IB Math AI HL — Topic 1 [Set 1]

Financial Applications

Compound Interest · Depreciation · Amortisation · Annuities

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Marks each	3	4	5
Total Marks	30	40	50

Name:_____ Date:_____

Score:_____/120

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Instructions: Answer all questions. Show all working. Give all currency answers correct to 2 decimal places unless otherwise stated. A GDC (TVM Solver / Finance App) is required throughout.

Key Formulae

Compound Interest (Future Value): $FV = PV\left(1 + \frac{r}{100k}\right)^{kn}$ where PV = present value, r = nominal annual rate (%), k = compounding periods per year, n = years.

Depreciation (reducing-balance): $V_n = V_0\left(1 - \frac{r}{100}\right)^n$

Savings Annuity (end-of-period payments, $i = r/(100k)$): $FV = PMT \cdot \frac{(1+i)^N - 1}{i}$

Loan Amortisation: Use GDC Finance App: enter N, $I\%$, PV, PMT, $FV = 0$, P/Y, C/Y.

Effective Annual Rate: $\text{EAR} = \left(1 + \frac{r}{100k}\right)^k - 1$

Real Interest Rate (Fisher): $1 + r_{\text{real}} = \frac{1 + r_{\text{nominal}}}{1 + r_{\text{inflation}}}$

GDC Tips

- **TVM Solver fields:** N = total periods; $I\%$ = nominal annual rate; PV = present value; PMT = payment per period; FV = future value; P/Y = payments per year; C/Y = compoundings per year.
- **Lump-sum investment** (no regular payments): set $PMT = 0$ and solve for FV or $I\%$.
- **Loan** (find monthly payment): enter $PV > 0$, $FV = 0$, solve for PMT (result is negative — report as positive).
- **Savings annuity** (find accumulated amount): $PV = 0$, enter $PMT < 0$, solve for FV.
- **Outstanding balance** after k payments: set $N = k$ and solve for FV.

- **Sign convention:** money leaving your pocket is negative; money received is positive.
- Always set $P/Y = C/Y$ to match the compounding frequency, unless told otherwise.

Common Mistakes to Avoid

1. **Not matching P/Y and C/Y:** Both must equal the number of compounding periods per year (e.g. 12 for monthly).
2. **Sign errors in TVM:** PV and PMT must have *opposite* signs for a loan. A sign error produces a nonsensical answer.
3. **Substituting a decimal for $r\%$:** In $FV = PV(1 + r/100k)^{kn}$, r is the percentage (e.g. 5.4, not 0.054).
4. **Rounding intermediate values:** Carry full GDC precision between parts; only round the *final* answer. Premature rounding loses accuracy marks.
5. **Depreciation sign:** The factor is $(1 - r/100)$, *not* $(1 + r/100)$.
6. **EAR versus nominal rate:** The EAR is always *higher* than the nominal rate when $k > 1$; never report the nominal rate as the EAR.

Section A — Easy EASY (10 questions × 3 marks = 30 marks)

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1. EASY Calculator [3 marks]

Sofia deposits \$5 000 into a savings account that pays a nominal annual interest rate of 4.2%, compounded quarterly.

(a) Find the value of Sofia's account after 3 years. Give your answer correct to 2 decimal places. [3]

2. EASY Calculator [3 marks]

A car is purchased for £18 500. Its value depreciates by 14% per year.

(a) Write down the value of the car after 1 year. [1]

(b) Find the value of the car after 4 years. Give your answer correct to 2 decimal places. [2]

3. EASY Calculator **[3 marks]**

An investment of \$3 200 grows to \$4 000 over exactly 5 years, with interest compounded monthly.

(a) Find the nominal annual interest rate. Give your answer correct to 2 decimal places. [3]

4. EASY Calculator **[3 marks]**

Layla deposits £400 at the end of each month into a savings account paying a nominal annual interest rate of 3.6%, compounded monthly.

(a) Find the total amount in Layla's account after 2 years. Give your answer correct to 2 decimal places. [3]

5. EASY Calculator **[3 marks]**

Marcus borrows \$12 000 to pay for home improvements. The loan carries a nominal annual interest rate of 5.4%, compounded monthly, and is repaid in equal monthly instalments over 3 years.

(a) Find the amount of each monthly instalment. Give your answer correct to 2 decimal places. [3]

6. EASY Calculator **[3 marks]**

Give all answers in this question correct to 2 decimal places.

Elena invests €6 500 in a fixed-term bond paying 3.8% per annum, compounded annually.

(a) Find the value of Elena's investment after 6 years. [2]

(b) Find the total interest earned over the 6 years. [1]

7. EASY Calculator **[3 marks]**

A printing machine is purchased for \$45 000 and loses 18% of its value at the end of each year.

