IB Math AI HL — Topic 4

Normal Distribution

The Normal Distribution · Probability Calculations · Inverse Normal · Unknown Parameters · Linear Combinations

Section	**EASY**	**MEDIUM**	**HARD**
Questions	10	10	10
Marks each	3	4	5
Section Total	30	40	50

Grand Total: 120 marks

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Name:______________________________ Date:_______________

Score:__________/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. A GDC is required throughout.

Key Formulae & Facts — Normal Distribution

Notation: $X \sim N(\mu, \sigma^2)$ means X is normally distributed with mean μ and variance σ^2.

Standardisation (z-score): $z = \dfrac{x - \mu}{\sigma}$ where $Z \sim N(0, 1)$

Symmetry: $P(X > \mu) = P(X < \mu) = 0.5$ and $P(X < \mu - k\sigma) = P(X > \mu + k\sigma)$

Empirical rule (68-95-99.7): $P(\mu - \sigma < X < \mu + \sigma) \approx 0.683$; $P(\mu - 2\sigma < X < \mu + 2\sigma) \approx 0.954$; $P(\mu - 3\sigma < X < \mu + 3\sigma) \approx 0.997$

Linear combinations: If $X \sim N(\mu_X, \sigma_X^2)$ and $Y \sim N(\mu_Y, \sigma_Y^2)$ independently, then

$$aX + bY \sim N(a\mu_X + b\mu_Y,\ a^2\sigma_X^2 + b^2\sigma_Y^2)$$

Sample mean: $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ by the Central Limit Theorem (or exact for normal populations)

GDC Tips for Normal Distribution

- **Normal CDF** (cumulative probability): `normalcdf(lower, upper,` μ, σ`)`
  For $P(X < a)$: use lower $= -10^{99}$ (or $-\infty$). For $P(X > a)$: use upper $= 10^{99}$.
- **Inverse Normal** (find x given probability): `invNorm(area,` μ, σ`)`
  Area is always the area to the **left** (left-tail probability).
- **Tip:** If $P(X > k) = p$, then $P(X < k) = 1 - p$; enter `invNorm(`$1-p$, μ, σ`)`.

- **Linear combination:** enter $E(W)$ and $\text{SD}(W) = \sqrt{\text{Var}(W)}$ directly into `normalcdf` or `invNorm`.
- **Always sketch** the shaded region before computing — it catches left/right tail errors.

Section A — Easy EASY GDC (10 questions × 3 marks = 30 marks)

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1. EASY GDC **[3 marks]**

The daily rainfall, R mm, at a mountain weather station is modelled by $R \sim N(18, 4^2)$.

(a) Find $P(R < 20)$. **[2]**

(b) Write down $P(R \geq 18)$. **[1]**

2. EASY GDC **[3 marks]**

The mass, M kg, of bags of topsoil produced at a garden centre follows $M \sim N(25, 1.5^2)$.

(a) Find $P(M > 27)$. **[2]**

(b) Hence write down $P(M < 23)$. **[1]**

3. EASY GDC **[3 marks]**

The time, T minutes, taken by students to read a short article follows $T \sim N(12, 2.5^2)$.
Find $P(10 < T < 15)$.

4. EASY GDC **[3 marks]**

The volume, V mL, of juice dispensed by a machine follows $V \sim N(330, 6^2)$.
The machine is recalibrated if a dispensed volume falls below the 5th percentile.
Find the value below which recalibration is triggered.

5. EASY GDC **[3 marks]**

The wingspan, W cm, of a species of butterfly follows $W \sim N(6.4, 0.8^2)$.

(a) Find $P(W > 7.5)$. **[2]**

(b) The widest 10% of butterflies are classified as "large". Find the minimum wingspan for a butterfly to be classified as large. **[1]**

6. EASY GDC **[3 marks]**

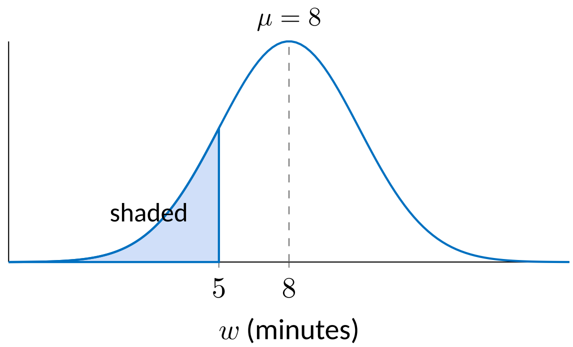
The length, L cm, of a type of screw produced in a factory follows $L \sim N(4.2, 0.15^2)$.

(a) Write down $P(L > 4.2)$. **[1]**

(b) Find $P(3.9 < L < 4.5)$. **[2]**

7. EASY GDC **[3 marks]**

The diagram shows the normal distribution of waiting times W minutes at a pharmacy, where $W \sim N(8, 3^2)$. The shaded region represents $P(W < 5)$.



