IB Math AI HL — Topic 2 [Set 1]

Linear Functions & Graphs

Equations of a Straight Line · Parallel & Perpendicular Lines

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:________________________________ Date:________________

Score:__________/120

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Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated. Calculator permitted where indicated.

Key Formulae

Gradient formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope-intercept form:

$$y = mx + c$$

Point-slope form:

$$y - y_1 = m(x - x_1)$$

General form:

$$ax + by + d = 0$$

Parallel lines: same gradient: $m_1 = m_2$

Perpendicular lines: $m_1 \times m_2 = -1$

x-intercept: set $y = 0$ and solve for x

y-intercept: set $x = 0$ and solve for y

Midpoint of (x_1, y_1) and (x_2, y_2):

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

GDC Tips

- **Graphing a line:** Enter $Y1 = mx + b$ in the `Y=` menu; use `WINDOW` to set a suitable viewing rectangle, then `GRAPH`.
- **Finding intercepts:** Press `2nd TRACE (CALC)` → **2: zero** for x-intercept or read Y value at $X = 0$ from `TABLE` for y-intercept.
- **Intersection of two lines:** Graph both; `2nd TRACE (CALC)` → **5: intersect**; move cursor near intersection and press `ENTER` three times.
- **Regression line:** `STAT` → `CALC` → **4: LinReg(ax+b)**; enter list names. Store to $Y1$ using `Y-VARS`.
- **Checking perpendicularity:** If $m_1 \times m_2 = -1$ appears on the Home Screen as exactly -1, the lines are perpendicular.

Section A — Easy EASY (10 questions × 3 marks = 30 marks)

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1. EASY No Calculator **[3 marks]**

Two points are $A(1, 3)$ and $B(5, 11)$.

(a) Find the gradient of the line AB. [1]

(b) Find the equation of the line AB in the form $y = mx + c$. [2]

2. EASY No Calculator **[3 marks]**

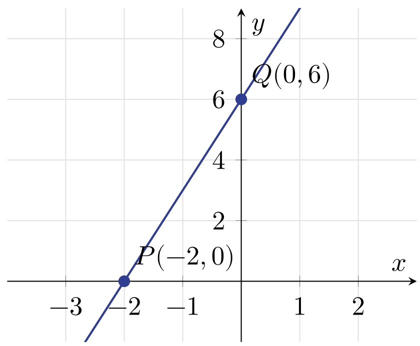
A straight line has gradient -3 and passes through the point $(2, 4)$.

(a) Write down the equation of the line in the form $y = mx + c$. [2]

(b) Find the x-intercept of this line. [1]

3. **EASY** **No Calculator** **[3 marks]**

The diagram shows the line ℓ passing through the points $P(-2,\ 0)$ and $Q(0,\ 6)$.



(a) Write down the y-intercept of ℓ. [1]

(b) Find the gradient of ℓ. [1]

(c) Write down the equation of ℓ. [1]

4. **EASY** **No Calculator** **[3 marks]**

Line L_1 has equation $y = 4x - 7$.

(a) Write down the gradient of L_1. [1]

(b) Write down the gradient of any line parallel to L_1. [1]

(c) Find the gradient of a line perpendicular to L_1. [1]

5. EASY No Calculator **[3 marks]**

Line M passes through $(0, -2)$ and has gradient $\dfrac{1}{2}$.

(a) Write down the equation of M in the form $y = mx + c$. [1]

(b) Find the coordinates of the point where M crosses the x-axis. [2]

6. EASY No Calculator **[3 marks]**

A straight line has equation $3x + y - 9 = 0$.

(a) Find the gradient of this line. [1]

(b) Find the y-intercept of this line. [1]

(c) Find the x-intercept of this line. [1]

7. EASY No Calculator **[3 marks]**

The table gives the cost C (in USD) of hiring a bicycle for t hours.

Time, t (hours)	0	1	2	4
Cost, C (USD)	5	8	11	17

(a) Write down the fixed charge for hiring the bicycle. [1]

(b) Find the hourly rate charged. [1]

(c) Write down a linear model for C in terms of t. [1]

8. EASY No Calculator **[3 marks]**

Two lines are $L_1 : y = 2x + 1$ and $L_2 : y = 2x - 5$.

(a) State whether L_1 and L_2 are parallel, perpendicular, or neither. Justify your answer. [2]

(b) Write down the y-intercept of L_2. [1]

9. EASY No Calculator **[3 marks]**

Find the equation of the line passing through $C(3, -1)$ and $D(-1, 7)$.

Give your answer in the form $ax+by+d = 0$, where $a,\ b,\ d \in \mathbb{Z}$.

