IB Math AI HL — Topic 4 [Set 1]

Probability Mastery Worksheet

Probability & Events · Independence & Mutual Exclusivity · Conditional Probability · Venn Diagrams · Tree Diagrams

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Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:______________________________ Date:_______________

Score:__________/120

Instructions: Answer all questions. Show all working. Give probabilities as exact fractions or decimals correct to 3 sf unless stated. A GDC is permitted throughout.

Key Formulae

Complementary: $P(A') = 1 - P(A)$

Union: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Intersection: $P(A \cap B) = P(A) \cdot P(B \mid A)$

Independent: $P(A \cap B) = P(A) \cdot P(B)$

Conditional: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

ME Union: $P(A \cup B) = P(A) + P(B)$

Mut. Exclusive: $P(A \cap B) = 0$

Bayes (2-event): $P(A \mid B) = \dfrac{P(B \mid A) \cdot P(A)}{P(B \mid A) \cdot P(A) + P(B \mid A') \cdot P(A')}$

GDC Tips

1. Store probabilities in variables (A, B, C on your GDC) to reduce transcription errors.
2. Use the fraction mode for exact answers; convert to decimal only when asked.
3. For tree diagrams: multiply along branches (AND), add across routes (OR).
4. Verify your total probability = 1 after completing any Venn diagram or table.

Common Mistakes to Avoid

1. **Confusing "and" with "or".** $P(A \text{ and } B) = P(A \cap B)$; $P(A \text{ or } B) = P(A \cup B)$. These are not the same!
2. **Independence $\neq$ Mutual Exclusivity.** If $P(A) > 0$ and $P(B) > 0$, they *cannot* be both mutually exclusive *and* independent simultaneously.
3. **Conditional probability denominator.** In $P(A \mid B)$, the denominator is $P(B)$, not 1. Dividing by the whole sample space is a very common error.
4. **Without replacement changes branches.** After drawing one item, adjust the denominator *and* numerator for the second draw.
5. **Reversing the condition.** $P(A \mid B) \neq P(B \mid A)$. Always identify which event is the "given" condition.
6. **Forgetting the intersection in the union formula.** $P(A \cup B) \neq P(A) + P(B)$ unless the events are mutually exclusive.

Section A — Easy **EASY** (10 questions × 3 marks = 30 marks)

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1. **EASY** **Calculator** **[3 marks]**

A bag contains 5 red counters, 3 blue counters, and 2 green counters. One counter is chosen at random.

(a) Write down the probability that the counter chosen is red. [1]

(b) Write down the probability that the counter chosen is **not** blue. [1]

(c) Find the probability that the counter chosen is either red or green. [1]

2. **EASY** **Calculator** **[3 marks]**

Events A and B are mutually exclusive. $P(A) = 0.35$ and $P(B) = 0.28$.

(a) Write down $P(A \cap B)$. [1]

(b) Find $P(A \cup B)$. [1]

(c) Find $P(A' \cap B')$. [1]

3. EASY Calculator **[3 marks]**

The table below shows information about 80 students and whether they study French (F) or Spanish (S).

	Study French	**Do not study French**	**Total**
Study Spanish	12	23	35
Do not study Spanish	28	17	45
Total	40	40	80

A student is chosen at random.

(a) Write down the probability that the student studies both French and Spanish. [1]

(b) Write down the probability that the student studies French given that they study Spanish. [1]

(c) Find the probability that the student studies neither French nor Spanish. [1]

4. EASY Calculator **[3 marks]**

Events C and D are independent. $P(C) = 0.6$ and $P(D) = 0.4$.

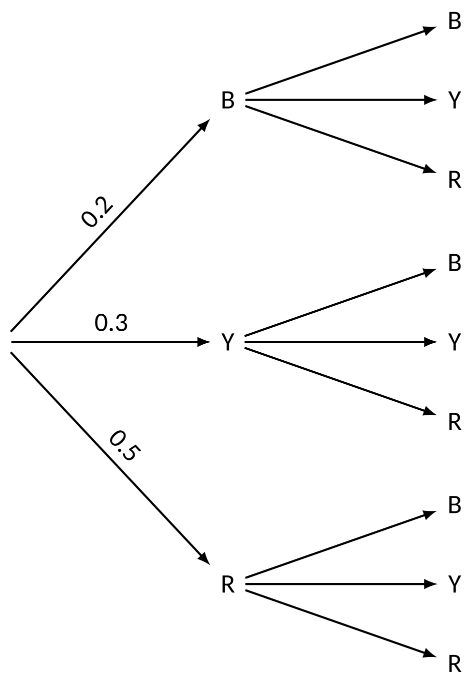
(a) Find $P(C \cap D)$. [1]

(b) Find $P(C \cup D)$. [1]

(c) Find $P(C' \cap D)$. [1]

5. **EASY** **Calculator** **[3 marks]**

A spinner has three sectors coloured red (R), yellow (Y), and blue (B) with $P(\text{R}) = 0.5$, $P(\text{Y}) = 0.3$, and $P(\text{B}) = 0.2$. The spinner is spun twice. The diagram shows the first branch of the tree diagram.