(a) Write down the annual depreciation multiplier. [1]

(b) Find the first year in which the machine's value falls below \$20 000. [2]

8. EASY Calculator **[3 marks]**

James saves \$250 at the end of each month in an account paying a nominal annual interest rate of 2.4%, compounded monthly.

(a) Write down the monthly interest rate as a decimal. [1]

(b) Find the total amount in James's account after 18 months. Give your answer correct to 2 decimal places. [2]

9. EASY Calculator **[3 marks]**

Valentina borrows \$8 000 at a nominal annual interest rate of 6.0%, compounded monthly. She makes monthly repayments of \$243.32 over 3 years.

(a) Find the outstanding balance of Valentina's loan after she has made 12 repayments. Give your answer correct to 2 decimal places. [3]

10. EASY Calculator **[3 marks]**

A pair of trainers costs \$350 today. The price is expected to rise by 2.9% per year due to inflation.

(a) Find the expected price of the trainers in 4 years. Give your answer correct to 2 decimal places. [2]

(b) Write down the total increase in price over the 4 years. [1]

Section B — Medium MEDIUM (10 questions × 4 marks = 40 marks)

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11. MEDIUM Calculator [4 marks]

Ravi invests \$7 500 at a nominal annual interest rate of 5.1%, compounded quarterly.

(a) Find the value of Ravi's investment after 3 years. Give your answer correct to 2 decimal places. [2]

(b) Find the minimum number of complete years for Ravi's investment to exceed \$11 000. [2]

12. MEDIUM Calculator [4 marks]

Priya needs to borrow £20 000. She is offered two loan options, both repaid in monthly instalments.

Option	Nominal annual rate (compounded monthly)	Term
A	4.8%	5 years
B	5.2%	4 years

(a) Find the monthly repayment under each option. Give your answers correct to 2 decimal places. [2]

(b) Find the total amount paid under each option, and determine which option costs less overall. [2]

13. MEDIUM Calculator **[4 marks]**

A bank offers two savings products, each requiring a lump-sum deposit of \$5 000 for 6 years.

- **Product X:** 4.4% per annum, compounded annually.
- **Product Y:** 4.2% per annum, compounded monthly.

(a) Find the value of the investment under each product after 6 years. Give your answers correct to 2 decimal places. [2]

(b) State which product gives the greater return and find the difference in final value. [2]

14. MEDIUM Calculator [4 marks]

Chen deposits €500 at the end of each month into a savings account paying a nominal annual interest rate of 3.0%, compounded monthly.

The table below shows the value of Chen's account at selected months.

Month (n)	Account value (€)
0	0.00
6	3 018.81
12	6 083.19
n	15 000.00

(a) Verify the value €3 018.81 shown in the table at month 6. [2]

(b) Find the minimum number of complete months, n, for Chen's account to first reach €15 000. [2]

15. MEDIUM Calculator [4 marks]

A country has a nominal annual interest rate of 6.0% and an annual inflation rate of 2.5%. Isabelle invests \$4 000 for 5 years, compounded annually.

(a) Find the nominal future value of Isabelle's investment after 5 years. [1]

(b) Find the real annual interest rate. [1]

(c) Find the real value of Isabelle's investment after 5 years (in today's purchasing power). Give your answer correct to 2 decimal places. [2]

16. MEDIUM Calculator **[4 marks]**

Two laptops are purchased simultaneously. Laptop A costs €1 200 and depreciates at 25% per year. Laptop B costs €900 and depreciates at 20% per year.

(a) Find the value of each laptop after 3 years. Give your answers correct to 2 decimal places. [2]

(b) Find the first year in which Laptop B is worth more than Laptop A. [2]

17. MEDIUM Calculator **[4 marks]**

Dani borrows \$15 000 at a nominal annual interest rate of 6.0%, compounded monthly, to be repaid in monthly instalments of \$289.99 over 5 years.

The table below shows part of Dani's amortisation schedule. Complete all missing values.

Payment	Payment (\$)	Interest (\$)	Principal (\$)	Balance (\$)
0	—	—	—	15 000.00
1	289.99	**(i)**	**(ii)**	**(iii)**
2	289.99	**(iv)**	216.06	14 568.95
3	289.99	**(v)**	**(vi)**	**(vii)**

(a) Find all missing values (i) to (vii) in the table. [3]

(b) Find the total interest paid over the full 5-year loan. [1]

18. MEDIUM Calculator **[4 marks]**

Kenji invests \$12 000 and compares two accounts over 8 years.

- **Account P:** 3.6% per annum, compounded quarterly.
- **Account Q:** $r\%$ per annum, compounded monthly.

