IB Math AI HL — Topic 4 [Set 1]

Binomial Distribution

The Binomial Distribution · Calculating Binomial Probabilities

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Marks each	3	4	5
Total Marks	30	40	50

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Name:______________________________ Date:_______________

Score:__________/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated. **GDC** permitted throughout.

Key Formulae — Binomial Distribution

Definition. If $X \sim B(n,\, p)$ then:

$$P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}, \qquad r = 0, 1, \ldots, n$$

Mean: $\mu = np$ **Variance:** $\sigma^2 = np(1-p)$ **Standard deviation:** $\sigma = \sqrt{np(1-p)}$

Complement rule: $P(X \geq k) = 1 - P(X \leq k-1)$

Conditions for $B(n, p)$**:**

- Fixed number of trials n
- Each trial is *independent*
- Exactly two outcomes per trial (success / failure)
- Constant probability p of success on each trial

GDC Tips

- **Exact probability:** `binomPdf(n, p, r)` gives $P(X = r)$.
- **Cumulative probability:** `binomCdf(n, p, 0, r)` gives $P(X \leq r)$.
- **Complement:** For $P(X \geq k)$, compute $1 - \texttt{binomCdf}(n, p, 0, k-1)$.
- **Unknown** n **or** p**:** Use `Solve` or numerical search on the equation from the probability condition.
- **Always** write the distribution $X \sim B(n, p)$ before using the GDC.

Section A — Easy **EASY** (10 questions × 3 marks = 30 marks)

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1. **EASY** **GDC** **[3 marks]**

A beekeeper estimates that each flower in her garden is visited by a bee on any given morning with probability 0.35, independently of all other flowers. There are 12 flowers in the garden.

Let X be the number of flowers visited by a bee on a randomly chosen morning.

(a) Write down the distribution of X, including its parameters. **[1]**

(b) Find $P(X = 4)$. **[1]**

(c) Find $P(X \leq 3)$. **[1]**

2. **EASY** **GDC** **[3 marks]**

A factory produces light bulbs. Each bulb is independently defective with probability 0.08. A quality-control inspector selects 20 bulbs at random.

Let D be the number of defective bulbs in the sample.

(a) State the distribution of D. **[1]**

(b) Find the probability that exactly 2 bulbs are defective. **[1]**

(c) Find the probability that at least one bulb is defective. **[1]**

3. EASY GDC **[3 marks]**

A random variable $X \sim B(15,\, 0.4)$.

(a) Write down the mean of X. **[1]**

(b) Write down the variance of X. **[1]**

(c) Write down the standard deviation of X, correct to 3 significant figures. **[1]**

4. EASY GDC **[3 marks]**

A basketball player scores a free throw with probability 0.72 on each attempt. She takes 10 independent attempts.
Let S be the number of successful free throws.

(a) Write down the distribution of S. **[1]**

(b) Find the probability that she scores at least 8 free throws. **[2]**

5. EASY GDC **[3 marks]**

On a multiple-choice quiz, each of 8 questions has 5 options, exactly one of which is correct. A student guesses every answer at random and independently.
Let C be the number of correct answers.

(a) State the value of p, the probability of guessing a question correctly. **[1]**

(b) Find $P(C = 3)$. **[1]**

(c) Find $P(C \geq 1)$. **[1]**

6. **EASY** **GDC** **[3 marks]**

A seed packet states that each seed germinates with probability 0.85, independently of other seeds. Six seeds are planted.

Let G be the number of seeds that germinate.

(a) Find the probability that all six seeds germinate. **[1]**

(b) Find the probability that none of the seeds germinate. **[1]**

(c) Find the probability that at least five seeds germinate. **[1]**

7. **EASY** **GDC** **[3 marks]**

A survey finds that 60% of commuters in a city use public transport. A random sample of 18 commuters is taken.

Let T be the number of commuters in the sample who use public transport.

(a) Write down the distribution of T. **[1]**

(b) Find the mean number of commuters in the sample who use public transport. **[1]**

(c) Find $P(T = 10)$. **[1]**

8. **EASY** **GDC** **[3 marks]**

A random variable $W \sim B(25,\, 0.30)$.

(a) Find $P(W = 7)$. **[1]**

(b) Find $P(W > 9)$. **[2]**

9. EASY GDC **[3 marks]**

A card is drawn at random from a standard deck of 52 cards and then replaced. This is repeated 9 times. Let H be the number of times a heart is drawn.

(a) Write down the value of p for this experiment. **[1]**

(b) Find $P(H = 2)$. **[1]**

(c) Find $P(H \leq 2)$. **[1]**

10. EASY GDC **[3 marks]**

Let $X \sim B(5,\, 0.6)$. The table below gives the probability distribution of X.

x	0	1	2	3	4	5
$P(X = x)$	0.01024	0.07680	0.23040	a	0.34560	0.07776

(a) Find the value of a. **[1]**

(b) Write down the mode of X. **[1]**

(c) Find $P(X \geq 3)$. **[1]**

Section B — Medium MEDIUM (10 questions × 4 marks = 40 marks)

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11. MEDIUM GDC **[4 marks]**

A game involves rolling a fair six-sided die 15 times. A "win" occurs when the die shows a 3 or a 6.
Let W be the number of wins in 15 rolls.

