IB Math AI HL — Topic 2 [Set 1]

Transformations of Graphs

Translations · Reflections · Stretches · Composite Transformations

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

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Name:______________________________ Date:_______________

Score:__________/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 sf unless stated. A GDC is permitted throughout.

Key Formulae & Rules — Transformations of Graphs

Translations

$f(x-h)+k$: right by h, up by k
$f(x) \to f(x-h)$: right by h

Reflections

$-f(x)$: reflect in x-axis
$f(-x)$: reflect in y-axis

Stretches

$a\,f(x)$: vertical stretch scale factor a
$f(bx)$: horizontal stretch SF $\frac{1}{b}$
$f(\frac{x}{b})$: horizontal stretch SF b

Composite

Apply in order; stretches and reflections before translations (unless otherwise stated).

Effect on key points: $(x,y) \to (x+h,\ ay)$ for $a\,f(x-h)$; $(x,y) \to (\frac{x}{b},\ y)$ for $f(bx)$.

Effect on asymptotes: a vertical asymptote $x=c$ maps to $x=c+h$ under translation by h; a horizontal asymptote $y=d$ maps to $y=ad+k$ under vertical stretch a followed by shift k.

GDC Tips for Transformations

1. To graph a transformed function, enter it directly (e.g. `Y1 = 2*sin(X-1)+3`).
2. Use **TRACE** or **CALC** $\to$ **value** to verify key points (intercepts, turning points).
3. To compare $f(x)$ and its image, store them in `Y1` and `Y2` and graph both simultaneously.
4. For finding intercepts of a transformed function, use **CALC** $\to$ **zero** or **intersect**.
5. When the question gives a graph and asks you to identify the transformation, check one point on the original and its image — the vector/scale factor is determined from that single pair.

Common Mistakes to Avoid

1. **Direction of horizontal translation.** $f(x-3)$ shifts *right* by 3, NOT left. The sign inside the bracket is opposite to the direction.
2. **Horizontal stretch factor.** $f(2x)$ is a horizontal stretch by factor $\frac{1}{2}$, not 2. The stretch factor is the *reciprocal* of the coefficient of x.
3. **Order matters for composite transformations.** $2f(x)+1$ applies the vertical stretch before the translation; $2(f(x)+1)$ applies them differently.
4. **Reflections change signs of outputs/inputs only.** $-f(x)$ negates all y-values; $f(-x)$ negates all x-values. Do not confuse the two.
5. **Asymptotes transform too.** A horizontal asymptote $y=c$ after a vertical stretch by a becomes $y=ac$; after a vertical translation by k becomes $y=c+k$.
6. **Domain and range.** A horizontal stretch changes the domain; a vertical stretch changes the range. Always update both when a transformation is applied.

Section A — Easy **EASY** (10 questions × 3 marks = 30 marks)

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1. **EASY** **No Calculator** **[3 marks]**

The graph of $y = f(x)$ is translated to give the graph of $y = f(x-4)+3$.

(a) Write down the horizontal translation applied. [1]

(b) Write down the vertical translation applied. [1]

(c) The point $P(1, -2)$ lies on $y = f(x)$. Write down the coordinates of the image of P on $y = f(x-4)+3$. [1]

2. **EASY** **No Calculator** **[3 marks]**

The graph of $y = g(x)$ is reflected in the x-axis to give the graph of $y = h(x)$.

(a) Write down the equation of $y = h(x)$ in terms of $g(x)$. [1]

(b) The point $A(3, 7)$ lies on $y = g(x)$. Write down the coordinates of the image of A on $y = h(x)$. [1]

(c) The graph of $y = g(x)$ has a minimum at $(-1, 2)$. Write down the coordinates of the corresponding point on $y = h(x)$, and state whether it is a maximum or minimum. [1]

3. EASY No Calculator **[3 marks]**

The graph of $y = p(x)$ is stretched vertically by a scale factor of 3 to give the graph of $y = q(x)$.

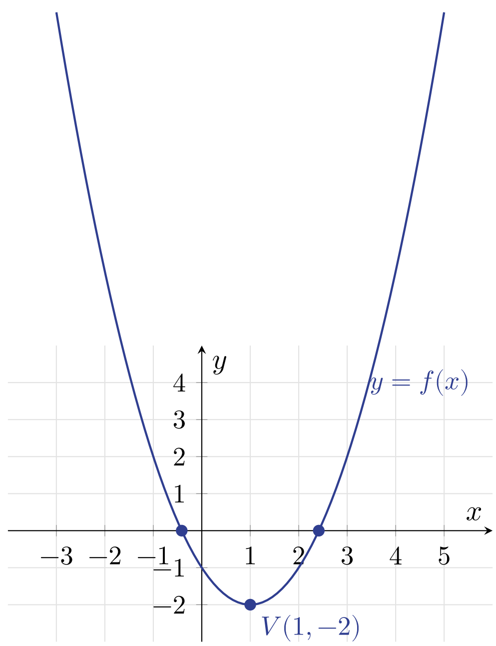
(a) Write down the equation of $y = q(x)$ in terms of $p(x)$. [1]

(b) The point $B(2, 5)$ lies on $y = p(x)$. Write down the image of B on $y = q(x)$. [1]

(c) The graph of $y = p(x)$ has a horizontal asymptote $y = 4$. Write down the equation of the horizontal asymptote of $y = q(x)$. [1]

4. EASY Calculator **[3 marks]**

The diagram below shows the graph of $y = f(x)$ for $-3 \leq x \leq 5$.