(a) Find the value of Kenji's investment in Account P after 8 years. Give your answer correct to 2 decimal places. [2]

(b) Find the value of r such that both accounts produce the same final value after 8 years. Give your answer correct to 2 decimal places. [2]

19. MEDIUM Calculator **[4 marks]**

A bank offers an account at a nominal annual interest rate of 5.4%, compounded monthly.

(a) Find the effective annual rate (EAR) of this account. Give your answer as a percentage correct to 2 decimal places. [2]

(b) A second account offers 5.5% per annum, compounded annually. Determine which account gives the higher return and justify your answer. [2]

20. MEDIUM Calculator **[4 marks]**

Felix pays \$600 at the end of each month into a retirement fund at a nominal annual interest rate of 4.8%, compounded monthly.

(a) Find the value of Felix's fund after 25 years. Give your answer correct to 2 decimal places. [2]

(b) Felix instead decides to reach the same accumulated amount by making larger monthly deposits of \$800. Find the minimum number of complete months required. [2]

Section C — Hard HARD (10 questions × 5 marks = 50 marks)

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21. HARD Calculator **[5 marks]**

A house is priced at \$280 000. The buyer must pay a deposit of 15% of the purchase price. She currently has \$22 000 saved, and will save an additional \$900 at the end of each month in an account paying 3.6% per annum, compounded monthly.

(a) Find the additional amount she still needs to save for the deposit. [1]

(b) Find the minimum number of complete months she must continue saving to reach the required deposit. [2]

(c) Once the deposit is paid, she borrows the remaining \$238 000 at a nominal annual interest rate of 4.5%, compounded monthly, over 25 years. Find the monthly repayment. Give your answer correct to 2 decimal places. [2]

22. HARD Calculator [5 marks]

Mia takes out a \$40 000 loan to be repaid in monthly instalments of \$450 over 10 years at a nominal annual interest rate of $r\%$, compounded monthly.

(a) Find the value of r. Give your answer correct to 2 decimal places. [3]

(b) Find the outstanding balance of the loan after Mia has made 5 years of repayments. Give your answer correct to 2 decimal places. [2]

23. HARD Calculator [5 marks]

Priya invests €8 000 at 4.2% per annum, compounded quarterly, for 3 years. At the end of 3 years she immediately transfers the full amount into an account paying 3.8% per annum, compounded monthly, for a further 4 years.

(a) Find the amount in Priya's account at the end of the 3-year period. Give your answer correct to 2 decimal places. [2]

(b) Find the total amount at the end of 7 years. Give your answer correct to 2 decimal places. [2]

(c) Find the single equivalent annual interest rate for the full 7-year period. Give your answer correct to 2 decimal places. [1]

24. HARD Calculator **[5 marks]**

An investor has \$10 000 and compares accounts in two countries over 6 years.

Country	Nominal annual rate	Annual inflation rate
X	7.2% (compounded annually)	3.1%
Y	5.8% (compounded monthly)	1.4%

(a) Find the effective annual rate (EAR) for Country Y's account. Give your answer as a percentage correct to 2 decimal places. [1]

(b) Find the real annual interest rate for each country. Give your answers correct to 2 decimal places. [2]

(c) Determine which country gives the higher real value after 6 years, and find the difference. Give your answer correct to 2 decimal places. [2]

25. HARD Calculator **[5 marks]**

A piece of industrial equipment is purchased for \$60 000 and depreciates by $r\%$ per year. After 8 years its value is \$18 500.

(a) Find the annual depreciation rate r. Give your answer correct to 2 decimal places. [2]

(b) The equipment must be replaced when its value first falls below \$8 000. Find the year in which this occurs. [2]

(c) State one limitation of using a constant-percentage depreciation model for this equipment. [1]

26. HARD Calculator [5 marks]

Hamid deposits \$350 at the end of each month into a savings plan. After exactly 10 years the account contains \$55 000. The nominal annual interest rate is $r\%$, compounded monthly.

(a) Find the value of r. Give your answer correct to 2 decimal places. [3]

(b) Using the rate found in (a), find the monthly deposit required to accumulate \$70 000 over the same 10-year period. Give your answer correct to 2 decimal places. [2]

27. HARD Calculator [5 marks]

Anya borrows \$25 000 at a nominal annual interest rate of 5.4%, compounded monthly, to be repaid over 4 years.

(a) Find the standard monthly repayment. Give your answer correct to 2 decimal places. [1]

(b) Find the total interest paid under the standard repayment plan. [1]

(c) Anya decides to pay an additional \$100 per month above the standard repayment. Find the number of complete months by which the loan is repaid early. [2]

(d) Find the saving in total interest from making the additional payments. Give your answer correct to 2 decimal places. [1]

28. **HARD** **Calculator** **[5 marks]**

Nadia invests \$$P$ at a nominal annual rate of 4.8%, compounded monthly. After 6 years the investment is worth exactly \$15 000.

