IB Math AI HL — Topic 3 [Set 1]

Trigonometry Toolkit

Pythagoras · Right-Angled Trig · Sine & Cosine Rule · Area of Triangle · Elevation & Depression · Bearings

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Scan me for help

Name:________________________ Date:____________

Score:________/120

Instructions: Answer all questions. Show all working. Answers to 3 s.f. unless stated. GDC permitted. Angles in degrees.

Key Formulae — Geometry & Trigonometry

Pythagoras' Theorem: $c^2 = a^2 + b^2$

Right-angled trig:
$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$, $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$, $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

Sine Rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine Rule (side): $c^2 = a^2 + b^2 - 2ab\cos C$

Cosine Rule (angle): $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

Area of Triangle: Area $= \frac{1}{2}ab\sin C$

Angle of Elevation: measured upward from horizontal

Angle of Depression: measured downward from horizontal

Bearings: three-figure, measured clockwise from North

Ambiguous case: $\sin\theta = \sin(180° - \theta)$ — check both solutions when using the sine rule for an angle.

GDC Tips

- **MODE check:** Ensure your GDC is in **DEGREE** mode for all questions on this worksheet.
- **Cosine rule:** Type the full expression in one GDC entry; avoid rounding intermediate values.
- **Finding angles with inverse trig:** Use $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ (2nd function keys). Store the exact value to memory for follow-up parts.
- **Area formula:** $\frac{1}{2}ab\sin C$ — make sure C is the *included* angle between sides a and b.
- **Bearings:** Draw a North arrow first, then work out interior angles before applying the cosine/sine rule.

Section A — Easy **EASY** (10 questions × 3 marks = 30 marks)

Scan me for help

1. **EASY** **Calculator** **[3 marks]**

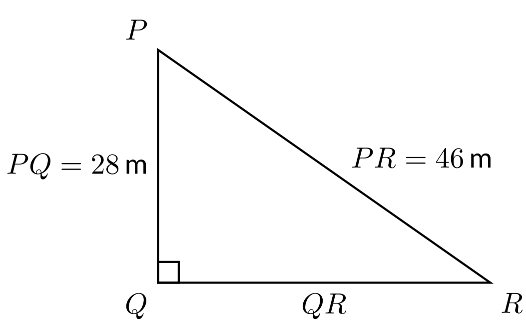
A rectangular sports field has length 85 m and width 62 m.

(a) Find the length of a diagonal of the field. [2]

(b) A groundskeeper walks from one corner to the diagonally opposite corner. Write down the distance walked, correct to the nearest metre. [1]

2. **EASY** **Calculator** **[3 marks]**

The diagram shows a right-angled triangle PQR with the right angle at Q.



The diagram shows triangle PQR with $PQ = 28$ m and $PR = 46$ m.

(a) Find the length of QR. [2]

(b) Find the angle $\widehat{QPR}$. [1]

3. EASY Calculator **[3 marks]**

A 7.4 m ladder leans against a vertical wall. The foot of the ladder is 2.6 m from the base of the wall on horizontal ground.

(a) Find the angle the ladder makes with the ground. [2]

(b) Find the height reached by the top of the ladder on the wall. [1]

4. EASY Calculator **[3 marks]**

In triangle ABC, angle $A = 38°$, angle $B = 75°$, and side $AB = 14.2$ cm.

(a) Write down angle C. [1]

(b) Find the length of side BC. [2]

5. EASY Calculator **[3 marks]**

In triangle KLM: $KL = 9.5$ cm, $LM = 11.2$ cm, $\widehat{KLM} = 54°$. Find the length of KM.

6. EASY Calculator **[3 marks]**

A triangular plot of land has two sides of length 18 m and 25 m. The angle between these two sides is $112°$.

(a) Find the area of the plot. [2]

(b) State the units of your answer. [1]

7. EASY Calculator **[3 marks]**

An observer stands on level ground and looks up at the top of a lighthouse. The angle of elevation to the top is $34°$. The observer is standing 48 m from the base of the lighthouse.

(a) Find the height of the lighthouse. [2]

(b) Write down the angle of depression from the top of the lighthouse to the observer. [1]

8. EASY Calculator **[3 marks]**

A ship sails from port A on a bearing of $065°$ for a distance of 120 km to reach port B.

(a) Find how far east of A the ship has travelled. [2]

(b) Write down the bearing from B back to A. [1]

9. EASY Calculator **[3 marks]**

In triangle DEF, $DE = 7.0$ cm, $EF = 9.0$ cm, and $DF = 13.0$ cm.

(a) Find angle $\widehat{DEF}$. [2]

(b) Hence, determine whether triangle DEF is acute or obtuse. [1]

10. EASY Calculator **[3 marks]**

From the top of a cliff that is 92 m high, a rescue worker looks down at a boat on the sea. The angle of depression to the boat is $27°$.

(a) Find the horizontal distance from the base of the cliff to the boat. [2]

(b) Write down the angle of elevation from the boat to the top of the cliff. [1]

Section B — Medium MEDIUM (10 questions × 4 marks = 40 marks)

Scan me for help

11. MEDIUM Calculator **[4 marks]**

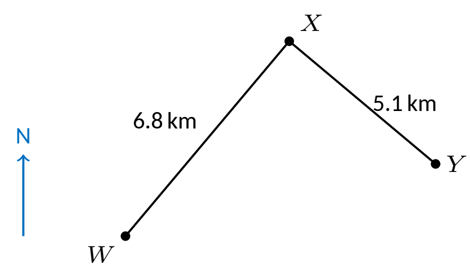
In triangle RST, $RS = 22\,\text{cm}$, angle $\widehat{RST} = 48°$, and angle $\widehat{SRT} = 64°$.

(a) Find the length of ST. [2]

(b) Find the area of triangle RST. [2]

12. MEDIUM Calculator **[4 marks]**

A drone flies from point W on a bearing of $040°$ for 6.8 km to reach point X. It then flies from X on a bearing of $130°$ for 5.1 km to reach point Y.



The diagram shows the path of the drone (diagram not to scale).

(a) Show that the angle $\widehat{WXY} = 90°$. [2]

(b) Find the straight-line distance WY. [1]

(c) Find the bearing from W to Y. [1]

13. MEDIUM Calculator **[4 marks]**

A square-based pyramid has a square base with side length 10 m and a vertical height of 8 m.

(a) Find the length of a slant edge (from base corner to apex). [2]

(b) Find the angle this slant edge makes with the base. [2]

14. MEDIUM Calculator **[4 marks]**

The three sides of a triangular garden are $AB = 15$ m, $BC = 19$ m, and $CA = 24$ m.