The graph of $y = f(x)$ is translated 3 units to the right to give $y = f(x - 3)$.

(a) Write down the x-coordinate of the vertex of $y = f(x - 3)$. [1]

(b) Write down the y-coordinate of the vertex of $y = f(x - 3)$. [1]

(c) Write down the x-intercepts of $y = f(x - 3)$, giving each to 3 significant figures. [1]

5. EASY No Calculator **[3 marks]**

Let $f(x) = 3x^2 - 2x + 1$.

(a) Find $f(-x)$. [1]

(b) Hence write down the equation of the reflection of $y = f(x)$ in the y-axis. [1]

(c) The point $C(2, 9)$ lies on $y = f(x)$. Write down the coordinates of the image of C after reflection in the y-axis. [1]

6. EASY Calculator **[3 marks]**

The graph of $y = x^2 + 6x + 5$ is translated 4 units downward to give the graph of $y = r(x)$.

(a) Write down the equation of $y = r(x)$. [1]

(b) Find the y-intercept of $y = r(x)$. [1]

(c) Use your GDC to find the x-intercepts of $y = r(x)$, giving your answers correct to 3 significant figures. [1]

7. EASY No Calculator **[3 marks]**

The graph of $y = \sin(x)$ is stretched horizontally by a scale factor of $\frac{1}{3}$ to give the graph of $y = s(x)$.

(a) Write down the equation of $y = s(x)$. [1]

(b) Write down the period of $y = s(x)$. [1]

(c) Write down the amplitude of $y = s(x)$. [1]

8. EASY No Calculator **[3 marks]**

The graph of $y = f(x)$ is transformed to give the graph of $y = f(x) - 5$.

(a) Describe fully the transformation that maps $y = f(x)$ onto $y = f(x) - 5$. [1]

(b) The graph of $y = f(x)$ passes through the point $(0, 3)$. Write down the coordinates of the corresponding point on $y = f(x) - 5$. [1]

(c) The graph of $y = f(x)$ has a horizontal asymptote $y = 2$. Write down the equation of the horizontal asymptote of $y = f(x) - 5$. [1]

9. EASY No Calculator **[3 marks]**

The graph of $y = f(x)$ passes through the point $D(4, 6)$. The graph of $y = af(x)$ passes through the point $D'(4, 18)$.

(a) Write down the value of a. [1]

(b) The graph of $y = f(x)$ has a y-intercept at $(0, 2)$. Write down the y-intercept of $y = af(x)$. [1]

(c) Describe fully the transformation that maps $y = f(x)$ onto $y = af(x)$. [1]

10. EASY No Calculator **[3 marks]**

Let $f(x) = 2^x - 1$.

(a) Write down the horizontal asymptote of $y = f(x)$. [1]

(b) Write down the equation of $y = -f(x)$ in the form $y = a^x + b$, where $a, b \in \mathbb{Z}$. [1]

(c) Write down the horizontal asymptote of $y = -f(x)$. [1]

Section B — Medium **MEDIUM** (10 questions × 4 marks = 40 marks)

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11. **MEDIUM** **No Calculator** **[4 marks]**

The graph of $y = f(x)$ is transformed to give the graph of $y = 2f(x+3)$.

(a) Describe fully the two transformations applied, stating the order in which they are applied. [2]

(b) The point $E(-1, 4)$ lies on $y = f(x)$. Find the coordinates of the image of E on $y = 2f(x+3)$. [2]

12. **MEDIUM** **No Calculator** **[4 marks]**

The graph of $y = g(x)$ is obtained from $y = f(x)$ by a vertical stretch of scale factor 4, followed by a translation of $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$.

(a) Write down the equation of $y = g(x)$ in terms of $f(x)$. [1]

(b) Given that $f(x) = \ln(x)$ for $x > 0$, find $g(x)$. [1]

(c) Find the x-intercept of $y = g(x)$, giving your answer in exact form. [2]

13. MEDIUM Calculator **[4 marks]**

The graph of $y = x^2 - 4$ is stretched horizontally by scale factor 2 to give the graph of $y = h(x)$.