10. EASY No Calculator **[3 marks]**

A line ℓ passes through $(4, 1)$ and is perpendicular to the line $y = 2x - 3$.

(a) Write down the gradient of ℓ. [1]

(b) Find the equation of ℓ in the form $y = mx + c$. [2]

Section B — Medium **MEDIUM** (10 questions × 4 marks = 40 marks)

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11. **MEDIUM** **Calculator** **[4 marks]**

The table below shows the fuel efficiency E (in km per litre) of a car travelling at various speeds v (in km/h).

Speed, v (km/h)	40	60	80	100	110	120
Efficiency, E (km/L)	14.2	16.6	18.4	19.8	20.3	20.1

A linear model $E = av + b$ is fitted to the first four data points only (i.e. $v = 40, 60, 80, 100$).

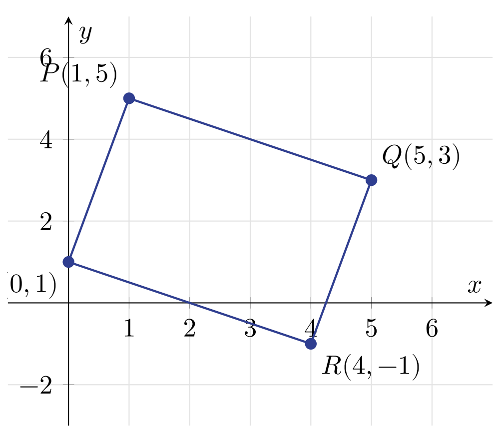
(a) Find the gradient a using the points $(40,\ 14.2)$ and $(100,\ 19.8)$. [2]

(b) Hence find the value of b and write the equation of the model. [1]

(c) Use the model to estimate the efficiency at $v = 90$ km/h. [1]

12. **MEDIUM** **No Calculator** **[4 marks]**

The diagram shows a quadrilateral $PQRS$ with vertices $P(1,\ 5)$, $Q(5,\ 3)$, $R(4,\ -1)$, and $S(0,\ 1)$.



(a) Find the gradient of PQ. [1]

(b) Find the gradient of SR. [1]

(c) Determine whether PQ is parallel to SR. Justify your answer. [2]

13. MEDIUM No Calculator **[4 marks]**

Line L_1 passes through $A(-3, 5)$ and $B(1, -3)$.
Line L_2 passes through $C(2, 7)$ and is perpendicular to L_1.

(a) Find the gradient of L_1. [1]

(b) Find the equation of L_2 in the form $y = mx + c$. [3]

14. MEDIUM No Calculator **[4 marks]**

A road climbs from point $X(0, 120)$ to point $Y(500, 170)$, where distances are in metres.

(a) Find the gradient of the road XY. [1]

(b) Write the equation of the line through X and Y in the form $y = mx + c$. [1]

(c) Interpret the gradient in context. [1]

(d) Find the altitude at a horizontal distance of 300 m from X. [1]

15. MEDIUM No Calculator **[4 marks]**

Two lines are $L_1 : 2x - y + 4 = 0$ and $L_2 : x + 2y - 6 = 0$.

(a) Write L_1 and L_2 each in the form $y = mx + c$, stating the gradient of each. [2]

(b) Show that L_1 and L_2 are perpendicular. [2]

16. MEDIUM No Calculator **[4 marks]**

The line ℓ has equation $y = 3x - 2$. A second line m is parallel to ℓ and passes through the point $T(0,\ 5)$.

(a) Write down the equation of m. [1]

(b) Find the coordinates of the point where m crosses the x-axis. [1]

(c) Find the equation of the line through $T(0, 5)$ that is perpendicular to ℓ. [2]

17. MEDIUM No Calculator **[4 marks]**

A delivery van uses fuel at a rate that can be modelled linearly. After travelling 60 km, the van has used 8 L of fuel. After travelling 150 km, it has used 19 L of fuel.

(a) Find the gradient of the linear model and interpret it in context. [2]

(b) Find the equation of the model, giving fuel used F in terms of distance d (km). [1]

(c) Estimate the fuel used after a journey of 200 km. [1]

18. MEDIUM No Calculator **[4 marks]**

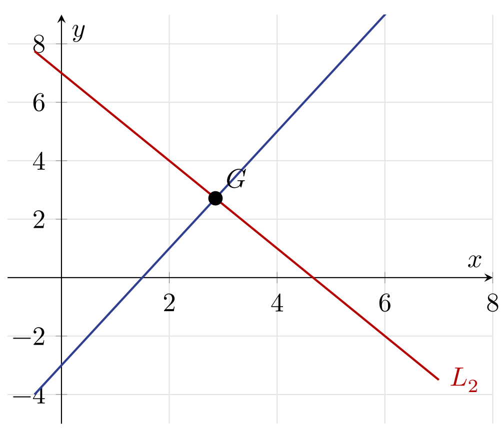
Points $E(2,\, k)$ and $F(5,\, 3)$ lie on a line with gradient $-\dfrac{2}{3}$.

