IB Math AI HL — Topic 3 [Set 1]

Trigonometric Identities & Equations

Unit Circle · Simple Identities · Graphs of Trig Functions · Solving Equations Using Graphs

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

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Name:____________________________ Date:______________

Score:__________/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated. Calculator permitted on all questions unless marked **No Calculator**.

Key Formulae — Trigonometric Identities & Equations

Pythagorean Identity:

$$\sin^2\theta + \cos^2\theta = 1$$

Derived forms:

$$\tan^2\theta + 1 = \sec^2\theta \quad \text{(HL)}$$

$$\cos^2\theta = 1 - \sin^2\theta$$

Tangent:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Exact values (degrees):

$$\sin 30^\circ = \tfrac{1}{2},\ \cos 30^\circ = \tfrac{\sqrt{3}}{2},\ \tan 30^\circ = \tfrac{1}{\sqrt{3}}$$

$$\sin 45^\circ = \cos 45^\circ = \tfrac{\sqrt{3}}{2},\ \tan 45^\circ = 1$$

$$\sin 60^\circ = \tfrac{\sqrt{3}}{2},\ \cos 60^\circ = \tfrac{1}{2},\ \tan 60^\circ = \sqrt{3}$$

General sinusoidal model:

$$f(x) = a\sin(b(x-c)) + d \quad \text{or} \quad a\cos(b(x-c)) + d$$

Parameters: amplitude $= |a|$; period $= \dfrac{2\pi}{b}$ (rad) $= \dfrac{360^\circ}{b}$ (deg); midline $y = d$

Symmetry of unit circle:

$$\sin(\pi - \theta) = \sin\theta, \quad \cos(\pi - \theta) = -\cos\theta$$

$$\sin(-\theta) = -\sin\theta, \quad \cos(-\theta) = \cos\theta$$

Quadrant signs (CAST):

$$Q1 : \text{all}+;\ Q2 : \sin+;\ Q3 : \tan+;\ Q4 : \cos+$$

Radian–degree: $180^\circ = \pi$ rad

GDC Tips

1. **Solving trig equations graphically:** Enter both sides of the equation as Y_1 and Y_2, then use `INTERSECT` (2nd $\to$ CALC $\to$ 5) to find solutions. Always set an appropriate window matching the domain given.
2. **Mode:** Always check whether the question uses **degrees** or **radians** before entering any trig calculation.

Switch via MODE menu on TI or Settings on Casio.

3. **Reading off parameters:** Use `TRACE` and `CALC` $\rightarrow$ Maximum/Minimum to read amplitude and midline from a plotted function.
4. **Sinreg / FitEq:** Use STAT $\rightarrow$ CALC $\rightarrow$ SinReg to fit a sinusoidal model to data directly.
5. **Counting solutions:** After graphing, always scroll along the full domain window to find *all* intersection points — it is easy to miss one outside the default view.

Section A — Easy **EASY** (10 questions × 3 marks = 30 marks)

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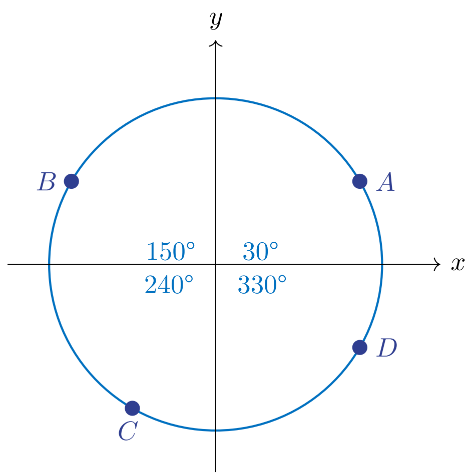
1. **EASY** **No Calculator** **[3 marks]**

The diagram shows the unit circle with four points labelled.



(a) Write down the exact coordinates of point A. **[1]**

(b) Write down the exact value of cos $150°$. **[1]**

(c) Write down the exact value of sin $240°$. **[1]**

2. EASY No Calculator **[3 marks]**

Given that sin $\theta = \dfrac{3}{5}$ and $0° < \theta < 90°$,

(a) find the exact value of cos θ, **[2]**

(b) write down the exact value of tan θ. **[1]**

3. EASY No Calculator **[3 marks]**

A function is defined by $f(x) = 4\sin(3x) - 1$, where x is in radians.