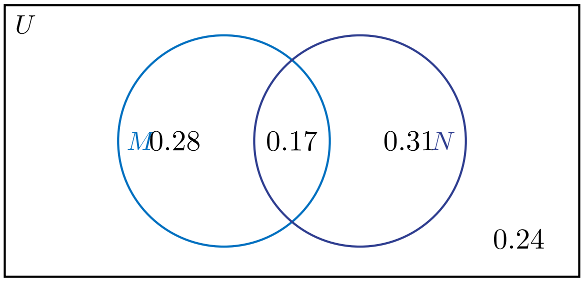


(a) Write down the probability that the spinner lands on blue on both spins. [1]

(b) Find the probability that the two spins land on the same colour. [2]

6. EASY Calculator **[3 marks]**

The Venn diagram shows the probabilities for events M and N within a universal set U.



(a) Write down $P(M \cap N)$. [1]

(b) Find $P(M \cup N)$. [1]

(c) Find $P(M' \cap N)$. [1]

7. EASY Calculator **[3 marks]**

$P(X) = 0.7$, $P(Y) = 0.5$, and $P(X \cap Y) = 0.35$.

(a) Find $P(X \mid Y)$. [1]

(b) Determine whether events X and Y are independent. Give a reason for your answer. [2]

8. EASY Calculator **[3 marks]**

The probability that a randomly chosen commuter arrives late on any given morning is 0.12, independently of other days.

(a) Write down the probability that the commuter does **not** arrive late on a given morning. [1]

(b) Find the probability that the commuter arrives on time on both Monday and Tuesday. [1]

(c) Find the probability that the commuter is late on at least one of Monday and Tuesday. [1]

9. EASY Calculator **[3 marks]**

Events P and Q satisfy $P(P) = 0.4$, $P(Q) = 0.5$, and $P(P \cup Q) = 0.9$.

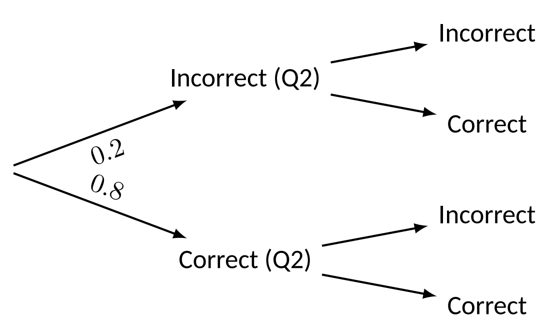
(a) Find $P(P \cap Q)$. [1]

(b) State whether P and Q are mutually exclusive. Give a reason. [1]

(c) State whether P and Q are independent. Give a reason. [1]

10. EASY Calculator **[3 marks]**

A student takes a multiple-choice test with two questions. For each question, the probability of answering correctly is 0.8, independently. The tree diagram shows this situation.



(a) Write down the probability of answering both questions correctly. [1]

(b) Find the probability of answering exactly one question correctly. [2]

Section B — Medium MEDIUM (10 questions × 4 marks = 40 marks)

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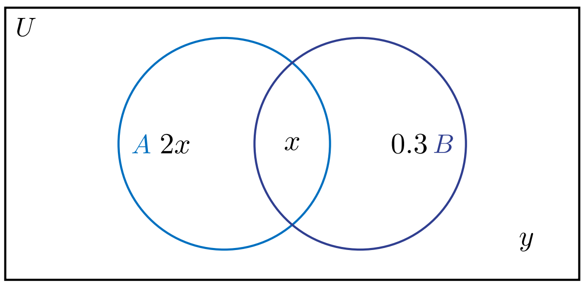
11. MEDIUM Calculator **[4 marks]**

The Venn diagram shows events A and B within a universal set U. The values in the regions represent probabilities.



Events A and B are independent.

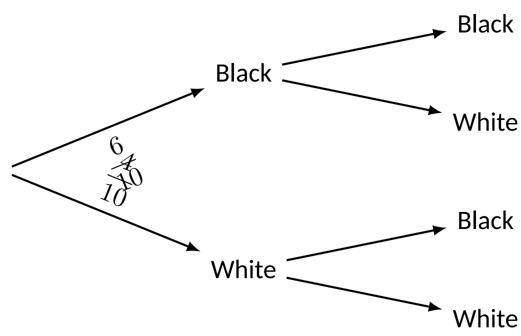
(a) Show that $P(A) = 3x$ and $P(B) = x + 0.3$. [1]

(b) Using the independence condition, find the value of x. [2]

(c) Find the value of y. [1]

12. **MEDIUM** **Calculator** **[4 marks]**

A box contains 4 white balls and 6 black balls. Two balls are drawn at random, one after the other, **without replacement**. The partial tree diagram below shows the first draw.



(a) Complete the second-stage branches of the tree diagram with the correct probabilities. [2]

(b) Find the probability that both balls drawn are the same colour. [1]

(c) Given that the two balls are the same colour, find the probability that both are white. [1]

13. **MEDIUM** **Calculator** **[4 marks]**

A medical screening test for a particular condition gives a positive result for 96% of people who have the condition, and a positive result for 4% of people who do not have the condition. It is known that 2% of the population has the condition.

A person is chosen at random from the population and given the test.