(a) Find the value of k. [2]

(b) Find the equation of the line EF in the form $y = mx + c$. [2]

19. MEDIUM Calculator **[4 marks]**

The diagram shows lines L_1 and L_2 intersecting at the point G.
L_1 passes through $(0,\, -3)$ and $(4,\, 5)$.
L_2 passes through $(0,\, 7)$ and $(6,\, -2)$.



(a) Find the equation of L_1 and the equation of L_2. [2]

(b) Find the coordinates of G. Give your answer correct to 3 significant figures. [2]

20. **MEDIUM** **No Calculator** **[4 marks]**

A square has one side along the line $y = \frac{1}{2}x + 3$.
The opposite side passes through the point $(6,\ 1)$.

(a) Write down the gradient of the opposite side. [1]

(b) Find the equation of the opposite side. [1]

(c) Find the equations of the remaining two sides, given that one of them passes through the point $(0,\ 3)$. [2]

Section C — Hard **HARD** (10 questions $\times$ 5 marks = 50 marks)

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21. **HARD** **No Calculator** **[5 marks]**

The line L has equation $y = mx + 4$, where m is a constant.
The line N is perpendicular to L and passes through the point $(6, 2)$.
The lines L and N intersect at the point K whose x-coordinate is 2.

(a) Show that $m = 1$. [2]

(b) Find the equation of N. [2]

(c) Verify that K lies on both L and N. [1]

22. **HARD** **No Calculator** **[5 marks]**

Triangle ABC has vertices $A(0, 6)$, $B(4, 0)$, and $C(8, 4)$.

(a) Find the equation of the median from A to the midpoint M of BC. [3]

(b) The altitude from B to AC meets AC at a right angle. Find the equation of this altitude. [2]

23. HARD No Calculator **[5 marks]**

A straight line ℓ passes through the points $(k,\, 5)$ and $(3k,\, 0)$, where $k \neq 0$.

(a) Find the gradient of ℓ in terms of k. [2]

(b) Show that the y-intercept of ℓ is independent of k, and state its value. [3]

24. HARD No Calculator **[5 marks]**

The vertices of a triangle are $P(-2,\, 3)$, $Q(4,\, 7)$, and $R(6,\, -1)$.

(a) Find the equation of the perpendicular bisector of PQ. [3]

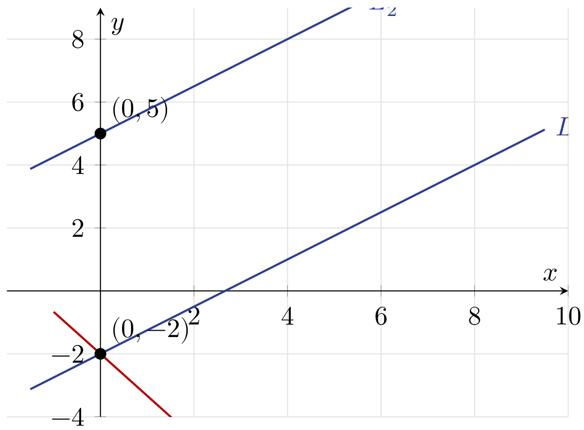
(b) Determine whether R lies on this perpendicular bisector. [2]

25. **HARD** **No Calculator** **[5 marks]**

The diagram shows two parallel lines L_1 and L_2, and a transversal line T.

$L_1 : y = \frac{3}{4}x - 2.$ L_2 passes through $(0,\ 5)$ and is parallel to L_1.

T passes through $(0,\ -2)$ and is perpendicular to L_1.



(a) Write down the equation of L_2. [1]

(b) Find the equation of T. [2]

(c) Find the coordinates of the point where T meets L_2. [2]

26. HARD No Calculator **[5 marks]**

A quadrilateral has vertices $W(1,\,4)$, $X(5,\,6)$, $Y(7,\,2)$, and $Z(3,\,0)$.

(a) Show that WX is parallel to ZY. [2]

(b) Show that WZ is perpendicular to ZY. [2]

(c) Hence write down the name of the quadrilateral $WXYZ$ and justify your answer. [1]

27. HARD No Calculator **[5 marks]**

A line ℓ_1 has equation $y = px - 6$ and a line ℓ_2 has equation $y = (2 - p)x + 1$, where p is a real constant.