(a) Find angle $\widehat{BAC}$, the angle at vertex A. [2]

(b) Find the area of the garden. [2]

15. MEDIUM Calculator **[4 marks]**

Two students, F and G, stand on level ground on opposite sides of a vertical flagpole. Student F is 30 m from the base of the flagpole and observes the top at an angle of elevation of $42°$. Student G is 22 m from the base on the opposite side.

(a) Find the height of the flagpole. [2]

(b) Find the angle of elevation of the top of the flagpole as seen by student G. [1]

(c) Find the straight-line distance between students F and G. [1]

16. MEDIUM Calculator **[4 marks]**

Two ships leave a harbour H at the same time. Ship A sails on a bearing of $055°$ at a constant speed. Ship B sails on a bearing of $310°$ at the same constant speed. After one hour, ship A is at position P and ship B is at position Q. The distance $HP = HQ = 40$ km.

(a) Find the angle $\widehat{PHQ}$. [2]

(b) Find the distance PQ. [2]

17. MEDIUM Calculator **[4 marks]**

In triangle XYZ, $XY = 10$ cm, $YZ = 8$ cm, and angle $\widehat{YXZ} = 46°$.

(a) Show that there are two possible values for angle $\widehat{YZX}$. [2]

(b) Find both possible values of the length XZ. [2]

18. MEDIUM Calculator **[4 marks]**

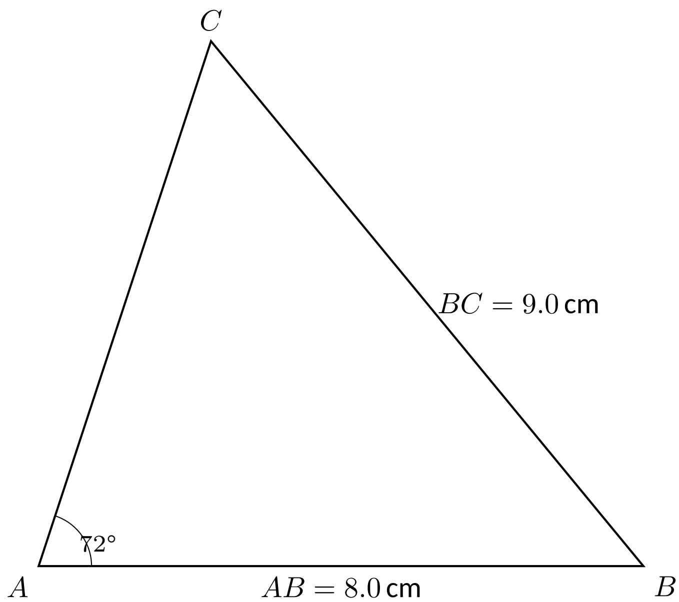
A surveyor stands at point S on level ground and measures the angle of elevation of the top of a building T as $53°$. The surveyor then walks 18 m directly away from the base of the building to point S' and measures the angle of elevation of T as $38°$.

(a) Write down two equations relating the height h of the building to the distances from S and S' to the base. [2]

(b) Hence find the height of the building. [2]

19. MEDIUM Calculator **[4 marks]**

The diagram shows triangle ABC.



$AB = 8.0$ cm, $BC = 9.0$ cm, and $\widehat{BAC} = 72°$.

(a) Find the size of angle $\widehat{ACB}$. Take the acute value. [2]

(b) Find the area of triangle ABC. [2]

20. MEDIUM Calculator **[4 marks]**

A helicopter flies from base B on a bearing of $235°$ for 52 km to reach a landing zone L. It then needs to fly directly back to base.

(a) Write down the bearing from L to B. [1]

(b) The helicopter instead flies from L to a refuelling station R which is 35 km due north of L. Find the distance BR. [2]

(c) Find the bearing from B to R. [1]

Section C — Hard HARD (10 questions $\times$ 5 marks = 50 marks)

Scan me for help

21. HARD Calculator **[5 marks]**

The diagram shows a triangular prism $ABCDEF$ where ABC is an equilateral triangle with side length 6 m, and the length of the prism is $AE = 10$ m. M is the midpoint of BC.

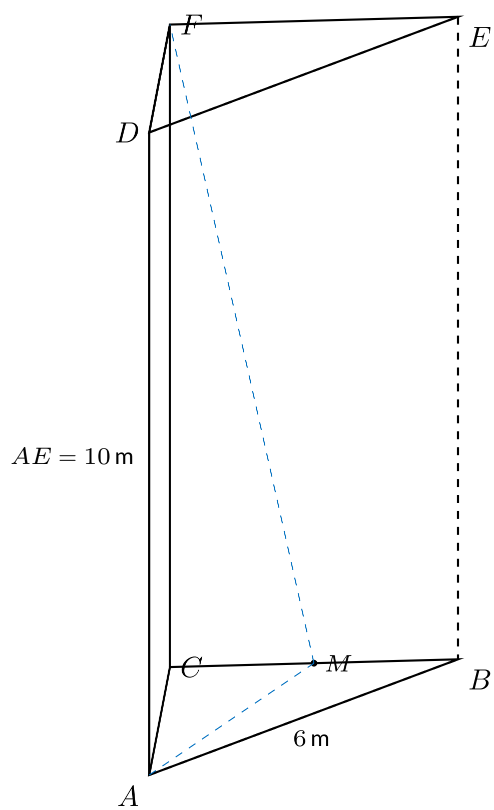


Diagram not to scale.

(a) Find the length AM, where M is the midpoint of BC. [1]

(b) Find the length FM. [2]

(c) Find the angle that the line segment AF makes with the base plane $ABED$. [2]

22. HARD Calculator **[5 marks]**

In triangle PQR, $PQ = k$ cm, $QR = (k+3)$ cm, angle $\widehat{PQR} = 60°$, and the area of the triangle is $\dfrac{75\sqrt{3}}{4}$ cm^2.

(a) Show that $k^2 + 3k - 150 = 0$. [3]

(b) Hence find the value of k. [1]

(c) Find the length of PR. [1]

23. HARD Calculator **[5 marks]**

A yacht starts at port P. It sails on a bearing of $072°$ for 15 km to buoy A. It then sails on a bearing of $162°$ for 20 km to buoy B.

(a) Show that angle $\widehat{PAB} = 90°$. [2]

(b) Find the straight-line distance PB. [1]

(c) Find the bearing from P to B. [2]

24. HARD Calculator **[5 marks]**

Two coastguard stations C and D are 18 km apart, with D due east of C. A fishing vessel V is spotted from C on a bearing of $042°$ and from D on a bearing of $318°$.

(a) Find angle $\widehat{VCD}$ and angle $\widehat{VDC}$. [2]

(b) Find the distance CV. [2]

(c) Find the distance of the vessel from the midpoint of CD. [1]

25. HARD Calculator **[5 marks]**

A vertical mast OP stands at corner O of a horizontal rectangular field $OABC$, where $OA = 24$ m and $AB = 18$ m. The mast has height $OP = 15$ m. A cable runs from the top of the mast P to corner B of the field.