(a) Write down the amplitude of f. **[1]**

(b) Write down the period of f. **[1]**

(c) Write down the equation of the midline of f. **[1]**

4. EASY No Calculator **[3 marks]**

(a) Write down the exact value of sin $60°$. **[1]**

(b) Write down the exact value of cos $30°$. **[1]**

(c) Hence show that $\sin^2 60° + \cos^2 30° = \dfrac{3}{2}$. **[1]**

5. **EASY** **No Calculator** **[3 marks]**

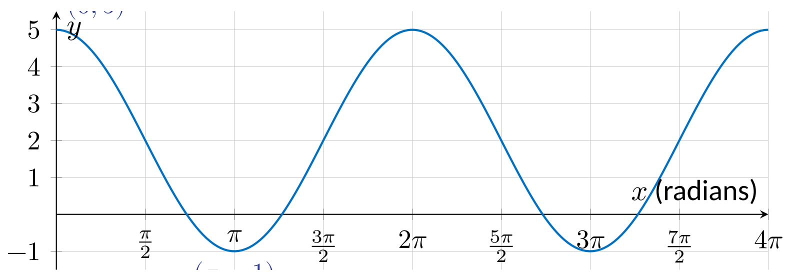
It is given that $\cos \alpha = -\dfrac{5}{13}$ and $90° < \alpha < 180°$.

(a) Show that $\sin \alpha = \dfrac{12}{13}$. [2]

(b) Write down the value of $\tan \alpha$ as a fraction. [1]

6. **EASY** **Calculator** **[3 marks]**

The graph of $y = f(x)$ is shown below.



(a) Write down the amplitude of f. [1]

(b) Write down the period of f. [1]

(c) Write down the equation of the midline of f. [1]

7. EASY No Calculator **[3 marks]**

(a) Convert $210°$ to radians. Give your answer as an exact multiple of π. [1]

(b) Convert $\dfrac{5\pi}{4}$ radians to degrees. [1]

(c) Write down the value of sin$\left(\dfrac{5\pi}{4}\right)$, giving an exact answer. [1]

8. EASY No Calculator **[3 marks]**

The graph of $g(x) = \sin(x) + k$ passes through the point $\left(\dfrac{\pi}{6}, 2\right)$.

(a) Find the value of k. [2]

(b) Write down the equation of the midline of g. [1]

9. EASY No Calculator **[3 marks]**

It is given that sin $\beta = -\dfrac{\sqrt{3}}{2}$ and cos $\beta = \dfrac{1}{2}$.

(a) Write down the exact value of tan β. [1]

(b) State the quadrant in which angle β lies, justifying your answer. [2]

10. EASY Calculator **[3 marks]**

Consider the equation $\sin(x) = 0.6$ where x is in radians.

(a) Write down one solution to this equation, giving your answer correct to 3 significant figures. **[1]**

(b) State the number of solutions of $\sin(x) = 0.6$ in the interval $0 \leq x \leq 4\pi$. **[1]**

(c) Write down the second positive solution, giving your answer correct to 3 significant figures. **[1]**

Section B — Medium **MEDIUM** (10 questions × 4 marks = 40 marks)

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11. **MEDIUM** **Calculator** **[4 marks]**

The depth of water at a harbour entrance, d metres, at time t hours after midnight is modelled by

$$d(t) = a\cos\left(\frac{2\pi}{12}t\right) + k.$$

The maximum depth is 9.4 m and the minimum depth is 2.6 m.

(a) Write down the value of a. [1]

(b) Write down the value of k. [1]

(c) Find the first time after midnight at which the depth equals 8 m. Give your answer in hours and minutes. [2]

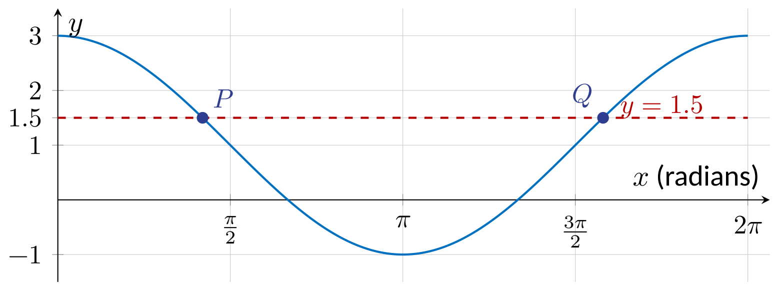
12. **MEDIUM** **No Calculator** **[4 marks]**

(a) Show that $\dfrac{\sin^2\theta}{1-\cos\theta} = 1 + \cos\theta$. [2]

(b) Hence, given that $\cos\theta = \dfrac{2}{7}$, find the exact value of $\dfrac{\sin^2\theta}{1-\cos\theta}$. [2]

13. MEDIUM Calculator **[4 marks]**

The diagram shows the graph of $y = 2\cos(x) + 1$ and the line $y = 1.5$ for $0 \le x \le 2\pi$.

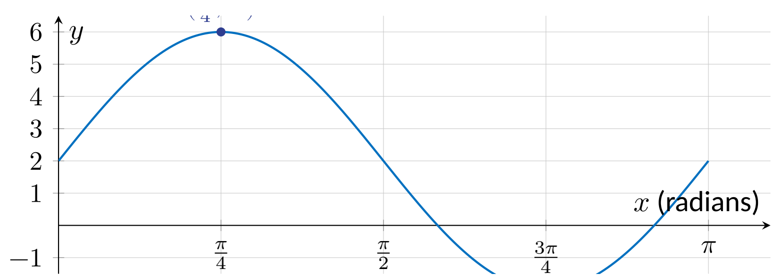


(a) Write down the x-coordinates of points P and Q, correct to 3 significant figures. **[2]**

(b) Hence state the values of x in $[0, 2\pi]$ for which $2\cos(x) + 1 > 1.5$. **[2]**

14. MEDIUM Calculator **[4 marks]**

The graph below shows one complete cycle of a sinusoidal function $f(x) = a\sin(bx) + d$.