(a) Write down the value of $P(W < 8)$. **[1]**

(b) Find the value of $P(W < 5)$. **[2]**

8. EASY GDC **[3 marks]**

The height, H cm, of sunflowers grown in a greenhouse follows $H \sim N(155, 12^2)$.

(a) Find $P(H > 170)$. **[2]**

(b) A nursery has 400 such sunflowers. Find the expected number with height greater than 170 cm. **[1]**

9. EASY GDC **[3 marks]**

The battery life, B hours, of a brand of wireless earbuds follows a normal distribution with mean 7.5 hours. It is known that $P(B < 6) = 0.1$.
Find the standard deviation of B.

10. EASY GDC **[3 marks]**

The mass, M g, of chocolate truffles produced by a confectioner follows $M \sim N(14, 1.2^2)$.

(a) Find $P(13 < M < 16)$. **[2]**

(b) Given that a truffle has mass greater than 13 g, find the probability its mass is also less than 16 g. **[1]**

Section B — Medium MEDIUM GDC (10 questions $\times$ 4 marks = 40 marks)

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11. MEDIUM GDC **[4 marks]**

The time, T seconds, taken by a robot arm to complete a task follows $T \sim N(\mu, \sigma^2)$.
It is given that $P(T < 4.2) = 0.25$ and $P(T < 5.8) = 0.85$.
Find the value of μ and the value of σ.

12. MEDIUM GDC **[4 marks]**

The diameter, D mm, of steel rods manufactured at a plant follows $D \sim N(12, 0.4^2)$.
Rods with diameter outside the range $[11.3,\ 12.7]$ mm are rejected.

(a) Find the probability that a randomly chosen rod is rejected. **[2]**

(b) In a batch of 500 rods, find the expected number that are rejected. **[1]**

(c) Find the probability that at least one rod from a sample of 4 is rejected. **[1]**

13. MEDIUM GDC **[4 marks]**

A packaging line fills boxes with two types of snack. The mass of type A snacks, A g, follows $A \sim N(180, 8^2)$ and the mass of type B snacks, B g, follows $B \sim N(120, 5^2)$. The masses are independent.
Let $W = A + B$ be the total mass of one box.

(a) Write down the distribution of W. **[2]**

(b) Find $P(W > 315)$. **[2]**

14. MEDIUM GDC **[4 marks]**

The score, S, on a cognitive test follows $S \sim N(72, 10^2)$.
Scores below 55 are classified "below average" and scores above 90 are "advanced".

(a) Find the probability that a randomly chosen candidate is classified "below average". **[2]**

(b) Find the probability that a randomly chosen candidate is classified neither "below average" nor "advanced". **[2]**

15. MEDIUM GDC **[4 marks]**

The lifetime, L hours, of a certain type of LED bulb follows $L \sim N(9500, 450^2)$.
A quality inspector tests a random sample of 36 bulbs and records the sample mean $\bar{L}$.

(a) Write down the distribution of $\bar{L}$, including its parameters. **[2]**

(b) Find $P(\bar{L} > 9600)$. **[2]**

16. MEDIUM GDC **[4 marks]**

The daily output, X units, of a production line follows $X \sim N(240, \sigma^2)$.
It is known that $P(220 < X < 260) = 0.90$.

(a) Show that $P(X < 260) = 0.95$. **[1]**

(b) Find the value of σ. **[3]**

17. MEDIUM GDC **[4 marks]**

The concentration, C mg/L, of a chemical in river water follows $C \sim N(3.4, 0.6^2)$.
Samples with concentration above 4.0 mg/L are sent for detailed analysis.

(a) Find $P(C > 4.0)$. **[2]**

(b) Given that a sample is sent for detailed analysis, find the probability that its concentration exceeds 4.5 mg/L. **[2]**

18. MEDIUM GDC **[4 marks]**

The masses of male and female starlings are independently normally distributed.
Male mass $M \sim N(84, 6^2)$ grams and female mass $F \sim N(78, 5^2)$ grams.
Let $D = M - F$.

(a) Write down the mean and variance of D. **[2]**

(b) Find the probability that a randomly chosen male starling is heavier than a randomly chosen female starling. **[2]**

19. MEDIUM GDC **[4 marks]**

The mass of a locally grown apple, A g, follows $A \sim N(185, 20^2)$.
A greengrocer packs apples into a crate. The mean mass of apples in a crate of n apples is $\bar{A}$.
The table shows the distribution of $\bar{A}$ for different values of n.

n	Mean of $\bar{A}$	Standard deviation of $\bar{A}$
9	185	p
25	185	4.00
q	185	2.50

(a) Write down the value of p. **[1]**

(b) Find the value of q. **[1]**

(c) Find the probability that the mean mass of a crate of 25 apples exceeds 190 g. **[2]**

20. MEDIUM GDC **[4 marks]**

The speed, V km/h, at which vehicles pass a checkpoint follows $V \sim N(\mu, 12^2)$.
It is given that 15% of vehicles travel at speeds below 68 km/h.