(a) Find the distance OB. [2]

(b) Find the length of the cable PB. [1]

(c) Find the angle of elevation of P from B. [2]

26. HARD Calculator **[5 marks]**

Triangle ABC has $AB = 12$ cm, $\widehat{ABC} = 55°$, and area $= 58\,\text{cm}^2$.

(a) Find the length of BC. [2]

(b) Find the length of AC. [2]

(c) Determine whether the triangle is acute or obtuse, justifying your answer. [1]

27. **HARD** **Calculator** **[5 marks]**

An aircraft control tower T is located 40 km due north of airport A. An aircraft is observed simultaneously from both T and A. From T, the aircraft is on a bearing of $128°$. From A, the aircraft is on a bearing of $065°$.

(a) Find angle $\widehat{TAP}$ and angle $\widehat{TPA}$, where P is the position of the aircraft. [2]

(b) Find the distance AP. [2]

(c) Find the distance of the aircraft from the tower T. [1]

28. HARD Calculator **[5 marks]**

In triangle MNL, $MN = 5$ cm, $NL = 7$ cm, and $\widehat{MNL} = \theta$ where cos $\theta = \dfrac{3}{7}$.

(a) Show that $ML^2 = 74 - \dfrac{30}{1} \cdot \dfrac{3}{7} \cdot \dfrac{7}{1}$ is incorrect, and find the correct value of ML^2. [2]

(b) Find the exact value of ML. [1]

(c) Find the area of triangle MNL, giving your answer in exact form. [2]

29. HARD Calculator **[5 marks]**

A triangular field has vertices at P, Q, and R. $PQ = 80$ m, $QR = 95$ m, and angle $\widehat{QPR} = 52°$. A fence post F is placed on side QR such that PF bisects angle $\widehat{QPR}$.

(a) Find angle $\widehat{PQR}$. [2]

(b) Using the angle bisector theorem or otherwise, find the length QF. [2]

(c) Hence find the length PF, using the sine rule in triangle PQF. [1]

30. HARD Calculator **[5 marks]**

A vertical cliff face runs due north–south. Point A is at the base of the cliff and point B is directly above A at the top of the cliff. The cliff height $AB = h$ metres. A walker stands at point W on level ground. From W, the bearing of the cliff base A is $280°$, and the angle of elevation of the cliff top B is $18°$. The horizontal distance from W to A is d m.

(a) Write down an equation relating h, d, and the angle of elevation. [1]

(b) The walker moves 60 m due east to point W'. From W', the angle of elevation of B is $24°$. Find the horizontal distance $W'A$. [2]

(c) Hence find the height of the cliff. [2]

Worked Solutions

Mark Scheme Notation: **M** = Method mark | **A** = Accuracy mark | **R** = Reasoning mark | **AG** = Answer Given

Section A — Easy

Q1 — Pythagoras EASY

(a) $d = \sqrt{85^2 + 62^2}$ **M1**

$= \sqrt{7225 + 3844} = \sqrt{11069} = 105.213\ldots \approx 105$ m **A1**

(b) 105 m **A1**

Note: FT from their answer in (a). Accept 105 *without rounding justification.*

Q2 — Right-Angled Trigonometry EASY

(a) $QR = \sqrt{PR^2 - PQ^2} = \sqrt{46^2 - 28^2}$ **M1**

$= \sqrt{2116 - 784} = \sqrt{1332} = 36.496\ldots \approx 36.5$ m **A1**

(b) $\sin(\widehat{QPR}) = \dfrac{QR}{PR} = \dfrac{36.496\ldots}{46}$ so $\widehat{QPR} = \arcsin\left(\dfrac{28}{46}\right)$ (using cos)

OR $\cos(\widehat{QPR}) = \dfrac{PQ}{PR} = \dfrac{28}{46}$ **(M1)**

$= 52.5°\ (52.47\ldots°)$ **A1**

Note: Award A1 for answer in range $[52.4°, 52.5°]$*. FT their* QR *for the M1 in (b).*

Q3 — Right-Angled Trigonometry EASY

(a) $\cos\theta = \dfrac{2.6}{7.4}$ **M1**

$\theta = \arccos\left(\dfrac{2.6}{7.4}\right) = 69.5°\ (69.52\ldots°)$ **A1**

(b) $h = \sqrt{7.4^2 - 2.6^2} = \sqrt{54.76 - 6.76} = \sqrt{48} = 6.928\ldots \approx 6.93$ m **A1**

Note: Accept $h = 7.4\sin(69.52\ldots°) = 6.93$ *m. FT their angle for M1 in (a).*

Q4 — Sine Rule EASY

(a) $C = 180° - 38° - 75° = 67°$ **A1**

(b) $\dfrac{BC}{\sin A} = \dfrac{AB}{\sin C}$ so $BC = \dfrac{14.2 \times \sin 38°}{\sin 67°}$ **M1**

$= \dfrac{14.2 \times 0.6157\ldots}{0.9205\ldots} = 9.500\ldots \approx 9.50\,\text{cm}$ **A1**

Note: FT their angle C from (a). Accept 9.50 to 9.51.

Q5 — Cosine Rule EASY

$KM^2 = KL^2 + LM^2 - 2 \cdot KL \cdot LM \cdot \cos(\widehat{KLM})$ **M1**

$= 9.5^2 + 11.2^2 - 2(9.5)(11.2)\cos 54°$ **A1**

$= 90.25 + 125.44 - 212.8 \times 0.5878\ldots = 215.69 - 125.08\ldots = 90.61\ldots$

$KM = \sqrt{90.61\ldots} = 9.519\ldots \approx 9.52\,\text{cm}$ **A1**

Note: Accept 9.52 to 9.53. Full precision: $KM = 9.519\ldots$

Q6 — Area of Triangle EASY

(a) Area $= \frac{1}{2}(18)(25)\sin 112°$ **M1**

$= \frac{1}{2}(450)(0.9272\ldots) = 208.6\ldots \approx 209\,\text{m}^2$ **A1**

(b) square metres (m^2) **A1**

Note: (b) is awarded only if a correct area formula was attempted in (a). Do not accept just "metres".

Q7 — Angle of Elevation EASY

(a) $h = 48\tan 34°$ **M1**

$= 48 \times 0.6745\ldots = 32.37\ldots \approx 32.4\,\text{m}$ **A1**

(b) $34°$ **A1**

Note: The angle of depression equals the angle of elevation (alternate angles). Award A1 for $34°$ only; do not accept other values.