(a) Write down the values of a, b, and d. **[3]**

(b) Hence write down $f(x)$. **[1]**

15. MEDIUM No Calculator **[4 marks]**

Solve the equation $2\sin\theta - 1 = 0$ for $0° \leq \theta \leq 360°$. Give exact answers.

(a) Write down the reference angle. [1]

(b) Hence find all solutions in $[0°, 360°]$. [2]

(c) State how many solutions exist in $[0°, 720°]$. [1]

16. MEDIUM No Calculator **[4 marks]**

(a) Show that $(1 - \sin\theta)(1 + \sin\theta) = \cos^2\theta$. [2]

(b) Given that θ is acute and $\sin\theta = \frac{\sqrt{5}}{4}$, find the exact value of $\cos\theta$. [2]

17. MEDIUM No Calculator **[4 marks]**

Solve the equation $2\cos(2x) = 1$ for $0° \leq x \leq 360°$.

(a) Show that $\cos(2x) = \frac{1}{2}$. [1]

(b) Find the two values of $2x$ in $[0°, 720°]$ for which $\cos(2x) = \frac{1}{2}$. [1]

(c) Hence write down all solutions for x in $[0°, 360°]$. [2]

18. MEDIUM Calculator **[4 marks]**

The temperature in a greenhouse, T°C, at time t hours after noon is modelled by

$$T(t) = 6\sin\left(\frac{\pi}{8}t\right) + 20.$$

(a) Write down the maximum temperature predicted by the model. [1]

(b) Find the first time after noon at which the temperature reaches 24 °C. Give your answer in hours and minutes. [2]

(c) State one limitation of this model for predicting temperatures over a 24-hour period. [1]

19. MEDIUM No Calculator **[4 marks]**

(a) Write down the period of the function $\tan(x)$, in radians. [1]

(b) Solve the equation $\tan(x) = \sqrt{3}$ for $0 \leq x \leq 2\pi$. Give exact answers. [2]

(c) State the values of x in $[0, 2\pi]$ for which $\tan(x)$ is undefined. [1]

20. MEDIUM Calculator **[4 marks]**

Consider the function $h(x) = 3\cos(x) - 2$ for $0 \leq x \leq 2\pi$.

(a) Write down the maximum and minimum values of $h(x)$. [2]

(b) Using your GDC, find the values of x in $[0, 2\pi]$ for which $h(x) = 0$. Give your answers correct to 3 significant figures. [2]

Section C — Hard **HARD** (10 questions × 5 marks = 50 marks)

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21. **HARD** **Calculator** **[5 marks]**

The height of a seat on a Ferris wheel above the ground, h metres, is modelled by

$$h(t) = a\sin(bt + c) + d$$

where t is the time in seconds after the ride begins. The following information is known.

Quantity	**Value**
Minimum height	1.5 m
Maximum height	21.5 m
Time for one complete revolution	40 s
Height at $t = 0$	1.5 m

(a) Write down the values of a, d, and b. **[3]**

(b) Explain why $c = -\dfrac{\pi}{2}$. **[1]**

(c) Find the first time after $t = 0$ at which the seat is at height 15 m. **[1]**

22. HARD No Calculator **[5 marks]**

(a) Show that $\dfrac{1}{\cos\theta} - \dfrac{\cos\theta}{1} = \dfrac{\sin^2\theta}{\cos\theta}$. [2]

(b) Hence prove that $\dfrac{\sin^2\theta}{\cos\theta} = \tan\theta \cdot \sin\theta$. [2]

(c) Given that $\sin\theta = \dfrac{2}{\sqrt{5}}$ and θ is acute, find the exact value of $\dfrac{1}{\cos\theta} - \cos\theta$. [1]

23. HARD Calculator **[5 marks]**

Consider the equation $\sin(3x) = 0.8$ where x is in radians.

(a) Find the smallest positive solution. Give your answer correct to 3 significant figures. [2]

(b) Find all solutions in the interval $0 \leq x \leq 2\pi$. [2]

(c) Determine the number of solutions in $[0, 6\pi]$. [1]

24. **HARD** **No Calculator** **[5 marks]**

A function is defined by $f(x) = k\sin(x) + 3$ where $k > 0$.