(a) Find the value of μ. **[3]**

(b) Hence find $P(V > 100)$. **[1]**

Section C — Hard HARD GDC (10 questions × 5 marks = 50 marks)

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21. HARD GDC **[5 marks]**

The mass, M kg, of a species of large tortoise follows $M \sim N(\mu, \sigma^2)$.
A wildlife researcher records that 30% of tortoises have mass below 62 kg and 10% have mass above 88 kg.
Find the value of μ and the value of σ.

22. HARD GDC **[5 marks]**

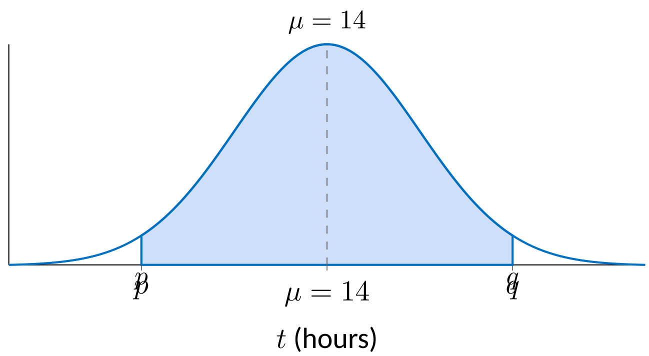
A drinks dispenser fills cups with volume V mL, where $V \sim N(250, 5^2)$.
A cafeteria orders n cups of the drink. The total volume dispensed is $T = V_1 + V_2 + \cdots + V_n$, where the cups are filled independently.

(a) Write down expressions for $E(T)$ and Var(T) in terms of n. **[2]**

(b) Find the smallest value of n such that $P(T > 5100) < 0.01$. **[3]**

23. HARD GDC **[5 marks]**

The length of time, T hours, a patient waits for elective surgery at a hospital follows $T \sim N(14, 3.5^2)$.
The diagram shows the distribution. The shaded region represents patients waiting between p hours and q hours, where $p < q$ and this region has total probability 0.80. It is also given that $P(T < p) = P(T > q)$.



(a) Explain why the shaded region is symmetrical about the mean. **[1]**

(b) Find the values of p and q. **[3]**

(c) Find the probability that a randomly chosen patient waits more than 20 hours. **[1]**

24. HARD GDC **[5 marks]**

The compressive strength, S MPa, of concrete blocks from manufacturer A follows $S \sim N(48, \sigma^2)$, where σ is unknown.
It is given that $P(S > 51) = 0.12$.

(a) Find the value of σ. **[2]**

Concrete blocks from manufacturer B follow $B \sim N(46, 2.8^2)$ MPa.
A structural engineer tests one block from A and one block from B and considers the average strength $\bar{Q} = \dfrac{S+B}{2}$.

(b) Write down the mean and variance of $\bar{Q}$. **[2]**

(c) Find $P(\bar{Q} > 48)$. **[1]**

25. HARD GDC **[5 marks]**

The reaction time, R ms, of a laboratory sample of volunteers follows $R \sim N(280, \sigma^2)$.
The researchers know that exactly 95% of volunteers have reaction times within ± 35 ms of the mean.

(a) Show that $\sigma = 17.9$ ms (correct to 3 significant figures). **[2]**

A new training programme aims to reduce reaction times. After the programme, a random sample of 40 volunteers has mean reaction time $\bar{R}$.

(b) Assuming σ remains unchanged, write down the distribution of $\bar{R}$. **[1]**

(c) Find $P(\bar{R} < 275)$. **[2]**

26. HARD GDC **[5 marks]**

In a blended coffee product, the barista adds c mL of espresso and $(250 - c)$ mL of steamed milk, where c is a fixed constant.
The volume of espresso $E \sim N(c, 3^2)$ mL and the volume of steamed milk $K \sim N(250 - c, 7^2)$ mL, independently.
Let $W = E + K$ be the total volume of the drink.

(a) Write down $E(W)$ and Var(W). **[2]**

(b) Hence show that SD(W) does not depend on c. **[1]**

(c) The barista wants $P(W > 265) < 0.05$. Find the largest integer value of c such that this condition is satisfied. **[2]**

27. HARD GDC **[5 marks]**

The yield, Y kg, of wheat from a single plant follows $Y \sim N(0.45, 0.08^2)$.
A "high-yield" plant is one for which $Y > 0.60$ kg.
A botanist selects 20 plants at random.

(a) Find the probability that a single plant is high-yield. **[2]**

(b) Find the probability that at most 2 of the 20 plants are high-yield. **[2]**

(c) Find the expected number of high-yield plants in the 20. **[1]**

28. HARD GDC **[5 marks]**

The speed, V km/h, of vehicles on a motorway follows $V \sim N(\mu, \sigma^2)$.
Traffic data shows that $P(V < 90) = 0.20$ and $P(V < 130) = 0.95$.