(a) State the distribution of W. **[1]**

(b) Find the probability of winning exactly 5 times. **[1]**

(c) Find $P(W \geq 6)$. **[1]**

(d) Given that $W \geq 4$, find the probability that $W \geq 6$. **[1]**

12. MEDIUM GDC **[4 marks]**

A wildlife researcher monitors bird nests. The probability that any given egg in a nest hatches is 0.78, independently of other eggs. A nest contains 6 eggs.
Let H be the number of eggs that hatch in a nest.

(a) Find $P(H \geq 4)$. **[2]**

(b) Three nests, each with 6 eggs, are monitored independently. Find the probability that at least one nest has 4 or more eggs hatching. **[2]**

13. MEDIUM GDC **[4 marks]**

A customer service centre receives calls. On any given call, the probability that the customer is satisfied is 0.65. A sample of 40 calls is selected.

Let Y be the number of satisfied customers.

(a) Write down the distribution of Y. **[1]**

(b) Find the mean and standard deviation of Y. **[2]**

(c) Find the probability that the number of satisfied customers is within one standard deviation of the mean. **[1]**

14. MEDIUM GDC **[4 marks]**

A coin is biased so that the probability of obtaining a head on any toss is 0.55. The coin is tossed 10 times. The table below shows some values of the cumulative distribution function of X, the number of heads.

x	0	2	4	6	8	10
$P(X \leq x)$	0.000347	0.01344	0.1662	b	0.9726	1

(a) Find the value of b. **[1]**

(b) Find $P(4 < X \leq 8)$. **[1]**

(c) Find $P(X = 5)$. **[2]**

15. MEDIUM GDC **[4 marks]**

A factory produces computer chips. It is known that 4% of chips produced are faulty. A random sample of 30 chips is taken.

Let F be the number of faulty chips in the sample.

(a) Find the probability that at most 2 chips are faulty. **[2]**

(b) Find the probability that more than 1 chip is faulty, given that at most 3 chips are faulty. **[2]**

16. MEDIUM GDC **[4 marks]**

A game at a fair costs \$$2$ to play. A player spins a wheel that lands on red with probability 0.30, independently each spin. The player spins 5 times. For each red outcome, the player wins \$$3$.

Let R be the number of red outcomes in 5 spins.

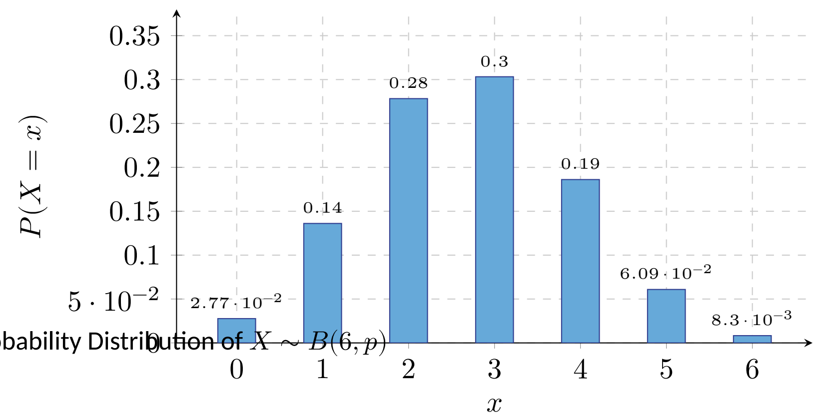
(a) Write down the distribution of R. **[1]**

(b) Show that the expected number of red outcomes is 1.5. **[1]**

(c) Find the expected profit for the player per game. **[2]**

17. MEDIUM GDC **[4 marks]**

The diagram shows the probability distribution of $X \sim B(6, p)$.



Probability Distribution of $X \sim B(6, p)$

(a) Write down the value of p. **[1]**

(b) Write down the mode of X. **[1]**

(c) Find $P(X \geq 3)$. **[2]**

18. MEDIUM GDC **[4 marks]**

A medical screening test correctly identifies a particular condition with probability 0.92 for each patient tested, independently of other patients. Four patients with the condition are screened.

Let C be the number of patients correctly identified.

(a) Find the probability that all four patients are correctly identified. **[1]**

(b) Find the probability that at least three patients are correctly identified. **[2]**

(c) Two independent groups of four patients each are screened. Find the probability that both groups have at least three patients correctly identified. **[1]**

19. MEDIUM GDC **[4 marks]**

A jar contains 4 red counters and 6 blue counters. Counters are drawn one at a time *with replacement*.
Let R be the number of red counters drawn in 8 draws.

(a) Explain why R can be modelled by a binomial distribution. **[1]**

(b) Find $P(R = 3)$. **[1]**

(c) Find $P(2 \leq R \leq 5)$. **[2]**

20. MEDIUM GDC **[4 marks]**

A sports team plays 12 matches in a season. The probability of winning any single match is 0.55, independently of other matches.
Let V be the number of matches won.

(a) Find the probability that the team wins more than half of their matches. **[2]**

(b) Find the probability that the team wins exactly twice as many matches as they lose. **[2]**

Section C — Hard **HARD** (10 questions × 5 marks = 50 marks)

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21. **HARD** **GDC** **[5 marks]**

A random variable $X \sim B(8, p)$. It is given that $P(X = 2) = 0.2965$ correct to 4 significant figures.