(a) The graph of f passes through the point $\left(\frac{\pi}{3}, 3 + \frac{3\sqrt{3}}{2}\right)$. Find the value of k. **[2]**

(b) Write down the maximum value of $f(x)$. **[1]**

(c) Find all values of x in $[0, 2\pi]$ for which $f(x) = 3$. **[2]**

25. **HARD** **No Calculator** **[5 marks]**

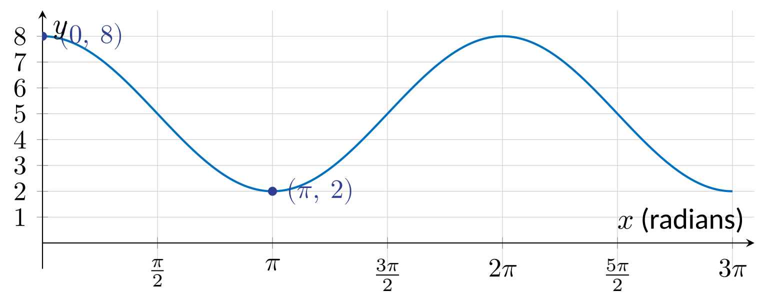
Consider the equation $2\sin^2\theta + \cos\theta - 1 = 0$ for $0° \leq \theta \leq 360°$.

(a) Show that the equation can be written as $2\cos^2\theta - \cos\theta - 1 = 0$. **[2]**

(b) Hence find all values of θ in $[0°, 360°]$. **[3]**

26. HARD Calculator **[5 marks]**

The diagram shows the graph of $y = g(x) = a\cos(bx) + d$ for $0 \le x \le 3\pi$.



(a) Write down the values of a, b, and d. **[3]**

(b) Find the values of x in $[0, 3\pi]$ for which $g(x) = 4$. Give your answers correct to 3 sf. **[2]**

27. HARD No Calculator **[5 marks]**

The function $p(x) = m\cos(x) + n$ has a maximum value of 7 and a minimum value of -3, where $m > 0$.

(a) Find the values of m and n. **[2]**

(b) Find all solutions to $p(x) = 4$ in the interval $0 \le x \le 2\pi$. Give exact answers. **[3]**

28. HARD No Calculator **[5 marks]**

Solve the equation $2\sin^2\theta - 5\sin\theta + 2 = 0$ for $0° \leq \theta \leq 360°$.

(a) Show that the equation is a quadratic in $\sin\theta$ and find the values of $\sin\theta$. **[2]**

(b) Hence find all values of θ in $[0°, 360°]$. **[3]**

29. HARD Calculator **[5 marks]**

An electrical signal is modelled by $V(t) = 12\sin(bt) + 5$, where V is voltage in volts, t is time in milliseconds, and $b > 0$.

The signal first reaches a voltage of 17 V at $t = 2.5$ milliseconds.

(a) Write down the maximum voltage of the signal. **[1]**

(b) Show that $\sin(2.5b) = 1$. **[1]**

(c) Find the value of b. **[2]**

(d) Find the period of the signal. **[1]**

30. HARD Calculator **[5 marks]**

Consider the function $f(x) = 2\sin(x) - 1$ for $0 \leq x \leq 2\pi$.

(a) Find the values of x in $[0, 2\pi]$ for which $f(x) = 0$. Give exact answers. **[2]**

(b) The graph of f lies above the x-axis for $x_1 < x < x_2$, where x_1 and x_2 are your answers from part (a). State the values of x in $[0, 2\pi]$ for which $f(x) < -1$. **[2]**

(c) Find the maximum value of $f(x)$ and the value of x at which it occurs. **[1]**

Worked Solutions

Mark scheme notation: **M** = method mark; **A** = accuracy mark; **R** = reasoning mark; **AG** = answer given (no mark). FT = follow-through.

Section A — Easy

Q1 — Unit Circle EASY

(a) A is at angle $30°$, so coordinates $= (\cos 30°,\ \sin 30°) = \left(\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$ **A1**

(b) $\cos 150° = -\cos 30° = -\dfrac{\sqrt{3}}{2}$ **A1**

(c) $\sin 240° = -\sin 60° = -\dfrac{\sqrt{3}}{2}$ **A1**

Q2 — Pythagorean Identity EASY

(a) $\sin^2\theta + \cos^2\theta = 1 \implies \cos^2\theta = 1 - \dfrac{9}{25} = \dfrac{16}{25}$ **M1**

Since θ is in Q1, $\cos\theta = \dfrac{4}{5}$ **A1**

(b) $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{3/5}{4/5} = \dfrac{3}{4}$ **A1**

Q3 — Amplitude, Period, Midline EASY

(a) Amplitude $= 4$ **A1**

(b) Period $= \dfrac{2\pi}{3}$ **A1**

(c) Midline: $y = -1$ **A1**

Q4 — Exact Values EASY

(a) $\sin 60° = \dfrac{\sqrt{3}}{2}$ **A1**

(b) $\cos 30° = \dfrac{\sqrt{3}}{2}$ **A1**

(c) $\sin^2 60° + \cos^2 30° = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4} + \frac{3}{4} = \frac{3}{2}$ **AG**

Q5 — Pythagorean Identity & Quadrant EASY

(a) $\sin^2 \alpha = 1 - \cos^2 \alpha = 1 - \frac{25}{169} = \frac{144}{169}$ **M1**

Since $90° < \alpha < 180°$, $\sin \alpha > 0$, so $\sin \alpha = \frac{12}{13}$ **AG**

(b) $\tan \alpha = \frac{12/13}{-5/13} = -\frac{12}{5}$ **A1**

Q6 — Reading Graph Parameters EASY

The graph shows $f(x) = 3\cos(x) + 2$: maximum $= 5$, minimum $= -1$.