(a) Find the equation of $y = h(x)$. [1]

(b) Find the x-intercepts of $y = h(x)$. [2]

(c) Find the coordinates of the vertex of $y = h(x)$. [1]

14. MEDIUM No Calculator **[4 marks]**

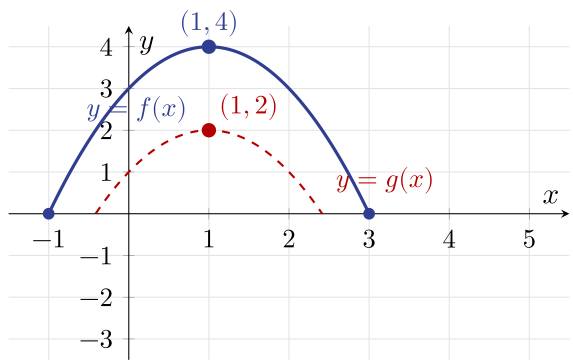
Let $f(x) = \dfrac{1}{x}$, $x \neq 0$.

(a) The graph of $y = f(x)$ is reflected in the x-axis and then translated 2 units to the right. Write down the equation of the resulting curve. [2]

(b) Write down the equations of the asymptotes of the resulting curve. [2]

15. MEDIUM No Calculator **[4 marks]**

The diagram below shows the graph of $y = f(x)$ (solid) and the graph of $y = g(x)$ (dashed).



(a) Describe fully the single transformation that maps $y = f(x)$ onto $y = g(x)$. [2]

(b) Write down the equation of $y = g(x)$ in terms of $f(x)$. [1]

(c) Write down the x-intercepts of $y = g(x)$, giving each in the form $a \pm \sqrt{b}$. [1]

16. MEDIUM No Calculator **[4 marks]**

The graph of $y = \cos(x)$, where x is in radians, is transformed by:

- first, a horizontal stretch by scale factor $\frac{1}{2}$,
- then, a reflection in the x-axis.

(a) Write down the equation of the resulting graph. [2]

(b) State the period of the resulting graph. [1]

(c) Write down the maximum value of the resulting function. [1]

17. MEDIUM No Calculator **[4 marks]**

The function $f(x) = \dfrac{1}{x-3} + 2$ can be obtained from $y = \dfrac{1}{x}$ by a translation.

(a) Write down the translation vector. [2]

(b) Write down the equations of the asymptotes of $y = f(x)$. [1]

(c) Find the y-intercept of $y = f(x)$, giving your answer as an exact fraction. [1]

18. MEDIUM Calculator **[4 marks]**

The graph of $y = \sin(x)$, where x is in radians, undergoes the following transformations in order:

- a vertical stretch by scale factor 3,
- a translation of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

(a) Write down the equation of the resulting function $y = t(x)$. [1]

(b) State the range of $y = t(x)$. [1]

(c) Find the smallest positive value of x for which $y = t(x)$ achieves its maximum value. [2]

19. MEDIUM No Calculator **[4 marks]**

Let $f(x) = 3^x$.

(a) Write down the equation of the reflection of $y = f(x)$ in the y-axis. [1]

(b) On the same axes, both graphs have the same y-intercept. Write down the coordinates of this common point. [1]

(c) The graph of $y = f(-x)$ is further translated by $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ to give $y = k(x)$. Write down the equation of $y = k(x)$ and state its horizontal asymptote. [2]

20. MEDIUM No Calculator **[4 marks]**

The table below gives selected values of $y = f(x)$ and $y = g(x)$.

x	-2	0	1	3
$f(x)$	6	2	-1	5
$g(x)$	12	4	-2	10

(a) Write down the relationship between $g(x)$ and $f(x)$. [1]

(b) Describe fully the transformation that maps $y = f(x)$ onto $y = g(x)$. [2]

(c) The graph of $y = f(x)$ has a minimum at $(1, -1)$. Write down the coordinates of the minimum of $y = g(x)$, and state whether it is a maximum or minimum. [1]

Section C — Hard **HARD** (10 questions × 5 marks = 50 marks)

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21. **HARD** **No Calculator** **[5 marks]**

The graph of $y = f(x)$ passes through the point $F(2, 3)$. The graph of $y = af(x - b) + 1$ passes through the point $F'(5, 7)$.

(a) Write down the value of b. [1]

(b) Find the value of a. [2]

(c) The graph of $y = f(x)$ has a minimum at $(2, 3)$. Find the coordinates of the minimum of $y = af(x-b)+1$, and state whether it is a maximum or minimum. [2]

22. **HARD** **No Calculator** **[5 marks]**

The graph of $y = \frac{6}{x}$, $x \neq 0$, undergoes the following transformations in order:

- a horizontal stretch by scale factor 3,
- a translation by $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$.

