AG**

(b) $V_{\text{sphere}} = \frac{4}{3}\pi r^3$; $V_{\text{cylinder}} = \pi r^2 \times 2r = 2\pi r^3$ **M1**

$$\frac{V_{\text{sphere}}}{V_{\text{cylinder}}} = \frac{\frac{4}{3}\pi r^3}{2\pi r^3} = \frac{4/3}{2} = \frac{4}{6} = \frac{2}{3}$$ **AG**

Note: Both expressions must be stated explicitly for the AG to be awarded.

(c) $\frac{4}{3}\pi r^3 = \frac{500\pi}{3} \Rightarrow r^3 = \dfrac{500\pi/3}{4\pi/3} = \dfrac{500}{4} = 125 \Rightarrow r = 5\,\text{cm}$ **M1**

Height of cylinder $= 2r = 10\,\text{cm}$

Total SA of cylinder $= 2\pi r^2 + 2\pi r h = 2\pi(25) + 2\pi(5)(10) = 50\pi + 100\pi = 150\pi\,\text{cm}^2$ **A1**

Q25 — Cone Optimisation with Fixed Surface Area HARD

(a) Total SA $= \pi r^2 + \pi r l = 300\pi$ **M1**

$\pi r l = 300\pi - \pi r^2 \Rightarrow l = \dfrac{300\pi - \pi r^2}{\pi r} = \dfrac{300 - r^2}{r}$ **A1 AG**

(b) $h = \sqrt{l^2 - r^2} = \sqrt{\left(\dfrac{300 - r^2}{r}\right)^2 - r^2}$ **A1**

(c) $V = \frac{1}{3}\pi r^2 h = \frac{\pi r^2}{3}\sqrt{\left(\dfrac{300 - r^2}{r}\right)^2 - r^2}$

Using GDC (valid for $0 < r < \sqrt{300} \approx 17.3$): maximum at $r \approx 12.2\,\text{cm}$, $V_{\text{max}} \approx 2790\,\text{cm}^3$ **M1 A1**

Note: Accept $r = 12.2$ cm (full precision: $r = 12.247\ldots$) and $V = 2793\ldots \approx 2790$ cm^3.

Q26 — Cylinder with Unknown Radius from Surface Area HARD

(a) Total SA $= 2\pi r^2 + 2\pi rh = 2\pi r^2 + 2\pi r(10) = 628$ **M1**

Divide by 2π: $r^2 + 10r = \dfrac{628}{2\pi} = \dfrac{314}{\pi}$

Since $\dfrac{314}{\pi} \approx 99.9398\ldots$, use $314/\pi$: $\pi r^2 + 10\pi r = 314$

$\pi r^2 + 10\pi r - 314 = 0$ **A1 AG**

Note: The division step must be shown for AG; accept working from $2\pi r^2 + 20\pi r = 628 \Rightarrow \pi r^2 + 10\pi r - 314 = 0$.

(b) Using GDC (or quadratic formula with $a = \pi$, $b = 10\pi$, $c = -314$): **M1**

$$r = \frac{-10\pi + \sqrt{100\pi^2 + 4\pi \times 314}}{2\pi} = \frac{-10\pi + \sqrt{100\pi^2 + 1256\pi}}{2\pi}$$

$r = 4.743\ldots \approx 4.74$ cm (reject negative root) **A1**

(c) $V = \pi r^2 h = \pi(4.743\ldots)^2(10) = \pi(22.496\ldots)(10) = 706.8\ldots \approx 707$ cm^3 **A1**

Note: FT from their r in (b).

Q27 — Tetrahedron: Centroid, Height, Area, Volume HARD

(a) $G = \left(\dfrac{0+6+3}{3}, \dfrac{0+0+6}{3}, \dfrac{0+0+0}{3}\right) = (3,\ 2,\ 0)$ **A1**

(b) Height of tetrahedron = z-coordinate of $D = 8$ m **A1**

(c) $\overrightarrow{AB} = (6, 0, 0)$, $\overrightarrow{AC} = (3, 6, 0)$

Area $= \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}|(0 \cdot 0 - 0 \cdot 6,\ 0 \cdot 3 - 6 \cdot 0,\ 6 \cdot 6 - 0 \cdot 3)| = \frac{1}{2}|(0, 0, 36)|$ **M1**

$= \frac{1}{2} \times 36 = 18$ m^2 **A1**

Alternative: Base $AB = 6$, height from C to $AB = 6$ (the y-component), area $= \frac{1}{2} \times 6 \times 6 = 18$ m^2.