(a) Find the value of P. Give your answer correct to 2 decimal places. [2]

(b) If Nadia had instead invested P at 5.0% per annum, compounded annually, find by how much more or less she would have after 6 years. Give your answer correct to 2 decimal places. [2]

(c) State the effective annual rate for the 4.8% monthly account. Give your answer as a percentage correct to 2 decimal places. [1]

29. HARD Calculator **[5 marks]**

Jasmine takes a loan of \$18 000 at $r\%$ per annum, compounded monthly, over a 3-year term. The table below shows the first two rows of her amortisation schedule.

Payment	Payment (\$)	Interest (\$)	Principal (\$)	Balance (\$)
0	—	—	—	18 000.00
1	p	105.00	q	17 549.21
2	p	s	t	17 095.79

(a) Use the interest charged on Payment 1 to find r. [2]

(b) Write down the values of p, q, s, and t. [2]

(c) Find the outstanding balance after 2 full years (24 payments). Give your answer correct to 2 decimal places. [1]

30. HARD Calculator **[5 marks]**

Kai needs \$38 000 in exactly 5 years for a property deposit. He plans to combine two strategies:

- Invest a lump sum of \$15 000 today at $r\%$ per annum, compounded quarterly.
- Deposit \$$k$ at the end of each month into a separate account paying 3.0% per annum, compounded monthly.

(a) Write an equation in r and k. [1]

(b) Given that $r = 6.0$, find the required value of k. Give your answer correct to 2 decimal places. [2]

(c) Given instead that $k = 250$, find the minimum annual rate r needed. Give your answer correct to 2 decimal places. [2]

Worked Solutions

Notation: **M** method mark **A** accuracy mark **R** reasoning mark **AG** answer given (implied mark)

Section A — Easy

Q1 — Compound Interest — EASY

Method 1 (TVM): $N = 12,\ I\% = 4.2,\ PV = -5000,\ PMT = 0,\ P/Y = 4,\ C/Y = 4$ **(M1)(A1)**

$FV = \$5\,667.69$ **A1**

Method 2 (formula): $FV = 5000\left(1 + \dfrac{4.2}{400}\right)^{12} = 5000 \times 1.133538\ldots = 5667.69$ **(M1)(A1)A1**

Note: Award (M1) for correct method; (A1) for correct entries or substitution; A1 for answer to 2 d.p.

Q2 — Depreciation — EASY

(a) $V_1 = 18500 \times 0.86 = £15\,910.00$ **A1**

(b) $V_4 = 18500 \times (0.86)^4 = 18500 \times 0.54700\ldots = £10\,119.65$ **M1 A1**

Note: Award M1 for substituting into $V_0(1 - r/100)^n$ with correct values; A1 for answer to 2 d.p.

Q3 — Finding the Nominal Rate — EASY

TVM: $N = 60,\ PV = -3200,\ FV = 4000,\ PMT = 0,\ P/Y = 12,\ C/Y = 12$ **(M1)(A1)**

$I\% = 4.47\%$ $(4.4712\ldots\%)$ **A1**

Note: Award (M1) for correct TVM setup or attempt to solve $4000 = 3200(1 + r/1200)^{60}$; A1 for answer to 2 d.p.

Q4 — Savings Annuity — EASY

TVM: $N = 24,\ I\% = 3.6,\ PV = 0,\ PMT = -400,\ P/Y = 12,\ C/Y = 12$ **(M1)(A1)**

$FV = £9\,938.60$ $(9938.602\ldots)$ **A1**

Note: Award (M1) for correct TVM setup with PV=0; (A1) for all entries correct; A1 for answer to 2 d.p.

Q5 — Loan Monthly Repayment — EASY

TVM: $N = 36,\ I\% = 5.4,\ PV = 12000,\ FV = 0,\ P/Y = 12,\ C/Y = 12$ **(M1)(A1)**

$PMT = \$361.81$ $(361.8108\ldots)$ **A1**

Note: Award (M1) for correct TVM with FV=0; A1 for answer to 2 d.p. Accept negative PMT if working

shown.

Q6 — Compound Interest - 2 d.p. Directive — EASY

(a) $FV = 6500 \times (1.038)^6 = 6500 \times 1.24925\ldots = €8\,130.13$ **M1 A1**

(b) Interest $= 8130.13 - 6500 = €1\,630.13$ **A1**

Note: FT from (a). Award M1 for using $PV \times (1+r)^n$; A1 each for answer to 2 d.p.

