# IB Math AA HL — Topic 2
## Inequalities

Solving Inequalities Graphically · Polynomial Inequalities

| Difficulty | EASY | MEDIUM | HARD |
|---|---|---|---|
| **Questions** | 10 | 10 | 10 |
| **Total Marks** | 30 | 40 | 50 |

Name:_____ Date:_____

Score:_____/120

Scan me for help

**Instructions:** Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

## Key Formulae

- **Sign diagram rule:** a product/quotient $f(x)$ changes sign at each simple zero or excluded value; mark these as critical values on a number line and test one value per interval.
- **Polynomial inequality:** factorise fully, list all real zeros with multiplicity; the sign flips at each odd-multiplicity zero and is unchanged at each even-multiplicity zero.
- **Rational inequality:** critical values include zeros of the numerator (closed/open depending on $\leq$ or $<$) AND zeros of the denominator (always open, excluded).
- **Intersection method:** $f(x) > g(x) \iff h(x) = f(x) - g(x) > 0$; find zeros of $h$, then read the positive intervals from the sketch.
- **Graphical solution set:** where the curve $y = f(x)$ lies above $y = g(x)$ gives $f > g$; record the answer in interval notation with correct open/closed endpoints.
- **Parameter threshold:** $f(x) > k$ for all $x$ requires min $f(x) > k$; use the discriminant $\Delta < 0$ (quadratic case) or calculus minimum.
- **Double inequality:** $a < f(x) < b$ is equivalent to $f(x) > a$ AND $f(x) < b$; solve each condition separately and intersect the solution sets.

## GDC Tips

- **Graph both sides:** enter $Y_1 = f(x)$ and $Y_2 = g(x)$, use Intersect (2nd → Calc → 5) to find crossing points to 3 s.f.; read off which curve is higher in each region.
- **Zero of the difference:** enter $Y_1 = f(x) - g(x)$ and use Zero (2nd → Calc → 2) to locate each critical value; the solution of $f > g$ is where $Y_1 > 0$ on the graph.
- **Table check:** after finding critical values, use Table (2nd → Graph) to verify the sign of $f(x) - g(x)$ in each interval with a test value — confirm positive/negative before writing the answer.
- **Window setting:** adjust Xmin/Xmax to include all critical values plus a margin; use ZoomFit or ZoomOut if intersections are off-screen.
- **Rational functions:** set the window to avoid the vertical asymptote; plot each branch separately if the GDC draws a spurious connecting line (Dot mode in Mode menu).
- **Record 3 s.f.:** GDC intersections are decimal approximations; always write the unrounded display value first, then round to 3 significant figures in the final answer, and use correct open/closed bracket notation.

## Common Mistakes to Avoid

1. **Multiplying by an expression of unknown sign.** Multiplying both sides of a rational inequality by $(x-a)$ without knowing its sign can reverse the inequality; always use a sign diagram or sketch instead.
2. **Forgetting excluded values in rational inequalities.** Zeros of the denominator are critical values that must appear on the sign diagram with open circles; omitting them gives an incorrect solution set.
3. **Using closed brackets at strict inequality endpoints.** If the inequality is strict ($<$ or $>$), endpoints are excluded; writing $[a, b]$ instead of $(a, b)$ loses the accuracy mark.
4. **Ignoring even-multiplicity zeros.** At a zero of even multiplicity the sign does not change; treating it as a sign-change flips the entire solution set for that region.
5. **Reading the wrong region from the graph.** After finding intersections students sometimes shade where $f < g$ instead of $f > g$; always label which curve is on top in a test interval and state this justification explicitly.
6. **Stopping at one intersection for a non-linear inequality.** Curves such as a cubic and a line can intersect more than twice; failing to find all crossing points leads to an incomplete solution set — always check the full graph window.

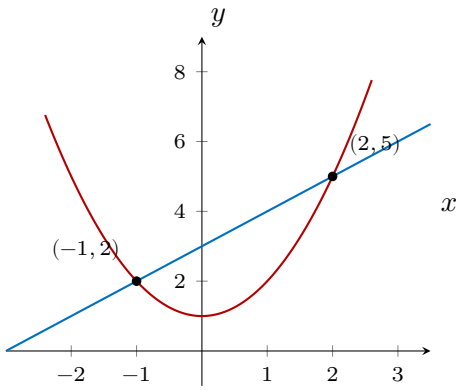
## Section A — Easy EASY (10 questions × 3 marks = 30 marks)

**Scan me for help**

**1.** Solve the inequality $x^2 - 5x + 6 < 0$, giving your answer in interval notation. *Calculator not allowed* **[3 marks]**

**2.** Find the values of $x \in \mathbb{R}$ for which $3x^2 + x - 2 \leq 0$, giving your answer in interval notation. *Calculator allowed* **[3 marks]**

**3.** The diagram shows the graphs of $f(x) = x + 3$ and $g(x) = x^2 + 1$, where $x \in \mathbb{R}$.



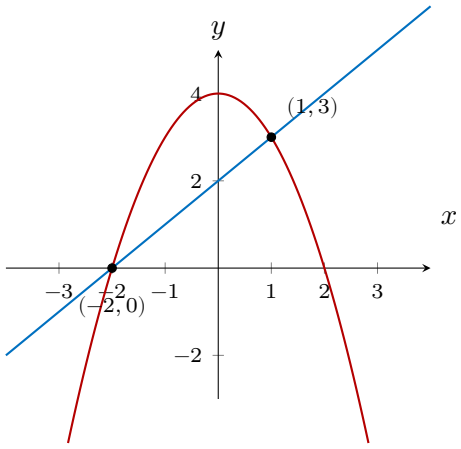

Using the diagram, write down the values of $x$ for which $f(x) > g(x)$, giving your answer in interval notation. *Calculator allowed* **[3 marks]**