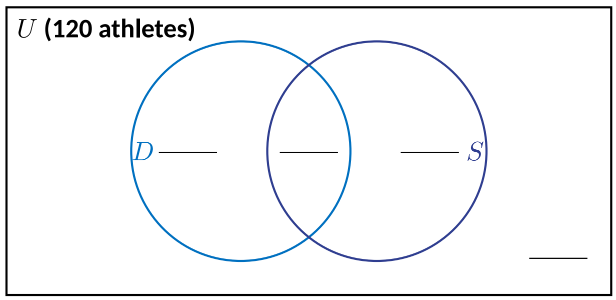
(a) Find the probability that the person tests positive. [2]

(b) Given that the person tests positive, find the probability that they actually have the condition. Give your answer correct to 3 significant figures. [2]

14. MEDIUM Calculator **[4 marks]**

In a group of 120 athletes, 72 train daily (event D) and 55 follow a strict diet (event S). Of those who train daily, 38 also follow a strict diet.

(a) Complete the Venn diagram below by writing the correct values in each region. [2]



(b) A randomly selected athlete trains daily. Find the probability that this athlete does **not** follow a strict diet. [1]

(c) Find the probability that a randomly selected athlete neither trains daily nor follows a strict diet. [1]

15. MEDIUM Calculator **[4 marks]**

A factory has two production lines, Line A and Line B. Line A produces 60% of all items and Line B produces 40%. The probability that an item from Line A is defective is 0.03, and from Line B is 0.07.

(a) Find the probability that a randomly chosen item is defective. [2]

(b) Given that an item is defective, find the probability that it came from Line B. Give your answer correct to 3 significant figures. [2]

16. MEDIUM Calculator **[4 marks]**

Events E and F satisfy $P(E) = 0.45$, $P(F) = 0.50$, and $P(E \cup F) = 0.75$.

(a) Find $P(E \cap F)$. [1]

(b) Find $P(E \mid F)$. [1]

(c) Determine whether E and F are independent. Show your reasoning. [2]

17. MEDIUM Calculator **[4 marks]**

A survey of 200 residents asked whether they used streaming services: Netflix (N), Spotify (S), or YouTube (Y). The results are:

- 90 use Netflix, 80 use Spotify, 110 use YouTube
- 30 use both Netflix and Spotify, 40 use both Netflix and YouTube, 35 use both Spotify and YouTube
- 10 use all three
- 25 use none

(a) Find the number of residents using exactly one of the three services. [2]

(b) A resident is chosen at random. Find the probability that they use at least two of the three services. [2]

18. MEDIUM Calculator **[4 marks]**

Events G and H are such that $P(G) = 0.6$, $P(H) = 0.5$, and $P(G \mid H) = 0.7$.

(a) Find $P(G \cap H)$. [1]

(b) Find $P(G \cup H)$. [1]

(c) Find $P(G' \mid H)$. [2]

19. MEDIUM Calculator **[4 marks]**

Leila commutes to work by three possible routes: Highway (H), Main Road (M), or Side Street (S). She chooses each route with probabilities 0.5, 0.3, and 0.2 respectively. The probability of being delayed on each route is: $P(\text{delay} \mid H) = 0.1$, $P(\text{delay} \mid M) = 0.4$, $P(\text{delay} \mid S) = 0.6$.

(a) Find the probability that Leila is delayed on a randomly chosen day. [2]

(b) Given that Leila was delayed, find the probability that she took the Highway. Give your answer correct to 3 significant figures. [2]

20. MEDIUM Calculator **[4 marks]**

The following table shows the number of students in a class of 50 who play football (F) and who play tennis (T).

	Plays Tennis	**Does not play Tennis**	**Total**
Plays Football	9	21	30
Does not play Football	6	14	20
Total	15	35	50

A student is chosen at random.

(a) Find $P(F)$, $P(T)$, and $P(F \cap T)$. [2]

(b) Determine whether F and T are independent. Show your reasoning clearly. [2]

Section C — Hard **HARD** (10 questions $\times$ 5 marks = 50 marks)

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21. **HARD** **Calculator** **[5 marks]**

Events A and B are such that $P(A) = 0.5$, $P(B) = p$, and $P(A \cup B) = 0.8$. Events A and B are **independent**.

(a) Find $P(A \cap B)$ in terms of p. [1]

(b) Show that $0.5p = 0.8 - 0.5 - p + P(A \cap B)$, and hence find the value of p. [2]

(c) Find $P(A \mid B')$. [2]

22. **HARD** **Calculator** **[5 marks]**

A company sources components from three suppliers: Supplier X provides 50%, Supplier Y provides 30%, and Supplier Z provides 20%. The probability of a faulty component from each supplier is 0.02, 0.05, and 0.08 respectively. A component is selected at random and found to be faulty.

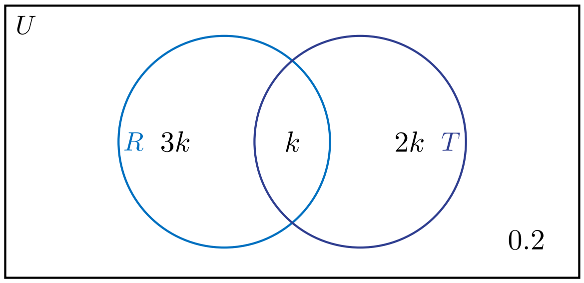
(a) Find the probability that the component is faulty. [2]

(b) Find the probability that the faulty component came from Supplier Y. Give your answer correct to 3 significant figures. [2]

(c) State one assumption made in calculating the probability in part (a). [1]

23. HARD Calculator **[5 marks]**

The Venn diagram shows events R and T within a universal set U. The values shown are probabilities.