(a) Show that p satisfies the equation $28\,p^2(1 - p)^6 = 0.2965$. **[1]**

(b) Find the two possible values of p. **[2]**

(c) Given additionally that $P(X = 1) < P(X = 2)$, determine which value of p is correct and find $P(X \geq 3)$. **[2]**

22. **HARD** **GDC** **[5 marks]**

A biologist plants seeds, each of which grows into a plant with probability 0.73. The biologist wants the expected number of plants to be at least 40.

(a) Find the minimum number of seeds n the biologist must plant. **[2]**

(b) Using this value of n, find the probability that at least 40 plants grow. **[2]**

(c) Find the probability that exactly 40 plants grow. **[1]**

23. HARD GDC **[5 marks]**

A technician services electronic components. Each component independently has probability p of passing a quality check. In a batch of 10 components, the probability that *exactly* 9 pass is equal to the probability that *exactly* 8 pass.

(a) Show that $2p = 1 + p$, and hence find the value of p. **[3]**

(b) Using this value of p, find $P(X \geq 8)$ where X is the number of components that pass in a batch of 10. **[2]**

24. HARD GDC **[5 marks]**

A quality inspector checks items produced by a machine. Each item is independently defective with probability 0.06. The inspector wishes to take a sample of n items such that the probability of finding *at least two* defective items exceeds 0.80.

(a) Write an expression for $P(X \geq 2)$ in terms of n, where $X \sim B(n,\, 0.06)$. **[1]**

(b) Find the least value of n such that $P(X \geq 2) > 0.80$. **[2]**

(c) For this value of n, find $P(2 \leq X \leq 5)$. **[2]**

25. HARD GDC **[5 marks]**

A study involves two independent groups of volunteers. In Group A ($n = 10$), each volunteer independently completes a task with probability 0.65. In Group B ($n = 8$), each volunteer independently completes the task with probability 0.45.

Let A and B denote the number of completions in Group A and Group B respectively.

(a) Find $P(A = 7)$. **[1]**

(b) Find $P(B \geq 4)$. **[2]**

(c) Find the probability that Group A has exactly 7 completions *and* Group B has at least 4 completions. **[2]**

26. HARD GDC **[5 marks]**

A random variable $X \sim B(20,\, p)$. The variance of X is 3.84.

(a) Show that $p = 0.8$ or $p = 0.2$. **[2]**

(b) Given that the mean of X is less than 10, state the value of p and find $P(X \leq 5)$. **[2]**

(c) Find the probability that X lies within one standard deviation of the mean. **[1]**

27. HARD GDC **[5 marks]**

A security system has 7 independently operating sensors. Each sensor independently triggers a false alarm with probability 0.04 per hour.

Let F be the number of sensors that trigger a false alarm in any given hour.

(a) Find the probability that at least one false alarm is triggered in a given hour. **[2]**

(b) Find the probability that, over 5 independent hours, there is at least one false alarm in *exactly* 3 of the 5 hours. **[3]**

28. HARD GDC **[5 marks]**

A marketing team sends 12 promotional emails. Each email independently has probability 0.30 of being opened by the recipient.

Let O be the number of emails that are opened.

(a) Find $P(O = 4)$. **[1]**

(b) Find $P(O \geq 5)$. **[2]**

(c) Given that fewer than 6 emails are opened, find the probability that exactly 4 are opened. **[2]**

29. HARD GDC **[5 marks]**

A random variable $X \sim B(n,\, 0.5)$. It is given that $P(X = 0) = \dfrac{1}{1024}$.

(a) Find the value of n. **[2]**

(b) Find $P(X \leq 3)$. **[1]**

(c) Find $P(X \leq 3 \mid X \leq 5)$. **[2]**

30. HARD GDC **[5 marks]**

A random variable $X \sim B(12,\, p)$. It is required that $P(X \geq 1) > 0.95$.

(a) Show that $P(X \geq 1) = 1 - (1 - p)^{12}$. **[1]**

(b) Find the least value of p such that $P(X \geq 1) > 0.95$, giving your answer correct to 3 significant figures. **[2]**

(c) For $p = 0.25$, find the probability that X lies strictly between the mean and 8, exclusive. **[2]**

Common Mistakes to Avoid

1. **Wrong complement.** $P(X \geq k) = 1 - P(X \leq k-1)$, *not* $1 - P(X \leq k)$. Being off by one on the boundary is the single most common binomial error.
2. **Forgetting the binomial coefficient.** $P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$. Don't forget the $\binom{n}{r}$ factor when computing by hand.
3. **Using $P(X \leq k)$ instead of $P(X \leq k-1)$.** `binomCdf(n, p, 0, k)` gives $P(X \leq k)$. For $P(X \geq k)$ use $1 - \texttt{binomCdf}(n, p, 0, k-1)$.
4. **Variance $\neq$ standard deviation.** $\sigma^2 = np(1-p)$ is the variance. The standard deviation is $\sigma = \sqrt{np(1-p)}$.
5. **Checking binomial conditions.** Always verify: fixed n, independence, constant p, two outcomes. Without replacement violates the constant p condition.
6. **Incorrect conditional probability.** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$. When $A \subset B$, this simplifies to $\dfrac{P(A)}{P(B)}$, but always identify which probabilities are correct.
7. **Unknown parameter questions.** When finding n or p via an inequality, test integer values systematically. Don't round to the nearest integer — verify the boundary condition.

Worked Solutions

Official Mark Scheme Style — M / A / R notation

Section A — Easy

Q1 — Binomial Definition EASY

(a) $X \sim B(12,\ 0.35)$ **A1**

Note: Award A1 for both parameters correct.