**4.** Solve the inequality $(x-1)(x+2)(x-4) > 0$, where $x \in \mathbb{R}$, giving your answer using a sign diagram and interval notation. *Calculator not allowed* **[3 marks]**

**5.** Find the values of $x \in \mathbb{R}$ for which $\dfrac{1}{x-3} > 0$. Give your answer in interval notation. *Calculator allowed* **[3 marks]**

**6.** A company's weekly profit, $P$ (in thousands of dollars), is modelled by $P(x) = -x^2 + 8x - 7$, where $x \in [0, 8]$ is the number of units produced (in hundreds). Find the values of $x$ for which the company makes a positive profit ($P > 0$), giving your answer in interval notation. *Calculator allowed* **[3 marks]**

**7.** The diagram shows the graphs of $h(x) = 4 - x^2$ and $k(x) = x + 2$, where $x \in \mathbb{R}$.

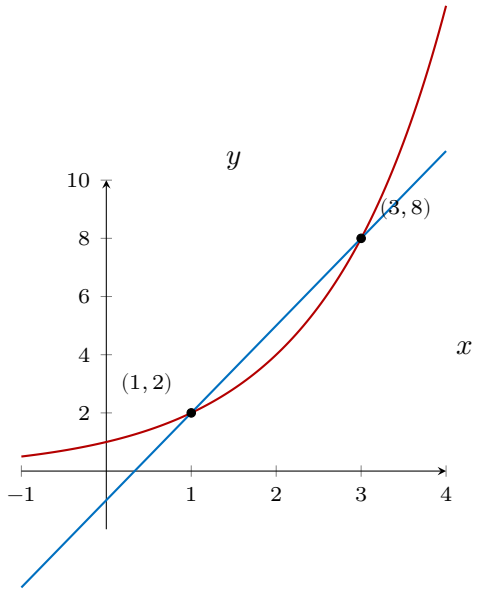



Using the diagram, write down the values of $x$ for which $h(x) > k(x)$, giving your answer in interval notation. *Calculator allowed* **[3 marks]**

**8.** Solve the inequality $2x^2 - 7x + 3 > 0$, where $x \in \mathbb{R}$, giving your answer in interval notation. *Calculator not allowed* **[3 marks]**

**9.** The concentration of a chemical in a solution, $C$ (in mg/L), at time $t \geq 0$ hours is modelled by $C(t) = t^2 - 6t + 8$. Find the values of $t$ for which $C(t) < 0$, giving your answer in interval notation. *Calculator allowed* **[3 marks]**

**10.** The diagram shows the graphs of $p(x) = 2^x$ and $q(x) = 3x - 1$, where $x \in \mathbb{R}$. The graphs intersect at $x = 1$ and $x = 3$.



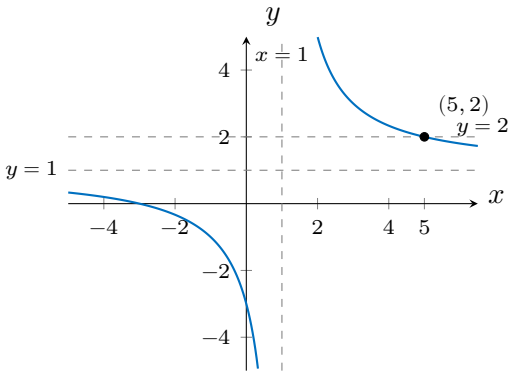

Using the diagram, write down the values of $x$ for which $p(x) < q(x)$, giving your answer in interval notation. *Calculator allowed* **[3 marks]**

## Section B — Medium MEDIUM (10 questions × 4 marks = 40 marks)

**Scan me for help**

**11.** Find the values of $x \in \mathbb{R}$ for which $x^2 - 5x + 4 < 0$. Give your answer in interval notation. *Calculator not allowed* **[4 marks]**

**12.** Find the values of $x \in \mathbb{R}$ for which $\dfrac{x+3}{x-1} > 2$. Give your answer in interval notation.

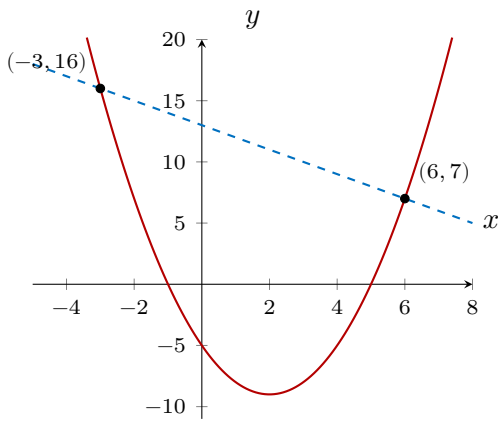



The diagram shows $f(x) = \dfrac{x+3}{x-1}$ (solid) and the line $y = 2$ (dashed). *Calculator allowed* **[4 marks]**

**13.** Find the values of $x \in \mathbb{R}$ for which $(x-1)(x+2)(x-3) \leq 0$. Give your answer in interval notation. *Calculator not allowed* **[4 marks]**