(a) Show that μ and σ satisfy the simultaneous equations:

$$90 = \mu - 0.8416\,\sigma \qquad 130 = \mu + 1.6449\,\sigma$$

giving z-scores correct to 4 significant figures. **[2]**

(b) Solve these equations to find μ and σ, giving your answers correct to 3 significant figures. **[3]**

29. HARD GDC **[5 marks]**

The protein content, P g per 100 g, of a food product follows $P \sim N(18.5, 1.8^2)$.
A nutritionist takes a random sample of n packets and records the sample mean $\bar{P}$.

(a) Write down the distribution of $\bar{P}$ in terms of n. **[1]**

(b) The nutritionist requires that $P(\bar{P} < 19) \geq 0.99$. Find the minimum value of n. **[4]**

30. HARD GDC **[5 marks]**

A factory produces two components, X and Y, independently.
$X \sim N(50, 2.5^2)$ mm and $Y \sim N(30, 1.5^2)$ mm.
A device requires that the combined length $L = X + Y$ satisfy $45 \leq L \leq 85$ mm for the device to function correctly.
A total clearance $G = X - Y$ is also measured; the device is categorised "tight" if $G < 17$ mm.

(a) Find the probability that a randomly assembled device functions correctly. **[2]**

(b) Find $P(G < 17)$. **[2]**

(c) Given that the device functions correctly, find the probability that it is also "tight". **[1]**

Worked Solutions

Normal Distribution — All 30 Questions

Q1 — Normal Probability, Left Tail EASY

$R \sim N(18, 4^2)$

(a) $P(R < 20)$ **M1**
$= 0.691\ (0.69146\ldots)$ **A1**

(b) $P(R \geq 18) = 0.5$ **A1**

Note: Award A1 for answer of 0.5 or equivalent. No working required.

Q2 — Normal Probability, Symmetry EASY

$M \sim N(25, 1.5^2)$

(a) $P(M > 27)$ **M1**
$= 0.0912\ (0.09121\ldots)$ **A1**

(b) $27 = 25 + 2$ and $23 = 25 - 2$, so by symmetry of the normal distribution
$P(M < 23) = P(M > 27) = 0.0912$ **A1**

Note: Accept answer following from their (a) by symmetry. Award A1 for "by symmetry" or equivalent reasoning stated explicitly.

Q3 — Normal Probability, Between Two Values EASY

$T \sim N(12, 2.5^2)$

$P(10 < T < 15)$ **M1**
$= 0.673\ (0.67277\ldots)$ **A1A1**

Note: M1 for correct bounds used in normalcdf. A1A1: first A1 for method set up correctly, second A1 for the answer. Accept 0.673.

Q4 — Inverse Normal, 5th Percentile EASY

$V \sim N(330, 6^2)$

$P(V < k) = 0.05$ **M1**
$k = \text{invNorm}(0.05, 330, 6) = 320.133\ldots \approx 320$ **A1A1**

Note: M1 for setting up invNorm with area 0.05. A1 for unrounded value. A1 for 320 (or $320.$ rounded to 3 s.f.).

Q5 — Normal Probability + Inverse Normal, Right Tail **EASY**

$W \sim N(6.4, 0.8^2)$

(a) $P(W > 7.5)$ **M1**

$= 0.0840\ (0.08398\ldots)$ **A1**

(b) $P(W > k) = 0.10$, so $P(W < k) = 0.90$

$k =$ invNorm$(0.90, 6.4, 0.8) = 7.43\ (7.4253\ldots)$ **A1**

Note: FT from their (a) for (b). Award A1 if correct invNorm used.

Q6 — Symmetry + Between-Values Probability **EASY**

$L \sim N(4.2, 0.15^2)$

(a) $P(L > 4.2) = 0.5$ **A1**

(b) $P(3.9 < L < 4.5)$ **M1**

$= 0.954\ (0.95450\ldots)$ **A1**

Note: 3.9 = 4.2 - 2(0.15) and 4.5 = 4.2 + 2(0.15); the empirical rule gives ≈ 0.954. *Accept GDC value 0.954 to 3 s.f.*

Q7 — Reading Bell-Curve Diagram **EASY**

$W \sim N(8, 3^2)$

(a) $P(W < 8) = 0.5$ **A1**

(b) $P(W < 5)$ **M1**

$= 0.159\ (0.15866\ldots)$ **A1**

Note: M1 for correct normalcdf expression. Accept 0.159.

Q8 — Normal Probability + Expected Frequency **EASY**

$H \sim N(155, 12^2)$

(a) $P(H > 170)$ **M1**

$= 0.106\ (0.10564\ldots)$ **A1**

(b) Expected number $= 400 \times 0.10564\ldots = 42.3\ (42.257\ldots)$ **A1**

Note: FT from their (a). Award A1 for their $p \times 400$ *evaluated correctly.*

Q9 — Find Standard Deviation from Percentile EASY

$B \sim N(7.5, \sigma^2); \quad P(B < 6) = 0.1$

Using inverse normal: z-score for area 0.10 is $z = -1.2816\ldots$ **M1**

$\frac{6 - 7.5}{\sigma} = -1.2816\ldots$ **A1**

$\sigma = \frac{1.5}{1.2816\ldots} = 1.17\ (1.1700\ldots)$ **A1**

Note: M1 for standardising and using $z = \pm 1.28$. A1 for correct equation. A1 for $\sigma = 1.17$ (accept 1.170).