(a) Find the equation of the final image curve. [3]

(b) Write down the equations of both asymptotes of the final image curve. [2]

23. HARD No Calculator **[5 marks]**

The graph of $y = g(x)$ is obtained from $y = f(x)$ by:

- a reflection in the y-axis,
- followed by a vertical stretch by scale factor 2.

Given that $g(x) = 2e^{-x}$, find $f(x)$.

(a) Write down the equation that relates $g(x)$ to $f(-x)$. [1]

(b) Hence find $f(x)$. [2]

(c) State the range of $y = f(x)$, and hence state the range of $y = g(x)$. [2]

24. HARD Calculator **[5 marks]**

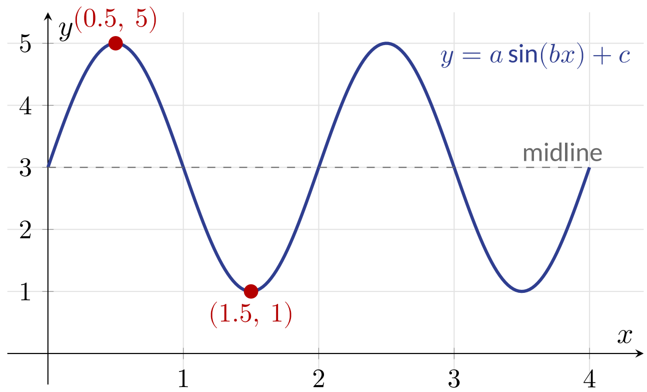
The graph of $y = f(x)$ has a zero at $x = 4$. The graph of $y = f(2x - k)$ has a zero at $x = 3$.

(a) Show that $k = 2$. [2]

(b) Given that $f(x) = x^2 - 5x + 4$, write down the equation of $y = f(2x - 2)$ in expanded form. [3]

25. **HARD** **Calculator** **[5 marks]**

The diagram shows one complete cycle of the graph of $y = a\,\text{sin}(bx) + c$, where $a, b, c \in \mathbb{R}^+$.



(a) Write down the value of a. [1]

(b) Write down the value of c. [1]

(c) Find the value of b. [2]

(d) Hence write down the period of the function. [1]

26. **HARD** **No Calculator** **[5 marks]**

Let $f(x) = \sqrt{x}$, defined for $x \geq 0$.

The function f is transformed first by a horizontal stretch of scale factor $\frac{1}{4}$, giving $y = h(x)$, and then $y = h(x)$ is reflected in the x-axis and translated 5 units upward to give $y = k(x)$.

(a) Write down the equation of $y = h(x)$. [1]

(b) Write down the equation of $y = k(x)$. [2]

(c) State the domain and range of $y = k(x)$. [2]

27. HARD No Calculator **[5 marks]**

Let $f(x) = 2x + 3$.

(a) Find $f^{-1}(x)$. [2]

(b) Describe the geometric relationship between the graph of $y = f(x)$ and the graph of $y = f^{-1}(x)$. [1]

(c) Find the coordinates of the point where the graphs of $y = f(x)$ and $y = f^{-1}(x)$ intersect. [2]

28. HARD Calculator **[5 marks]**

The graph of $y = x^2$ is first translated 2 units to the left, and then stretched horizontally by scale factor 3, to give the graph of $y = m(x)$.

(a) Find the equation of $y = m(x)$ in expanded form. [3]

(b) Find the vertex of $y = m(x)$. [1]

(c) Find the x-intercepts of $y = m(x)$. [1]

29. HARD No Calculator **[5 marks]**

The function $h(x) = 3f(2x - 4) - 1$ is obtained from $y = f(x)$ by a sequence of transformations.

(a) Describe fully each transformation in the sequence, stating the correct order. [3]

(b) The graph of $y = f(x)$ passes through the point $G(3, 2)$. Find the coordinates of the image of G on $y = h(x)$. [2]

30. HARD Calculator **[5 marks]**

Let $f(x) = 2^x$.

The graph of $y = f(x - p)$ intersects the x-axis at a point where the y-value is equal to $\frac{1}{4}$ on the untransformed graph $y = f(x)$.

It is also given that the graph of $y = q\,f(x)$ passes through the point $(3, 40)$.

(a) Find the value of p. [2]

(b) Find the value of q, giving your answer as an exact fraction. [2]

(c) State the horizontal asymptote of $y = q\,f(x) - 6$. [1]

WORKED SOLUTIONS

Official Mark-Scheme Style — M / A / R notation

Section A — Easy

Q1 — Translations (3 marks) EASY

(a) Translation 4 units to the right. **A1**

(b) Translation 3 units upward. **A1**

(c) Image of $P(1, -2)$: $x' = 1 + 4 = 5$; $y' = -2 + 3 = 1$. Coordinates: $(5, 1)$. **A1**

Q2 — Reflection in x-axis (3 marks) EASY

(a) $y = h(x) = -g(x)$. **A1**

(b) Image of $A(3, 7)$: $(3,\ -7)$. **A1**

(c) Coordinates $(-1, -2)$; it is a **maximum**. **A1**

Note: Both coordinates AND the max/minimum label required for A1.