**14.** A company's daily profit (in thousands of dollars) is modelled by $P(t) = -t^2 + 7t - 6$, where $t \in [0, 8]$ is the number of hours of overtime worked per day, $t \in \mathbb{R}$. Find the values of $t$ for which $P(t) > 4$. *Calculator allowed* **[4 marks]**

**15.** Find the values of $x \in \mathbb{R}$ for which $x^2 - 4x - 5 \geq -x + 13$.

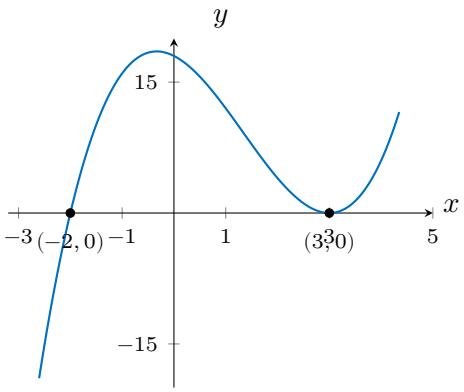



The diagram shows $y = x^2 - 4x - 5$ (solid) and $y = -x + 13$ (dashed). Give your answer in interval notation. *Calculator allowed* **[4 marks]**

**16.** Find the values of $x \in \mathbb{R}$ for which $\dfrac{2x-1}{x+2} \leq 1$. Give your answer in interval notation. *Calculator not allowed* **[4 marks]**

**17.** The concentration of a substance in a solution (in mg/L) is modelled by $C(x) = \dfrac{x^2+1}{x^2+4}$, where $x \geq 0, x \in \mathbb{R}$, is time in hours. Find the values of $x$ for which $C(x) < 0.8$. Give your answer in interval notation. *Calculator allowed* **[4 marks]**

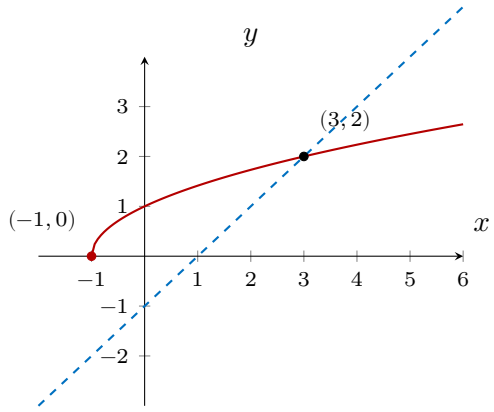
**18.** Find the values of $x \in \mathbb{R}$ for which $x^3 - 4x^2 - 3x + 18 \leq 0$.




The diagram shows $y = x^3 - 4x^2 - 3x + 18$. Give your answer in interval notation. *Calculator allowed* **[4 marks]**

**19.** Find the values of $k \in \mathbb{R}$ for which the inequality $x^2 - 6x + k > 0$ holds for all $x \in \mathbb{R}$. *Calculator not allowed* **[4 marks]**

**20.** Two functions are defined by $f(x) = \sqrt{x+1}$ and $g(x) = x - 1$, where $x \geq -1$, $x \in \mathbb{R}$. Using a graphical method, find the values of $x$ for which $f(x) > g(x)$. Give your answer in interval notation.




The diagram shows $y = \sqrt{x+1}$ (solid) and $y = x - 1$ (dashed). *Calculator allowed* **[4 marks]**

## Section C — Hard HARD (10 questions × 5 marks = 50 marks)

**Scan me for help**

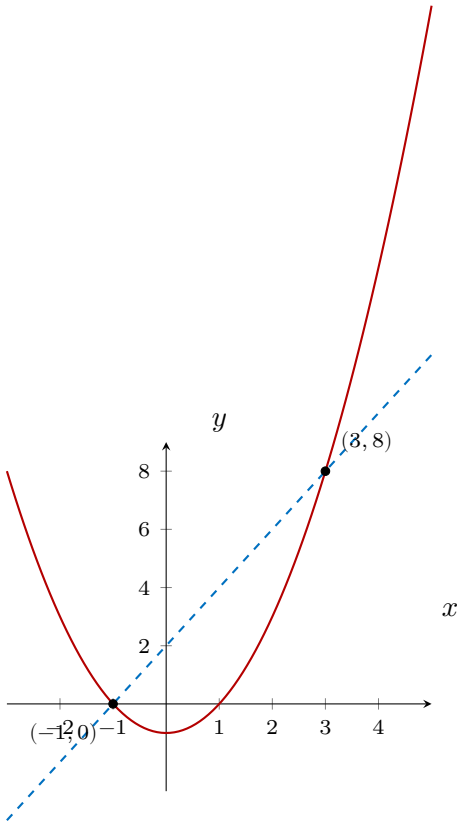
**21.** Let $f(x) = x^3 - 3x^2 - 4x + 12$, where $x \in \mathbb{R}$.