Q10 — Normal Probability + Conditional EASY

$M \sim N(14, 1.2^2)$

(a) $P(13 < M < 16)$ **M1**
$= 0.796\ (0.79585\ldots)$ **A1**

(b) $P(M > 13) = 1 - P(M < 13)$
$P(M < 13) = 0.2023\ldots$, so $P(M > 13) = 0.7977\ldots$

$P(13 < M < 16 \mid M > 13) = \frac{P(13 < M < 16)}{P(M > 13)} = \frac{0.79585\ldots}{0.79774\ldots}$ **(M1)**

$= 0.998\ (0.99763\ldots)$ **A1**

Note: Award final A1 for answer in range [0.997, 0.999]. FT from (a).

Q11 — Find μ and σ from Two Equations MEDIUM

$T \sim N(\mu, \sigma^2); \quad P(T < 4.2) = 0.25; \quad P(T < 5.8) = 0.85$

METHOD 1 (z-scores):
$z_1 = \text{invNorm}(0.25, 0, 1) = -0.6745\ldots$ **(A1)**
$z_2 = \text{invNorm}(0.85, 0, 1) = 1.0364\ldots$ **(A1)**

Setting up equations: **M1**
$4.2 = \mu - 0.6745\,\sigma$
$5.8 = \mu + 1.0364\,\sigma$

Subtracting: $1.6 = 1.7109\,\sigma \implies \sigma = 0.935\ (0.9352\ldots)$ **A1**
$\mu = 4.2 + 0.6745 \times 0.9352 = 4.83\ (4.830\ldots)$ **A1**

Q12 — Normal Probability + Count + Binomial MEDIUM

$D \sim N(12, 0.4^2)$

(a) $P(\text{rejected}) = P(D < 11.3) + P(D > 12.7)$ **M1**
$= 0.04006\ldots + 0.04006\ldots = 0.0801\ (0.08012\ldots)$ **A1**

(b) Expected rejected $= 500 \times 0.0801\ldots = 40.1\ (40.06\ldots)$ **A1**

(c) Let $X \sim B(4, 0.0801\ldots)$
$P(X \geq 1) = 1 - P(X = 0) = 1 - (1 - 0.0801)^4$ **(M1)**
$= 0.283\ (0.2834\ldots)$ **A1**

Note: FT from (a) for (b) and (c). For (c), award (M1) for correct binomial approach.

Q13 — Sum of Independent Normals MEDIUM

$A \sim N(180, 64); \quad B \sim N(120, 25); \quad W = A + B$

(a) $E(W) = 180 + 120 = 300$ **A1**
Var$(W) = 64 + 25 = 89$ **A1**
$W \sim N(300, 89)$

(b) $P(W > 315)$ **M1**
$= 0.0561\ (0.05606\ldots)$ (using SD$(W) = \sqrt{89} = 9.434\ldots$) **A1**

Note: A common error is to add SDs instead of variances: SD$(W) \neq 8 + 5$. Deduct A1 in (a) if this error appears.

Q14 — Two Threshold Classification MEDIUM

$S \sim N(72, 100)$

(a) $P(S < 55)$ **M1**
$= 0.0446\ (0.04457\ldots)$ **A1**

(b) $P(55 \leq S \leq 90) = P(S < 90) - P(S < 55)$ **M1**
$= 0.9641\ldots - 0.04457\ldots = 0.920\ (0.9195\ldots)$ **A1**

Note: FT from their (a). Award M1 for correct subtraction approach.

Q15 — Sampling Distribution of the Mean MEDIUM

$L \sim N(9500, 450^2); \quad n = 36$

(a) $\bar{L} \sim N\left(9500,\ \frac{450^2}{36}\right) = N(9500,\ 75^2)$ **A1A1**

(A1 for mean = 9500; A1 for SD = 75 or variance = 5625)

(b) $P(\bar{L} > 9600)$ using $N(9500, 75^2)$ **M1**
$= 0.0912\ (0.09121\ldots)$ **A1**

Note: A common error is to use $\sigma = 450$ instead of $\sigma/\sqrt{36} = 75$. M1 requires correct SD.

Q16 — Unknown σ from Symmetric Interval MEDIUM

$X \sim N(240, \sigma^2)$; $P(220 < X < 260) = 0.90$

(a) By symmetry of the normal distribution about $\mu = 240$: **R1**

$P(X < 260) = \dfrac{1 + 0.90}{2} = 0.95$ **AG**

(b) $P(X < 260) = 0.95$

$z = \text{invNorm}(0.95, 0, 1) = 1.6449\ldots$ **M1**

$\dfrac{260 - 240}{\sigma} = 1.6449\ldots$ **A1**

$\sigma = \dfrac{20}{1.6449} = 12.2\ (12.158\ldots)$ **A1**

Note: (a) award R1 for explicit use of symmetry. (b) M1 requires standardising equation.