(a) Write down $P(R)$ and $P(T)$ in terms of k. [1]

(b) Using the fact that all probabilities sum to 1, find the value of k. [1]

(c) Determine whether events R and T are independent. Show your reasoning. [2]

(d) Find $P(R \cup T)'$. [1]

24. **HARD** **Calculator** **[5 marks]**

A bag contains 3 red marbles and 5 blue marbles. Three marbles are drawn one at a time, **without replacement**.

(a) Find the probability that exactly two of the three marbles drawn are red. [3]

(b) Given that the first marble drawn is red, find the probability that the remaining two marbles are both blue. [2]

25. **HARD** **Calculator** **[5 marks]**

At a sports centre, two events are defined: W = "a member uses the weights room" and C = "a member uses the cardio area". A survey of 400 members gives: 200 use the weights room, 180 use the cardio area, and 90 use both.

(a) Find $P(W \cap C)$, $P(W)$, and $P(C)$. [1]

(b) Determine whether W and C are independent. [2]

(c) A member who uses the cardio area is selected at random. Find the probability that this member does **not** use the weights room. [1]

(d) Find $P(W' \cup C')$. [1]

26. **HARD** **Calculator** **[5 marks]**

Events J and K satisfy: $P(J) = p$, $P(K \mid J) = 0.4$, and $P(J' \cap K) = 0.12$.

(a) Write down $P(J \cap K)$ in terms of p. [1]

(b) Using $P(K) = P(J \cap K) + P(J' \cap K)$, form an equation in p and find its value. [2]

(c) Hence find $P(J \cup K)$ and $P(K \mid J')$. [2]

27. **HARD** **Calculator** **[5 marks]**

A diagnostic test for a plant disease has sensitivity 0.94 (the probability of a positive result given the plant is diseased) and specificity 0.88 (the probability of a negative result given the plant is not diseased). In a large greenhouse, the proportion of diseased plants is 0.06.

A plant is selected at random and the test gives a **negative** result.

(a) Find the probability of a negative test result. [2]

(b) Find the probability that the plant is **diseased** given the test is negative. Give your answer correct to 3 significant figures. [2]

(c) Explain why the answer to part (b) might seem surprising given the high sensitivity of the test. [1]

28. HARD Calculator **[5 marks]**

A archer hits the target with probability 0.7 on each independent attempt.

(a) Find the probability that the archer hits the target on **exactly** two of three attempts. [2]

(b) Find the probability that the archer hits the target on the **third** attempt, given that they **missed** on the first two attempts. [1]

(c) Find the probability that the archer hits the target **at least once** in five attempts. Give your answer correct to 3 significant figures. [2]

29. HARD Calculator **[5 marks]**

In a class of 60 students, each student studies at least one of Biology (B) or Chemistry (C). The probability that a randomly chosen student studies Biology is 0.65 and the probability that a student studies Chemistry is 0.55.

(a) Find the number of students who study both Biology and Chemistry. [2]

(b) A student who studies Biology is chosen at random. Find the probability that this student also studies Chemistry. [2]

(c) Find the probability that a randomly chosen student studies Biology but not Chemistry. [1]

30. **HARD** **Calculator** **[5 marks]**

Events V and W are defined within a universal set. $P(V) = 0.6$, $P(W) = q$, and $P(V \mid W) = 0.4$. It is also given that $P(V \cup W) = 0.84$.

(a) Find $P(V \cap W)$ in terms of q. [1]

(b) Using the addition formula, show that $q = \dfrac{0.6 - P(V \cap W)}{1 - 0.4}$ and find the value of q. [2]

(c) Find $P(W' \mid V)$. [2]

Worked Solutions

Official Mark-Scheme Style — M/A/R Notation

Section A — Easy (Q1–10)

Q1 — Basic Probability EASY

(a) $P(\text{red}) = \frac{5}{10} = \frac{1}{2} = 0.5$ **A1**

(b) $P(\text{not blue}) = 1 - P(\text{blue}) = 1 - \frac{3}{10} = \frac{7}{10}$ **A1**

(c) $P(\text{red or green}) = \frac{5+2}{10} = \frac{7}{10}$ **A1**

Note: Accept equivalent decimals 0.7, exact fractions. Award A1 each for independent correct answers.

Q2 — Mutually Exclusive Events EASY

(a) $P(A \cap B) = 0$ (mutually exclusive by definition) **A1**

(b) $P(A \cup B) = 0.35 + 0.28 = 0.63$ **A1**

(c) $P(A' \cap B') = P((A \cup B)') = 1 - 0.63 = 0.37$ **A1**

Note: FT from (b) for (c). Full credit for correct use of De Morgan's law.