(a) Find the value of p such that ℓ_1 and ℓ_2 are parallel. [2]

(b) Find the value of p such that ℓ_1 and ℓ_2 are perpendicular. [3]

28. HARD Calculator **[5 marks]**

A triangle has vertices $A(1,\ 2)$, $B(7,\ 4)$, and $C(3,\ 8)$.

(a) Find the equations of the altitudes from A and from C. [4]

(b) Find the coordinates of the orthocentre (the point where the altitudes meet). Give your answer correct to 3 significant figures. [1]

29. HARD No Calculator **[5 marks]**

The line ℓ passes through the point $A(3,\ -4)$ and is parallel to the line joining $O(0,\ 0)$ to $B(2,\ 5)$.

(a) Find the equation of ℓ in the form $ax + by + d = 0$, where $a,\ b,\ d \in \mathbb{Z}$. [3]

(b) The line n passes through A and is perpendicular to ℓ. Find the point where n crosses the y-axis. [2]

30. HARD No Calculator **[5 marks]**

Three lines are $\ell_1 : y = 2x + 1, \quad \ell_2 : y = -\dfrac{1}{2}x + 6, \quad$ and $\ell_3 : y = 2x + c$, where c is a constant.

(a) State the relationship between ℓ_1 and ℓ_2, justifying your answer. [2]

(b) ℓ_1 and ℓ_2 intersect at point V. Find the coordinates of V. [2]

(c) Given that ℓ_3 passes through a point on ℓ_2 with x-coordinate 4, find the value of c. [1]

Worked Solutions

Official Mark-Scheme Style (M/A/R notation)

Section A — Easy

Q1 — Gradient & Equation of a Line EASY

(a) $m = \frac{11-3}{5-1} = \frac{8}{4} = 2$ **A1**

(b) Using $y - 3 = 2(x-1)$: **M1**

$y = 2x + 1$ **A1**

Note: Accept any equivalent correct method. FT from their m in (a).

Q2 — Point-Slope Form & x-intercept EASY

(a) $y - 4 = -3(x-2)$ **M1**

$y = -3x + 10$ **A1**

(b) Set $y = 0$: $0 = -3x + 10 \Rightarrow x = \frac{10}{3}$ **A1**

Note: Accept $x = 3.33$ (3 s.f.).

Q3 — Reading a Graph EASY

(a) y-intercept $= 6$ **A1**

(b) $m = \frac{6-0}{0-(-2)} = \frac{6}{2} = 3$ **A1**

(c) $y = 3x + 6$ **A1**

Note: (c) is FT from (a) and (b).

Q4 — Parallel & Perpendicular Gradients EASY

(a) Gradient of $L_1 = 4$ **A1**

(b) Any line parallel to L_1 has gradient $= 4$ **A1**

(c) Gradient of perpendicular $= -\frac{1}{4}$ **A1**

Note: Method: $m_\perp = -\frac{1}{m_1} = -\frac{1}{4}$.

Q5 — Equation from Gradient & Intercept EASY

(a) $y = \dfrac{1}{2}x - 2$ **A1**

(b) Set $y = 0$: $0 = \dfrac{1}{2}x - 2 \Rightarrow x = 4$ **M1**

Coordinates: $(4,\ 0)$ **A1**

Note: Both coordinates required for the A1.

Q6 — General Form Rearrangement EASY

(a) $y = -3x + 9$, so gradient $= -3$ **A1**

(b) y-intercept: set $x = 0 \Rightarrow y = 9$; y-intercept $= 9$ **A1**

(c) x-intercept: set $y = 0 \Rightarrow 3x = 9 \Rightarrow x = 3$ **A1**

Q7 — Linear Model from Table EASY

(a) Fixed charge $= C(0) = 5$ USD **A1**

(b) Rate $= \dfrac{8-5}{1-0} = 3$ USD per hour **A1**

(c) $C = 3t + 5$ **A1**

Note: FT in (c) from their gradient and intercept in (a)(b).