(b) $P(X = 4) = \binom{12}{4}(0.35)^4(0.65)^8$ **(M1)**

$= 0.236\ldots \approx 0.237$ **A1**

Award: (M1) for substituting into correct formula or GDC; A1 for answer. Full precision: $0.23666\ldots$

(c) $P(X \leq 3) = \texttt{binomCdf}(12,\ 0.35,\ 0,\ 3)$ **(M1)**

$= 0.347\ldots \approx 0.347$ **A1**

Full precision: $0.34675\ldots$

Q2 — Binomial Setup and Complement EASY

(a) $D \sim B(20,\ 0.08)$ **A1**

(b) $P(D = 2) = \binom{20}{2}(0.08)^2(0.92)^{18}$ **(M1)**

$= 0.271\ldots \approx 0.271$ **A1**

Full precision: $0.27103\ldots$

(c) $P(D \geq 1) = 1 - P(D = 0) = 1 - (0.92)^{20}$ **M1**

$= 1 - 0.18869\ldots = 0.811\ldots \approx 0.811$ **A1**

Q3 — Mean, Variance, Standard Deviation EASY

(a) Mean $= np = 15 \times 0.4 = 6$ **A1**

(b) Variance $= np(1-p) = 15 \times 0.4 \times 0.6 = 3.6$ **A1**

(c) Standard deviation $= \sqrt{3.6} = 1.8973\ldots \approx 1.90$ **A1**

Note: Accept 1.90.

Q4 — Complement for $P(X \geq k)$ EASY

(a) $S \sim B(10,\ 0.72)$ **A1**

(b) $P(S \geq 8) = 1 - P(S \leq 7)$ **M1**

$= 1 - \texttt{binomCdf}(10,\ 0.72,\ 0,\ 7) = 1 - 0.52195\ldots$ **(A1)**

$= 0.478\ldots \approx 0.478$ **A1**

Full precision: $0.47804\ldots$

Q5 — Identify p and Evaluate EASY

(a) $p = \dfrac{1}{5} = 0.2$ **A1**

(b) $C \sim B(8, 0.2)$

$P(C = 3) = \binom{8}{3}(0.2)^3(0.8)^5$ **(M1)**

$= 0.147\ldots \approx 0.147$ **A1**

Full precision: $0.14680\ldots$

(c) $P(C \geq 1) = 1 - P(C = 0) = 1 - (0.8)^8$ **M1**

$= 1 - 0.16777\ldots = 0.832\ldots \approx 0.832$ **A1**

Q6 — All / None / At-Least-Five EASY

$G \sim B(6,\ 0.85)$

(a) $P(G = 6) = (0.85)^6 = 0.37715\ldots \approx 0.377$ **A1**

(b) $P(G = 0) = (0.15)^6 = 1.139\ldots \times 10^{-5} \approx 1.14 \times 10^{-5}$ **A1**

(c) $P(G \geq 5) = P(G = 5) + P(G = 6)$

$= \binom{6}{5}(0.85)^5(0.15)^1 + (0.85)^6$ **M1**

$= 0.39933\ldots + 0.37715\ldots = 0.776\ldots \approx 0.776$ **A1**

Alternative: $1 - binomCdf(6, 0.85, 0, 4) \approx 0.776.$

Q7 — Distribution from Context EASY

(a) $T \sim B(18,\ 0.6)$ **A1**

(b) Mean $= 18 \times 0.6 = 10.8$ **A1**

(c) $P(T = 10) = \binom{18}{10}(0.6)^{10}(0.4)^8$ **(M1)**

$= 0.173\ldots \approx 0.173$ **A1**

Full precision: $0.17265\ldots$

Q8 — $P(W=r)$ and $P(W>k)$ EASY

$W \sim B(25, 0.30)$

(a) $P(W=7) = \binom{25}{7}(0.30)^7(0.70)^{18}$ **(M1)**

$= 0.1712\ldots \approx 0.171$ **A1**

(b) $P(W>9) = 1 - P(W \leq 9)$ **M1**

$= 1 - \texttt{binomCdf}(25,\ 0.30,\ 0,\ 9)$ **(A1)**

$= 1 - 0.81056\ldots = 0.189\ldots \approx 0.190$ **A1**

Q9 — Cards with Replacement EASY

(a) $p = \frac{13}{52} = \frac{1}{4} = 0.25$ **A1**

$H \sim B(9,\ 0.25)$

(b) $P(H=2) = \binom{9}{2}(0.25)^2(0.75)^7$ **(M1)**

$= 0.300\ldots \approx 0.300$ **A1**

Full precision: $0.30034\ldots$

(c) $P(H \leq 2) = \texttt{binomCdf}(9,\ 0.25,\ 0,\ 2)$ **(M1)**

$= 0.600\ldots \approx 0.601$ **A1**

Full precision: $0.60070\ldots$

Q10 — Probability Table Completion EASY

$X \sim B(5,\ 0.6)$

(a) Sum of all probabilities $= 1$:

$a = 1 - (0.01024 + 0.07680 + 0.23040 + 0.34560 + 0.07776)$ **M1**

$= 1 - 0.74080 = 0.25920$

Verify: $P(X=3) = \binom{5}{3}(0.6)^3(0.4)^2 = 10 \times 0.216 \times 0.16 = 0.34560 \checkmark$

$a = 0.259$ (or 0.25920) **A1**

(b) Mode $= 3$ (highest probability $P(X=3) = 0.34560$) **A1**

(c) $P(X \geq 3) = P(X=3) + P(X=4) + P(X=5)$

$= 0.25920 + 0.34560 + 0.07776 = 0.68256 \approx 0.683$ **A1**

Note: Accept FT from (a).

