IB Math AA HL — Topic 2
Inequalities

Solving Inequalities Graphically · Polynomial Inequalities

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

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Name:________________________ Date:______________

Score:__________/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

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Key Formulae

- **Sign diagram rule:** a product/quotient $f(x)$ changes sign at each simple zero or excluded value; mark these as critical values on a number line and test one value per interval.
- **Polynomial inequality:** factorise fully, list all real zeros with multiplicity; the sign flips at each odd-multiplicity zero and is unchanged at each even-multiplicity zero.
- **Rational inequality:** critical values include zeros of the *numerator* (closed/open depending on $\leq$ or $<$) AND zeros of the *denominator* (always open, excluded).
- **Intersection method:** $f(x) > g(x) \iff h(x) = f(x) - g(x) > 0$; find zeros of h, then read the positive intervals from the sketch.
- **Graphical solution set:** where the curve $y = f(x)$ lies *above* $y = g(x)$ gives $f > g$; record the answer in interval notation with correct open/closed endpoints.
- **Parameter threshold:** $f(x) > k$ for *all* x requires min $f(x) > k$; use the discriminant $\Delta < 0$ (quadratic case) or calculus minimum.
- **Double inequality:** $a < f(x) < b$ is equivalent to $f(x) > a$ **AND** $f(x) < b$; solve each condition separately and intersect the solution sets.

GDC Tips

- **Graph both sides:** enter $Y_1 = f(x)$ and $Y_2 = g(x)$, use `Intersect` (2nd $\to$ Calc $\to$ 5) to find crossing points to 3 s.f.; read off which curve is higher in each region.
- **Zero of the difference:** enter $Y_1 = f(x) - g(x)$ and use `Zero` (2nd $\to$ Calc $\to$ 2) to locate each critical value; the solution of $f > g$ is where $Y_1 > 0$ on the graph.
- **Table check:** after finding critical values, use `Table` (2nd $\to$ Graph) to verify the sign of $f(x) - g(x)$ in each interval with a test value — confirm positive/negative before writing the answer.
- **Window setting:** adjust `Xmin`/`Xmax` to include *all* critical values plus a margin; use `ZoomFit` or `ZoomOut` if intersections are off-screen.
- **Rational functions:** set the window to avoid the vertical asymptote; plot each branch separately if the GDC draws a spurious connecting line (`Dot` mode in `Mode` menu).
- **Record 3 s.f.:** GDC intersections are decimal approximations; always write unrounded display value first, then round to 3 significant figures in the final answer, and use correct open/closed bracket notation.

Common Mistakes to Avoid

1. **Multiplying by an expression of unknown sign.** Multiplying both sides of a rational inequality by $(x—a)$ without knowing its sign can reverse the inequality; always use a sign diagram or sketch instead.
2. **Forgetting excluded values in rational inequalities.** Zeros of the denominator are critical values that must appear on the sign diagram with *open* circles; omitting them gives an incorrect solution set.
3. **Using closed brackets at strict inequality endpoints.** If the inequality is strict ($<$ or $>$), endpoints are excluded; writing $[a, b]$ instead of (a, b) loses the accuracy mark.
4. **Ignoring even-multiplicity zeros.** At a zero of even multiplicity the sign does *not* change; treating it as a sign-change flips the entire solution set for that region.
5. **Reading the wrong region from the graph.** After finding intersections students sometimes shade where $f < g$ instead of $f > g$; always label which curve is on top in a test interval and state this justification explicitly.
6. **Stopping at one intersection for a non-linear inequality.** Curves such as a cubic and a line can intersect more than twice; failing to find all crossing points leads to an incomplete solution set — always check the full graph window.