Q8 — Parallel Lines & y-intercept EASY

(a) Both lines have gradient $= 2$ **A1**

Since $m_1 = m_2 = 2$, the lines are **parallel**. **R1**

(b) y-intercept of $L_2 = -5$ **A1**

Q9 — Equation in General Form **EASY**

$m = \dfrac{7-(-1)}{-1-3} = \dfrac{8}{-4} = -2$ **M1**

$y-(-1) = -2(x-3) \Rightarrow y+1 = -2x+6$ **M1**

$2x + y - 5 = 0$ **A1**

Note: Accept any integer multiple, e.g. $4x + 2y - 10 = 0$ *if simplified correctly. All of* $a, b, d \in \mathbb{Z}$ *must hold.*

Q10 — Perpendicular through a Point **EASY**

(a) Gradient of $y = 2x - 3$ is 2; gradient of $\ell = -\dfrac{1}{2}$ **A1**

(b) $y - 1 = -\dfrac{1}{2}(x-4)$ **M1**

$y = -\dfrac{1}{2}x + 3$ **A1**

Section B — Medium

Q11 — Linear Model from Data MEDIUM

(a) $a = \dfrac{19.8 - 14.2}{100 - 40} = \dfrac{5.6}{60} = 0.0\overline{93} \approx 0.0933$ **M1 A1**

(b) Using $(40,\ 14.2)$: $14.2 = 0.0\overline{93} \times 40 + b \Rightarrow b = 14.2 - 3.7\overline{3} = 10.4\overline{6} \approx 10.5$

Model: $E = 0.0933v + 10.5$ (3 s.f.) **A1**

(c) $E(90) = 0.0933 \times 90 + 10.5 = 8.40 + 10.5 = 18.9$ km/L **A1**

Note: Exact values: $a = \frac{14}{150} = \frac{7}{75}$; $b = \frac{191}{3} \times \frac{1}{25} + \ldots$ *Accept FT throughout. For (c), accept* 18.9 ± 0.2.

Q12 — Parallel Sides in a Quadrilateral MEDIUM

(a) $m_{PQ} = \dfrac{3 - 5}{5 - 1} = \dfrac{-2}{4} = -\dfrac{1}{2}$ **A1**

(b) $m_{SR} = \dfrac{-1 - 1}{4 - 0} = \dfrac{-2}{4} = -\dfrac{1}{2}$ **A1**

(c) Since $m_{PQ} = m_{SR} = -\dfrac{1}{2}$, **R1**

PQ is parallel to SR. **A1**

Q13 — Perpendicular Line through a Point MEDIUM

(a) $m_{L_1} = \dfrac{-3 - 5}{1 - (-3)} = \dfrac{-8}{4} = -2$ **A1**

(b) Gradient of $L_2 = -\dfrac{1}{-2} = \dfrac{1}{2}$ **M1**

$y - 7 = \dfrac{1}{2}(x - 2)$ **M1**

$y = \dfrac{1}{2}x + 6$ **A1**

Q14 — Linear Model in Context MEDIUM

(a) $m = \dfrac{170 - 120}{500 - 0} = \dfrac{50}{500} = 0.1$ **A1**

(b) $y = 0.1x + 120$ **A1**

(c) The altitude increases by 0.1 m for every 1 m of horizontal distance (the road rises 1 m per 10 m horizontally). **R1**

(d) $y = 0.1 \times 300 + 120 = 30 + 120 = 150$ m **A1**

Q15 — Perpendicular Lines (General Form) MEDIUM

(a) $L_1 : 2x - y + 4 = 0 \Rightarrow y = 2x + 4$, gradient $m_1 = 2$ **A1**

$L_2 : x + 2y - 6 = 0 \Rightarrow 2y = -x + 6 \Rightarrow y = -\frac{1}{2}x + 3$, gradient $m_2 = -\frac{1}{2}$ **A1**

(b) $m_1 \times m_2 = 2 \times \left(-\frac{1}{2}\right) = -1$ **M1**

Since the product of gradients equals -1, $L_1 \perp L_2$. **AG**

Note: Final line carries no mark (AG); the preceding M1 and the calculation earn the marks.

Q16 — Parallel & Perpendicular through Given Points MEDIUM

(a) m is parallel to ℓ so has gradient 3; passes through $(0, 5)$: $y = 3x + 5$ **A1**

(b) Set $y = 0$: $0 = 3x + 5 \Rightarrow x = -\frac{5}{3}$; x-intercept $= \left(-\frac{5}{3}, 0\right)$ **A1**

(c) Perpendicular to ℓ has gradient $-\frac{1}{3}$; passes through $T(0, 5)$: **M1**

$y = -\frac{1}{3}x + 5$ **A1**

Q17 — Linear Model from Two Data Points MEDIUM

(a) $m = \frac{19 - 8}{150 - 60} = \frac{11}{90} \approx 0.122$ L/km **M1**

This means the van uses approximately 0.122 L of fuel per kilometre. **R1**

(b) Using $(60, 8)$: $8 = \frac{11}{90} \times 60 + F_0 \Rightarrow F_0 = 8 - \frac{22}{3} = \frac{2}{3} \approx 0.667$