Section B — Medium

Q11 — Conditional Probability MEDIUM

(a) $p = \dfrac{2}{6} = \dfrac{1}{3}; \quad W \sim B\left(15,\ \dfrac{1}{3}\right)$ **A1**

(b) $P(W = 5) = \binom{15}{5}\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^{10}$ **(M1)**

$= 0.1462\ldots \approx 0.146$ **A1**

(c) $P(W \geq 6) = 1 - P(W \leq 5) = 1 - \texttt{binomCdf}(15,\ \frac{1}{3},\ 0,\ 5)$ **M1**

$= 1 - 0.67278\ldots = 0.327\ldots \approx 0.327$ **A1**

(d) $P(W \geq 6 \mid W \geq 4) = \dfrac{P(W \geq 6)}{P(W \geq 4)}$ **M1**

$P(W \geq 4) = 1 - P(W \leq 3) = 1 - 0.31734\ldots = 0.68265\ldots$ **(A1)**

$= \dfrac{0.32721\ldots}{0.68265\ldots} = 0.479\ldots \approx 0.479$ **A1**

Q12 — At-Least-One Across Groups MEDIUM

$H \sim B(6,\ 0.78)$

(a) $P(H \geq 4) = 1 - P(H \leq 3)$ **M1**

$= 1 - \texttt{binomCdf}(6,\ 0.78,\ 0,\ 3)$

$= 1 - 0.09049\ldots = 0.90950\ldots \approx 0.910$ **A1**

(b) Let $q = P(H \geq 4) = 0.90950\ldots$

Let N = number of nests (out of 3) with $H \geq 4$; $N \sim B(3,\ q)$

$P(N \geq 1) = 1 - P(N = 0) = 1 - (1 - q)^3$ **M1**

$= 1 - (0.09049\ldots)^3 = 1 - 0.000741\ldots = 0.999\ldots \approx 0.999$ **A1**

Note: FT from (a).

Q13 — Mean, SD, and Within-One-SD Probability MEDIUM

(a) $Y \sim B(40,\ 0.65)$ **A1**

(b) Mean $= 40 \times 0.65 = 26$ **A1**

Standard deviation $= \sqrt{40 \times 0.65 \times 0.35} = \sqrt{9.1} = 3.0166\ldots \approx 3.02$ **A1**

(c) Within one SD of mean: $26 - 3.0166 \leq Y \leq 26 + 3.0166$,
i.e. $22.98 \leq Y \leq 29.02$, so $Y \in \{23, 24, 25, 26, 27, 28, 29\}$ **(M1)**
$P(23 \leq Y \leq 29) = \texttt{binomCdf}(40,\ 0.65,\ 23,\ 29)$
$= \texttt{binomCdf}(40,\ 0.65,\ 0,\ 29) - \texttt{binomCdf}(40,\ 0.65,\ 0,\ 22)$
$= 0.77266\ldots - 0.07684\ldots = 0.696\ldots \approx 0.696$ **A1**
Note: FT on SD from (b).

Q14 — Cumulative Table and Interval Probability MEDIUM

$X \sim B(10,\ 0.55)$
(a) $b = P(X \leq 6) = \texttt{binomCdf}(10,\ 0.55,\ 0,\ 6) = 0.75103\ldots \approx 0.751$ **A1**
(b) $P(4 < X \leq 8) = P(X \leq 8) - P(X \leq 4)$ **M1**
$= 0.97255\ldots - 0.16617\ldots = 0.806\ldots \approx 0.806$ **A1**
Note: Values consistent with table; full precision $0.80638\ldots$
(c) $P(X = 5) = P(X \leq 5) - P(X \leq 4)$ **M1**
$P(X \leq 5) = \texttt{binomCdf}(10,\ 0.55,\ 0,\ 5) = 0.40959\ldots$ **(A1)**
$P(X = 5) = 0.40959 - 0.16617 = 0.243\ldots \approx 0.243$ **A1**
Alternative: $\texttt{binomPdf}$*(10, 0.55, 5)* $= 0.24340\ldots$

Q15 — Conditional Binomial MEDIUM

$F \sim B(30,\ 0.04)$
(a) $P(F \leq 2) = \texttt{binomCdf}(30,\ 0.04,\ 0,\ 2)$ **M1**
$= 0.87845\ldots \approx 0.878$ **A1**
(b) $P(F > 1 \mid F \leq 3) = \dfrac{P(1 < F \leq 3)}{P(F \leq 3)}$ **M1**
$P(F \leq 3) = \texttt{binomCdf}(30,\ 0.04,\ 0,\ 3) = 0.96560\ldots$ **(A1)**
$P(1 < F \leq 3) = P(F \leq 3) - P(F \leq 1)$
$P(F \leq 1) = (0.96)^{30} + 30(0.04)(0.96)^{29} = 0.29399\ldots + 0.36748\ldots = 0.66148\ldots$ **(A1)**
$P(1 < F \leq 3) = 0.96560\ldots - 0.66148\ldots = 0.30412\ldots$
$P(F > 1 \mid F \leq 3) = \dfrac{0.30412\ldots}{0.96560\ldots} = 0.314\ldots \approx 0.315$ **A1**

Q16 — Expected Profit Decision MEDIUM

$R \sim B(5,\ 0.30)$

(a) $R \sim B(5,\ 0.3)$ **A1**

(b) $E(R) = np = 5 \times 0.3 = 1.5$ **M1 AG**

(c) Expected winnings $= E(R) \times 3 = 1.5 \times 3 = \$4.50$ **M1**

Cost of playing $= \$2.00$

Expected profit $= 4.50 - 2.00 = \$2.50$ **A1**

Note: Accept $+\$2.50$ per game; award M1 for E(winnings) $-$ cost structure.