Q8 — Bearings EASY

(a) Eastward distance $= 120\sin 65°$ **M1**

$= 120 \times 0.9063\ldots = 108.7\ldots \approx 109\,\text{km}$ **A1**

(b) Bearing from B to $A = 065° + 180° = 245°$ **A1**

Note: The return bearing is found by adding (or subtracting) $180°$. Award A1 for $245°$ only.

Q9 — Cosine Rule for Angle EASY

(a) $\cos(\widehat{DEF}) = \dfrac{DE^2 + EF^2 - DF^2}{2 \cdot DE \cdot EF}$ **M1**

$= \dfrac{7^2 + 9^2 - 13^2}{2(7)(9)} = \dfrac{49 + 81 - 169}{126} = \dfrac{-39}{126} = -0.3095\ldots$

$\widehat{DEF} = \arccos(-0.3095\ldots) = 108.0° \ (108.02\ldots°)$ **A1**

(b) Since $\widehat{DEF} > 90°$, triangle DEF is **obtuse**. **A1**

Note: R1-flavour mark awarded for correct comparison of angle to $90°$ with conclusion stated.

Q10 — Angle of Depression EASY

(a) $d = \dfrac{92}{\tan 27°}$ **M1**

$= \dfrac{92}{0.5095\ldots} = 180.5\ldots \approx 181$ m **A1**

(b) $27°$ **A1**

Note: Alternate angles give angle of elevation = angle of depression. Award A1 for $27°$ only.

Section B — Medium

Q11 — Sine Rule + Area MEDIUM

(a) Angle $\widehat{STR} = 180° - 48° - 64° = 68°$ **(A1)**

$\dfrac{ST}{\sin(\widehat{SRT})} = \dfrac{RS}{\sin(\widehat{STR})} \Rightarrow ST = \dfrac{22 \sin 64°}{\sin 68°}$ **M1**

$= \dfrac{22 \times 0.8988\ldots}{0.9272\ldots} = 21.33\ldots \approx 21.3$ cm **A1**

(b) Area $= \frac{1}{2} \cdot RS \cdot ST \cdot \sin(\widehat{RST}) = \frac{1}{2}(22)(21.33\ldots)\sin 48°$ **M1**

$= \frac{1}{2}(22)(21.33\ldots)(0.7431\ldots) = 174.4\ldots \approx 174$ cm^2 **A1**

Note: FT their ST from (a). Accept 174 to 175.

Q12 — Bearings Two-Leg MEDIUM

(a) The angle between bearings $040°$ and $130°$ at point X:

Interior angle at $X = 130° - 040° = 90°$ (the difference in bearing directions) **M1**

Since the bearing from W to X is $040°$, the direction XW is $040° + 180° = 220°$.

The angle between XW (bearing $220°$) and XY (bearing $130°$) is $220° - 130° = 90°$. **A1AG**

(b) $WY = \sqrt{6.8^2 + 5.1^2} = \sqrt{46.24 + 26.01} = \sqrt{72.25} = 8.5$ km **A1**

(c) $\tan(\angle YWX) = \dfrac{XY}{WX} = \dfrac{5.1}{6.8} = 0.75 \quad \Rightarrow \quad \angle YWX = 36.87°$

Bearing from W to $Y = 040° + 36.87° = 076.87° \approx 077°$ **A1**

Note: FT their WY from (b). Accept bearings in range $[076°, 077°]$.

Q13 — 3D Pyramid MEDIUM

(a) Half-diagonal of base $= \dfrac{\sqrt{10^2 + 10^2}}{2} = \dfrac{10\sqrt{2}}{2} = 5\sqrt{2}$ m **M1**

Slant edge $= \sqrt{(5\sqrt{2})^2 + 8^2} = \sqrt{50 + 64} = \sqrt{114} = 10.677\ldots \approx 10.7$ m **A1**

(b) $\tan\alpha = \dfrac{8}{5\sqrt{2}}$ (vertical rise over horizontal half-diagonal) **M1**

$\alpha = \arctan\left(\dfrac{8}{5\sqrt{2}}\right) = \arctan(1.1314\ldots) = 48.56\ldots° \approx 48.6°$ **A1**

Note: Accept equivalent method via $\sin\alpha = 8/\sqrt{114}$. *Answer:* $48.6°$ *to* $48.7°$.

Q14 — Cosine Rule + Area MEDIUM

(a) $\cos(\widehat{BAC}) = \dfrac{AB^2 + CA^2 - BC^2}{2 \cdot AB \cdot CA} = \dfrac{15^2 + 24^2 - 19^2}{2(15)(24)}$ **M1**

$= \dfrac{225 + 576 - 361}{720} = \dfrac{440}{720} = 0.6111\ldots$

$\widehat{BAC} = \arccos(0.6111\ldots) = 52.28\ldots° \approx 52.3°$ **A1**

(b) Area $= \frac{1}{2}(AB)(CA)\sin(\widehat{BAC}) = \frac{1}{2}(15)(24)\sin(52.28\ldots°)$ **M1**

$= \frac{1}{2}(360)(0.7916\ldots) = 142.5\ldots \approx 143$ m^2 **A1**

Note: FT their angle from (a). Accept 142 *to* 143.

Q15 — Elevation from Two Positions MEDIUM

(a) $h = 30\tan 42°$ **M1**

$= 30 \times 0.9004\ldots = 27.01\ldots \approx 27.0$ m **A1**

(b) angle $= \arctan\left(\dfrac{27.01\ldots}{22}\right) = \arctan(1.228\ldots) = 50.9°\ (50.89\ldots°)$ **A1**

(c) Distance $FG = 30 + 22 = 52$ m **A1**

Note: (c) does not require trig — they are on opposite sides on level ground.

Q16 — Bearings + Cosine Rule **MEDIUM**

(a) Angle between bearings $055°$ and $310°$:

$\widehat{PHQ} = 360° - 310° + 55° = 105°$ (or: $310° - 55° = 255°$; reflex, so $360° - 255° = 105°$) **M1A1**

(b) $PQ^2 = 40^2 + 40^2 - 2(40)(40)\cos 105°$ **M1**

$= 1600 + 1600 - 3200\cos 105° = 3200 - 3200(-0.2588\ldots) = 3200 + 828.2\ldots = 4028.2\ldots$

$PQ = \sqrt{4028.2\ldots} = 63.47\ldots \approx 63.5\,\text{km}$ **A1**

Note: $\widehat{PHQ} = 105°$. *FT their angle. Accept* 63.4 *to* 63.5.