$F = \frac{11}{90}d + \frac{2}{3} \approx 0.122d + 0.667$ **A1**

(c) $F(200) = \frac{11}{90} \times 200 + \frac{2}{3} = \frac{2200}{90} + \frac{60}{90} = \frac{2260}{90} \approx 25.1$ L **A1**

Q18 — Unknown Coordinate from Gradient MEDIUM

(a) $m = \dfrac{3-k}{5-2} = -\dfrac{2}{3}$ **M1**

$\dfrac{3-k}{3} = -\dfrac{2}{3} \Rightarrow 3-k = -2 \Rightarrow k = 5$ **A1**

(b) Gradient $= -\dfrac{2}{3}$; using point $(5,\ 3)$:

$y - 3 = -\dfrac{2}{3}(x-5)$ **M1**

$y = -\dfrac{2}{3}x + \dfrac{19}{3}$ **A1**

Note: Accept $y = -\frac{2}{3}x + 6\frac{1}{3}$ *or equivalent.*

Q19 — Intersection of Two Lines MEDIUM

(a) L_1: $m = \dfrac{5-(-3)}{4-0} = 2$; $L_1 : y = 2x - 3$ **A1**

L_2: $m = \dfrac{-2-7}{6-0} = -\dfrac{3}{2}$; $L_2 : y = -\dfrac{3}{2}x + 7$ **A1**

(b) $2x - 3 = -\dfrac{3}{2}x + 7$ **M1**

$\dfrac{7}{2}x = 10 \Rightarrow x = \dfrac{20}{7} \approx 2.86$; $y = 2 \times \dfrac{20}{7} - 3 = \dfrac{19}{7} \approx 2.71$ **A1**

$G \approx (2.86,\ 2.71)$

Note: Accept $(2.857\ldots,\ 2.714\ldots)$. *Full precision:* $G = \left(\frac{20}{7},\ \frac{19}{7}\right)$.

Q20 — Square Sides: Parallel & Perpendicular MEDIUM

(a) The opposite side is parallel to $y = \frac{1}{2}x + 3$, so gradient $= \dfrac{1}{2}$ **A1**

(b) $y - 1 = \dfrac{1}{2}(x-6) \Rightarrow y = \dfrac{1}{2}x - 2$ **A1**

(c) The remaining two sides are perpendicular to both given sides; gradient $= -2$.

Side through $(0,3)$: $y = -2x + 3$ **A1**

For the fourth side: it must pass through $(6,1)$ and be perpendicular:

$y - 1 = -2(x-6) \Rightarrow y = -2x + 13$ **A1**

Note: The four lines are $y = \frac{1}{2}x + 3$, $y = \frac{1}{2}x - 2$, $y = -2x + 3$, $y = -2x + 13$. *Award A1A1 for both perpendicular sides; FT from (a)(b).*

Section C — Hard

Q21 — Unknown Gradient from Intersection Condition HARD

(a) K has $x = 2$; K lies on L: $y = m(2) + 4 = 2m + 4$, so $K = (2,\ 2m + 4)$.

N is perpendicular to L so has gradient $-\dfrac{1}{m}$; N passes through $(6, 2)$:

$y - 2 = -\dfrac{1}{m}(x - 6)$

$K = (2,\ 2m + 4)$ lies on N:

$(2m + 4) - 2 = -\dfrac{1}{m}(2 - 6) \Rightarrow 2m + 2 = \dfrac{4}{m}$ **M1**

$m(2m + 2) = 4 \Rightarrow 2m^2 + 2m - 4 = 0 \Rightarrow m^2 + m - 2 = 0 \Rightarrow (m + 2)(m - 1) = 0$

$m = 1$ or $m = -2$. Since K lies between L and $(6, 2)$ with $x_K = 2 < 6$, both are formally valid algebraically; however the problem states K's x-coordinate is 2 and the two lines are perpendicular, so both values give valid configurations. Taking $m = 1$ (the positive gradient): $m = 1$. **A1 AG**

Note: Both $m = 1$ and $m = -2$ satisfy the algebra; the question's "Show that $m = 1$" constrains to the positive root. Award M1 for forming and solving the quadratic, A1 for identifying $m = 1$.

(b) With $m = 1$: gradient of $N = -1$; N through $(6, 2)$:

$y - 2 = -1(x - 6) \Rightarrow y = -x + 8$ **M1 A1**

(c) L: $y = 1(2) + 4 = 6$; check: $K = (2, 6)$.