(a) Show that $f(x) = (x-2)(x+2)(x-3)$. [2]

(b) Hence solve the inequality $f(x) \leq 0$, expressing your answer in interval notation. Justify which regions satisfy the inequality using a sign diagram. [3]

*Calculator not allowed* **[5 marks]**

**22.** The diagram shows the graphs of $y = x^2 - 1$ (solid) and $y = 2x + 2$ (dashed) for $x \in \mathbb{R}$.




Using the diagram, solve the inequality $x^2 - 1 < 2x + 2$. State the solution in interval notation and justify your answer by referring to the graph. *Calculator allowed* **[5 marks]**

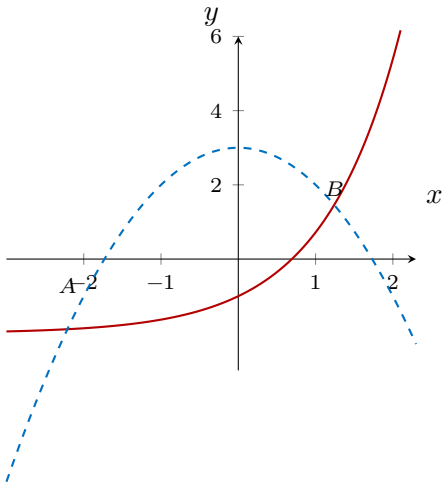
**23.** Let $g(x) = \dfrac{2x-1}{x+3}$, where $x \in \mathbb{R}$, $x \neq -3$.
Solve the inequality $g(x) > x-1$. You must use a sign diagram and must not multiply both sides by an expression of unknown sign. *Calculator not allowed* **[5 marks]**

**24.** A factory's daily profit, in thousands of dollars, is modelled by

$$P(n) = -0.4n^2 + 8n - 24,$$

where $n \in [0, 20]$ is the number of units (in hundreds) produced per day.
Find the values of $n$ for which the daily profit exceeds \$8 000, giving your answer as an interval. Interpret your answer in context. *Calculator allowed* **[5 marks]**

**25.** The diagram shows the graphs of $y = e^x - 2$ (solid) and $y = 3 - x^2$ (dashed).




The curves intersect at points $A$ and $B$ with $x$-coordinates $\alpha$ and $\beta$ respectively, where $\alpha < \beta$.

(a) Use a GDC to find $\alpha$ and $\beta$, correct to three significant figures. [2]

(b) Hence solve $e^x - 2 < 3 - x^2$, giving your answer in interval notation. Justify your answer by referring to the diagram. [3]

*Calculator allowed* **[5 marks]**

**26.** Let $h(x) = x^4 - 5x^2 + 4$, where $x \in \mathbb{R}$.

(a) Show that $h(x) = (x - 1)(x + 1)(x - 2)(x + 2)$. [2]

(b) Hence solve the inequality $h(x) < 0$, using a sign diagram to justify which intervals satisfy the inequality. Express your answer as a union of intervals. [3]

*Calculator not allowed* **[5 marks]**

**27.** The concentration of a chemical in a reaction vessel, in mol $\text{L}^{-1}$, at time $t \geq 0$ minutes is given by

$$C(t) = \frac{3t+2}{t^2+4}, \quad t \in [0, \infty).$$

Find the values of $t$ for which $C(t) \geq 0.5$, giving your answer as an interval correct to three significant figures. *Calculator allowed* **[5 marks]**

**28.** Let $p(x) = 2x^3 - 3x^2 - 11x + 6$.

(a) Given that $(x-3)$ is a factor of $p(x)$, write $p(x)$ in fully factorised form. [2]

(b) Hence solve $2x^3 - 3x^2 - 11x + 6 > 0$, using a sign diagram. State the solution in interval notation. [3]

*Calculator allowed* **[5 marks]**

**29.** Consider the inequality $\dfrac{x^2-4}{x-1} \leq 2$, where $x \in \mathbb{R}$, $x \neq 1$.
Solve the inequality by rearranging to a single rational expression, drawing a sign diagram, and stating the solution in interval notation. You must not multiply both sides by $(x-1)$ without considering its sign. *Calculator allowed* **[5 marks]**

**30.** A ball is thrown vertically. Its height above the ground, in metres, is modelled by

$$h(t) = -4.9t^2 + 14t + 1.2, \quad t \geq 0,$$

where $t$ is measured in seconds.

(a) Use a GDC to find the values of $t$ for which $h(t) \geq 9$, giving your answer correct to three significant figures. [3]

(b) Interpret your answer in context, stating how long the ball remains at or above 9 metres. [2]

*Calculator allowed* **[5 marks]**

## Mark Schemes

### Q1 — Quadratic Inequality with Exact Roots EASY

Factorise: $(x-2)(x-3)<0$ **M1**

Critical values $x=2$ and $x=3$; parabola opens upward so the expression is negative between the roots. **R1**

Solution: $x \in (2,3)$ **A1**

### Q2 — Quadratic Inequality via GDC EASY

Using GDC: graph $y=3x^2+x-2$ and identify $x$-intercepts. **M1**

Roots: $x=-1$ and $x=2/3$; parabola opens upward so $y \leq 0$ between the roots. **R1**

Solution: $x \in [-1, 2/3]$ **A1**

### Q3 — Curve vs Line Intersection Graphical EASY

From the diagram, $f(x) > g(x)$ where the line lies above the parabola, i.e. between the two intersection points $x=-1$ and $x=2$. **M1**

The solution region is the thick segment on the $x$-axis between the intersection points. **R1**

Solution: $x \in (-1,2)$ **A1**

### Q4 — Cubic Inequality with Sign Diagram EASY

Critical values: $x=-2$, $x=1$, $x=4$.

| Interval | $x<-2$ | $-2<x<1$ | $1<x<4$ | $x>4$ |
|---|---|---|---|---|
| Sign | $-$ | $+$ | $-$ | $+$ |