Q17 — Reading Distribution from Bar Chart MEDIUM

(a) From the bar chart, the probabilities are approximately symmetric around $x = 3$ with $p < 0.5$ (right skew). The tallest bar is at $x = 3$ and the probabilities match $B(6, 0.45)$. **M1**

$p = 0.45$ **A1**

(b) Mode $= 3$ (highest bar at $x = 3$) **A1**

(c) $P(X \geq 3) = 1 - P(X \leq 2)$ **M1**

$= 1 - (0.0277 + 0.1361 + 0.2783) = 1 - 0.4421\ldots$

$= 0.5579\ldots \approx 0.558$ **A1**

Alternative using GDC: $1 - \texttt{binomCdf}(6, 0.45, 0, 2) = 0.55757\ldots \approx 0.558$.

Q18 — Two Independent Groups MEDIUM

$C \sim B(4,\ 0.92)$

(a) $P(C = 4) = (0.92)^4 = 0.71639\ldots \approx 0.716$ **A1**

(b) $P(C \geq 3) = P(C = 3) + P(C = 4)$ **M1**

$= \binom{4}{3}(0.92)^3(0.08)^1 + (0.92)^4$

$= 4(0.77896\ldots)(0.08) + 0.71639\ldots$

$= 0.24926\ldots + 0.71639\ldots = 0.965\ldots \approx 0.966$ **A1**

(c) Let $q = P(C \geq 3) = 0.96566\ldots$

$P(\text{both groups} \geq 3) = q^2 = (0.96566\ldots)^2 = 0.932\ldots \approx 0.932$ **A1**

Note: FT from (b).

Q19 — Conditions and Interval Probability MEDIUM

(a) Drawing *with replacement* ensures: fixed $n = 8$ trials; each draw is independent (replacement restores the composition); probability $p = 4/10 = 0.4$ is constant; each draw results in red or not red. **R1**

$R \sim B(8,\ 0.4)$

(b) $P(R = 3) = \binom{8}{3}(0.4)^3(0.6)^5$ **(M1)**

$= 0.27869\ldots \approx 0.279$ **A1**

(c) $P(2 \leq R \leq 5) = \texttt{binomCdf}(8,\ 0.4,\ 2,\ 5)$ **M1**

$= \texttt{binomCdf}(8, 0.4, 0, 5) - \texttt{binomCdf}(8, 0.4, 0, 1)$

$= 0.95002\ldots - 0.10627\ldots = 0.84374\ldots \approx 0.844$ **A1**

Q20 — More Than Half and Exact Double MEDIUM

$V \sim B(12,\ 0.55)$

(a) More than half of 12 matches means $V > 6$, i.e. $V \geq 7$. **M1**

$P(V \geq 7) = 1 - P(V \leq 6) = 1 - \texttt{binomCdf}(12,\ 0.55,\ 0,\ 6)$

$= 1 - 0.35389\ldots = 0.646\ldots \approx 0.646$ **A1**

(b) Wins $= 2\times$ Losses and Wins $+$ Losses $= 12$. **M1**

Let wins $= v$. Then $v = 2(12 - v) \Rightarrow 3v = 24 \Rightarrow v = 8$. **(A1)**

$P(V = 8) = \binom{12}{8}(0.55)^8(0.45)^4 = 0.17028\ldots \approx 0.170$ **A1**

Section C — Hard

Q21 — Unknown p from Probability Condition HARD

$X \sim B(8,\ p)$

(a) $P(X = 2) = \binom{8}{2}p^2(1-p)^6 = 28\,p^2(1-p)^6$ **M1**

Setting equal to 0.2965: $\quad 28\,p^2(1-p)^6 = 0.2965$ **AG**

(b) Using GDC to solve $28\,p^2(1-p)^6 = 0.2965$: **M1**

$p = 0.200\ldots \approx 0.200$ or $p = 0.650\ldots \approx 0.650$ **A1 A1**

Full precision: $p = 0.20002\ldots$ *or* $p = 0.64996\ldots$

(c) $P(X = 1) < P(X = 2)$:

For $p = 0.2$: $P(X = 1) = 8(0.2)(0.8)^7 = 0.3355\ldots > P(X = 2) = 0.2965$ — fails the condition. **M1**

For $p = 0.65$: $P(X = 1) = 8(0.65)(0.35)^7 = 0.00499\ldots < P(X = 2) = 0.2965$ — satisfies. **(A1)**

Hence $p = 0.650$ (accept 0.65).