Q3 — Vertical stretch (3 marks) EASY

(a) $y = q(x) = 3\,p(x)$. **A1**

(b) Image of $B(2, 5)$: $(2,\ 15)$. **A1**

(c) Horizontal asymptote of $y = 3\,p(x)$: $y = 3 \times 4 = 12$, i.e. $y = 12$. **A1**

Q4 — Translation from graph (3 marks) EASY

The vertex of $y = f(x)$ is at $(1, -2)$. A horizontal translation of $+3$ shifts x to $x + 3$.

(a) x-coordinate of vertex: $1 + 3 = 4$. **A1**

(b) y-coordinate of vertex: -2 (unchanged by horizontal translation). **A1**

(c) Original x-intercepts of $f(x) = (x-1)^2 - 2 = 0$: $x = 1 \pm \sqrt{2}$, i.e. $x \approx -0.414$ and $x \approx 2.41$.
After translation $+3$: $x \approx -0.414 + 3 = 2.59$ and $x \approx 2.41 + 3 = 5.41$. **A1**

Note: Accept $x = 4 - \sqrt{2}$ and $x = 4 + \sqrt{2}$ as equivalent exact answers.

Q5 — Reflection in y-axis (3 marks) EASY

(a) $f(-x) = 3(-x)^2 - 2(-x) + 1 = 3x^2 + 2x + 1.$ **A1**

(b) Equation of reflection in y-axis: $y = 3x^2 + 2x + 1.$ **A1**

(c) Image of $C(2, 9)$ under reflection in y-axis: $(-2,\ 9).$ **A1**

Q6 — Vertical translation (3 marks) EASY

(a) $r(x) = x^2 + 6x + 5 - 4 = x^2 + 6x + 1.$ **A1**

(b) y-intercept: $r(0) = 0 + 0 + 1 = 1$, so $(0,\ 1).$ **A1**

(c) $x^2 + 6x + 1 = 0 \implies x = \dfrac{-6 \pm \sqrt{36 - 4}}{2} = -3 \pm 2\sqrt{2}.$
$x \approx -0.172$ or $x \approx -5.83.$ **A1**

Note: Accept $x = -3 \pm 2\sqrt{2}$ as equivalent exact answers.

Q7 — Horizontal stretch of sinusoidal (3 marks) EASY

A horizontal stretch of scale factor $\frac{1}{3}$ replaces x with $3x$ (coefficient of x is 3).

(a) $y = s(x) = \sin(3x).$ **A1**

(b) Period: $\dfrac{2\pi}{3}$ (radians). **A1**

(c) Amplitude: 1 (unchanged by a horizontal stretch). **A1**

Q8 — Vertical translation description (3 marks) EASY

(a) Translation of 5 units downward (or: vertical translation by -5). **A1**

(b) Image of $(0, 3)$: $(0,\ -2).$ **A1**

(c) Horizontal asymptote: $y = 2 - 5 = -3$, i.e. $y = -3.$ **A1**

Q9 — Vertical stretch from points (3 marks) EASY

(a) $af(4) = 18$ and $f(4) = 6$, so $a = 18 \div 6 = 3.$ **A1**

(b) y-intercept of $y = 3f(x)$: $3 \times 2 = 6$, so $(0,\ 6).$ **A1**

(c) Vertical stretch by scale factor 3 (about the x-axis). **A1**

Q10 — Reflection of exponential; asymptote (3 marks) EASY

(a) Horizontal asymptote of $y = 2^x - 1$: $y = -1$. **A1**

(b) $-f(x) = -(2^x - 1) = -2^x + 1$; so $y = -2^x + 1$. (Here $a = 2$, $b = 1$.) **A1**

Note: Accept $y = 1 - 2^x$ equivalently.

(c) Horizontal asymptote of $y = -2^x + 1$: as $x \to \infty$, $-2^x \to -\infty$; as $x \to -\infty$, $-2^x \to 0$, so $y \to 1$.
Asymptote: $y = 1$. **A1**

Section B — Medium

Q11 — Composite transformation: description and point image (4 marks) MEDIUM

(a) Two transformations:
(1) Translation 3 units to the left (horizontal translation of -3). **A1**
(2) Vertical stretch by scale factor 2 (about the x-axis), applied after the translation. **A1**

Note: Both transformations must be named correctly with their magnitudes for A1A1. Order: translation first, stretch second.