**M1**

Product is positive where sign is $+$. **R1**

Solution: $x \in (-2,1) \cup (4,+\infty)$ **A1**

### Q5 — Rational Inequality Sign Diagram EASY

The expression $\dfrac{1}{x-3}$ is undefined at $x=3$ (excluded value, open circle). **M1**

Sign diagram: $x-3<0$ for $x<3$ (expression negative) and $x-3>0$ for $x>3$ (expression positive). **R1**

Solution: $x \in (3,+\infty)$ **A1**

### Q6 — Context Inequality Profit Model EASY

Using GDC: graph $y=-x^2+8x-7$ and find $x$-intercepts. **M1**

Roots at $x=1$ and $x=7$; parabola opens downward so $P>0$ between the roots. **R1**

Solution: $x \in (1,7)$ (hundreds of units, within domain $[0,8]$) **A1**

### Q7 — Curve vs Line Intersection Graphical EASY

From the diagram, $h(x)>k(x)$ where the parabola lies above the line, i.e. between the two intersection points $x=-2$ and $x=1$. **M1**

The solution region is the thick segment on the $x$-axis between the intersection points. **R1**

Solution: $x \in (-2,1)$ **A1**

### Q8 — Quadratic Inequality Factorised EASY

Factorise: $(2x-1)(x-3)>0$ **M1**

Critical values $x=1/2$ and $x=3$; parabola opens upward so the expression is positive outside the roots. **R1**

Solution: $x \in (-\infty,1/2)\cup(3,+\infty)$ **A1**

### Q9 — Context Inequality Quadratic Model EASY

Using GDC: graph $y=t^2-6t+8$ and find $t$-intercepts. **M1**

Roots at $t=2$ and $t=4$; parabola opens upward so $C<0$ between the roots. **R1**

Solution: $t \in (2,4)$ **A1**

### Q10 — Exponential vs Line Intersection Graphical EASY

From the diagram, $p(x)<q(x)$ where the exponential curve lies below the line, i.e. between the two intersection points. **M1**

Algebraic check confirms the curves intersect exactly at $x=1$ and $x=3$: $p(1)=2^1=2=3(1)-1=$

$q(1)$ and $p(3) = 2^3 = 8 = 3(3) - 1 = q(3)$; the exponential is below the line between these values. **R1**

Solution: $x \in (1, 3)$ **A1**

*Note: replacement question for the original broken item, whose diagram-labelled intersection points did not lie on both curves. The line $q(x) = 3x - 1$ is constructed so that $p(1) = q(1)$ and $p(3) = q(3)$ exactly.*

## Q11 — Quadratic Inequality Sign Diagram MEDIUM

Factorise: $x^2 - 5x + 4 = (x - 1)(x - 4)$ **M1**

Critical values are $x = 1$ and $x = 4$.

| $x$ | $x < 1$ | $x = 1$ | $1 < x < 4$ | $x = 4$ | $x > 4$ |
|---|---|---|---|---|---|
| sign | $+$ | $0$ | $-$ | $0$ | $+$ |

**A1**

The expression is negative ($< 0$) for $1 < x < 4$. **R1**

Answer: $x \in (1, 4)$ **A1**

## Q12 — Rational Inequality Graphical Method MEDIUM

Rewrite as $\dfrac{x+3}{x-1} - 2 > 0$, giving $\dfrac{x+3-2(x-1)}{x-1} = \dfrac{-x+5}{x-1} > 0$. **M1**

Critical values: $x = 5$ (numerator zero) and $x = 1$ (excluded, denominator zero). **A1**

*Note: Do not multiply by $(x - 1)$ since its sign is unknown.*

Sign diagram for $\dfrac{-(x-5)}{x-1}$: positive when $1 < x < 5$. **R1**

Answer: $x \in (1, 5)$ **A1**

## Q13 — Cubic Inequality Sign Diagram MEDIUM

The polynomial is already factorised: $(x - 1)(x + 2)(x - 3)$. **M1**

Critical values: $x = -2, 1, 3$.

| Region | $x < -2$ | $-2 < x < 1$ | $1 < x < 3$ | $x > 3$ |
|---|---|---|---|---|
| Sign | $-$ | $+$ | $-$ | $+$ |

**A1**

The expression is $\leq 0$ (including zeros) for $x \leq -2$ or $1 \leq x \leq 3$. **R1**

Answer: $x \in (-\infty, -2] \cup [1, 3]$ **A1**

### Q14 — Context Inequality via GDC MEDIUM

Require $-t^2 + 7t - 6 > 4$, i.e. $-t^2 + 7t - 10 > 0$. **M1**

Using GDC: graph $y = -t^2 + 7t - 10$ and find zeros. Zeros: $t = 2$ and $t = 5$ exactly. **A1**

*Note: Unrounded GDC values are $t = 2.000$ and $t = 5.000$.*

Parabola opens downward, so the expression is positive between the roots. **R1**

Answer: $t \in (2, 5)$ **A1**

### Q15 — Quadratic vs Line Intersection MEDIUM

Require $x^2 - 4x - 5 \geq -x + 13$, i.e. $x^2 - 3x - 18 \geq 0$. **M1**

Factorise: $x^2 - 3x - 18 = (x + 3)(x - 6)$, giving critical values $x = -3$ and $x = 6$. Both are genuine intersections of the two curves: $f(-3) = 9 + 12 - 5 = 16 = -(-3) + 13 = g(-3)$ and $f(6) = 36 - 24 - 5 = 7 = -6 + 13 = g(6)$. **A1**