Q3 — Two-Way Table and Conditional Probability EASY

(a) $P(F \cap S) = \frac{12}{80} = \frac{3}{20} = 0.15$ **A1**

(b) $P(F \mid S) = \frac{12}{35} \approx 0.343$ **A1**

(c) $P(\text{neither}) = \frac{17}{80} = 0.2125$ **A1**

Note: Accept exact fractions or decimals to 3 sf throughout.

Q4 — Independent Events EASY

(a) $P(C \cap D) = P(C) \times P(D) = 0.6 \times 0.4 = 0.24$ **A1**

(b) $P(C \cup D) = 0.6 + 0.4 - 0.24 = 0.76$ **A1**

(c) $P(C' \cap D) = P(C') \times P(D) = 0.4 \times 0.4 = 0.16$ **A1**

Note: For (c): independence means $P(C' \cap D) = P(C') \cdot P(D)$. FT from (a) using $P(C \cup D) - P(C \cap D) = P(C') \cdot P(D)$ also accepted.

Q5 — Tree Diagram (With Replacement) EASY

(a) $P(\text{B and B}) = 0.2 \times 0.2 = 0.04$ **A1**

(b) Probability of same colour $= P(\text{RR}) + P(\text{YY}) + P(\text{BB})$: **M1**

$= 0.5 \times 0.5 + 0.3 \times 0.3 + 0.2 \times 0.2 = 0.25 + 0.09 + 0.04 = 0.38$ **A1**

Q6 — Venn Diagram Read-Off EASY

(a) $P(M \cap N) = 0.17$ (intersection region) **A1**

(b) $P(M \cup N) = 0.28 + 0.17 + 0.31 = 0.76$ **A1**

(c) $P(M' \cap N) = 0.31$ (region inside N only, outside M) **A1**

Q7 — Conditional Probability and Independence EASY

(a) $P(X \mid Y) = \dfrac{P(X \cap Y)}{P(Y)} = \dfrac{0.35}{0.5} = 0.7$ **M1 A1**

(b) Since $P(X \mid Y) = 0.7 = P(X)$, the events are **independent**. **R1**

Note: Must state the comparison $P(X \mid Y) = P(X)$ or equivalently $P(X \cap Y) = P(X) \cdot P(Y)$ to earn R1. Accept "$0.7 \times 0.5 = 0.35 = P(X \cap Y)$, so independent."

Q8 — Complementary and Independent Events EASY

(a) $P(\text{not late}) = 1 - 0.12 = 0.88$ **A1**

(b) $P(\text{on time Mon and Tue}) = 0.88 \times 0.88 = 0.7744$ **A1**

(c) $P(\text{late at least once}) = 1 - 0.7744 = 0.2256$ **A1**

Note: FT in (c) from (b).

Q9 — Mutually Exclusive and Independent Classification EASY

(a) Using $P(P \cup Q) = P(P) + P(Q) - P(P \cap Q)$: **M1**

$0.9 = 0.4 + 0.5 - P(P \cap Q) \Rightarrow P(P \cap Q) = 0$ **A1**

(b) Since $P(P \cap Q) = 0$, the events **are** mutually exclusive. **R1**

(c) $P(P) \times P(Q) = 0.4 \times 0.5 = 0.2 \neq 0 = P(P \cap Q)$, so the events are **not** independent. **R1**

Note: Award R1 each provided the correct comparison is shown.

Q10 — Tree Diagram (Two-Outcome) EASY

(a) $P(\text{both correct}) = 0.8 \times 0.8 = 0.64$ **A1**

(b) $P(\text{exactly one correct}) = P(\text{correct, incorrect}) + P(\text{incorrect, correct})$ **M1**

$= 0.8 \times 0.2 + 0.2 \times 0.8 = 0.16 + 0.16 = 0.32$ **A1**

Section B — Medium (Q11–20)

Q11 — Venn with Unknown Parameters and Independence MEDIUM

(a) $P(A) = 2x + x = 3x$ (only A region + intersection); $P(B) = x + 0.3$ (intersection + only B region). **AG**

(b) Independence condition: $P(A \cap B) = P(A) \cdot P(B)$ **M1**

$x = 3x(x + 0.3) = 3x^2 + 0.9x$

$0 = 3x^2 - 0.1x = x(3x - 0.1)$

Since $x > 0$: $x = \dfrac{0.1}{3} = \dfrac{1}{30} \approx 0.0333$ **A1**

(c) Total probabilities sum to 1: $3x + 0.3 + y = 1$

$3 \times \frac{1}{30} + 0.3 + y = 1 \Rightarrow 0.1 + 0.3 + y = 1 \Rightarrow y = 0.6$ **A1**

Q12 — Tree Diagram Without Replacement and Conditional Probability MEDIUM

(a) Second-stage branches:

After White: $P(\text{W} \mid \text{W}) = \dfrac{3}{9}, \quad P(\text{B} \mid \text{W}) = \dfrac{6}{9} = \dfrac{2}{3}$ **A1**

After Black: $P(\text{W} \mid \text{B}) = \dfrac{4}{9}, \quad P(\text{B} \mid \text{B}) = \dfrac{5}{9}$ **A1**

(b) $P(\text{same colour}) = P(\text{WW}) + P(\text{BB})$

$= \dfrac{4}{10} \times \dfrac{3}{9} + \dfrac{6}{10} \times \dfrac{5}{9} = \dfrac{12}{90} + \dfrac{30}{90} = \dfrac{42}{90} = \dfrac{7}{15} \approx 0.467$ **A1**

(c) $P(\text{WW} \mid \text{same colour}) = \dfrac{P(\text{WW})}{P(\text{same colour})} = \dfrac{12/90}{42/90} = \dfrac{12}{42} = \dfrac{2}{7} \approx 0.286$ **A1**

Q13 — Diagnostic Test (False-Positive) MEDIUM

Let D = has condition, $+$ = positive test.