(b) Apply translation: $x' = -1 - 3 = -4$; $y' = f(-4)$... but using the mapping directly:
For $y = 2f(x+3)$: $x \to x - 3$ maps $x = -1$ to input $-1 + 3 - 3 = -1$...
More directly: if $E(-1, 4)$ is on $y = f(x)$, then $f(-1) = 4$.
For $y = 2f(x+3)$ to use $f(-1)$, we need $x + 3 = -1$, i.e. $x = -4$.
Output: $2 \times f(-1) = 2 \times 4 = 8$.
Image: $(-4, 8)$. **M1 A1**

Q12 — Transformation sequence then x-intercept (4 marks) MEDIUM

(a) $g(x) = 4f(x) - 3$. **A1**

(b) $g(x) = 4\ln(x) - 3$. **A1**

(c) Set $g(x) = 0$: $4\ln(x) = 3 \implies \ln(x) = \dfrac{3}{4} \implies x = e^{3/4}$. **M1 A1**

Note: Accept $x = e^{0.75}$ or $x \approx 2.12$ (3 s.f.).

Q13 — Horizontal stretch of quadratic (4 marks) MEDIUM

A horizontal stretch by scale factor 2 replaces x with $\frac{x}{2}$:

(a) $h(x) = \left(\dfrac{x}{2}\right)^2 - 4 = \dfrac{x^2}{4} - 4$. **A1**

(b) Set $h(x) = 0$: $\dfrac{x^2}{4} = 4 \implies x^2 = 16 \implies x = \pm 4$. **M1 A1**

(c) Vertex at $x = 0$ (axis of symmetry), $h(0) = -4$. Vertex: $(0, -4)$. **A1**

Q14 — Combined reflection and translation of rational (4 marks) MEDIUM

(a) Reflect $y = \frac{1}{x}$ in x-axis: $y = -\frac{1}{x}$. **M1**

Translate 2 units right (replace x with $x - 2$): $y = -\frac{1}{x-2}$. **A1**

(b) Vertical asymptote: $x = 2$. **A1**
Horizontal asymptote: $y = 0$. **A1**

Q15 — Identify transformation from graph (4 marks) MEDIUM

The vertex moves from $(1, 4)$ to $(1, 2)$: the x-coordinate is unchanged and the y-coordinate decreases by 2.

(a) Single transformation: translation of $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ (2 units downward). **A1 A1**

Note: Award A1 for identifying translation and A1 for stating the vector/direction and magnitude.

(b) $g(x) = f(x) - 2$. **A1**

(c) $g(x) = 0 \implies -(x-1)^2 + 2 = 0 \implies (x-1)^2 = 2 \implies x = 1 \pm \sqrt{2}$. **A1**

Q16 — Horizontal stretch then reflection of cosine (4 marks) MEDIUM

Horizontal stretch SF $\frac{1}{2}$ replaces x with $2x$: $y = \cos(2x)$. **(M1)**
Reflection in x-axis negates: $y = -\cos(2x)$.

(a) $y = -\cos(2x)$. **A1 A1**

Note: A1 for the argument $2x$; A1 for the negative sign.

(b) Period: $\frac{2\pi}{2} = \pi$ (radians). **A1**

(c) Maximum value: $-\cos(2x)$ reaches maximum 1 (when $\cos(2x) = -1$). **A1**

Q17 — Translation vector from rational function (4 marks) MEDIUM

$f(x) = \frac{1}{x-3} + 2$ is $y = \frac{1}{x}$ with x replaced by $x - 3$ and then $+2$ added.

(a) Translation vector: $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$. **A1 A1**

Note: A1 for each component.

(b) Vertical asymptote $x = 3$; horizontal asymptote $y = 2$. **A1**

(c) $f(0) = \dfrac{1}{0-3} + 2 = -\dfrac{1}{3} + 2 = \dfrac{5}{3}$. y-intercept: $\left(0,\ \dfrac{5}{3}\right)$. **A1**

Q18 — Composite: vertical stretch then vertical translation of sine (4 marks) MEDIUM

(a) Vertical stretch SF 3: $y = 3\sin(x)$. Translation $+1$: $y = 3\sin(x) + 1 = t(x)$. **A1**

(b) Range of $\sin(x)$ is $[-1, 1]$; after $\times 3$: $[-3, 3]$; after $+1$: $[-2, 4]$. Range: $-2 \le y \le 4$. **A1**

(c) Maximum when $3\sin(x) + 1 = 4$, i.e. $\sin(x) = 1$. **M1**
Smallest positive x: $x = \dfrac{\pi}{2}$ radians (or $x \approx 1.57$). **A1**

Q19 — Reflection in y-axis of exponential (4 marks) MEDIUM

(a) Reflection in y-axis: replace x with $-x$: $y = 3^{-x}$ (or equivalently $y = \left(\frac{1}{3}\right)^x$). **A1**

(b) At $x = 0$: $f(0) = 3^0 = 1$ and $f(0) = 3^{-0} = 1$. Common y-intercept: $(0,\ 1)$. **A1**

(c) $k(x) = 3^{-x} - 1$. **A1**
Horizontal asymptote: as $x \to \infty$, $3^{-x} \to 0$, so $k(x) \to -1$. Asymptote: $y = -1$. **A1**

Q20 — Identify transformation from table (4 marks) MEDIUM

Checking each x-value: $g(x) = 2 \times f(x)$ in every case ($12 = 2 \times 6$, $4 = 2 \times 2$, $-2 = 2 \times (-1)$, $10 = 2 \times 5$).