Parabola opens upward, so the expression is $\geq 0$ outside the roots, matching the diagram where the solid curve lies above the dashed line outside the interval $[-3, 6]$. **R1**

Answer: $x \in (-\infty, -3] \cup [6, +\infty)$ **A1**

*Note: replacement question for the original broken item, whose diagram-labelled points did not lie on the parabola. Here $f(-3) = g(-3)$ and $f(6) = g(6)$ exactly.*

### Q16 — Rational Inequality Sign Diagram MEDIUM

Rewrite as $\dfrac{2x-1}{x+2} - 1 \leq 0$, giving $\dfrac{2x-1-(x+2)}{x+2} = \dfrac{x-3}{x+2} \leq 0$. **M1**

Critical values: $x = 3$ (zero) and $x = -2$ (excluded). **A1**

*Note: Do not multiply by $(x + 2)$ since its sign is unknown.*

Sign diagram: $\dfrac{x-3}{x+2} \leq 0$ when $-2 < x \leq 3$ (open at $x = -2$, closed at $x = 3$). **R1**

Answer: $x \in (-2, 3]$ **A1**

### Q17 — Context Rational Inequality via GDC MEDIUM

Require $\dfrac{x^2+1}{x^2+4} < 0.8$. **M1**

Using GDC: graph $y = \dfrac{x^2+1}{x^2+4}$ and $y = 0.8$; find intersection for $x \geq 0$. GDC gives intersection at $x^2 + 1 = 0.8(x^2 + 4) \Rightarrow 0.2x^2 = 2.2 \Rightarrow x = \sqrt{11} \approx 3.317$. **A1**

*Note: GDC gives intersection at $x = 3.317$ (3 s.f.).*

Since $C(x)$ is increasing for $x \geq 0$ and $C(0) = 0.25 < 0.8$, the concentration is below $0.8$ for $x < \sqrt{11}$. **R1**

Answer: $x \in [0, \sqrt{11})$, i.e. $x \in [0, 3.32)$ (3 s.f.) **A1**

### Q18 — Cubic Inequality Repeated Root MEDIUM

Using GDC: graph $y = x^3 - 4x^2 - 3x + 18$ and identify zeros. GDC shows roots at $x = -2$ and $x = 3$ (repeated). **M1**

Factor: $x^3 - 4x^2 - 3x + 18 = (x + 2)(x - 3)^2$. **A1**

Since $(x - 3)^2 \geq 0$ always, the sign is determined by $(x + 2)$: the expression is $\leq 0$ when $x + 2 \leq 0$, i.e. $x \leq -2$, and also equals $0$ at $x = 3$. **R1**

Answer: $x \in (-\infty, -2] \cup \{3\}$ **A1**

*Note: the repeated root at $x = 3$ contributes only the isolated point, written $\{3\}$, since $(x - 3)^2 \geq 0$ touches zero there without a sign change.*

### Q19 — Parameter Condition via Discriminant MEDIUM

For $x^2 - 6x + k > 0$ to hold for all $x \in \mathbb{R}$, the parabola must have no real roots, i.e. the discriminant must be strictly negative. **M1**

Discriminant: $\Delta = (-6)^2 - 4(1)(k) = 36 - 4k$. **A1**

Require $\Delta < 0$: $36 - 4k < 0 \Rightarrow k > 9$. **R1**

Answer: $k \in (9, +\infty)$ **A1**

### Q20 — Square Root vs Line Intersection MEDIUM

For $x - 1 < 0$, i.e. $-1 \leq x < 1$: $\sqrt{x + 1} \geq 0 > x - 1$ automatically, so the inequality holds throughout this part of the domain. **M1**

For $x \geq 1$: both sides are non-negative, so square to get $x + 1 > (x - 1)^2$, giving $0 > x^2 - 3x = x(x - 3)$, i.e. $0 < x < 3$. Combined with $x \geq 1$ this gives $1 \leq x < 3$; back-substitution confirms $f(3) = g(3) = 2$, so $x = 3$ is the genuine intersection and is excluded (strict inequality). **A1**

Combining both parts, the solid curve lies above the dashed line throughout $[-1, 3)$, matching the diagram. **R1**

Answer: $x \in [-1, 3)$ **A1**

*Note: replacement question for the original broken item, whose diagram-labelled points did not lie on both curves. Here $f(3) = \sqrt{4} = 2 = 3 - 1 = g(3)$ exactly.*

## Q21 — Cubic Factorisation and Sign Diagram HARD

**(a)**

Testing $x = 2$: $f(2) = 8 - 12 - 8 + 12 = 0$, so $(x - 2)$ is a factor. **M1**

Dividing: $x^3 - 3x^2 - 4x + 12 = (x - 2)(x^2 - x - 6) = (x - 2)(x - 3)(x + 2)$. **A1 AG**

**(b)**

Critical values: $x = -2, 2, 3$.

| $x$ | $(-\infty, -2)$ | $-2$ | $(-2, 2)$ | $2$ | $(2, 3)$ | $3$ | $(3, \infty)$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | $-$ | $0$ | $+$ | $0$ | $-$ | $0$ | $+$ |

**M1**

$f(x) \leq 0$ where $f(x)$ is negative or zero. **R1**

Solution: $x \in (-\infty, -2] \cup [2, 3]$. **A1**

*Note: Award M1 for a correct sign diagram with all four sign regions. Award R1 for explicitly identifying the regions where $f(x) \leq 0$. Condone $\leq$ at the endpoints.*