Q17 — Normal Probability + Conditional MEDIUM

$C \sim N(3.4, 0.36)$

(a) $P(C > 4.0)$ **M1**

$= 0.159\ (0.15866\ldots)$ **A1**

(b) $P(C > 4.5 \mid C > 4.0) = \dfrac{P(C > 4.5)}{P(C > 4.0)}$ **M1**

$P(C > 4.5) = 0.02275\ldots$

$= \dfrac{0.02275\ldots}{0.15866\ldots} = 0.143\ (0.14337\ldots)$ **A1**

Note: M1 for correct conditional probability fraction. FT from (a).

Q18 — Difference of Two Normals MEDIUM

$M \sim N(84, 36)$; $F \sim N(78, 25)$; $D = M - F$

(a) $E(D) = 84 - 78 = 6$ **A1**

$\text{Var}(D) = 36 + 25 = 61$ **A1**

Note: Variance of a difference adds variances (not subtracts). Deduct A1 if Var(D) = $36 - 25 = 11$.

(b) $P(M > F) = P(D > 0)$ **M1**

$D \sim N(6, 61)$; $P(D > 0) = 0.778\ (0.77789\ldots)$ **A1**

Q19 — Sampling Distribution Table MEDIUM

$A \sim N(185, 400)$

(a) $\text{SD}(\bar{A}) = \dfrac{20}{\sqrt{9}} = \dfrac{20}{3} = 6.67\ (6.666\ldots)$, so $p = 6.67$ **A1**

(b) $2.50 = \dfrac{20}{\sqrt{q}} \implies \sqrt{q} = 8 \implies q = 64$ **A1**

(c) $\bar{A} \sim N(185, 4^2)$ for $n = 25$ **(M1)**
$P(\bar{A} > 190) = 0.106\ (0.10565\ldots)$ **A1**

Q20 — Find μ from Percentile MEDIUM

$V \sim N(\mu, 144); \quad P(V < 68) = 0.15$

(a) $z = \text{invNorm}(0.15, 0, 1) = -1.0364\ldots$ **M1**

$\dfrac{68-\mu}{12} = -1.0364\ldots \implies 68 - \mu = -12.437\ldots$ **A1**

$\mu = 80.4\ (80.437\ldots)$ **A1**

(b) $P(V > 100)$ with $\mu = 80.4, \sigma = 12$
$= 0.0454\ (0.04541\ldots)$ **A1**

Note: FT from their μ in (a) for (b). Accept answers in range [0.0453, 0.0455].

Q21 — Two Unknowns, Unsignposted HARD

$M \sim N(\mu, \sigma^2); \quad P(M < 62) = 0.30; \quad P(M > 88) = 0.10$

$z_1 = \text{invNorm}(0.30, 0, 1) = -0.5244\ldots$ **(A1)**
$z_2 = \text{invNorm}(0.90, 0, 1) = 1.2816\ldots$ **(A1)**

Equations: **M1**
$62 = \mu - 0.5244\,\sigma$
$88 = \mu + 1.2816\,\sigma$

Subtracting: $26 = 1.8060\,\sigma \implies \sigma = 14.4\ (14.397\ldots)$ **A1**
$\mu = 62 + 0.5244 \times 14.397 = 69.6\ (69.55\ldots)$ **A1**

Q22 — Sum of n Normals, Find Minimum n HARD

$V \sim N(250, 25); \quad T = \sum_{i=1}^{n} V_i$

(a) $E(T) = 250n$ **A1**
$\text{Var}(T) = 25n$ **A1**

(b) $T \sim N(250n,\ 25n)$, so $\text{SD}(T) = 5\sqrt{n}$

Require $P(T > 5100) < 0.01$: **M1**

$P\left(Z > \dfrac{5100 - 250n}{5\sqrt{n}}\right) < 0.01$

For the inequality to hold: $\dfrac{5100-250n}{5\sqrt{n}} > 2.3263\ldots$ **(A1)**

Testing $n=19$: $E(T)=4750$, $5\sqrt{19}=21.79$, $z=\dfrac{5100-4750}{21.79}=16.1>2.326$ ✓

Testing $n=20$: $E(T)=5000$, $5\sqrt{20}=22.36$, $z=\dfrac{5100-5000}{22.36}=4.47>2.326$ ✓

Testing $n=21$: $E(T)=5250>5100$, so $P(T>5100)>0.5>0.01$ ×

Smallest $n=20$. **A1**

Note: M1 for correct z-score inequality. (A1) for $z=2.326$ or equivalent. A1 for $n=20$. Accept systematic GDC search.