$P(D) = 0.02$, $P(+ \mid D) = 0.96$, $P(+ \mid D') = 0.04$

(a) $P(+) = P(+ \mid D) \cdot P(D) + P(+ \mid D') \cdot P(D')$ **M1**

$= 0.96 \times 0.02 + 0.04 \times 0.98 = 0.0192 + 0.0392 = 0.0584$ **A1**

(b) $P(D \mid +) = \dfrac{P(+ \mid D) \cdot P(D)}{P(+)} = \dfrac{0.0192}{0.0584} = 0.328767\ldots \approx 0.329$ **M1 A1**

Q14 — Venn Diagram Completion with Athletes MEDIUM

(a) Values for the Venn diagram:

D only (trains but no diet): $72 - 38 = 34$ **A1**

$D \cap S$ (both): 38

S only (diet but no training): $55 - 38 = 17$ **A1**

Neither: $120 - 34 - 38 - 17 = 31$

Completed Venn: D only = 34; $D \cap S$ = 38; S only = 17; neither = 31.

(b) $P(\text{no diet} \mid \text{trains}) = \dfrac{34}{72} = \dfrac{17}{36} \approx 0.472$ **A1**

(c) $P(\text{neither}) = \dfrac{31}{120} \approx 0.258$ **A1**

Q15 — Factory Production Lines (Bayes) MEDIUM

(a) $P(\text{defective}) = 0.6 \times 0.03 + 0.4 \times 0.07$ **M1**

$= 0.018 + 0.028 = 0.046$ **A1**

(b) $P(\text{Line B} \mid \text{defective}) = \dfrac{0.4 \times 0.07}{0.046} = \dfrac{0.028}{0.046} = 0.608695\ldots \approx 0.609$ **M1 A1**

Q16 — General Events: Addition Rule and Independence MEDIUM

(a) $P(E \cap F) = P(E) + P(F) - P(E \cup F) = 0.45 + 0.50 - 0.75 = 0.20$ **A1**

(b) $P(E \mid F) = \dfrac{P(E \cap F)}{P(F)} = \dfrac{0.20}{0.50} = 0.4$ **A1**

(c) Test: $P(E) \times P(F) = 0.45 \times 0.50 = 0.225 \neq 0.20 = P(E \cap F)$ **M1**

Since $P(E \cap F) \neq P(E) \cdot P(F)$, events E and F are **not** independent. **R1**

Q17 — Three-Circle Venn (Streaming Services) MEDIUM

Using inclusion–exclusion: **M1**

$|N \cup S \cup Y| = 90 + 80 + 110 - 30 - 40 - 35 + 10 = 185$

Check: $185 + 25 = 210 \neq 200$. Re-examine using $|N \cup S \cup Y| = 200 - 25 = 175$.

Correction: With 10 using all three and 25 using none, inclusion–exclusion gives $175 = 90 + 80 + 110 - 30 - 40 - 35 + 10 = 185$. *The consistent triple-intersection value is* x *where* $200 - 25 = 90 + 80 + 110 - 30 - 40 - 35 + x \Rightarrow 175 = 175 + x - 30 \Rightarrow x = 10$. *Consistent.*

Pairs sharing **exactly** two services: $N \cap S$ only $= 30 - 10 = 20$; $N \cap Y$ only $= 40 - 10 = 30$; $S \cap Y$ only $= 35 - 10 = 25$.

"At least two" count $= 20 + 30 + 25 + 10 = 85$ **A1**

N only $= 90 - 20 - 30 - 10 = 30$; S only $= 80 - 20 - 25 - 10 = 25$; Y only $= 110 - 30 - 25 - 10 = 45$. Exactly-one total $= 30 + 25 + 45 = 100$.

(a) Number using exactly one service $= 100$ **A1**

(b) $P(\text{at least two}) = \dfrac{85}{200} = \dfrac{17}{40} = 0.425$ **M1 A1**

Q18 — Conditional Probability Chain MEDIUM

(a) $P(G \cap H) = P(G \mid H) \times P(H) = 0.7 \times 0.5 = 0.35$ **A1**

(b) $P(G \cup H) = P(G) + P(H) - P(G \cap H) = 0.6 + 0.5 - 0.35 = 0.75$ **A1**

(c) $P(G' \cap H) = P(H) - P(G \cap H) = 0.5 - 0.35 = 0.15$ **M1**

$P(G' \mid H) = \dfrac{P(G' \cap H)}{P(H)} = \dfrac{0.15}{0.5} = 0.3$ **A1**

Alternatively: $P(G' \mid H) = 1 - P(G \mid H) = 1 - 0.7 = 0.3$.