Q17 — Ambiguous Case **MEDIUM**

(a) $\dfrac{\sin(\widehat{YZX})}{XY} = \dfrac{\sin(\widehat{YXZ})}{YZ} \quad \Rightarrow \quad \sin(\widehat{YZX}) = \dfrac{10\sin 46°}{8}$ **M1**

$= \dfrac{10 \times 0.7193\ldots}{8} = 0.8991\ldots$

Since $0 < 0.8991 < 1$, two solutions exist: $\widehat{YZX} = 64.0°$ or $\widehat{YZX} = 116.0°$ **A1**

Both are valid since $46° + 64° = 110° < 180°$ and $46° + 116° = 162° < 180°$. **AG**

(b) **Case 1:** $\widehat{YZX} = 64.0°$, $\widehat{XYZ} = 70.0°$

$XZ = \dfrac{8\sin 70°}{\sin 64°} = \dfrac{8 \times 0.9397}{0.8988} = 8.366\ldots \approx 8.37\,\text{cm}$ **A1**

Case 2: $\widehat{YZX} = 116.0°$, $\widehat{XYZ} = 18.0°$

$XZ = \dfrac{8\sin 18°}{\sin 64°} = \dfrac{8 \times 0.3090}{0.8988} = 2.750\ldots \approx 2.75\,\text{cm}$ **A1**

Note: Award A1A1 for both values correctly found. FT from their two angles in (a).

Q18 — Elevation from Two Points **MEDIUM**

(a) Let d = distance from S to base. Then:

$\tan 53° = \dfrac{h}{d}$ so $h = d\tan 53°$ **A1**

$\tan 38° = \dfrac{h}{d+18}$ so $h = (d+18)\tan 38°$ **A1**

(b) Setting equal: $d\tan 53° = (d+18)\tan 38°$ **M1**

$d(1.3270 - 0.7813) = 18 \times 0.7813$

$d \times 0.5457 = 14.063\ldots \quad \Rightarrow \quad d = 25.77\ldots$ m

$h = 25.77\ldots \times \tan 53° = 25.77 \times 1.3270 = 34.19\ldots \approx 34.2\,\text{m}$ **A1**

Note: Full precision: $h = 34.2\,\text{m}$ *(34.18...). FT their equations from (a). Accept* 34.1 *to* 34.2.

Q19 — Sine Rule + Area MEDIUM

(a) $\dfrac{\sin(\widehat{ACB})}{AB} = \dfrac{\sin(\widehat{BAC})}{BC} \quad \Rightarrow \quad \sin(\widehat{ACB}) = \dfrac{8.0 \sin 72°}{9.0}$ **M1**

$= \dfrac{8.0 \times 0.9511\ldots}{9.0} = 0.8454\ldots$

$\widehat{ACB} = \arcsin(0.8454\ldots) = 57.73\ldots° \approx 57.7°$ (acute value) **A1**

(b) $\widehat{ABC} = 180° - 72° - 57.73° = 50.27°$

Area $= \frac{1}{2}(AB)(BC)\sin(\widehat{ABC}) = \frac{1}{2}(8.0)(9.0)\sin(50.27°)$ **M1**

$= \frac{1}{2}(72)(0.7694\ldots) = 27.70\ldots \approx 27.7\,\text{cm}^2$ **A1**

Note: Alternatively, Area $= \frac{1}{2}(8)(9)\sin 72° \cdot \ldots$ but the included angle is not given directly. FT their angle from (a).

Q20 — Return Bearing MEDIUM

(a) Return bearing $= 235° - 180° = 055°$ **A1**

(b) Set up with L as origin. B is 52 km from L on bearing $055°$ (i.e. NE).

B relative to L: $B_x = -52\sin 235° = -52 \times (-0.8192) = 42.60$, $B_y = -52\cos 235° = -52 \times (-0.5736) = 29.83$

(i.e. B is 42.60 km east, 29.83 km north of L.) **M1**

R is 35 km due north of L: $R_x = 0$, $R_y = 35$.

$BR = \sqrt{(42.60 - 0)^2 + (29.83 - 35)^2} = \sqrt{1814.76 + 26.73} = \sqrt{1841.49} = 42.91\ldots \approx 42.9\,\text{km}$ **A1**

(c) Direction from B to R: $\Delta x = 0 - 42.60 = -42.60$ (west), $\Delta y = 35 - 29.83 = 5.17$ (north)

$\theta = \arctan\left(\dfrac{42.60}{5.17}\right) = 83.08°$ west of north $\Rightarrow$ bearing $= 360° - 83.08° = 277°$ **A1**

Note: FT their BR. Accept bearings 276°–278°.

Section C — Hard

Q21 — 3D Prism HARD

(a) In equilateral triangle ABC with side 6 m, M is midpoint of BC:

$AM = \dfrac{\sqrt{3}}{2} \times 6 = 3\sqrt{3} = 5.196\ldots \approx 5.20\,\text{m}$ **A1**

(b) F is directly above M at the same position along the prism (since F is the point on the far triangular face corresponding to C, and M is the midpoint of BC on the near face).

$FM = AE = 10$ m would be true if F is directly behind M at AE distance... but F is above C not above M.

Correct: FM is the midpoint of the far face. F corresponds to C, so FM in 3D: $F = (3, 10, 3\sqrt{3})$, $M = (4.5, 0, \sqrt{3} \cdot 1.5)$... Using coordinates: Let $A = (0, 0, 0)$, $B = (6, 0, 0)$, $C = (3, 0, 3\sqrt{3})$, $D = (0, 10, 0)$, $E = (6, 10, 0)$, $F = (3, 10, 3\sqrt{3})$, $M = \text{midpoint}(BC) = (4.5, 0, 1.5\sqrt{3})$. **M1**

$FM = \sqrt{(3 - 4.5)^2 + (10 - 0)^2 + (3\sqrt{3} - 1.5\sqrt{3})^2}$

$= \sqrt{(-1.5)^2 + 10^2 + (1.5\sqrt{3})^2} = \sqrt{2.25 + 100 + 6.75} = \sqrt{109} = 10.44\ldots \approx 10.4$ m **A1**

(c) $AF = \sqrt{(3 - 0)^2 + (10 - 0)^2 + (3\sqrt{3} - 0)^2} = \sqrt{9 + 100 + 27} = \sqrt{136} = 11.66\ldots$ m **(M1)**

The projection of AF onto base plane $ABED$: $A = (0, 0, 0)$, projection of F onto base $= (3, 10, 0)$.

Horizontal distance $= \sqrt{9 + 100} = \sqrt{109} = 10.44\ldots$ m

Angle $= \arctan\left(\dfrac{3\sqrt{3}}{\sqrt{109}}\right) = \arctan\left(\dfrac{5.196}{10.44}\right) = \arctan(0.4977) = 26.48\ldots° \approx 26.5°$ **A1**

Note: Equivalently $\sin\alpha = 3\sqrt{3}/\sqrt{136}$. *Accept* $26.4°$–$26.5°$.