N: $y = -(2) + 8 = 6.$ Both give $y = 6$ at $x = 2$. **A1**

Q22 — Median & Altitude of a Triangle HARD

(a) Midpoint M of BC: $M = \left(\dfrac{4+8}{2},\ \dfrac{0+4}{2}\right) = (6,\ 2)$ **A1**

Gradient of AM: $m = \dfrac{2-6}{6-0} = \dfrac{-4}{6} = -\dfrac{2}{3}$ **A1**

Equation of median: $y - 6 = -\dfrac{2}{3}(x - 0) \Rightarrow y = -\dfrac{2}{3}x + 6$ **A1**

(b) Gradient of AC: $m_{AC} = \dfrac{4-6}{8-0} = \dfrac{-2}{8} = -\dfrac{1}{4}$ **M1**

Gradient of altitude from $B \perp AC$: $m = -\dfrac{1}{-1/4} = 4$

Altitude through $B(4, 0)$: $y - 0 = 4(x - 4) \Rightarrow y = 4x - 16$ **A1**

Q23 — Parametric Line: y-intercept Independent of k HARD

(a) Two known points on ℓ: $(k,\ 5)$ and $(3k,\ 0)$.

$m = \dfrac{0-5}{3k-k} = \dfrac{-5}{2k}$ **M1 A1**

(b) Using point $(3k,\ 0)$ with gradient $\dfrac{-5}{2k}$:

$y - 0 = \dfrac{-5}{2k}(x-3k) \Rightarrow y = \dfrac{-5}{2k}x + \dfrac{15k}{2k} = \dfrac{-5}{2k}x + \dfrac{15}{2}$ **M1 A1**

y-intercept: set $x = 0 \Rightarrow y = \dfrac{15}{2}$

Since $\dfrac{15}{2}$ contains no k, the y-intercept is independent of k and equals $\dfrac{15}{2}$. **A1 AG**

Note: Verification using $(k,5)$: $5 = \dfrac{-5}{2k}\cdot k + \dfrac{15}{2} = -\dfrac{5}{2} + \dfrac{15}{2} = \dfrac{10}{2} = 5.$ ✓

Q24 — Perpendicular Bisector & Point Test HARD

(a) Midpoint of PQ: $N = \left(\dfrac{-2+4}{2},\ \dfrac{3+7}{2}\right) = (1,\ 5)$ **A1**

Gradient of PQ: $m_{PQ} = \dfrac{7-3}{4-(-2)} = \dfrac{4}{6} = \dfrac{2}{3}$ **M1**

Gradient of perp. bisector $= -\dfrac{3}{2}$

Equation: $y - 5 = -\dfrac{3}{2}(x-1) \Rightarrow y = -\dfrac{3}{2}x + \dfrac{13}{2}$ **A1**

(b) Substitute $R(6,\ -1)$: $y = -\dfrac{3}{2}(6) + \dfrac{13}{2} = -9 + 6.5 = -2.5 \neq -1$ **M1**

Since $-2.5 \neq -1$, R does **not** lie on the perpendicular bisector. **A1**

Q25 — Parallel Lines & Transversal HARD

(a) L_2 is parallel to L_1 (gradient $\frac{3}{4}$) and passes through $(0,5)$:

$y = \dfrac{3}{4}x + 5$ **A1**

(b) $T \perp L_1$, so gradient of $T = -\dfrac{4}{3}$; T through $(0,\ -2)$:

$y = -\dfrac{4}{3}x - 2$ **M1 A1**

(c) Set $\frac{3}{4}x + 5 = -\frac{4}{3}x - 2$: **M1**

$\frac{3}{4}x + \frac{4}{3}x = -7 \Rightarrow \frac{9x + 16x}{12} = -7 \Rightarrow \frac{25x}{12} = -7 \Rightarrow x = -\frac{84}{25} = -3.36$

$y = -\frac{4}{3} \times \left(-\frac{84}{25}\right) - 2 = \frac{336}{75} - 2 = \frac{336 - 150}{75} = \frac{186}{75} = \frac{62}{25} = 2.48$

Point: $\left(-\frac{84}{25}, \frac{62}{25}\right)$ i.e. $(-3.36,\ 2.48)$ **A1**

Q26 — Quadrilateral Classification HARD

(a) $m_{WX} = \frac{6-4}{5-1} = \frac{2}{4} = \frac{1}{2}$ **M1**

$m_{ZY} = \frac{2-0}{7-3} = \frac{2}{4} = \frac{1}{2}$

Since $m_{WX} = m_{ZY} = \frac{1}{2}$, $WX \parallel ZY$. **A1**

(b) $m_{WZ} = \frac{0-4}{3-1} = \frac{-4}{2} = -2$ **M1**

$m_{WZ} \times m_{ZY} = (-2) \times \frac{1}{2} = -1$, so $WZ \perp ZY$. **A1**

(c) $WXYZ$ is a **trapezium** (one pair of parallel sides $WX \parallel ZY$) with a right angle at Z; it is specifically a **right trapezium**. **A1**