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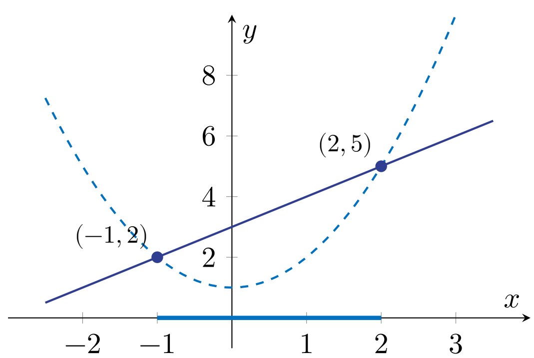
Section A — Easy

EASY

1. Solve the inequality $x^2 - 5x + 6 < 0$, giving your answer in interval notation. *Calculator not allowed* **[3 marks]**

2. Find the values of $x \in \mathbb{R}$ for which $3x^2 + x - 2 \leq 0$, giving your answer in interval notation. *Calculator allowed* **[3 marks]**

3. The diagram shows the graphs of $f(x) = x + 3$ and $g(x) = x^2 + 1$, where $x \in \mathbb{R}$.



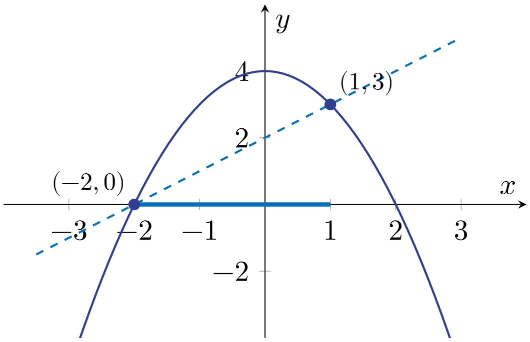
Using the diagram, write down the values of x for which $f(x) > g(x)$, giving your answer in interval notation. *Calculator allowed* **[3 marks]**

4. Solve the inequality $(x-1)(x+2)(x-4) > 0$, where $x \in \mathbb{R}$, giving your answer using a sign diagram and interval notation. *Calculator not allowed* **[3 marks]**

5. Find the values of $x \in \mathbb{R}$ for which $\dfrac{1}{x-3} > 0$. Give your answer in interval notation. *Calculator allowed* **[3 marks]**

6. A company's weekly profit, P (in thousands of dollars), is modelled by $P(x) = -x^2 + 8x - 7$, where $x \in [0, 8]$ is the number of units produced (in hundreds). Find the values of x for which the company makes a positive profit ($P > 0$), giving your answer in interval notation. *Calculator allowed* **[3 marks]**

7. The diagram shows the graphs of $h(x) = 4 - x^2$ and $k(x) = x + 2$, where $x \in \mathbb{R}$.

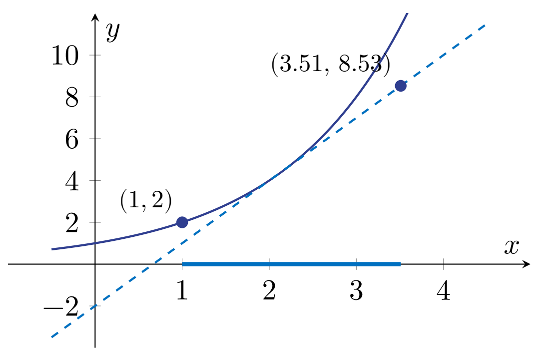


Using the diagram, write down the values of x for which $h(x) > k(x)$, giving your answer in interval notation. *Calculator allowed* **[3 marks]**

8. Solve the inequality $2x^2 - 7x + 3 > 0$, where $x \in \mathbb{R}$, giving your answer in interval notation. *Calculator not allowed* **[3 marks]**

9. The concentration of a chemical in a solution, C (in mg/L), at time $t \geq 0$ hours is modelled by $C(t) = t^2 - 6t + 8$. Find the values of t for which $C(t) < 0$, giving your answer in interval notation. *Calculator allowed* **[3 marks]**

10. The diagram shows the graphs of $p(x) = 2^x$ and $q(x) = 3x - 2$, where $x \in \mathbb{R}$. The graphs intersect at $x = 1.00$ and $x = 3.51$ (correct to 3 significant figures).