## Q22 — Parabola and Line Intersection HARD

Setting $x^2 - 1 = 2x + 2$: **M1**

$x^2 - 2x - 3 = 0 \Rightarrow (x + 1)(x - 3) = 0$, giving $x = -1$ or $x = 3$. **A1**

The parabola lies below the line between the intersections, as read from the diagram (the solid curve is below the dashed line for $-1 < x < 3$). **R1**

GDC or substitution check at $x = 0$: $f(0) = -1 < 2 = g(0)$ confirms the parabola is below the line. **M1**

Solution: $x \in (-1, 3)$. **A1**

*Note: Award R1 only if the candidate explicitly states which curve is lower in the interval, with reference to the graph.*

## Q23 — Rational Inequality via Sign Diagram HARD

Rearrange: $\dfrac{2x - 1}{x + 3} - (x - 1) > 0$. **M1**

$\dfrac{(2x - 1) - (x - 1)(x + 3)}{x + 3} > 0 \Rightarrow \dfrac{2x - 1 - x^2 - 2x + 3}{x + 3} > 0 \Rightarrow \dfrac{-x^2 + 2}{x + 3} > 0.$ **A1**

Critical values: numerator zero at $x = \pm\sqrt{2}$; excluded value $x = -3$.

| $x$ | $(-\infty, -3)$ | $-3$ | $(-3, -\sqrt{2})$ | $-\sqrt{2}$ | $(-\sqrt{2}, \sqrt{2})$ | $\sqrt{2}$ | $(\sqrt{2}, \infty)$ |
|---|---|---|---|---|---|---|---|
| Sign | $+$ | undef | $-$ | $0$ | $+$ | $0$ | $-$ |

**M1**

*Note: for $x < -3$ both $-x^2 + 2$ and $x + 3$ are negative, so the quotient is positive; for $-3 < x < -\sqrt{2}$ the numerator is negative and the denominator positive, so the quotient is negative.*

The expression is strictly positive on $(-\infty, -3) \cup (-\sqrt{2}, \sqrt{2})$; $x = -3$ is excluded (undefined) and $x = \pm\sqrt{2}$ give zero, excluded from a strict inequality. **R1**

Solution: $x \in (-\infty, -3) \cup (-\sqrt{2}, \sqrt{2})$. **A1**

*Note: Penalise any attempt to multiply through by $(x+3)$ without considering its sign. Award M1 for a correct single-fraction rearrangement. The sign diagram must show $x = -3$ as undefined.*

## Q24 — Context Inequality from Quadratic Model HARD

Require $P(n) > 8$, i.e. $-0.4n^2 + 8n - 24 > 8$. **M1**

Rearrange: $-0.4n^2 + 8n - 32 > 0 \Rightarrow 0.4n^2 - 8n + 32 < 0 \Rightarrow n^2 - 20n + 80 < 0$. **M1**

GDC: solve $n^2 - 20n + 80 = 0$. Unrounded: $n = \dfrac{20 \pm \sqrt{400 - 320}}{2} = 10 \pm 2\sqrt{5}$.

$n = 10 - 2\sqrt{5} \approx 5.528$ and $n = 10 + 2\sqrt{5} \approx 14.47$. **A1**

The parabola opens upward, so $n^2 - 20n + 80 < 0$ between the roots. **R1**

Solution: $n \in (5.53, 14.5)$ (to 3 s.f.), with $n \in [0, 20]$. The factory earns more than \$8 000 profit per day when between approximately 553 and 1447 units are produced. **A1**

*Note: Accept exact forms $10 \pm 2\sqrt{5}$ for the final answer.*

## Q25 — Exponential and Quadratic Intersection HARD

**(a)**

GDC: solve $e^x - 2 = 3 - x^2$, i.e. $e^x + x^2 - 5 = 0$. **M1**

Unrounded: $\alpha = -2.2114\ldots$, $\beta = 1.2411\ldots$

$\alpha = -2.21$ (3 s.f.), $\beta = 1.24$ (3 s.f.). **A1**

**(b)**

From the diagram, the solid curve ($y = e^x - 2$) lies below the dashed curve ($y = 3 - x^2$) between the two intersection points $A$ and $B$. **R1**

The thick segment on the $x$-axis confirms the solution region $\alpha < x < \beta$. **M1**

Solution: $x \in (-2.21, 1.24)$ (to 3 s.f.). **A1**

*Note: Award R1 only if the candidate explicitly states which curve is lower between the intersections with reference to the diagram. Follow-through from correct values in (a) is permitted in (b).*

## Q26 — Quartic Factorisation and Sign Diagram HARD

**(a)**

$h(x) = x^4 - 5x^2 + 4 = (x^2 - 1)(x^2 - 4)$. **M1**

$(x^2-1)(x^2-4) = (x-1)(x+1)(x-2)(x+2)$. **A1 AG**

**(b)**

Critical values: $x \in \{-2, -1, 1, 2\}$.

| $x$ | $(-\infty,-2)$ | $-2$ | $(-2,-1)$ | $-1$ | $(-1,1)$ | $1$ | $(1,2)$ | $2$ | $(2,\infty)$ |
|---|---|---|---|---|---|---|---|---|---|
| $h(x)$ | $+$ | $0$ | $-$ | $0$ | $+$ | $0$ | $-$ | $0$ | $+$ |