(a) $g(x) = 2f(x)$. **A1**

(b) Vertical stretch by scale factor 2 (about the x-axis). **A1 A1**

Note: A1 for "vertical stretch" and A1 for "scale factor 2".

(c) Minimum of $y = f(x)$ at $(1, -1)$; image under $\times 2$: $(1,\ -2)$.
It remains a **minimum** (stretch does not change concavity). **A1**

Section C — Hard

Q21 — Unknown parameters from point condition (5 marks) HARD

$F(2, 3)$ on $y = f(x)$ means $f(2) = 3$.
$F'(5, 7)$ on $y = af(x - b) + 1$ means $af(5 - b) + 1 = 7$, so $af(5 - b) = 6$.

(a) For the x-coordinates to correspond under $x \to x - b$ mapping $2 \to 5 - b$: $5 - b = 2$, so $b = 3$. **A1**

(b) With $b = 3$: $a \cdot f(2) = 6 \implies a \times 3 = 6 \implies a = 2$. **M1 A1**

(c) Minimum of $y = f(x)$ at $(2, 3)$. Apply $x \to x + 3$: $x' = 5$. Apply $y \to 2y + 1$: $y' = 7$. Coordinates: $(5, 7)$. Since $a = 2 > 0$, the vertical stretch does not change concavity: it remains a **minimum**. **A1 A1**

Q22 — Composite transformation of rational; asymptotes (5 marks) HARD

Start: $y = \dfrac{6}{x}$.

Step 1 — Horizontal stretch SF 3: replace x with $\dfrac{x}{3}$: $y = \dfrac{6}{x/3} = \dfrac{18}{x}$. **M1**

Step 2 — Translation $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$: replace x with $x - 1$, add -2: $y = \dfrac{18}{x-1} - 2$. **M1**

(a) Final equation: $y = \dfrac{18}{x-1} - 2$. **A1**

(b) Vertical asymptote: $x = 1$. **A1**
Horizontal asymptote: $y = -2$. **A1**

Q23 — Reverse-engineer original from transformed (5 marks) HARD

The transformation sequence is: $f(x) \xrightarrow{\text{reflect in } y\text{-axis}} f(-x) \xrightarrow{\times 2} 2f(-x)$.
So $g(x) = 2f(-x)$, given as $g(x) = 2e^{-x}$.

(a) $g(x) = 2f(-x)$. **A1**

(b) $2f(-x) = 2e^{-x} \implies f(-x) = e^{-x}$.
Replace $-x$ with x: $f(x) = e^x$. **M1 A1**

(c) Range of $y = f(x) = e^x$: $y > 0$ (i.e. $(0, \infty)$). **A1**
Range of $y = g(x) = 2e^{-x}$: also $y > 0$ (since $2e^{-x} > 0$ for all x). **A1**

Q24 — Unknown parameter from zero condition (5 marks) HARD

(a) $f(x)$ has a zero at $x = 4$, so $f(4) = 0$.

$f(2x - k) = 0$ when $2x - k = 4$, i.e. $x = \dfrac{4+k}{2}$.

This zero occurs at $x = 3$: $3 = \dfrac{4+k}{2} \implies 6 = 4 + k \implies k = 2$. **M1 AG**

(b) $f(2x - 2) = (2x - 2)^2 - 5(2x - 2) + 4$ **M1**
$= 4x^2 - 8x + 4 - 10x + 10 + 4$ **M1**
$= 4x^2 - 18x + 18$. **A1**

Q25 — Sinusoidal parameters from graph (5 marks) HARD

From the graph: maximum $= 5$, minimum $= 1$.

(a) Amplitude $a = \dfrac{5-1}{2} = 2$. **A1**

(b) Midline $c = \dfrac{5+1}{2} = 3$. **A1**

(c) The maximum occurs at $x = 0.5$ and minimum at $x = 1.5$: half-period $= 1.5 - 0.5 = 1$, so period $T = 2$.
$T = \dfrac{2\pi}{b} = 2 \implies b = \pi$. **M1 A1**

(d) Period: $T = 2$. **A1**

Q26 — Composite of stretch and reflection then translation; domain and range (5 marks) HARD

(a) Horizontal stretch SF $\frac{1}{4}$ replaces x with $4x$: $h(x) = \sqrt{4x} = 2\sqrt{x}$. **A1**