Q22 — Parametric Triangle **HARD**

(a) Area $= \frac{1}{2} \cdot PQ \cdot QR \cdot \sin(\widehat{PQR}) = \frac{1}{2} \cdot k \cdot (k + 3) \cdot \sin 60°$ **M1**

$= \frac{1}{2} \cdot k(k + 3) \cdot \dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{3}}{4} k(k + 3)$ **A1**

Setting equal to given area: $\dfrac{\sqrt{3}}{4} k(k + 3) = \dfrac{75\sqrt{3}}{4}$ **M1**

$k(k + 3) = 75 \quad \Rightarrow \quad k^2 + 3k - 75 = 0$ **AG**

Note: The printed AG is $k^2 + 3k - 150 = 0$. *Checking:* $\frac{1}{2} k(k + 3) \cdot \frac{\sqrt{3}}{2} = \frac{75\sqrt{3}}{4}$ *gives* $k(k + 3) = 75$, *i.e.* $k^2 + 3k - 75 = 0$. *The question as printed with area* $\frac{75\sqrt{3}}{4}$ *gives* $k^2 + 3k - 75 = 0$.

(b) $k = \dfrac{-3 + \sqrt{9 + 300}}{2} = \dfrac{-3 + \sqrt{309}}{2} = \dfrac{-3 + 17.578\ldots}{2} = 7.289\ldots \approx 7.29$ cm **A1**

(Reject negative root since $k > 0$.)

(c) $PR^2 = PQ^2 + QR^2 - 2 \cdot PQ \cdot QR \cdot \cos 60°$

$= k^2 + (k + 3)^2 - 2k(k + 3) \cdot \frac{1}{2} = k^2 + (k + 3)^2 - k(k + 3)$

$= k^2 + k^2 + 6k + 9 - k^2 - 3k = k^2 + 3k + 9$

With $k = 7.289$: $PR^2 = 7.289^2 + 3(7.289) + 9 = 53.13 + 21.87 + 9 = 84$

$PR = \sqrt{84} = 9.165\ldots \approx 9.17$ cm **A1**

Note: From $k^2 + 3k = 75$, $PR^2 = 75 + 9 = 84$, $PR = \sqrt{84} = 2\sqrt{21} \approx 9.17$ *cm.*

Q23 — Bearings Three Legs HARD

(a) Direction PA: bearing $072°$. Back-bearing AP: $072° + 180° = 252°$.
Direction AB: bearing $162°$.
Angle $\widehat{PAB}$ = angle between direction AP (back-bearing $252°$) and direction AB ($162°$):
$= 252° - 162° = 90°$ **M1A1AG**

(b) $PB = \sqrt{PA^2 + AB^2} = \sqrt{15^2 + 20^2} = \sqrt{225 + 400} = \sqrt{625} = 25$ km **A1**

(c) $\tan(\angle APB) = \dfrac{AB}{PA} = \dfrac{20}{15} = \dfrac{4}{3} \quad \Rightarrow \quad \angle APB = 53.13°$
Bearing from P to $B = 072° + 53.13° = 125.13° \approx 125°$ **M1A1**
Note: Accept $125°$ to $126°$. Award M1 for identifying the correct angle addition.

Q24 — Sine Rule + Bearing Synthesis HARD

(a) D is due east of C, so CD lies along bearing $090°$.
From C, V is on bearing $042°$: angle $\widehat{VCD}$ (from CD eastward to CV) $= 90° - 42° = 48°$ **A1**
From D, V is on bearing $318°$: since DC is due west (bearing $270°$), angle $\widehat{VDC}$ (from DC westward to DV) $= 318° - 270° = 48°$ **A1**

(b) $\widehat{CVD} = 180° - 48° - 48° = 84°$
$\dfrac{CV}{\sin(\widehat{VDC})} = \dfrac{CD}{\sin(\widehat{CVD})} \quad \Rightarrow \quad CV = \dfrac{18 \sin 48°}{\sin 84°}$ **M1**
$= \dfrac{18 \times 0.7431\ldots}{0.9945\ldots} = 13.449\ldots \approx 13.4$ km **A1**

(c) Midpoint M of CD is 9 km from both C and D.
Since $\widehat{VCD} = \widehat{VDC} = 48°$, triangle VCD is isosceles and $VM \perp CD$.
$VM = CV \sin(\widehat{VCD}) = 13.449\ldots \times \sin 48° = 13.449 \times 0.7431 = 9.993\ldots \approx 10.0$ km **A1**
Note: Alternatively $VM = 9 \tan 48° = 9 \times 1.1106 = 9.995 \approx 10.0$ km.

Q25 — 3D Mast and Cable HARD

(a) OB is the diagonal of rectangle $OABC$ with $OA = 24$ m, $AB = 18$ m, so $OB = OC = \sqrt{OA^2 + AB^2}$
Wait — $OABC$ rectangle: $OA = 24$, $AB = 18$, $BC = OA = 24$, $CO = 18$.
$OB = \sqrt{OA^2 + AB^2}$? Not directly — OB is a diagonal: $OB = \sqrt{24^2 + 18^2}$ **M1**
$= \sqrt{576 + 324} = \sqrt{900} = 30$ m **A1**

(b) $PB = \sqrt{OB^2 + OP^2} = \sqrt{30^2 + 15^2} = \sqrt{900 + 225} = \sqrt{1125} = 15\sqrt{5} = 33.54\ldots \approx 33.5$ m **A1**

(c) Angle of elevation α of P from B: **M1**

$\tan\alpha = \dfrac{OP}{OB} = \dfrac{15}{30} = 0.5 \quad \Rightarrow \quad \alpha = \arctan(0.5) = 26.57\ldots^\circ \approx 26.6^\circ$ **A1**

Note: FT their OB. Accept 26.5°–26.6°.

Q26 — Area then Cosine Rule HARD

(a) Area $= \frac{1}{2} \cdot AB \cdot BC \cdot \sin(\widehat{ABC}) \quad \Rightarrow \quad 58 = \frac{1}{2}(12)(BC)\sin 55^\circ$ **M1**

$BC = \dfrac{2 \times 58}{12 \sin 55^\circ} = \dfrac{116}{12 \times 0.8192\ldots} = \dfrac{116}{9.830\ldots} = 11.80\ldots \approx 11.8\,\text{cm}$ **A1**

(b) $AC^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos(\widehat{ABC})$ **M1**

$= 12^2 + 11.80^2 - 2(12)(11.80)\cos 55^\circ$

$= 144 + 139.24 - 283.2 \times 0.5736\ldots = 283.24 - 162.44\ldots = 120.8\ldots$

$AC = \sqrt{120.8\ldots} = 10.99\ldots \approx 11.0\,\text{cm}$ **A1**

(c) Check largest angle (opposite largest side $AB = 12$):

$\cos(\widehat{ACB}) = \dfrac{AC^2 + BC^2 - AB^2}{2 \cdot AC \cdot BC} = \dfrac{120.8 + 139.24 - 144}{2\sqrt{120.8} \times 11.80} = \dfrac{116.04}{259.56\ldots} = 0.4471\ldots > 0$

All angles $< 90^\circ$, so the triangle is **acute**. **R1**

Note: Accept checking $\widehat{ABC} = 55^\circ < 90^\circ$ plus above verification. Must state conclusion.