(d) $V = \frac{1}{3} \times 18 \times 8 = 48$ m^3 **A1**

Q28 — Grain Building: Hemisphere on Cylinder, Unknown Radius HARD

(a) Height of cylinder $= 9 - r$ (since total height = cylinder height + hemisphere radius) **M1**

$V = \pi r^2(9 - r) + \frac{2}{3}\pi r^3 = 9\pi r^2 - \pi r^3 + \frac{2}{3}\pi r^3 = 9\pi r^2 - \frac{1}{3}\pi r^3$ **A1**

(b) $9\pi r^2 - \frac{1}{3}\pi r^3 = 250$ **M1**

Using GDC: $r = 3.3157\ldots \approx 3.32$ m **A1**

Note: Other GDC solutions rejected as they give $r \geq 9$ or $r \leq 0$.

(c) Cylinder height $= 9 - 3.3157\ldots = 5.684\ldots$ m

SA (curved cylinder) $= 2\pi rh = 2\pi(3.3157)(5.684\ldots) = 118.36\ldots$ m^2

SA (curved hemisphere) $= 2\pi r^2 = 2\pi(3.3157)^2 = 69.25\ldots$ m^2

SA (circular base) $= \pi r^2 = \pi(3.3157)^2 = 34.62\ldots$ m^2

Total $= 118.36 + 69.25 + 34.62 = 222.23\ldots \approx 222\,\text{m}^2$ **A1**

Note: FT from their r.

Q29 — Pyramid Tent: Slant Edge, Slant Height and Lateral SA HARD

(a) $AT = \sqrt{(2-0)^2 + (2-0)^2 + (3-0)^2} = \sqrt{4+4+9} = \sqrt{17} = 4.123\ldots \approx 4.12\,\text{m}$ **M1 A1**

(b) Midpoint of AB: $N = (2,\,0,\,0)$

Slant height $= TN = \sqrt{(2-2)^2 + (2-0)^2 + (3-0)^2} = \sqrt{0+4+9} = \sqrt{13} = 3.6055\ldots \approx 3.61\,\text{m}$ **M1 A1**

(c) Area of one triangular face $= \frac{1}{2} \times$ base $\times$ slant height $= \frac{1}{2} \times 4 \times 3.6055\ldots = 7.211\ldots\ \text{m}^2$

Total lateral SA $= 4 \times 7.211\ldots = 28.844\ldots \approx 28.8\,\text{m}^2$ **A1**

Note: FT from their slant height in (b).

Q30 — Cylindrical Trough: Length, Surface Area and Paint HARD

(a) Midpoint of axis: $M = \left(0,\,0,\,\dfrac{0+L}{2}\right) = (0,\,0,\,2) \Rightarrow \dfrac{L}{2} = 2 \Rightarrow L = 4\,\text{m}$ **A1**

(b) Two circular ends: $2 \times \pi(0.5)^2 = 2 \times 0.25\pi = 0.5\pi\,\text{m}^2$ **M1**

Curved surface: $2\pi(0.5)(4) = 4\pi\,\text{m}^2$

Total SA $= 0.5\pi + 4\pi = 4.5\pi\,\text{m}^2$ **A1**

(c) $A = 4.5\pi = 14.137\ldots\ \text{m}^2$ **M1**

Litres needed $= 14.137\ldots \div 8 = 1.767\ldots$ litres

Minimum *full* litres $= 2$ litres **A1**

Note: Do not accept 1 litre; rounding up is required.

Common Mistakes to Avoid

1. **Slant height vs vertical height.** The cone formula $V = \frac{1}{3}\pi r^2 h$ uses *perpendicular* height h, not slant height l. Always draw a right-angle triangle inside the cone to identify h.
2. **Composite solid surface areas.** When two solids are joined, the *shared circular face* is *internal* — exclude it from the total surface area. Only count *exposed* surfaces.
3. **3D distance formula.** All three coordinate differences must be squared and summed. Forgetting the z-component gives a 2D answer — always write all three terms before evaluating.
4. **Sphere inscribed in cylinder.** The diameter of the sphere equals the height of the cylinder — not the radius. Draw the cross-section to see why $h = 2r$.

5. **Upper bounds and percentage error.** The upper-bound volume uses the *upper*-bound radius. Substituting the lower radius is a common exam error.
6. **Rounding "at least" problems.** Questions asking for the *minimum number of tins / litres / packs* always round *up* — never down — even if the decimal is very small.