Q23 — Symmetric Interval and Diagram Reading HARD

$T \sim N(14, 3.5^2)$

(a) Since $P(T<p)=P(T>q)$, the unshaded tails are equal.
The normal distribution is symmetric about $\mu=14$, so p and q are equidistant from the mean.
Therefore the shaded region is symmetrical about the mean. **R1**

(b) $P(T<p)=\dfrac{1-0.80}{2}=0.10$ **M1**

$p=$ invNorm$(0.10, 14, 3.5)=9.51\ (9.513\ldots)$ **A1**
$q=28-p=18.5\ (18.487\ldots)$ **A1**

(c) $P(T>20)$
$=0.0447\ (0.04468\ldots)$ **A1**

Q24 — Find σ then Linear Combination HARD

$S\sim N(48,\sigma^2)$; $\quad P(S>51)=0.12$

(a) $P(S<51)=0.88$; $\quad z=$ invNorm$(0.88,0,1)=1.1750\ldots$ **M1**

$\dfrac{51-48}{\sigma}=1.1750 \implies \sigma=\dfrac{3}{1.1750}=2.55\ (2.5531\ldots)$ **A1**

$B\sim N(46, 7.84)$; $\quad \bar{Q}=\dfrac{S+B}{2}$

(b) $E(\bar{Q})=\dfrac{48+46}{2}=47$ **A1**

Var$(\bar{Q})=\dfrac{1}{4}(\sigma^2+7.84)=\dfrac{1}{4}(6.519\ldots+7.84)=\dfrac{14.359\ldots}{4}=3.590\ldots$ **A1**

(c) $\bar{Q}\sim N(47,\ 3.590)$; $\quad P(\bar{Q}>48)$ **(M1)**
$=0.299\ (0.29909\ldots)$ **A1**

Note: In (b), the variance uses σ^2 from (a); FT is permitted. In (c), (M1) is implied.

Q25 — Show-That Sigma, Then Sample Mean Distribution HARD

$R \sim N(280, \sigma^2)$; $\quad P(|R - 280| < 35) = 0.95$

(a) By symmetry: $P(R < 315) = 0.975$ **M1**

$z =$ invNorm$(0.975, 0, 1) = 1.9600$

$\dfrac{35}{\sigma} = 1.9600 \implies \sigma = \dfrac{35}{1.9600} = 17.857\ldots$

$\sigma = 17.9$ ms (correct to 3 s.f.) **A1 AG**

Note: Must show unrounded value $17.857\ldots$ before stating 17.9 to earn AG.

(b) $\bar{R} \sim N\left(280,\ \dfrac{17.9^2}{40}\right) = N(280,\ (2.830\ldots)^2)$ **A1**

(c) $P(\bar{R} < 275)$ with SD$(\bar{R}) = 17.9/\sqrt{40} = 2.830\ldots$ **M1**

$= 0.0386\ (0.03860\ldots)$ **A1**

Q26 — Weighted Combination, Show SD Constant, Find c HARD

$E \sim N(c, 9)$; $\quad K \sim N(250 - c, 49)$; $\quad W = E + K$

(a) $E(W) = c + (250 - c) = 250$ **A1**

Var$(W) = 9 + 49 = 58$ **A1**

(b) SD$(W) = \sqrt{58}$ which contains no c; therefore SD(W) does not depend on c. **R1 AG**

(c) $P(W > 265) < 0.05$; $W \sim N(250, 58)$, SD$(W) = \sqrt{58} = 7.616\ldots$ **M1**

$P(W > 265) = P\left(Z > \dfrac{265 - 250}{7.616}\right) = P(Z > 1.970\ldots) = 0.0245\ldots$

Since Var(W) is constant, $P(W > 265)$ does not depend on c.
The condition is always satisfied for any $c \in (0, 250)$.
The largest integer value of c in a physically meaningful range is $c = 249$. **A1**

Note: M1 for computing $P(W > 265)$ and recognising it is $0.0245 < 0.05$ regardless of c. A1 for concluding $c = 249$ (any integer from 1 to 249 is valid; the largest is 249).

Q27 — Normal then Binomial Synthesis HARD

$Y \sim N(0.45, 0.08^2)$

(a) $P(Y > 0.60)$ **M1**

$= 0.0304\ (0.03038\ldots)$ **A1**

(b) Let $X \sim B(20,\ 0.03038\ldots)$ **(M1)**

$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$= 0.548\ (0.54795\ldots)$ **A1**

(c) $E(X) = 20 \times 0.03038\ldots = 0.608\ (0.6076\ldots)$ **A1**

Note: (b) Award (M1) for recognising binomial. FT from (a) throughout. (c) FT from their probability in (a).