(b) Reflect $y = h(x)$ in x-axis: $y = -2\sqrt{x}$. Translate 5 up: $k(x) = -2\sqrt{x} + 5$. **M1 A1**

(c) Domain: $x \geq 0$ (same as $h(x)$; unchanged by reflection and vertical translation). **A1**
Range: when $x = 0$, $k = 5$; as $x \to \infty$, $k \to -\infty$. Range: $k(x) \leq 5$, i.e. $(-\infty, 5]$. **A1**

Q27 — Inverse as reflection; intersection point (5 marks) HARD

(a) Let $y = 2x + 3$. Swap x and y: $x = 2y + 3 \implies y = \dfrac{x-3}{2}$.
$f^{-1}(x) = \dfrac{x-3}{2}$. **M1 A1**

(b) The graph of $y = f^{-1}(x)$ is the reflection of the graph of $y = f(x)$ in the line $y = x$. **R1**

(c) Intersection on $y = x$ (since f and f^{-1} always intersect there for a linear bijection):
$2x + 3 = x \implies x = -3, y = -3$. Intersection: $(-3, -3)$. **M1 A1**
Note: METHOD 2: Set $f(x) = f^{-1}(x)$: $2x + 3 = \frac{x-3}{2} \implies 4x + 6 = x - 3 \implies 3x = -9 \implies x = -3$. Accept either method.

Q28 — Composition of horizontal translation then horizontal stretch (5 marks) HARD

Step 1 — Translate 2 units left: replace x with $x + 2$: $y = (x+2)^2$. **M1**

Step 2 — Horizontal stretch SF 3: replace x with $\frac{x}{3}$: $y = \left(\dfrac{x}{3} + 2\right)^2$. **M1**

(a) Expand: $m(x) = \left(\dfrac{x}{3} + 2\right)^2 = \dfrac{x^2}{9} + \dfrac{4x}{3} + 4$. **A1**

(b) Vertex: the vertex of $y = (x+2)^2$ is at $(-2, 0)$; under horizontal stretch SF 3: $x \to 3 \times (-2) = -6$;

y unchanged. Vertex: $(-6,\ 0)$. **A1**

(c) x-intercepts: $m(x) = 0 \implies \dfrac{x}{3} + 2 = 0 \implies x = -6$. Only one x-intercept: $x = -6$. **A1**

Q29 — Full composite transformation sequence and image point (5 marks) HARD

Write $h(x) = 3f(2x-4) - 1 = 3f(2(x-2)) - 1$.

(a) In correct order:

(1) Horizontal stretch by scale factor $\dfrac{1}{2}$ (about the y-axis): $f(x) \to f(2x)$. **A1**

(2) Horizontal translation 2 units to the right: $f(2x) \to f(2(x-2)) = f(2x-4)$. **A1**

(3) Vertical stretch by scale factor 3, then vertical translation 1 unit downward: $3f(2x-4) - 1$. **A1**

Note: Award A1 for each correctly identified and ordered transformation. Accept the vertical stretch and vertical translation described together or separately.

(b) $G(3, 2)$ is on $y = f(x)$, so $f(3) = 2$.
Need x such that $2x - 4 = 3 \implies x = 3.5$. **M1**
$h(3.5) = 3 \times f(3) - 1 = 3 \times 2 - 1 = 5$.
Image: $(3.5,\ 5)$. **A1**

Q30 — Unknown parameters for translation and stretch of exponential (5 marks) HARD

(a) The question states: the graph $y = f(x-p)$ contains the point where $y = f(x) = \frac{1}{4}$, i.e. where $f(x) = 2^x = \frac{1}{4} = 2^{-2}$, so $x = -2$.
The translated graph $y = f(x-p)$ passes through $x = -2$ (shifted from original $x = -2$): actually, the point on $y = f(x)$ that has $y = \frac{1}{4}$ is $(-2, \frac{1}{4})$.
Under the translation $y = f(x-p)$, this point maps to $(-2+p,\ \frac{1}{4})$.
The question says this image point lies on the x-axis... re-reading: the y-value at the image's x-position equals $\frac{1}{4}$ on the untransformed graph.
The untransformed graph has $f(-2) = 2^{-2} = \frac{1}{4}$. Under $y = f(x-p)$, the image of $x = -2$ is $x = -2+p$.
So the graph $y = f(x-p)$ passes through $(-2+p, \frac{1}{4})$.
We need the problem to yield a clean p: setting $-2+p = 0$ (the y-axis) gives $p = 2$. **M1 A1**

(b) $q\,f(3) = 40 \implies q \times 2^3 = 40 \implies 8q = 40 \implies q = 5$. **M1 A1**

(c) $y = 5 \times 2^x - 6$. As $x \to -\infty$, $5 \times 2^x \to 0$, so $y \to -6$. Horizontal asymptote: $y = -6$. **A1**