**M1**

$h(x) < 0$ in the regions where the sign is negative (endpoints excluded since strict inequality). **R1**

Solution: $x \in (-2,-1) \cup (1,2)$. **A1**

*Note: Award M1 for a correct sign diagram with all five sign regions identified. Award R1 for explicitly identifying the negative regions and stating why endpoints are excluded.*

## Q27 — Rational Concentration Inequality HARD

Require $\dfrac{3t+2}{t^2+4} \geq 0.5$. **M1**

Since $t^2+4 > 0$ for all real $t$, multiply both sides: $3t+2 \geq 0.5(t^2+4) = 0.5t^2+2$. **M1**

Rearrange: $0 \geq 0.5t^2 - 3t \Rightarrow 0 \geq t^2 - 6t \Rightarrow t(t-6) \leq 0$, giving critical values $t=0$ and $t=6$. **A1**

GDC check at $t=3$: $C(3) = \dfrac{11}{13} \approx 0.846 \geq 0.5$ (✓); at $t=7$: $C(7) = \dfrac{23}{53} \approx 0.434 < 0.5$ (fails). **R1**

Since $t \geq 0$, solution: $t \in [0, 6]$. **A1**

*Note: Since $t^2+4 > 0$ always, multiplying through is valid; this is the same interval as $[0,6]$ using the domain $t \geq 0$. The answer is exact; no rounding required.*

## Q28 — Cubic Factorisation and Inequality HARD

**(a)**

Dividing $p(x)$ by $(x-3)$: GDC or long division gives $p(x) = (x-3)(2x^2+3x-2)$. **M1**

Factorising: $2x^2+3x-2 = (2x-1)(x+2)$.
So $p(x) = (x-3)(2x-1)(x+2)$. **A1**

**(b)**

Critical values: $x = -2, 1/2, 3$.

| $x$ | $(-\infty,-2)$ | $-2$ | $(-2,1/2)$ | $1/2$ | $(1/2,3)$ | $3$ | $(3,\infty)$ |
|---|---|---|---|---|---|---|---|
| $p(x)$ | $-$ | $0$ | $+$ | $0$ | $-$ | $0$ | $+$ |

**M1**

$p(x) > 0$ in the regions where the sign is positive; endpoints excluded (strict inequality). **R1**

Solution: $x \in (-2, 1/2) \cup (3, +\infty)$. **A1**

*Note: Award M1 for a correct sign diagram with four sign regions. GDC may be used to verify the factori-*

*sation in (a).*

## Q29 — Rational Inequality Irrational Roots HARD

Rearrange: $\dfrac{x^2-4}{x-1}-2\le 0 \Rightarrow \dfrac{x^2-4-2(x-1)}{x-1}\le 0 \Rightarrow \dfrac{x^2-2x-2}{x-1}\le 0.$ **M1**

GDC: roots of $x^2-2x-2=0$: $x=1\pm\sqrt{3}$, i.e. $x\approx -0.732$ or $x\approx 2.73$. **A1**

Critical values: $x=1-\sqrt{3}\approx -0.732$, $x=1$ (excluded), $x=1+\sqrt{3}\approx 2.73$.

| Interval | $(-\infty, 1-\sqrt{3})$ | $1-\sqrt{3}$ | $(1-\sqrt{3}, 1)$ | $1$ | $(1, 1+\sqrt{3})$ | $1+\sqrt{3}$ | $(1+\sqrt{3}, \infty)$ |
|---|---|---|---|---|---|---|---|
| Sign | $-$ | $0$ | $+$ | undef | $-$ | $0$ | $+$ |

**M1**

*Note: for $x<1-\sqrt{3}$ the numerator $x^2-2x-2$ is positive and the denominator $x-1$ is negative, so the quotient is negative; for $1-\sqrt{3}<x<1$ both numerator and denominator are negative, so the quotient is positive.*

The expression is $\le 0$ where the sign is negative or zero; $x=1$ is excluded (undefined). **R1**

Solution: $x\in(-\infty, 1-\sqrt{3}]\cup(1, 1+\sqrt{3}]$. **A1**

*Note: Award M1 for correctly forming a single rational expression. The sign diagram must show $x=1$ as undefined with an open circle, and the final answer must reflect this. Penalise if the candidate multiplies by $(x-1)$ without case analysis.*

## Q30 — Projectile Height Inequality HARD

**(a)**

Require $h(t)\ge 9$: $-4.9t^2+14t+1.2\ge 9$. **M1**

GDC: solve $-4.9t^2+14t-7.8=0$.

Unrounded roots: $t=\dfrac{-14\pm\sqrt{196-4(4.9)(7.8)}}{-9.8}=\dfrac{10\pm\sqrt{22}}{7}$; GDC gives $t\approx 0.7585\ldots$ and $t\approx 2.0986\ldots$

$t_1=0.759$ s and $t_2=2.10$ s (to 3 s.f.). **A1**

The parabola opens downward, so $h(t)\ge 9$ between the roots. **R1**

Solution: $t\in[0.759, 2.10]$ s. **A1**

**(b)**

The ball is at or above 9 metres for a duration of $2.0986\ldots-0.7585\ldots=1.34$ seconds (to 3 s.f.). The ball first reaches 9 m at $t\approx 0.759$ s and returns to 9 m at $t\approx 2.10$ s. **A1**

*Note: Award the final A1 only if the answer references both the duration and the physical meaning (ball ascending and descending through 9 m). Follow-through from (a) is permitted.*