(a) Amplitude $= \frac{5-(-1)}{2} = 3$ **A1**

(b) Period $= 2\pi$ **A1**

(c) Midline: $y = \frac{5+(-1)}{2} = 2$, so $y = 2$ **A1**

Q7 — Radian–Degree Conversion EASY

(a) $210° \times \frac{\pi}{180°} = \frac{7\pi}{6}$ **A1**

(b) $\frac{5\pi}{4} \times \frac{180°}{\pi} = 225°$ **A1**

(c) $225°$ is in Q3, reference angle $= 45°$, $\sin\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ **A1**

Q8 — Graph Transformation EASY

(a) $g\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) + k = \frac{1}{2} + k = 2$ **M1**

$k = \frac{3}{2}$ **A1**

(b) Midline: $y = \frac{3}{2}$ (equivalently $y = 1.5$) **A1**

Q9 — Tangent Identity & Quadrant EASY

(a) $\tan\beta = \dfrac{\sin\beta}{\cos\beta} = \dfrac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$ **A1**

(b) Since $\sin\beta < 0$ and $\cos\beta > 0$, both coordinates on the unit circle satisfy $x > 0, y < 0$, placing β in **Quadrant 4**. **R1 A1**

Q10 — Counting Solutions EASY

(a) $x = \arcsin(0.6) = 0.6435\ldots \approx 0.644$ rad **A1**

(b) $\sin(x) = 0.6$ gives 2 solutions per period of 2π; in $[0, 4\pi]$ there are **4** solutions. **A1**

Note: Accept any integer from 3 to 4 only if the boundary values are handled correctly. Correct answer is 4.

(c) Second positive solution: $x = \pi - 0.6435\ldots = 2.498\ldots \approx 2.50$ rad **A1**

Section B — Medium

Q11 — Sinusoidal Harbour Model MEDIUM

(a) $a = \dfrac{9.4 - 2.6}{2} = 3.4$ **A1**

(b) $k = \dfrac{9.4 + 2.6}{2} = 6.0$ **A1**

(c) Solve $3.4\cos\left(\dfrac{\pi}{6}t\right) + 6 = 8$ **M1**

$\cos\left(\dfrac{\pi}{6}t\right) = \dfrac{2}{3.4} = 0.5882\ldots$

$\dfrac{\pi}{6}t = \arccos(0.5882\ldots) = 0.9403\ldots$ rad

$t = \dfrac{6 \times 0.9403\ldots}{\pi} = 1.795\ldots \approx 1$ h 48 min **A1**

Note: Accept $t \approx 1.80$ hours or "1 hour 48 minutes". FT from their a and k.

Q12 — Identity Proof & Evaluation MEDIUM

(a) $\dfrac{\sin^2\theta}{1-\cos\theta} = \dfrac{1-\cos^2\theta}{1-\cos\theta}$ (using $\sin^2\theta = 1 - \cos^2\theta$) **M1**

$= \dfrac{(1-\cos\theta)(1+\cos\theta)}{1-\cos\theta} = 1 + \cos\theta$ **AG**

(b) $\dfrac{\sin^2\theta}{1-\cos\theta} = 1 + \cos\theta = 1 + \dfrac{2}{7} = \dfrac{9}{7}$ **M1 A1**

Note: M1 for using the result of (a); A1 for $\frac{9}{7}$ (exact). Accept equivalent fractions.

Q13 — Solve from Graph **MEDIUM**

(a) From graph / GDC: $2\cos(x) + 1 = 1.5 \implies \cos(x) = 0.25$ **(M1)**

$x_P = \arccos(0.25) = 1.3181\ldots \approx 1.32$ rad; $\quad x_Q = 2\pi - 1.3181\ldots = 4.9650\ldots \approx 4.97$ rad **A1 A1**

(b) $2\cos(x) + 1 > 1.5$ when the graph is above the line $y = 1.5$, i.e. $0 \le x < 1.32$ or $4.97 < x \le 2\pi$ **A1**

Note: FT from part (a). Strict inequalities required for the open boundary at P and Q.

Q14 — Find Equation from Graph **MEDIUM**

(a) Amplitude $= \dfrac{6-(-2)}{2} = 4$, so $a = 4$ **A1**

Period $= \pi$, so $b = \dfrac{2\pi}{\pi} = 2$ **A1**

Midline $= \dfrac{6+(-2)}{2} = 2$, so $d = 2$ **A1**

(b) $f(x) = 4\sin(2x) + 2$ **A1**

Note: Award A1A1A1A1 only if all three parameters from (a) are correct and consistent.