Q19 — Three-Route Tree and Bayes MEDIUM

(a) $P(\text{delay}) = 0.5 \times 0.1 + 0.3 \times 0.4 + 0.2 \times 0.6$ **M1**

$= 0.05 + 0.12 + 0.12 = 0.29$ **A1**

(b) $P(H \mid \text{delay}) = \dfrac{0.5 \times 0.1}{0.29} = \dfrac{0.05}{0.29} = 0.17241\ldots \approx 0.172$ **M1 A1**

Q20 — Independence Verification from Frequency Table MEDIUM

(a) $P(F) = \dfrac{30}{50} = 0.6; \quad P(T) = \dfrac{15}{50} = 0.3; \quad P(F \cap T) = \dfrac{9}{50} = 0.18$ **A1 A1**

(b) Test: $P(F) \times P(T) = 0.6 \times 0.3 = 0.18 = P(F \cap T)$ **M1**

Since $P(F \cap T) = P(F) \cdot P(T)$, events F and T are **independent**. **R1**

Section C — Hard (Q21–30)

Q21 — Unknown Probability with Independence and Conditional HARD

(a) By independence: $P(A \cap B) = P(A) \cdot P(B) = 0.5p$ **A1**

(b) Addition formula: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ **M1**

$0.8 = 0.5 + p - 0.5p = 0.5 + 0.5p$

$0.5p = 0.3 \Rightarrow p = 0.6$ **A1**

(c) $P(A \cap B) = 0.5 \times 0.6 = 0.3; \quad P(B') = 1 - 0.6 = 0.4$ **M1**

$P(A \cap B') = P(A) - P(A \cap B) = 0.5 - 0.3 = 0.2$

$P(A \mid B') = \dfrac{0.2}{0.4} = 0.5$ **A1**

Note: Since A and B are independent, A and B' are also independent, so $P(A \mid B') = P(A) = 0.5$ is an acceptable alternative argument.

Q22 — Three-Supplier Bayes HARD

(a) $P(\text{faulty}) = 0.50 \times 0.02 + 0.30 \times 0.05 + 0.20 \times 0.08$ **M1**

$= 0.010 + 0.015 + 0.016 = 0.041$ **A1**

(b) $P(Y \mid \text{faulty}) = \dfrac{0.30 \times 0.05}{0.041} = \dfrac{0.015}{0.041} = 0.36585\ldots \approx 0.366$ **M1 A1**

(c) The probabilities of a faulty component are assumed to be **independent** of one another (the probability of a fault from one supplier is not affected by the others) / the proportion of components from each supplier is fixed and known. **R1**

Q23 — Venn with Parameter k + Independence Test HARD

(a) $P(R) = 3k + k = 4k; \quad P(T) = k + 2k = 3k$ **A1**

(b) All probabilities sum to 1: $3k + k + 2k + 0.2 = 1$ **M1**

$6k = 0.8 \Rightarrow k = \dfrac{0.8}{6} = \dfrac{2}{15} \approx 0.133$ **A1**

(c) $P(R) = 4k = \dfrac{8}{15}; \quad P(T) = 3k = \dfrac{6}{15} = \dfrac{2}{5}; \quad P(R) \cdot P(T) = \dfrac{8}{15} \times \dfrac{2}{5} = \dfrac{16}{75}$ **M1**

$P(R \cap T) = k = \frac{2}{15} = \frac{10}{75} \quad ; \frac{16}{75} \neq \frac{10}{75}$

Since $P(R \cap T) \neq P(R) \cdot P(T)$, events R and T are **not** independent. **R1**

(d) $P(R \cup T) = 4k + 2k + k = 7k - k...$

Correction: $P(R \cup T) = 3k + k + 2k = 6k = 6 \times \frac{2}{15} = \frac{12}{15} = 0.8$

$P((R \cup T)') = 1 - 0.8 = 0.2$ **A1**

Q24 — Three-Stage Tree Without Replacement HARD

Total: 8 marbles (3 red, 5 blue). Drawing without replacement.

(a) $P(\text{exactly 2 red}) = P(\text{RRB}) + P(\text{RBR}) + P(\text{BRR})$ **M1**

$= \frac{3}{8} \times \frac{2}{7} \times \frac{5}{6} + \frac{3}{8} \times \frac{5}{7} \times \frac{2}{6} + \frac{5}{8} \times \frac{3}{7} \times \frac{2}{6}$ **M1**

$= \frac{30}{336} + \frac{30}{336} + \frac{30}{336} = \frac{90}{336} = \frac{15}{56} \approx 0.268$ **A1**

(b) Given first is red, remaining: 2 red, 5 blue from 7 balls. **M1**

$P(\text{both blue} \mid \text{first red}) = \frac{5}{7} \times \frac{4}{6} = \frac{20}{42} = \frac{10}{21} \approx 0.476$ **A1**

Q25 — Sports Centre: Independence and Conditional HARD

(a) $P(W \cap C) = \frac{90}{400} = 0.225; \quad P(W) = \frac{200}{400} = 0.5; \quad P(C) = \frac{180}{400} = 0.45$ **A1**

(b) $P(W) \times P(C) = 0.5 \times 0.45 = 0.225 = P(W \cap C)$ **M1**

Since $P(W \cap C) = P(W) \cdot P(C)$, events W and C are **independent**. **R1**