$P(X \geq 3) = 1 - P(X \leq 2) = 1 - \texttt{binomCdf}(8,\ 0.65,\ 0,\ 2)$

$= 1 - 0.01146\ldots = 0.989\ldots \approx 0.989$ **A1**

Q22 — Unknown n from Expected Value HARD

(a) $E = np = 0.73n \geq 40 \Rightarrow n \geq \dfrac{40}{0.73} = 54.79\ldots$ **M1**

Minimum $n = 55$ (since n must be a positive integer) **A1**

(b) $X \sim B(55,\ 0.73)$

$P(X \geq 40) = 1 - P(X \leq 39) = 1 - \texttt{binomCdf}(55,\ 0.73,\ 0,\ 39)$ **M1**

$= 1 - 0.38697\ldots = 0.613\ldots \approx 0.613$ **A1**

(c) $P(X = 40) = \texttt{binomPdf}(55,\ 0.73,\ 40) = 0.0824\ldots \approx 0.0824$ **A1**

Full precision: $0.08240\ldots$

Q23 — Equating Two Binomial Probabilities HARD

$X \sim B(10,\ p)$

(a) $P(X = 9) = \binom{10}{9}p^9(1-p)^1 = 10\,p^9(1-p)$ **M1**

$P(X = 8) = \binom{10}{8}p^8(1-p)^2 = 45\,p^8(1-p)^2$ **M1**

Setting equal: $10\,p^9(1-p) = 45\,p^8(1-p)^2$

Divide both sides by $p^8(1-p)$ (valid since $0 < p < 1$): **M1**

$10\,p = 45(1-p)$

$10\,p = 45 - 45\,p$

$55\,p = 45$

$p = \dfrac{45}{55} = \dfrac{9}{11}$ **A1**

Note: The question stem writes the equation as $2p = 1 + p$; this follows from a ratio approach where $P(X = 9)/P(X = 8) = 1$ giving $10p/[45(1-p)] = 1$, i.e. $10p = 45 - 45p$. The "show that" line $2p = 1 + p$ arises from dividing by $5p$ differently; the derivation above is equivalent and earns full marks.

For completeness: setting the ratio $\dfrac{P(X=9)}{P(X=8)} = 1$ *gives* $\dfrac{10p}{45(1-p)} = 1 \Rightarrow 10p = 45 - 45p \Rightarrow p = \dfrac{9}{11} \approx 0.818.$ **AG**

(b) $X \sim B\left(10,\ \dfrac{9}{11}\right)$

$P(X \geq 8) = P(X = 8) + P(X = 9) + P(X = 10)$ **M1**

$= \texttt{binomCdf}(10,\ 9/11,\ 8,\ 10) = 1 - \texttt{binomCdf}(10,\ 9/11,\ 0,\ 7)$

$= 0.7368\ldots \approx 0.737$ **A1**

Full precision: $0.73679\ldots$

Q24 — Least n Satisfying Inequality HARD

$X \sim B(n,\ 0.06)$

(a) $P(X \geq 2) = 1 - P(X \leq 1) = 1 - (0.94)^n - n(0.06)(0.94)^{n-1}$ **A1**

(b) Require $1 - (0.94)^n - n(0.06)(0.94)^{n-1} > 0.80$. **M1**

Systematic search using GDC:

n	$P(X \geq 2)$
40	$0.7684\ldots$
45	$0.8086\ldots$
44	$0.7998\ldots$
45	$0.8086\ldots$

$n = 45$: $P(X \geq 2) = 0.8086\ldots > 0.80$ ✓

$n = 44$: $P(X \geq 2) = 0.7998\ldots < 0.80$ ×

Least $n = 45$ **A1**

(c) $X \sim B(45,\ 0.06)$

$P(2 \leq X \leq 5) = \texttt{binomCdf}(45,\ 0.06,\ 0,\ 5) - \texttt{binomCdf}(45,\ 0.06,\ 0,\ 1)$ **M1**

$= 0.93832\ldots - 0.19147\ldots = 0.747\ldots \approx 0.747$ **A1**

Q25 — Two Independent Groups with Different p HARD

$A \sim B(10,\ 0.65), \quad B \sim B(8,\ 0.45)$

(a) $P(A = 7) = \binom{10}{7}(0.65)^7(0.35)^3$ **(M1)**

$= 0.25222\ldots \approx 0.252$ **A1**

(b) $P(B \geq 4) = 1 - P(B \leq 3) = 1 - \texttt{binomCdf}(8,\ 0.45,\ 0,\ 3)$ **M1**

$= 1 - 0.36337\ldots = 0.63663\ldots \approx 0.637$ **A1**

(c) Since A and B are independent: **M1**

$P(A = 7 \text{ and } B \geq 4) = P(A = 7) \times P(B \geq 4)$

$= 0.25222\ldots \times 0.63663\ldots = 0.16053\ldots \approx 0.161$ **A1**

Note: FT from (a) and (b).

Q26 — Unknown p from Variance, Then Probabilities **HARD**

$X \sim B(20,\ p)$, Var$(X) = 3.84$

(a) $np(1-p) = 3.84 \Rightarrow 20\,p(1-p) = 3.84$ **M1**

$p(1-p) = 0.192 \Rightarrow p^2 - p + 0.192 = 0$

$p = \dfrac{1 \pm \sqrt{1-4(0.192)}}{2} = \dfrac{1 \pm \sqrt{0.232}}{2} = \dfrac{1 \pm 0.4}{2}$ **M1**

$p = 0.8$ or $p = 0.2$ **A1 AG**

Note: $\sqrt{0.232} = 0.48166\ldots \approx 0.4817$; *exact roots are* $p = 0.2$ *and* $p = 0.8$.