Using the diagram, write down the values of x for which $p(x) < q(x)$, giving your answer in interval notation.
Calculator allowed **[3 marks]**

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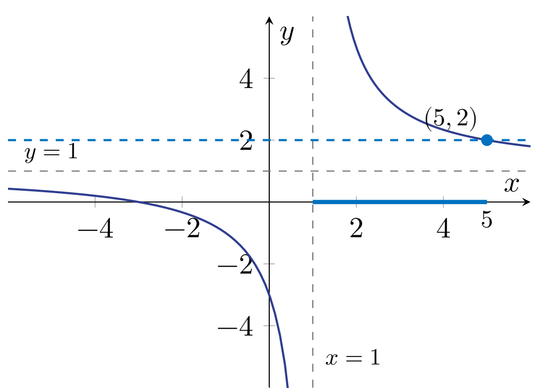
Section B — Medium

MEDIUM

11. Find the values of $x \in \mathbb{R}$ for which $x^2 - 5x + 4 < 0$. Give your answer in interval notation. *Calculator not allowed* **[4 marks]**

12. Find the values of $x \in \mathbb{R}$ for which $\dfrac{x+3}{x-1} > 2$. Give your answer in interval notation.

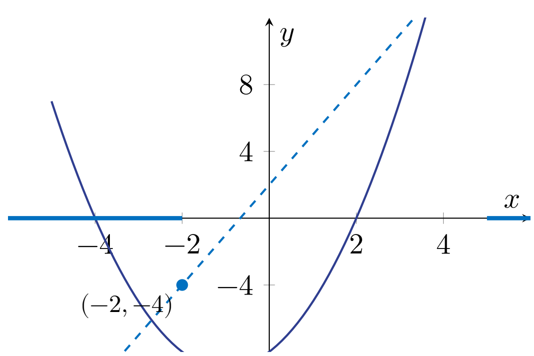


The diagram shows $f(x) = \dfrac{x+3}{x-1}$ (solid) and the line $y = 2$ (dashed). *Calculator allowed* **[4 marks]**

13. Find the values of $x \in \mathbb{R}$ for which $(x-1)(x+2)(x-3) \leq 0$. Give your answer in interval notation. *Calculator not allowed* **[4 marks]**

14. A company's daily profit (in thousands of dollars) is modelled by $P(t) = -t^2 + 7t - 6$, where $t \in [0, 8]$ is the number of hours of overtime worked per day, $t \in \mathbb{R}$. Find the values of t for which $P(t) > 4$. *Calculator allowed* **[4 marks]**

15. Find the values of $x \in \mathbb{R}$ for which $x^2 + 2x - 8 \geq 3x + 2$.

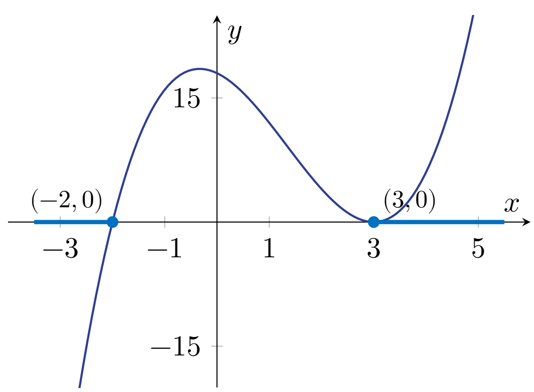


The diagram shows $y = x^2 + 2x - 8$ (solid) and $y = 3x + 2$ (dashed). Give your answer in interval notation. *Calculator allowed* **[4 marks]**

16. Find the values of $x \in \mathbb{R}$ for which $\dfrac{2x-1}{x+2} \leq 1$. Give your answer in interval notation. *Calculator not allowed* **[4 marks]**