Q7 — Depreciation and Threshold — EASY

(a) Multiplier $= 1 - 0.18 = 0.82$ **A1**

(b) $45000 \times (0.82)^n < 20000$ **M1**

$n > 4$ gives $V_4 = 45000 \times (0.82)^4 = \$20\,346.66$; $n = 5$ gives $V_5 = 45000 \times (0.82)^5 = \$16\,684.26$

First year value falls below \$20 000 is **Year 5**. **A1**

Note: Award M1 for evaluating $45000 \times (0.82)^n$ for consecutive values near the threshold; A1 for Year 5.

Q8 — Monthly Savings Rate and FV — EASY

(a) $i = \dfrac{2.4}{1200} = 0.002$ **A1**

(b) TVM: $N = 18,\ I\% = 2.4,\ PV = 0,\ PMT = -250,\ P/Y = 12,\ C/Y = 12$ **(M1)**

$FV = \$4\,577.32$ $\quad (4577.318\ldots)$ **A1**

Note: FT from (a) if wrong i used consistently. Award (M1) for correct TVM or annuity formula; A1 to 2 d.p.

Q9 — Outstanding Loan Balance — EASY

TVM (balance after 12 payments): $N = 12,\ I\% = 6.0,\ PV = 8000,\ PMT = -243.32,\ P/Y = 12,\ C/Y = 12$ **(M1)(A1)**

$FV = \$5\,491.93$ $\quad (5491.934\ldots)$ **A1**

Note: Award (M1) for correct TVM setup to find balance; (A1) for all entries correct with $N = 12$; A1 to 2 d.p.

Q10 — Inflation — EASY

(a) Price $= 350 \times (1.029)^4 = 350 \times 1.12115\ldots = \$392.40$ **M1 A1**

(b) Increase $= 392.40 - 350 = \$42.40$ **A1**

Note: FT from (a). Award M1 for correct compound growth formula.

Section B — Medium

Q11 — Investment: FV and Time to Target — MEDIUM

(a) TVM: $N = 12,\ I\% = 5.1,\ PV = -7500,\ PMT = 0,\ P/Y = 4,\ C/Y = 4$ **(M1)**

$FV = \$8\,731.49$ (8731.494 ...) **A1**

(b) Need $7500 \times (1.01275)^{4n} > 11000$ **M1**

$n > \dfrac{\ln(11000/7500)}{4\ln(1.01275)} = 7.558\ldots$ so minimum number of complete years $=$ **8** **A1**

Note: Award M1 for forming the correct inequality; A1 for 8 years.

Q12 — Loan Comparison — MEDIUM

(a) Option A: $N = 60,\ I\% = 4.8,\ PV = 20000,\ FV = 0,\ P/Y = 12,\ C/Y = 12 \Rightarrow PMT_A = £375.59$ **(M1)A1**

Option B: $N = 48,\ I\% = 5.2,\ PV = 20000,\ FV = 0,\ P/Y = 12,\ C/Y = 12 \Rightarrow PMT_B = £462.40$ **A1**

Note: Award (M1) for correct TVM for either option.

(b) Total A $= 60 \times 375.59 = £22\,535.69$; Total B $= 48 \times 462.40 = £22\,195.20$ **M1**

Option B costs less overall (by £340.49). **A1**

Note: FT from (a). Award M1 for computing both totals; A1 for correct identification.

Q13 — Two Savings Products — MEDIUM

(a) Product X: $FV = 5000 \times (1.044)^6 = 5000 \times 1.29480\ldots = \$6\,474.00$ **(M1)A1**

Product Y: $N = 72,\ I\% = 4.2,\ PV = -5000,\ P/Y = 12,\ C/Y = 12 \Rightarrow FV = \$6\,430.15$ **A1**

(b) **Product X** gives the greater return; difference $= 6474.00 - 6430.15 = \$43.85$ **A1 A1**

Note: FT from (a). Award A1 for correct identification; A1 for difference to 2 d.p.

Q14 — Savings Annuity with Table — MEDIUM

(a) $i = 3.0/1200 = 0.0025$; after 6 months:

$$FV = 500 \times \frac{(1.0025)^6 - 1}{0.0025} = 500 \times \frac{0.01503\ldots}{0.0025} = 500 \times 6.03763\ldots = €3\,018.81$$ **M1 A1**

(b) Need $500 \times \dfrac{(1.0025)^n - 1}{0.0025} \geq 15000$: $(1.0025)^n \geq 1.075$

$n \geq \dfrac{\ln 1.075}{\ln 1.0025} = 28.96\ldots$; minimum number of complete months = **29** **M1 A1**

Note: Award M1 for correct inequality or TVM setup with FV=15000; A1 for 29 months.