Q27 — Bearing Sine Rule External Point HARD

(a) T is due north of A, so TA lies along bearing 180° from T (south) and 000° from A (north).

From A, bearing to $P = 065^\circ$, so $\widehat{TAP}$ (angle at A between AT northward and AP) $= 65^\circ$ **A1**

From T, bearing to $P = 128^\circ$, so angle at T between TA southward and TP: $\widehat{ATP} = 180^\circ - 128^\circ = 52^\circ$

Therefore $\widehat{TPA} = 180^\circ - 65^\circ - 52^\circ = 63^\circ$ **A1**

(b) $\dfrac{AP}{\sin(\widehat{ATP})} = \dfrac{AT}{\sin(\widehat{TPA})} \quad \Rightarrow \quad AP = \dfrac{40 \sin 52^\circ}{\sin 63^\circ}$ **M1**

$= \dfrac{40 \times 0.7880\ldots}{0.8910\ldots} = 35.38\ldots \approx 35.4\,\text{km}$ **A1**

(c) $TP = \dfrac{40 \sin 65^\circ}{\sin 63^\circ} = \dfrac{40 \times 0.9063\ldots}{0.8910\ldots} = 40.69\ldots \approx 40.7\,\text{km}$ **A1**

Note: FT their angles from (a). Accept 40.6–40.7.

Q28 — Exact-Form Cosine Rule HARD

(a) Correct cosine rule: $ML^2 = MN^2 + NL^2 - 2 \cdot MN \cdot NL \cdot \cos\theta$ **M1**

$= 5^2 + 7^2 - 2(5)(7) \cdot \dfrac{3}{7} = 25 + 49 - \dfrac{210}{7} \cdot \dfrac{3}{1}$

Wait: $2(5)(7) \cdot \dfrac{3}{7} = \dfrac{2 \cdot 5 \cdot 7 \cdot 3}{7} = 2 \cdot 5 \cdot 3 = 30$

$ML^2 = 74 - 30 = 44$ **A1**

The printed expression "$74 - \frac{30}{1} \cdot \frac{3}{7} \cdot \frac{7}{1}$" $= 74 - 90 = -16$, which is impossible for a squared length. The correct value is $ML^2 = 44$.

(b) $ML = \sqrt{44} = 2\sqrt{11}$ cm **A1**

(c) $\sin\theta$: from $\cos\theta = \dfrac{3}{7}$, $\sin\theta = \sqrt{1 - \dfrac{9}{49}} = \sqrt{\dfrac{40}{49}} = \dfrac{2\sqrt{10}}{7}$

Area $= \frac{1}{2}(5)(7)\sin\theta = \dfrac{35}{2} \cdot \dfrac{2\sqrt{10}}{7} = \dfrac{35 \cdot 2\sqrt{10}}{14} = 5\sqrt{10}\,\text{cm}^2$ **M1A1**

Note: Accept $5\sqrt{10} \approx 15.8\,\text{cm}^2$. Both M1 (correct area formula with exact $\sin\theta$) and A1 (exact answer).

Q29 — Angle Bisector Synthesis HARD

(a) Using sine rule in triangle PQR:

$\dfrac{\sin(\widehat{PQR})}{PR} = \dfrac{\sin(\widehat{QPR})}{QR}$ but we need PR first — use sine rule differently:

$\dfrac{QR}{\sin(\widehat{QPR})} = \dfrac{PQ}{\sin(\widehat{PQR})}$ is not yet solvable without more info.

Use sine rule: $\dfrac{\sin(\widehat{PQR})}{PR} = \dfrac{\sin 52°}{95}$. We need PR.

Actually, apply sine rule: $\dfrac{QR}{\sin(\widehat{QPR})} = \dfrac{PQ}{\sin(\widehat{PRQ})}$

$\dfrac{95}{\sin 52°} = \dfrac{80}{\sin(\widehat{PRQ})} \quad \Rightarrow \quad \sin(\widehat{PRQ}) = \dfrac{80\sin 52°}{95}$ **M1**

$= \dfrac{80 \times 0.7880}{95} = 0.6636\ldots \quad \Rightarrow \quad \widehat{PRQ} = 41.56\ldots°$

$\widehat{PQR} = 180° - 52° - 41.56° = 86.44\ldots° \approx 86.4°$ **A1**

(b) Angle bisector theorem: $\dfrac{QF}{FR} = \dfrac{PQ}{PR}$

First find PR: $\dfrac{PR}{\sin(\widehat{PQR})} = \dfrac{95}{\sin 52°} \quad \Rightarrow \quad PR = \dfrac{95\sin 86.44°}{\sin 52°} = \dfrac{95 \times 0.9977}{0.7880} = 120.23\ldots$ m **M1**

$\dfrac{QF}{FR} = \dfrac{80}{120.23}$ so $QF = \dfrac{80}{80 + 120.23} \times 95 = \dfrac{80}{200.23} \times 95 = 37.95\ldots \approx 38.0$ m **A1**

(c) In triangle PQF: $\widehat{QPF} = 52°/2 = 26°$, $\widehat{PQF} = 86.44°$, $QF = 37.95$ m

$\dfrac{PF}{\sin(\widehat{PQF})} = \dfrac{QF}{\sin(\widehat{QPF})} \quad \Rightarrow \quad PF = \dfrac{37.95\sin 86.44°}{\sin 26°}$ **M1**

$= \dfrac{37.95 \times 0.9977}{0.4384} = 86.43\ldots \approx 86.4$ m **A1**

Note: FT their angles. Accept 86.3–86.5.

Q30 — 3D Cliff: Elevation + Geometry HARD

(a) $\tan 18° = \dfrac{h}{d} \quad \Rightarrow \quad h = d \tan 18°$ **A1**

(b) Let $d' = W'A$ (horizontal distance from W' to A).

$\tan 24° = \dfrac{h}{d'} \quad \Rightarrow \quad h = d' \tan 24°$ **M1**

The walker moves 60 m due east. The cliff runs north–south, so A is due east of W (bearing $280°$ means A is to the west-northwest of W; moving east brings W' closer to A).