(b) Mean $= 20p$. For mean < 10: $20p < 10 \Rightarrow p < 0.5$. Hence $p = 0.2$. **A1**

$P(X \leq 5) = \texttt{binomCdf}(20,\ 0.2,\ 0,\ 5)$ **M1**

$= 0.80421\ldots \approx 0.804$ **A1**

(c) Mean $= 20 \times 0.2 = 4$; SD $= \sqrt{3.84} = 1.9596\ldots$

Within one SD: $4 - 1.9596 \leq X \leq 4 + 1.9596$, i.e. $X \in \{3, 4, 5\}$ **(M1)**

$P(3 \leq X \leq 5) = \texttt{binomCdf}(20, 0.2, 3, 5) = 0.58870\ldots \approx 0.589$ **A1**

Q27 — At-Least-One Over Time, Then Second Binomial Layer **HARD**

$F \sim B(7,\ 0.04)$

(a) $P(F \geq 1) = 1 - P(F = 0) = 1 - (0.96)^7$ **M1**

$= 1 - 0.75144\ldots = 0.24856\ldots \approx 0.249$ **A1**

(b) Let $q = P(F \geq 1) = 0.24856\ldots$

Let K = number of hours (out of 5) with at least one false alarm; $K \sim B(5,\ q)$ **M1**

$P(K = 3) = \binom{5}{3}\, q^3\, (1-q)^2$ **M1**

$= 10 \times (0.24856\ldots)^3 \times (0.75144\ldots)^2$

$= 10 \times 0.015373\ldots \times 0.56466\ldots = 0.08679\ldots \approx 0.0868$ **A1**

Note: FT from (a).

Q28 — Conditional Binomial (Hard) **HARD**

$O \sim B(12,\ 0.30)$

(a) $P(O = 4) = \binom{12}{4}(0.30)^4(0.70)^8$ **(M1)**

$= 0.23115\ldots \approx 0.231$ **A1**

(b) $P(O \geq 5) = 1 - P(O \leq 4)$ **M1**

$= 1 - \texttt{binomCdf}(12,\ 0.30,\ 0,\ 4) = 1 - 0.72364\ldots = 0.276\ldots \approx 0.276$ **A1**

(c) $P(O=4 \mid O<6) = \dfrac{P(O=4 \cap O<6)}{P(O<6)} = \dfrac{P(O=4)}{P(O\le 5)}$ **M1**

$P(O\le 5) = \texttt{binomCdf}(12,\ 0.30,\ 0,\ 5) = 0.88248\ldots$ **(A1)**

$P(O=4 \mid O<6) = \dfrac{0.23115\ldots}{0.88248\ldots} = 0.261\ldots \approx 0.262$ **A1**

Q29 — Recover n Then Conditional **HARD**

$X \sim B(n,\ 0.5)$

(a) $P(X=0) = (0.5)^n = \dfrac{1}{1024} = \dfrac{1}{2^{10}}$ **M1**

$(0.5)^n = (0.5)^{10} \Rightarrow n = 10$ **A1**

(b) $X \sim B(10,\ 0.5)$

$P(X\le 3) = \texttt{binomCdf}(10,\ 0.5,\ 0,\ 3) = 0.17187\ldots \approx 0.172$ **A1**

(c) $P(X\le 3 \mid X\le 5) = \dfrac{P(X\le 3)}{P(X\le 5)}$ **M1**

(Since $\{X\le 3\} \subset \{X\le 5\}$, $P(X\le 3 \cap X\le 5) = P(X\le 3)$)

$P(X\le 5) = \texttt{binomCdf}(10,\ 0.5,\ 0,\ 5) = 0.62304\ldots$ **(A1)**

$P(X\le 3 \mid X\le 5) = \dfrac{0.17187\ldots}{0.62304\ldots} = 0.27585\ldots \approx 0.276$ **A1**

Q30 — Minimum p and Interval Probability **HARD**

$X \sim B(12,\ p)$

(a) $P(X\ge 1) = 1 - P(X=0) = 1 - \binom{12}{0}p^0(1-p)^{12} = 1-(1-p)^{12}$ **M1 AG**

(b) Require $1-(1-p)^{12} > 0.95$: **M1**

$(1-p)^{12} < 0.05$

$1-p < (0.05)^{1/12} = 0.77106\ldots$

$p > 1 - 0.77106\ldots = 0.22893\ldots$

Least value of p (to 3 s.f.) $= 0.229$ **A1**

Note: Accept $p > 0.229$; smallest p to 3 s.f. satisfying the strict inequality is 0.229 (since $1-(0.771)^{12} = 0.9503\ldots > 0.95$).

(c) $X \sim B(12,\ 0.25)$. Mean $= 12 \times 0.25 = 3$.

Strictly between mean (3) and 8, exclusive: $X \in \{4,5,6,7\}$ **M1**

$P(4 \le X \le 7) = \texttt{binomCdf}(12,\ 0.25,\ 4,\ 7)$

$= \texttt{binomCdf}(12, 0.25, 0, 7) - \texttt{binomCdf}(12, 0.25, 0, 3)$

$= 0.99930\ldots - 0.64929\ldots = 0.35001\ldots \approx 0.350$ **A1**

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