17. The concentration of a substance in a solution (in mg/L) is modelled by $C(x) = \dfrac{x^2+1}{x^2+4}$, where $x \geq 0$, $x \in \mathbb{R}$, is time in hours. Find the values of x for which $C(x) < 0.8$. Give your answer in interval notation. *Calculator allowed* **[4 marks]**

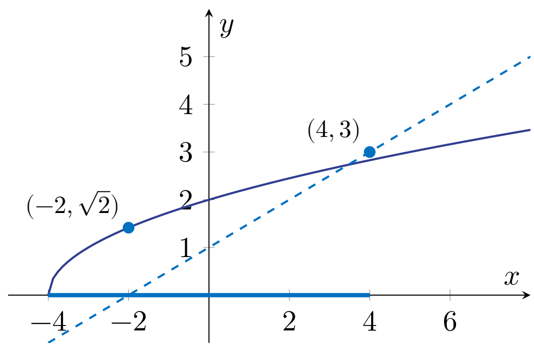
18. Find the values of $x \in \mathbb{R}$ for which $x^3 - 4x^2 - 3x + 18 \leq 0$.



The diagram shows $y = x^3 - 4x^2 - 3x + 18$. Give your answer in interval notation. *Calculator allowed* **[4 marks]**

19. Find the values of $k \in \mathbb{R}$ for which the inequality $x^2 - 6x + k > 0$ holds for all $x \in \mathbb{R}$. *Calculator not allowed* **[4 marks]**

20. Two functions are defined by $f(x) = \sqrt{x+4}$ and $g(x) = \dfrac{x}{2} + 1$, where $x \geq -4$, $x \in \mathbb{R}$. Using a graphical method, find the values of x for which $f(x) > g(x)$. Give your answer in interval notation.



The diagram shows $y = \sqrt{x+4}$ (solid) and $y = \dfrac{x}{2} + 1$ (dashed). *Calculator allowed* **[4 marks]**

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Section C — Hard

HARD

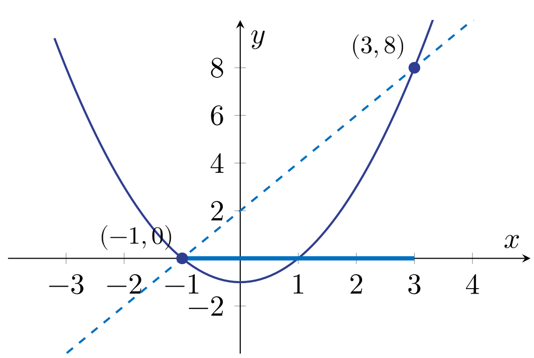
21. Let $f(x) = x^3 - 3x^2 - 4x + 12$, where $x \in \mathbb{R}$.

(a) Show that $f(x) = (x-2)(x+2)(x-3)$. **[2 marks]**

(b) Hence solve the inequality $f(x) \leq 0$, expressing your answer in interval notation. Justify which regions satisfy the inequality using a sign diagram. **[3 marks]**

Calculator not allowed **[5 marks]**

22. The diagram shows the graphs of $y = x^2 - 1$ (solid) and $y = 2x + 2$ (dashed) for $x \in \mathbb{R}$.



Using the diagram, solve the inequality $x^2 - 1 < 2x + 2$. State the solution in interval notation and justify your answer by referring to the graph. *Calculator allowed* **[5 marks]**

23. Let $g(x) = \dfrac{2x-1}{x+3}$, where $x \in \mathbb{R}$, $x \neq -3$.

Solve the inequality $g(x) > x - 1$. You must use a sign diagram and must not multiply both sides by an expression of unknown sign. *Calculator not allowed* **[5 marks]**