Q15 — Solve $2\sin\theta - 1 = 0$ Exactly **MEDIUM**

(a) $2\sin\theta = 1 \implies \sin\theta = \dfrac{1}{2}$; reference angle $= 30°$ **A1**

(b) $\sin\theta > 0$ in Q1 and Q2:

$\theta = 30°$ and $\theta = 180° - 30° = 150°$ **A1 A1**

(c) Each period $(360°)$ contributes 2 solutions; in $[0°, 720°]$ there are **4** solutions. **A1**

Q16 — Identity & Exact Value **MEDIUM**

(a) $(1 - \sin\theta)(1 + \sin\theta) = 1 - \sin^2\theta$ **M1**

$= \cos^2\theta$ (by $\sin^2\theta + \cos^2\theta = 1$) **AG**

(b) $\cos^2\theta = 1 - \sin^2\theta = 1 - \dfrac{5}{16} = \dfrac{11}{16}$ **M1**

Since θ is acute, $\cos\theta = \dfrac{\sqrt{11}}{4}$ **A1**

Q17 — Solve $2\cos(2x) = 1$ MEDIUM

(a) $2\cos(2x) = 1 \implies \cos(2x) = \dfrac{1}{2}$ **AG**

(b) $2x \in [0°, 720°]$; $\cos(2x) = \dfrac{1}{2} \implies 2x = 60°$ or $2x = 300°$; also $2x = 60° + 360° = 420°$ or $2x = 300° + 360° = 660°$ **A1**

(c) Dividing by 2: $x = 30°,\ 150°,\ 210°,\ 330°$ **A1 A1**

Note: Award A1 for any two correct values, A1 for all four. Award A0A0 if values outside $[0°, 360°]$ included without justification.

Q18 — Greenhouse Temperature Model MEDIUM

(a) Maximum temperature $= 6 + 20 = 26\,°\text{C}$ **A1**

(b) $6\sin\left(\dfrac{\pi}{8}t\right) + 20 = 24 \implies \sin\left(\dfrac{\pi}{8}t\right) = \dfrac{2}{3}$ **M1**

$\dfrac{\pi}{8}t = \arcsin\left(\dfrac{2}{3}\right) = 0.7297\ldots$

$t = \dfrac{8 \times 0.7297\ldots}{\pi} = 1.857\ldots \approx 1$ h 51 min **A1**

(c) E.g. "The sinusoidal model predicts temperatures that repeat exactly with a fixed period; in reality temperatures change over the seasons / do not follow a perfectly sinusoidal pattern over 24 hours." **R1**

Q19 — Tangent Equation MEDIUM

(a) Period of $\tan(x) = \pi$ radians **A1**

(b) $\tan(x) = \sqrt{3} \implies$ reference angle $= \dfrac{\pi}{3}$; tan is positive in Q1 and Q3:

$x = \dfrac{\pi}{3}$ and $x = \pi + \dfrac{\pi}{3} = \dfrac{4\pi}{3}$ **A1 A1**

(c) $\tan(x)$ is undefined at $x = \dfrac{\pi}{2}$ and $x = \dfrac{3\pi}{2}$ **A1**

Q20 — Max/Min and Zeros of $h(x) = 3\cos(x) - 2$ MEDIUM

(a) Maximum $= 3(1) - 2 = 1$; Minimum $= 3(-1) - 2 = -5$ **A1 A1**

(b) $3\cos(x) - 2 = 0 \implies \cos(x) = \dfrac{2}{3}$ **(M1)**

$x = \arccos\left(\dfrac{2}{3}\right) = 0.8411\ldots \approx 0.841$ rad **A1**

$x = 2\pi - 0.8411\ldots = 5.4420\ldots \approx 5.44$ rad **A1**

Note: Award A1A1 for both correct values to 3 sf. FT if a candidate uses $\cos(x) = 2/3$ *correctly.*

Section C — Hard

Q21 — Ferris Wheel: Full Parameter Recovery HARD

(a) $a = \dfrac{21.5 - 1.5}{2} = 10$ **A1**

$d = \dfrac{21.5 + 1.5}{2} = 11.5$ **A1**

Period $= 40$ s, so $b = \dfrac{2\pi}{40} = \dfrac{\pi}{20}$ **A1**

(b) At $t = 0$: $h(0) = 10\sin(c) + 11.5 = 1.5 \implies \sin(c) = -1 \implies c = -\dfrac{\pi}{2}$ **R1**

(c) $10\sin\left(\dfrac{\pi}{20}t - \dfrac{\pi}{2}\right) + 11.5 = 15$ **(M1)**

$\sin\left(\dfrac{\pi}{20}t - \dfrac{\pi}{2}\right) = 0.35$

$\dfrac{\pi}{20}t - \dfrac{\pi}{2} = \arcsin(0.35) = 0.3576\ldots$

$t = \dfrac{20(0.3576\ldots + \pi/2)}{\pi} = \dfrac{20 \times 1.9284\ldots}{\pi} = 12.27\ldots \approx 12.3$ s **A1**