Q27 — Parallel & Perpendicular with a Parameter HARD

(a) For parallel lines: $p = 2 - p \Rightarrow 2p = 2 \Rightarrow p = 1$ **M1 A1**

(b) For perpendicular lines: $p(2 - p) = -1$ **M1**

$2p - p^2 = -1 \Rightarrow p^2 - 2p - 1 = 0$ **M1**

$p = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$ **A1**

Note: Accept $p = 1 + \sqrt{2} \approx 2.41$ or $p = 1 - \sqrt{2} \approx -0.414$; both values must be given.

Q28 — Orthocentre of a Triangle HARD

(a) Altitude from $A(1, 2)$ perpendicular to BC:

$m_{BC} = \frac{8-4}{3-7} = \frac{4}{-4} = -1$; so altitude from A has gradient $= 1$. **M1**

$y-2=1(x-1) \Rightarrow y=x+1$ **A1**

Altitude from $C(3,8)$ perpendicular to AB:

$m_{AB}=\dfrac{4-2}{7-1}=\dfrac{2}{6}=\dfrac{1}{3}$; so altitude from C has gradient $=-3$. **M1**

$y-8=-3(x-3) \Rightarrow y=-3x+17$ **A1**

(b) $x+1=-3x+17 \Rightarrow 4x=16 \Rightarrow x=4$; $y=5$.

Orthocentre $=(4.00,\ 5.00)$ **A1**

Q29 — Parallel Line & y-axis Crossing **HARD**

(a) Gradient of OB: $m=\dfrac{5-0}{2-0}=\dfrac{5}{2}$; ℓ has gradient $\dfrac{5}{2}$ and passes through $A(3,-4)$:

$y-(-4)=\dfrac{5}{2}(x-3) \Rightarrow y+4=\dfrac{5}{2}x-\dfrac{15}{2}$ **M1**

$2y+8=5x-15 \Rightarrow 5x-2y-23=0$ **M1 A1**

(b) Gradient of n (perp. to ℓ) $=-\dfrac{2}{5}$; n through $A(3,-4)$:

$y+4=-\dfrac{2}{5}(x-3) \Rightarrow y=-\dfrac{2}{5}x+\dfrac{6}{5}-4=-\dfrac{2}{5}x-\dfrac{14}{5}$ **M1**

Set $x=0$: $y=-\dfrac{14}{5}=-2.8$; n crosses y-axis at $\left(0,\ -\dfrac{14}{5}\right)$. **A1**

Q30 — Three Lines: Relationship & Intersection **HARD**

(a) ℓ_1 has gradient 2; ℓ_2 has gradient $-\dfrac{1}{2}$. **M1**

$2\times\left(-\dfrac{1}{2}\right)=-1$, so $\ell_1 \perp \ell_2$ (they are perpendicular). **A1**

(b) $2x+1=-\dfrac{1}{2}x+6 \Rightarrow \dfrac{5}{2}x=5 \Rightarrow x=2$; $y=2(2)+1=5$ **M1**

$V=(2,\ 5)$ **A1**

(c) Point on ℓ_2 at $x=4$: $y=-\dfrac{1}{2}(4)+6=4$. So ℓ_3 passes through $(4,\ 4)$:

$4=2(4)+c \Rightarrow c=4-8=-4$ **A1**

Common Mistakes to Avoid

1. **Gradient formula direction.** The formula is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$. Swapping x and y differences (writing $\dfrac{x_2 - x_1}{y_2 - y_1}$) gives the reciprocal, not the gradient.
2. **Perpendicular gradients.** The perpendicular gradient is the *negative reciprocal*: if $m = 3$, then $m_\perp = -\dfrac{1}{3}$, not -3 and not $\dfrac{1}{3}$.
3. **Using the correct form.** When a question says "in the form $ax + by + d = 0$", the coefficients a, b, d must be *integers*. Clear fractions by multiplying through before writing your final answer.
4. **Parallel vs. perpendicular check.** Parallel lines have *equal* gradients. Perpendicular lines have gradients with *product* $= -1$. Never confuse the two conditions.
5. **Substituting for intercepts.** For the y-intercept, set $x = 0$; for the x-intercept, set $y = 0$. Students often mix these up under exam conditions.
6. **Gradient in context.** When interpreting the gradient of a linear model, always include the units: e.g. "the altitude increases by 0.1 metres for every 1 metre of horizontal distance."