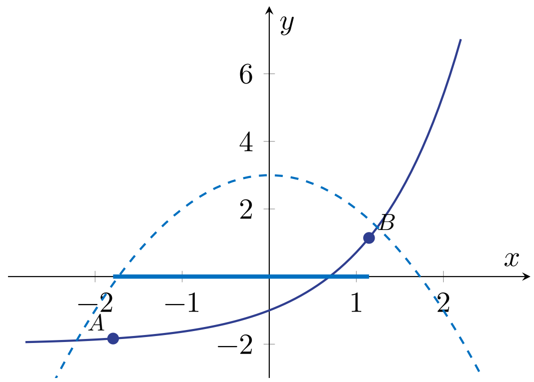
24. A factory's daily profit, in thousands of dollars, is modelled by

$$P(n) = -0.4n^2 + 8n - 24,$$

where $n \in [0, 20]$ is the number of units (in hundreds) produced per day.

Find the values of n for which the daily profit exceeds \$8 000, giving your answer as an interval. Interpret your answer in context. *Calculator allowed* **[5 marks]**

25. The diagram shows the graphs of $y = e^x - 2$ (solid) and $y = 3 - x^2$ (dashed).



The curves intersect at points A and B with x-coordinates α and β respectively, where $\alpha < \beta$.

(a) Use a GDC to find α and β, correct to three significant figures. **[2 marks]**

(b) Hence solve $e^x - 2 < 3 - x^2$, giving your answer in interval notation. Justify your answer by referring to the diagram. **[3 marks]**

Calculator allowed **[5 marks]**

26. Let $h(x) = x^4 - 5x^2 + 4$, where $x \in \mathbb{R}$.

(a) Show that $h(x) = (x-1)(x+1)(x-2)(x+2)$. **[2 marks]**

(b) Hence solve the inequality $h(x) < 0$, using a sign diagram to justify which intervals satisfy the inequality. Express your answer as a union of intervals. **[3 marks]**

Calculator not allowed **[5 marks]**

27. The concentration of a chemical in a reaction vessel, in mol L^{-1}, at time $t \geq 0$ minutes is given by

$$C(t) = \frac{3t+2}{t^2+4}, \quad t \in [0, \infty).$$

Find the values of t for which $C(t) \geq 0.5$, giving your answer as an interval correct to three significant figures.

Calculator allowed **[5 marks]**

28. Let $p(x) = 2x^3 - 3x^2 - 11x + 6$.

(a) Given that $(x-3)$ is a factor of $p(x)$, write $p(x)$ in fully factorised form. **[2 marks]**

(b) Hence solve $2x^3 - 3x^2 - 11x + 6 > 0$, using a sign diagram. State the solution in interval notation. **[3 marks]**

Calculator allowed **[5 marks]**

29. Consider the inequality $\dfrac{x^2-4}{x-1} \leq 2$, where $x \in \mathbb{R}$, $x \neq 1$.

Solve the inequality by rearranging to a single rational expression, drawing a sign diagram, and stating the solution in interval notation. You must not multiply both sides by $(x-1)$ without considering its sign. *Calculator allowed* **[5 marks]**

30. A ball is thrown vertically. Its height above the ground, in metres, is modelled by

$$h(t) = -4.9t^2 + 14t + 1.2, \quad t \geq 0,$$

where t is measured in seconds.

(a) Use a GDC to find the values of t for which $h(t) \geq 9$, giving your answer correct to three significant figures. **[3 marks]**

(b) Interpret your answer in context, stating how long the ball remains at or above 9 metres. **[2 marks]**

Calculator allowed **[5 marks]**

Worked Solutions

Q1 — Quadratic inequality, exact roots [EASY]

Factorise: $(x-2)(x-3) < 0$ **M1**

Critical values $x = 2$ and $x = 3$; parabola opens upward so the expression is negative *between* the roots. **R1**

Solution: $x \in (2,\, 3)$ **A1**

Q2 — Quadratic inequality, GDC [EASY]