Q15 — Real vs Nominal Value — MEDIUM

(a) Nominal FV $= 4000 \times (1.06)^5 = 4000 \times 1.33823\ldots = \$5\,352.90$ **A1**

(b) $r_{\text{real}} = \dfrac{1.06}{1.025} - 1 = 0.034146\ldots = 3.41\%$ **A1**

(c) Real value $= 4000 \times (1.034146\ldots)^5 = 4000 \times 1.18280\ldots = \$4\,731.19$ **M1 A1**

Note: Award M1 for applying $(1 + r_{real})^5$ to \$4000; A1 for answer to 2 d.p.

Q16 — Depreciation Comparison — MEDIUM

(a) Laptop A: $V_3 = 1200 \times (0.75)^3 = 1200 \times 0.421875 = €506.25$ **A1**

Laptop B: $V_3 = 900 \times (0.80)^3 = 900 \times 0.512 = €460.80$ **A1**

(b) Need $900 \times (0.80)^n > 1200 \times (0.75)^n \Rightarrow \left(\dfrac{0.80}{0.75}\right)^n > \dfrac{4}{3}$ **M1**

$n > \dfrac{\ln(4/3)}{\ln(16/15)} = 4.457\ldots$; first year B is worth more: **Year 5**

Verification: $V_5^A = 284.77$; $V_5^B = 294.91$ ✓ **A1**

Note: Award M1 for forming the inequality; A1 for Year 5.

Q17 — Amortisation Table — MEDIUM

Monthly rate $= 6.0/1200 = 0.005$.

Payment 1: (i) Interest $= 15000 \times 0.005 = \$75.00$; (ii) Principal $= 289.99 - 75.00 = \$214.99$; (iii) Balance $= 15000 - 214.99 = \$14\,785.01$ **A1**

Payment 2: (iv) Interest $= 14785.01 \times 0.005 = \$73.93$; (v) given as correct; Balance confirmed \$14 568.95 **A1**

Payment 3: (v) Interest $= 14568.95 \times 0.005 = \$72.84$; (vi) Principal $= 289.99 - 72.84 = \$217.15$; (vii) Balance $= 14568.95 - 217.15 = \$14\,351.80$ **A1**

(b) Total interest $= 60 \times 289.99 - 15000 = 17399.40 - 15000 = \$2\,399.40$ **A1**

Q18 — Finding a Matching Rate — MEDIUM

(a) $FV_P = 12000 \times \left(1 + \frac{3.6}{400}\right)^{32} = 12000 \times (1.009)^{32} = 12000 \times 1.33204\ldots = \$15\,984.48$ **M1 A1**

(b) Need $12000 \times \left(1 + \frac{r}{1200}\right)^{96} = 15984.48$ **M1**

TVM: $N = 96,\ PV = -12000,\ FV = 15984.48,\ PMT = 0,\ P/Y = 12,\ C/Y = 12 \Rightarrow r = 3.59\%$ $(3.5893\ldots\%)$ **A1**

Note: FT from (a). Award M1 for correct TVM setup or equation; A1 to 2 d.p.

Q19 — Effective Annual Rate — MEDIUM

(a) EAR $= \left(1 + \frac{5.4}{1200}\right)^{12} - 1 = (1.0045)^{12} - 1 = 0.055357\ldots = 5.54\%$ **M1 A1**

(b) EAR of first account $= 5.54\% > 5.5\%$ (the second account's rate, already compounded annually) **M1**

The first account gives a higher return, since its effective annual rate (5.54%) exceeds 5.5%. **R1**

Note: Award M1 for comparing EARs; R1 for contextual conclusion.

Q20 — Retirement Annuity — MEDIUM

(a) TVM: $N = 300,\ I\% = 4.8,\ PV = 0,\ PMT = -600,\ P/Y = 12,\ C/Y = 12$ **(M1)(A1)**

$FV = \$346\,826.90$ $(346826.898\ldots)$ **A1 (within 2-mark part)**

(b) TVM: $N = ?,\ I\% = 4.8,\ PV = 0,\ PMT = -800,\ FV = 346826.90,\ P/Y = 12,\ C/Y = 12$ **(M1)**

$N = 251.96\ldots$ months; minimum complete months = **252** **A1**

Note: FT from (a). Award (M1) for correct TVM; A1 for 252 months.