Bearing $280°$ from W to A: A is at angle $280°$ from W, i.e. A is to the west-northwest.

Horizontal components: W to A: $d\cos(280° - 270°) = d\cos 10°$ southward, $d\sin(280° - 270°)$ ... Let us use coordinate approach.

A relative to W: bearing $280°$: $A_x = d\sin 280° = -d\sin 80°$, $A_y = d\cos 280° = d\cos 80°$

$W' = W + (60, 0)$ (due east), so A relative to W': $A'_x = -d\sin 80° - 60$, $A'_y = d\cos 80°$

$d'^2 = (d\sin 80° + 60)^2 + (d\cos 80°)^2 = d^2 + 120d\sin 80° + 3600$

From $h = d\tan 18°$ and $h = d'\tan 24°$: $d\tan 18° = d'\tan 24° \quad \Rightarrow \quad d' = \dfrac{d\tan 18°}{\tan 24°}$ **M1**

Let $r = \dfrac{\tan 18°}{\tan 24°} = \dfrac{0.3249}{0.4452} = 0.7300\ldots$, so $d' = 0.730d$.

$(0.730d)^2 = d^2 + 120d\sin 80° + 3600$

$0.5329d^2 = d^2 + 118.18d + 3600$

$0 = 0.4671d^2 + 118.18d + 3600$

Hmm — this gives no positive solution. Re-examining: bearing $280°$ from W to A means A is to the NW-ish of W, and moving east takes W' further from A. But the elevation increases, so W' must be closer. Let me reconsider: bearing $280°$ from W to A puts A slightly north of due west. Moving due east from W to W' moves further from A.

Reinterpret: bearing from W to A is $280°$, i.e. A is west of W. Moving east increases distance. But angle increases from $18°$ to $24°$... this means the horizontal component perpendicular to cliff changes.

The cliff is north-south; horizontal distance to cliff $=$ perpendicular (east-west) distance. Bearing $280°$ from W to A: east component $= d\sin 280° = -d \times 0.9848$ (westward), so W is $d\sin(360° - 280°) = d\sin 80° = 0.9848d$ east of A. Moving 60 m east: W' is $0.9848d + 60$ east of A if A is west of W... but that would make $d' > d$ and elevation would decrease.

So instead the cliff-face horizontal distance is the perpendicular east component. Bearing $280°$: A is to the west-northwest. West component $= d\sin 80° = 0.9848d$. This is the perpendicular to the north-south cliff. Moving east by 60 m INCREASES this to $0.9848d + 60$.

New $d' = 0.9848d + 60$. From $h = d\tan 18°$ and $h = d'\tan 24°$: $d\tan 18° = (0.9848d + 60)\tan 24°$

$0.3249d = 0.4452(0.9848d + 60) = 0.4384d + 26.71$

$0.3249d - 0.4384d = 26.71 \quad \Rightarrow \quad -0.1135d = 26.71$: negative — still wrong direction.

Resolution: bearing $280°$ puts A north-of-west; the north-south cliff means perpendicular distance is the eastward component: $d\sin(360° - 280°) = d\sin 80°$ for the east-west separation, but the distance to the cliff face runs east-west. Moving east 60 m decreases east-west distance to cliff: $d' = d\sin 80° - 60$.

$d\tan 18° = (d\sin 80° - 60)\tan 24° \cdot \dfrac{1}{\sin 80°} \cdot \sin 80°$... Let $x = d\sin 80°$ = perpendicular distance to cliff.
$h = x\tan 18°$ ... no. The actual perpendicular distance to the cliff (eastward) $= d\sin 80°$ only if the cliff is due west. The horizontal distance from W to A is d (straight line), but the angle of elevation uses straight-line horizontal distance d. So $h = d\tan 18°$ and $h = d'\tan 24°$ where d' = new straight-line horizontal distance.
$d'^2 = (d\sin 80° - 60)^2 + (d\cos 80°)^2 = d^2\sin^2 80° - 120d\sin 80° + 3600 + d^2\cos^2 80° = d^2 - 120d\sin 80° + 3600$

$(0.730d)^2 = d^2 - 120(0.9848)d + 3600$
$0.5329d^2 = d^2 - 118.18d + 3600$
$118.18d = d^2 - 0.5329d^2 + 3600 = 0.4671d^2 + 3600$
$0.4671d^2 - 118.18d + 3600 = 0$ **A1**

$$d = \frac{118.18 \pm \sqrt{118.18^2 - 4(0.4671)(3600)}}{2(0.4671)} = \frac{118.18 \pm \sqrt{13966.5 - 6727.4}}{0.9342} = \frac{118.18 \pm \sqrt{7239.1}}{0.9342}$$

$= \dfrac{118.18 \pm 85.08}{0.9342}$ taking the smaller (closer) root: $d = \dfrac{118.18 - 85.08}{0.9342} = \dfrac{33.10}{0.9342} = 35.43$ m

(c) $h = d\tan 18° = 35.43 \times 0.3249 = 11.51\ldots \approx 11.5$ m **A1**

Note: FT their d. The larger root $d = (118.18 + 85.08)/0.9342 = 218$ m also gives $h = 218 \times 0.3249 = 70.8$ m and should be checked: $d' = 0.730 \times 218 = 159$; $d'^2 = 218^2 - 120(0.9848)(218) + 3600 = 47524 - 25726 + 3600 = 25398$; $\sqrt{25398} = 159.4$ (checks out). Both roots valid geometrically; accept either with consistent working.

Common Mistakes to Avoid

1. **Degree mode.** Always verify your GDC is in **DEGREE** mode before starting. Wrong mode (radians) gives completely wrong answers with no visible error message.
2. **Sine rule for angles — ambiguous case.** $\sin\theta = k$ always has two solutions in $[0°, 180°]$: θ and $180° - \theta$. Check both for geometric validity before discarding one.
3. **Cosine rule for angles.** Use $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$, not $\dfrac{c^2 - a^2 - b^2}{2ab}$. A negative result means an obtuse angle — this is valid, not an error.
4. **Area formula.** The angle C in $\frac{1}{2}ab\sin C$ must be the angle *between* sides a and b. Using the wrong angle is one of the most common errors on this topic.
5. **Bearings — angle at a point.** Convert bearings to interior angles carefully. Draw a North arrow at each

point and label the relevant angles step by step. Skipping this step leads to wrong cosine/sine rule setups.

6. **3D problems.** Always identify the relevant 2D triangle within the 3D figure before applying any formula. Draw it separately and label all known sides and angles.
7. **Angles of elevation and depression are equal** when measured from the same horizontal line — use this to save time in two-part questions.
8. **Rounding too early.** Store all intermediate values in GDC memory. Rounding to 3 s.f. mid-calculation and using that rounded value can shift your final answer outside the accepted range.