Q22 — Prove Identity & Evaluate HARD

(a) $\dfrac{1}{\cos\theta} - \cos\theta = \dfrac{1 - \cos^2\theta}{\cos\theta}$ **M1**

$= \dfrac{\sin^2\theta}{\cos\theta}$ (using $\sin^2\theta = 1 - \cos^2\theta$) **AG**

(b) $\dfrac{\sin^2\theta}{\cos\theta} = \dfrac{\sin\theta \cdot \sin\theta}{\cos\theta} = \sin\theta \cdot \dfrac{\sin\theta}{\cos\theta} = \tan\theta \cdot \sin\theta$ **M1 AG**

(c) $\sin\theta = \dfrac{2}{\sqrt{5}}$, so $\cos^2\theta = 1 - \dfrac{4}{5} = \dfrac{1}{5}$, so $\cos\theta = \dfrac{1}{\sqrt{5}}$ (acute).

$\dfrac{1}{\cos\theta} - \cos\theta = \dfrac{\sin^2\theta}{\cos\theta} = \dfrac{4/5}{1/\sqrt{5}} = \dfrac{4}{5} \times \sqrt{5} = \dfrac{4\sqrt{5}}{5}$ **M1 A1**

Q23 — $\sin(3x) = 0.8$: All Solutions **HARD**

(a) Let $u = 3x$. $\sin(u) = 0.8 \implies u = \arcsin(0.8) = 0.9272\ldots$ **M1**

$x = \dfrac{0.9272\ldots}{3} = 0.3090\ldots \approx 0.309$ rad **A1**

(b) $3x \in [0, 6\pi]$; for each period of sin, two solutions exist.

In $u \in [0, 6\pi]$: the general solutions are $u = 0.9272 + 2k\pi$ and $u = \pi - 0.9272 + 2k\pi$ for integer $k \geq 0$.

Solutions for $x \in [0, 2\pi]$ (i.e. $u \in [0, 6\pi]$): **M1**

$u_1 = 0.9272$: $x = 0.309$
$u_2 = \pi - 0.9272 = 2.2143$: $x = 0.738$
$u_3 = 0.9272 + 2\pi = 7.2104$: $x = 2.403$
$u_4 = 2.2143 + 2\pi = 8.4975$: $x = 2.833$
$u_5 = 0.9272 + 4\pi = 13.554$: $x = 4.518$
$u_6 = 2.2143 + 4\pi = 14.841$: $x = 4.947$

$x \approx 0.309,\ 0.738,\ 2.40,\ 2.83,\ 4.52,\ 4.95$ **A1**

Note: Award A1 for all six values correct to 3 sf.

(c) In $[0, 6\pi]$: $3x \in [0, 18\pi]$ gives 3 full periods of $[0, 6\pi]$, each with 6 solutions, total = **18** solutions. **A1**

Q24 — Find k from External Point **HARD**

(a) Substituting $\left(\dfrac{\pi}{3},\ 3 + \dfrac{3\sqrt{3}}{2}\right)$:

$k \sin\left(\dfrac{\pi}{3}\right) + 3 = 3 + \dfrac{3\sqrt{3}}{2}$ **M1**

$k \cdot \dfrac{\sqrt{3}}{2} = \dfrac{3\sqrt{3}}{2} \implies k = 3$ **A1**

(b) Maximum of $f(x) = k + 3 = 3 + 3 = 6$ **A1**

(c) $f(x) = 3\sin(x) + 3 = 3 \implies \sin(x) = 0$

$x = 0,\ \pi,\ 2\pi$ **A1 A1**

Note: Award A1 for any two correct values, A1 for all three.

Q25 — Quadratic in $\cos\theta$ **HARD**

(a) Replace $\sin^2\theta$ with $1 - \cos^2\theta$: **M1**

$2(1 - \cos^2\theta) + \cos\theta - 1 = 0$

$2 - 2\cos^2\theta + \cos\theta - 1 = 0$

$-2\cos^2\theta + \cos\theta + 1 = 0 \implies 2\cos^2\theta - \cos\theta - 1 = 0$ **AG**

(b) Let $u = \cos\theta$: $2u^2 - u - 1 = 0 \implies (2u+1)(u-1) = 0$ **M1**

$u = 1$ or $u = -\dfrac{1}{2}$

$\cos\theta = 1 \implies \theta = 0°$ or $360°$ **A1**

$\cos\theta = -\dfrac{1}{2} \implies \theta = 120°$ or $\theta = 240°$ **A1 A1**

Note: Award A1 for factoring correctly, A1 for $\{0°, 360°\}$, *A1 for* $\{120°, 240°\}$.