Section C — Hard

Q21 — Grand Tour: Savings + Loan — HARD

(a) Required deposit $= 0.15 \times 280000 = \$42\,000$; additional needed $= 42000 - 22000 = \$20\,000$ **A1**

(b) $22000 + 900 \times \frac{(1.003)^n - 1}{0.003} \geq 42000 \Rightarrow (1.003)^n \geq 1.06\overline{6}$ **M1**

$n \geq \frac{\ln(1.06\overline{6})}{\ln 1.003} = 21.54\ldots$; minimum = **22** months **A1**

[Check: after 22 months, total $= 22000 + 900 \times \frac{(1.003)^{22} - 1}{0.003} = \$42\,436.35 > \$42\,000$ ✓]

(c) TVM: $N = 300,\ I\% = 4.5,\ PV = 238000,\ FV = 0,\ P/Y = 12,\ C/Y = 12$ **(M1)**

$PMT = \$1\,322.88 \quad (1322.878\ldots)$ **A1**

Note: Award M1 for setting up the savings inequality; A1 for 22 months; (M1) for loan TVM; A1 to 2 d.p.

Q22 — Unknown Loan Rate and Balance — HARD

(a) TVM: $N = 120,\ PV = 40000,\ PMT = -450,\ FV = 0,\ P/Y = 12,\ C/Y = 12$ **(M1)(A1)**

$r = 6.29\% \quad (6.2935\ldots\%)$ **A1**

(b) TVM: $N = 60,\ I\% = 6.2935\ldots,\ PV = 40000,\ PMT = -450,\ P/Y = 12,\ C/Y = 12$ **(M1)**

$FV = \$23\,113.00 \quad (23113.003\ldots)$ **A1**

Note: Use unrounded rate from (a). Award (M1) for correct TVM with $N = 60$; A1 to 2 d.p.

Q23 — Two-Phase Investment — HARD

(a) $FV_1 = 8000 \times \left(1 + \dfrac{4.2}{400}\right)^{12} = 8000 \times (1.0105)^{12} = 8000 \times 1.13353\ldots = €9\,068.30$ **M1 A1**

(b) $FV_2 = 9068.30 \times \left(1 + \dfrac{3.8}{1200}\right)^{48} = 9068.30 \times (1.00\overline{31})^{48} = 9068.30 \times 1.16398\ldots = €10\,554.42$ **M1 A1**

(c) $8000(1+r)^7 = 10554.42 \Rightarrow r = \left(\dfrac{10554.42}{8000}\right)^{1/7} - 1 = 1.31930^{0.14286} - 1 = 0.04038\ldots = 4.04\%$ **A1**

Note: FT from (b). Award A1 to 2 d.p.

Q24 — Real Returns: Two Countries — HARD

(a) $\text{EAR}_Y = \left(1 + \dfrac{5.8}{1200}\right)^{12} - 1 = 5.96\% \quad (5.9567\ldots\%)$ **A1**

(b) Real rate X $= \dfrac{1.072}{1.031} - 1 = 3.98\% \quad (3.9767\ldots\%)$; Real rate Y $= \dfrac{1.059567}{1.014} - 1 = 4.49\%$ $(4.4938\ldots\%)$ **A1 A1**

(c) Real value X $= 10000 \times (1.03977)^6 = 10000 \times 1.26362\ldots = \$12\,636.21$ **M1**

Real value Y $= 10000 \times (1.04494)^6 = 10000 \times 1.30180\ldots = \$13\,018.02$

Country Y gives the higher real value; difference $= 13018.02 - 12636.21 = \$381.81$ **A1**

Note: FT from (a)(b). Award M1 for computing both real values; A1 for correct identification and difference to 2 d.p. Accept slight variation from rounding of rates.

Q25 — Unknown Depreciation Rate and Threshold — HARD

(a) $60000 \times (1-r/100)^8 = 18500 \Rightarrow (1-r/100) = (18500/60000)^{1/8} = 0.30833^{0.125} = 0.86323\ldots$ **M1**

$r = (1 - 0.86323) \times 100 = 13.68\%$ **A1**

(b) $60000 \times (0.86323)^n < 8000 \Rightarrow n > \dfrac{\ln(8000/60000)}{\ln(0.86323)} = \dfrac{-2.0149}{-0.14711} = 13.70\ldots$ **M1**

Equipment is replaced in **Year 14**. **A1**

[Check: $V_{13} = \$8\,867.67 > 8000$; $V_{14} = \$7\,654.86 < 8000$ ✓]

(c) e.g. The model assumes a constant annual depreciation rate, whereas in practice the rate may vary with age, usage, or changes in market conditions. **R1**

Note: Award R1 for any valid, contextual limitation.

Q26 — Unknown Rate in Savings Plan — HARD

(a) TVM: $N = 120,\ PV = 0,\ PMT = -350,\ FV = 55000,\ P/Y = 12,\ C/Y = 12$ **(M1)(A1)**

$r = 5.22\%$ $\quad (5.2223\ldots\%)$ **A1**

(b) TVM: $N = 120,\ I\% = 5.2223\ldots,\ PV = 0,\ FV = 70000,\ P/Y = 12,\ C/Y = 12$ **(M1)**

$PMT = \$445.45$ $\quad (445.454\ldots)$ **A1**

Note: FT from (a); use unrounded rate. Award A1 to 2 d.p.