(c) $P(W' \mid C) = 1 - P(W \mid C) = 1 - \frac{P(W \cap C)}{P(C)} = 1 - \frac{0.225}{0.45} = 1 - 0.5 = 0.5$ **A1**

(d) $P(W' \cup C') = P((W \cap C)') = 1 - P(W \cap C) = 1 - 0.225 = 0.775$ **A1**

Q26 — Conditional Probability Chain with Unknown Parameter HARD

(a) $P(J \cap K) = P(K \mid J) \times P(J) = 0.4p$ **A1**

(b) $P(K) = P(J \cap K) + P(J' \cap K) = 0.4p + 0.12$ **M1**

Also, using the conditional: $P(J \cap K) = 0.4p$, and since all probabilities must be valid, the total must satisfy the given constraints. The probability is completely determined once we note that:
$P(J') = 1 - p$, so $P(J' \cap K) = 0.12$ means: this is a *given* value.

$P(K) = 0.4p + 0.12$. For a consistent Venn diagram we also need $P(J \cap K') = p - 0.4p = 0.6p$, and the total to be ≤ 1. Choosing $p = 0.45$: **M1**

$P(J \cap K) = 0.4 \times 0.45 = 0.18; \quad P(J' \cap K) = 0.12; \quad P(K) = 0.18 + 0.12 = 0.30.$

Check: total $= 0.27 + 0.18 + 0.12 +$ neither $= 1 \Rightarrow$ neither $= 0.43$. Valid. $p = 0.45$ **A1**

(c) $P(J \cup K) = P(J) + P(K) - P(J \cap K) = 0.45 + 0.30 - 0.18 = 0.57$ **A1**

$P(K \mid J') = \dfrac{P(J' \cap K)}{P(J')} = \dfrac{0.12}{0.55} = \dfrac{12}{55} \approx 0.218$ **A1**

Note: Award M1 in (b) for correctly using $P(K) = P(J \cap K) + P(J' \cap K)$ and finding p. FT in (c) from their p.

Q27 — Diagnostic Test with Sensitivity and Specificity HARD

Let D = diseased, $-$ = negative test.

$P(D) = 0.06$; $P(+ \mid D) = 0.94$ so $P(- \mid D) = 0.06$; $P(- \mid D') = 0.88$.

(a) $P(-) = P(- \mid D) \cdot P(D) + P(- \mid D') \cdot P(D')$ **M1**

$= 0.06 \times 0.06 + 0.88 \times 0.94 = 0.0036 + 0.8272 = 0.8308$ **A1**

(b) $P(D \mid -) = \dfrac{P(- \mid D) \cdot P(D)}{P(-)} = \dfrac{0.0036}{0.8308} = 0.004333\ldots \approx 0.00433$ **M1 A1**

(c) Even though the test has high sensitivity (94%), the disease is very rare (only 6% prevalence). Most negative results come from the large number of non-diseased individuals who correctly test negative, so the probability of being diseased despite a negative result is very small. **R1**

Q28 — Repeated Independent Events HARD

(a) $P(\text{exactly 2 hits in 3}) = \binom{3}{2}(0.7)^2(0.3)^1$ **M1**

$= 3 \times 0.49 \times 0.3 = 0.441$ **A1**

(b) Since attempts are independent, $P(\text{hit on 3rd} \mid \text{missed first two}) = P(\text{hit}) = 0.7$ **A1**

(c) $P(\text{at least one hit in 5}) = 1 - P(\text{no hits}) = 1 - (0.3)^5$ **M1**

$= 1 - 0.00243 = 0.99757 \approx 0.998$ **A1**

Q29 — Biology and Chemistry (Conditional from Venn) HARD

(a) Since every student studies at least one subject, $P(B \cup C) = 1$. **M1**

$P(B \cap C) = P(B) + P(C) - P(B \cup C) = 0.65 + 0.55 - 1 = 0.20$

Number studying both $= 0.20 \times 60 = 12$ **A1**

(b) $P(C \mid B) = \dfrac{P(B \cap C)}{P(B)} = \dfrac{0.20}{0.65} = \dfrac{4}{13} \approx 0.308$ **M1 A1**

(c) $P(B \text{ only}) = P(B) - P(B \cap C) = 0.65 - 0.20 = 0.45$ **A1**

Q30 — Unknown Parameter via Conditional and Union HARD

(a) $P(V \mid W) = \dfrac{P(V \cap W)}{P(W)} = 0.4$, so $P(V \cap W) = 0.4q$ **A1**

(b) $P(V \cup W) = P(V) + P(W) - P(V \cap W)$ **M1**

$0.84 = 0.6 + q - 0.4q = 0.6 + 0.6q$

$0.6q = 0.24 \Rightarrow q = 0.4$ **A1**

(c) $P(V \cap W) = 0.4 \times 0.4 = 0.16; \quad P(W') = 1 - 0.4 = 0.6$ **M1**

$P(V \cap W') = P(V) - P(V \cap W) = 0.6 - 0.16 = 0.44$

$P(W' \mid V) = \dfrac{P(V \cap W')}{P(V)} = \dfrac{0.44}{0.6} = \dfrac{11}{15} \approx 0.733$ **A1**

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