Q26 — Find Equation from Graph & Solve HARD

(a) From graph: max $= 8$, min $= 2$:

$a = \dfrac{8-2}{2} = 3$ **A1**

$d = \dfrac{8+2}{2} = 5$ **A1**

Period $= 2\pi$ (from graph, one full cycle visible), so $b = \dfrac{2\pi}{2\pi} = 1$ **A1**

(b) $3\cos(x) + 5 = 4 \implies \cos(x) = -\dfrac{1}{3}$ **M1**

$x = \arccos(-\frac{1}{3}) = 1.9106\ldots \approx 1.91$ rad

$x = 2\pi - 1.9106\ldots = 4.3726\ldots \approx 4.37$ rad

$x = 2\pi + 1.9106\ldots = 8.1938\ldots \approx 8.19$ rad (within $[0, 3\pi]$ since $3\pi = 9.425$)

$x \approx 1.91,\ 4.37,\ 8.19$ rad **A1**

Note: Award A1 for any two correct values; final A1 requires all three (to 3 sf).

Q27 — Find Parameters, Solve Exactly HARD

(a) Max $= m + n = 7$; Min $= -m + n = -3$ **M1**

Adding: $2n = 4 \implies n = 2$; Subtracting: $2m = 10 \implies m = 5$ **A1**

(b) $p(x) = 5\cos(x) + 2 = 4 \implies \cos(x) = \dfrac{2}{5}$ **M1**

$x = \arccos\left(\dfrac{2}{5}\right)$; but for exact answers: $x = \arccos\left(\dfrac{2}{5}\right)$ and $x = 2\pi - \arccos\left(\dfrac{2}{5}\right)$ **A1 A1**

(Approx: $x \approx 1.16$ rad and $x \approx 5.12$ rad)

Note: "Exact" here means in the form $\arccos(2/5)$ *and* $2\pi - \arccos(2/5)$. *Award A1A1 for both. Accept decimal equivalents to 3 sf.*

Q28 — Quadratic in $\sin\theta$ HARD

(a) Let $u = \sin\theta$: $2u^2 - 5u + 2 = 0$ **M1**

$(2u-1)(u-2) = 0 \implies u = \frac{1}{2}$ or $u = 2$

Since $-1 \leq \sin\theta \leq 1$, reject $\sin\theta = 2$; so $\sin\theta = \frac{1}{2}$ **A1**

(b) $\sin\theta = \frac{1}{2}$: reference angle $= 30°$; $\sin > 0$ in Q1 and Q2:

$\theta = 30°$ **A1**

$\theta = 150°$ **A1**

No solutions from $\sin\theta = 2$ (rejected above). **R1**

Note: Award R1 for explicitly rejecting $\sin\theta = 2$ with a reason.

Q29 — Find Frequency Parameter b HARD

(a) Maximum voltage $= 12 + 5 = 17$ V **A1**

(b) At $t = 2.5$, $V = 17$: $12\sin(2.5b) + 5 = 17 \implies 12\sin(2.5b) = 12 \implies \sin(2.5b) = 1$ **AG**

(c) $\sin(2.5b) = 1 \implies 2.5b = \frac{\pi}{2}$ (first occurrence) **M1**

$b = \frac{\pi}{5} = 0.6283\ldots \approx 0.628$ **A1**

(d) Period $= \frac{2\pi}{b} = \frac{2\pi}{\pi/5} = 10$ ms **A1**

Q30 — Zeros, Sign Analysis & Maximum of $f(x) = 2\sin(x) - 1$ HARD

(a) $2\sin(x) - 1 = 0 \implies \sin(x) = \frac{1}{2}$ **M1**

$x_1 = \frac{\pi}{6}$ and $x_2 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ **A1**

(b) $f(x) < -1 \implies 2\sin(x) - 1 < -1 \implies \sin(x) < 0$ **M1**

$\sin(x) < 0$ in $(\pi, 2\pi)$, so $f(x) < -1$ for $\pi < x < 2\pi$ **A1**

Note: Accept open or closed interval at endpoints for full marks. The strict inequality $f < -1$ is met strictly inside $(\pi, 2\pi)$.

(c) Maximum of $f(x) = 2(1) - 1 = 1$, occurring at $x = \frac{\pi}{2}$ **A1**

Common Mistakes to Avoid

1. **Degree vs. radian mode.** Always check your GDC is set to the correct angle mode before any trig calculation. A common error is computing sin(30) in radian mode (getting -0.988) instead of 0.5.
2. **Pythagorean identity signs.** When finding cos θ from sin θ, always determine the correct sign using the given quadrant. $\cos^2\theta = \frac{16}{25}$ gives $\cos\theta = \pm\frac{4}{5}$; the quadrant decides which.
3. **Missing solutions.** When solving sin$(x) = k$, there are always *two* solutions per period (unless $k = \pm1$). Use the symmetry $x_2 = \pi - x_1$ (for sin) or $x_2 = 2\pi - x_1$ (for cos) to find the second solution.
4. **Period with horizontal stretch.** The period of sin(bx) is $\dfrac{2\pi}{b}$, NOT 2π. For sin$(3x)$, the period is $\dfrac{2\pi}{3}$.
5. **Quadratic trig equations.** Always check whether your values of sin θ or cos θ are in $[-1, 1]$. Reject any value outside this range and state the reason.
6. **"Show that" steps.** When proving an identity, always start from one side only. Do not manipulate both sides simultaneously.