Q27 — Loan and Overpayment — HARD

(a) TVM: $N = 48,\ I\% = 5.4,\ PV = 25000,\ FV = 0,\ P/Y = 12,\ C/Y = 12 \Rightarrow PMT = \$580.27$ **A1**

(b) Total interest $= 48 \times 580.27 - 25000 = 27852.96 - 25000 = \$2\,852.96$ **A1**

(c) New monthly payment $= 580.27 + 100 = \$680.27$

TVM: $I\% = 5.4,\ PV = 25000,\ PMT = -680.27,\ FV = 0,\ P/Y = 12,\ C/Y = 12 \Rightarrow N = 40.26\ldots$ **M1**

Needs 41 payments (last one partial); months saved $= 48 - 41 =$ **7** months **A1**

(d) Balance after 40 payments of \$680.27:

$$FV_{40} = 25000(1.0045)^{40} - 680.27 \times \frac{(1.0045)^{40} - 1}{0.0045} = \$177.71$$

Total paid $= 40 \times 680.27 + 177.71 = \$27\,388.51$; interest $= 27388.51 - 25000 = 2388.51$

Saving $= 2852.96 - 2388.51 = \$464.45$ **A1**

Note: FT from (a)(b). Award A1 for saving to 2 d.p. Accept answers in range \$460–\$468 from GDC rounding.

Q28 — Finding Principal and Comparison — HARD

(a) TVM: $N = 72,\ I\% = 4.8,\ PMT = 0,\ FV = 15000,\ P/Y = 12,\ C/Y = 12 \Rightarrow PV = P$ **(M1)**

$P = \$11\,252.89$ $\quad (11252.893\ldots)$ **A1**

(b) $FV_{\text{alt}} = 11252.89 \times (1.05)^6 = 11252.89 \times 1.34010\ldots = \$15\,079.94$ **M1**

Difference $= 15079.94 - 15000 = \$79.94$ more under the 5.0% annual account. **A1**

(c) EAR $= (1.004)^{12} - 1 = 0.04907\ldots = \mathbf{4.91}\%$ **A1**

Q29 — Amortisation Table with Unknown Rate — HARD

(a) Interest on Payment 1 $= 18000 \times \dfrac{r}{1200} = 105.00$ **M1**

$r = \dfrac{105 \times 1200}{18000} = 7.00\%$ **A1**

(b) $i = 7/1200$; TVM: $N = 36,\ I\% = 7,\ PV = 18000,\ FV = 0 \Rightarrow PMT = 555.79$

$p = \$555.79$; $q = 555.79 - 105.00 = \$450.79$; $s = 17549.21 \times (7/1200) = \$102.37$; $t = 555.79 - 102.37 = \$453.42$ **A1 A1**

Note: Award A1 for p and q correct; A1 for s and t correct.

(c) TVM: $N = 24,\ I\% = 7,\ PV = 18000,\ PMT = -555.79,\ P/Y = 12,\ C/Y = 12$ **(M1)**

$FV = \$6\,423.25$ $\quad (6423.251\ldots)$ **A1**

Note: Award (M1) for correct TVM with $N = 24$; A1 to 2 d.p.

Q30 — Synthesis: Lump Sum + Annuity — HARD

(a) Let $i = 0.0025$. The equation relating r and k is:

$$15000\left(1 + \frac{r}{400}\right)^{20} + k \cdot \frac{(1.0025)^{60} - 1}{0.0025} = 38000$$

A1

(b) $r = 6.0$: lump sum $= 15000 \times (1.015)^{20} = 15000 \times 1.34686\ldots = \$20\,202.83$ **M1**

Annuity factor $= \dfrac{(1.0025)^{60} - 1}{0.0025} = \dfrac{0.16162\ldots}{0.0025} = 64.647\ldots$

$k = \dfrac{38000 - 20202.83}{64.647} = \dfrac{17797.17}{64.647} = \$275.30$ **A1**

(c) $k = 250$: annuity contribution $= 250 \times 64.647 = \$16\,161.72$

Lump sum required $= 38000 - 16161.72 = \$21\,838.28$ **M1**

$15000\left(1 + \dfrac{r}{400}\right)^{20} = 21838.28 \Rightarrow \left(1 + \dfrac{r}{400}\right)^{20} = 1.45589\ldots$

$r = 400 \times \left[(1.45589)^{0.05} - 1\right] = 400 \times 0.018958\ldots = 7.58\%$ **A1**

Note: Award M1 for finding required lump-sum value; A1 for $r = 7.58\%$ to 2 d.p.

End of Worksheet

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