Q28 — Show-That System then Solve **HARD**

$V \sim N(\mu, \sigma^2); \quad P(V < 90) = 0.20; \quad P(V < 130) = 0.95$

(a) $z_1 =$ invNorm$(0.20, 0, 1) = -0.8416\ldots \approx -0.8416$ (4 s.f.) **A1**

$z_2 =$ invNorm$(0.95, 0, 1) = 1.6449\ldots \approx 1.6449$ (4 s.f.) **A1**

Standardising gives: $\dfrac{90-\mu}{\sigma} = -0.8416 \implies 90 = \mu - 0.8416\,\sigma$ **AG**

$\dfrac{130-\mu}{\sigma} = 1.6449 \implies 130 = \mu + 1.6449\,\sigma$ **AG**

(b) Subtracting the first from the second: **M1**

$40 = 2.4865\,\sigma \implies \sigma = 16.1\ (16.090\ldots)$ **A1**

$\mu = 90 + 0.8416 \times 16.090 = 103\ (103.54\ldots)$ **A1**

Q29 — Sample Mean, Find Minimum n **HARD**

$P \sim N(18.5, 1.8^2)$

(a) $\bar{P} \sim N\left(18.5, \dfrac{1.8^2}{n}\right) = N\left(18.5, \dfrac{3.24}{n}\right)$ **A1**

(b) Require $P(\bar{P} < 19) \geq 0.99$: **M1**

$\dfrac{19-18.5}{1.8/\sqrt{n}} \geq$ invNorm$(0.99, 0, 1) = 2.3263\ldots$ **(A1)**

$\dfrac{0.5\sqrt{n}}{1.8} \geq 2.3263 \implies \sqrt{n} \geq \dfrac{2.3263 \times 1.8}{0.5} = 8.3748\ldots$ **A1**

$n \geq 70.1\ldots \implies n = 71$ **A1**

Note: M1 for standardising. (A1) for $z = 2.326$. A1 for $\sqrt{n} \geq 8.37$. A1 for $n = 71$. Accept $n \geq 71$.

Q30 — Synthesis: Two Combinations and Conditional **HARD**

$X \sim N(50, 6.25); \quad Y \sim N(30, 2.25); \quad L = X + Y; \quad G = X - Y$

(a) $L \sim N(80, 6.25 + 2.25) = N(80, 8.5)$; SD$(L) = \sqrt{8.5} = 2.915\ldots$ **(M1)**

$P(45 \leq L \leq 85)$

$= 0.956\ (0.95612\ldots)$ **A1**

(b) $G = X - Y \sim N(50 - 30, 6.25 + 2.25) = N(20, 8.5)$; SD$(G) = 2.915\ldots$ **M1**

$P(G < 17) = 0.151\ (0.15083\ldots)$ **A1**

(c) P(functions correctly AND tight) $= P(45 \leq L \leq 85) \cap P(G < 17)$

Since L and G are *not* independent (L and G both depend on X and Y),
we need $P(X + Y \leq 85, X - Y < 17, X + Y \geq 45)$. **(M1)**

However, $P(X + Y < 45)$ is negligible (≈ 0), and the binding constraints are
$X + Y \leq 85$ and $X - Y < 17$.

Via GDC bivariate normal computation or noting that L and G are jointly normal with Cov(L, G) = Var(X) − Var$(Y) = 6.25 - 2.25 = 4$:

$$P(G < 17 \mid 45 \le L \le 85) = \frac{P(G < 17 \cap L \le 85)}{P(45 \le L \le 85)} \approx \frac{0.1508}{0.9561} = 0.158\ (0.1577\ldots)$$ **A1**

Note: Since $P(L < 45) \approx 0$ and events $G < 17$ and $L \le 85$ are correlated, the answer is $P(G < 17$ and $L \le 85)/P(45 \le L \le 85)$. Full working via joint normal: Corr$(L, G) = 4/8.5 = 0.471$; $P(L \le 85$ and $G < 17)$ from bivariate normal ≈ 0.1508. Award A1 for answer in range $[0.157, 0.159]$.

Common Mistakes to Avoid

1. **Adding standard deviations instead of variances.** If $W = X + Y$ with X, Y independent, then Var$(W) = \sigma_X^2 + \sigma_Y^2$. Do *not* write SD$(W) = \sigma_X + \sigma_Y$ — this is wrong.
2. **Inverse normal: always use the left-tail area.** For $P(X > k) = 0.10$, enter `invNorm(0.90,` μ`,` σ`)`, not 0.10. GDC input is *always* a left-tail probability.
3. **Sample mean standard deviation.** For $\bar{X}$, the standard deviation is $\frac{\sigma}{\sqrt{n}}$, *not* σ. Forgetting $\sqrt{n}$ makes the sampling distribution far too wide.
4. **Symmetry argument for finding σ.** When a symmetric interval $[\mu - k, \mu + k]$ has probability p, then $P(X < \mu + k) = \frac{1+p}{2}$ (not p itself). Always halve the tail correctly.
5. **Variance of a difference.** Var$(X - Y)$ = Var(X) + Var(Y) (variances always add for *independent* variables, even in a difference).
6. **Conditional probability setup.** $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$. Make sure the numerator is P(both conditions), not just $P(A)$.
7. **Normal + Binomial chain.** When the question involves "select n items from a normal population and count how many exceed a threshold", this is a *two-step* problem: first find p from the normal, then use B(n, p) for counting.