Using GDC: graph $y = 3x^2 + x - 2$ and identify x-intercepts. **M1**

Roots: $x = -1$ and $x = \frac{2}{3}$; parabola opens upward so $y \leq 0$ between the roots. **R1**

Solution: $x \in \left[-1,\, \frac{2}{3}\right]$ **A1**

Q3 — Inequality between two curves, graphical [EASY]

From the diagram, $f(x) > g(x)$ where the line lies *above* the parabola, i.e. between the two intersection points $x = -1$ and $x = 2$. **M1**

The solution region is the thick segment on the x-axis between the intersection points. **R1**

Solution: $x \in (-1,\, 2)$ **A1**

Q4 — Cubic inequality, sign diagram [EASY]

Critical values: $x = -2,\ x = 1,\ x = 4$. Sign diagram:

Interval	$x < -2$	$-2 < x < 1$	$1 < x < 4$	$x > 4$
Sign	$-$	$+$	$-$	$+$

M1

Product is positive where sign is $+$. **R1**

Solution: $x \in (-2,\, 1) \cup (4,\, +\infty)$ **A1**

Q5 — Rational inequality, sign diagram [EASY]

The expression $\dfrac{1}{x-3}$ is undefined at $x = 3$ (excluded value, open circle). **M1**

Sign diagram: $x - 3 < 0$ for $x < 3$ (expression negative) and $x - 3 > 0$ for $x > 3$ (expression positive). **R1**

Solution: $x \in (3, +\infty)$ **A1**

Q6 — Context inequality, profit model [EASY]

Using GDC: graph $y = -x^2 + 8x - 7$ and find x-intercepts. **M1**

Roots at $x = 1$ and $x = 7$; parabola opens downward so $P > 0$ between the roots. **R1**

Solution: $x \in (1, 7)$ (hundreds of units, within domain $[0, 8]$) **A1**

Q7 — Inequality between two curves, graphical [EASY]

From the diagram, $h(x) > k(x)$ where the parabola lies *above* the line, i.e. between the two intersection points $x = -2$ and $x = 1$. **M1**

The solution region is the thick segment on the x-axis between the intersection points. **R1**

Solution: $x \in (-2, 1)$ **A1**

Q8 — Quadratic inequality, factorised [EASY]

Factorise: $(2x - 1)(x - 3) > 0$ **M1**

Critical values $x = \dfrac{1}{2}$ and $x = 3$; parabola opens upward so the expression is positive *outside* the roots. **R1**

Solution: $x \in \left(-\infty, \dfrac{1}{2}\right) \cup (3, +\infty)$ **A1**

Q9 — Context inequality, quadratic model [EASY]

Using GDC: graph $y = t^2 - 6t + 8$ and find t-intercepts. **M1**

Roots at $t = 2$ and $t = 4$; parabola opens upward so $C < 0$ between the roots. **R1**

Solution: $t \in (2, 4)$ **A1**

Q10 — Inequality between curves, graphical/GDC [EASY]

From the diagram, $p(x) < q(x)$ where the exponential curve lies *below* the line, i.e. between the two intersection points. **M1**

GDC gives intersection points at $x = 1.00$ and $x = 3.51$ (to 3 s.f.); the exponential is below the line between these values. **R1**

Solution: $x \in (1.00, 3.51)$ **A1**

Q11 — Quadratic inequality, sign diagram [MEDIUM]

Factorise: $x^2 - 5x + 4 = (x-1)(x-4)$ **M1**

Critical values are $x = 1$ and $x = 4$. Sign diagram for $(x-1)(x-4)$: **A1**

x	$x<1$	$x=1$	$1<x<4$	$x=4$	$x>4$
sign	$+$	0	$-$	0	$+$

The expression is negative (<0) for $1 < x < 4$. **R1**

Answer: $x \in (1, 4)$ **A1**

Q12 — Rational inequality, graphical method [MEDIUM]

Rewrite as $\dfrac{x+3}{x-1} - 2 > 0$, giving $\dfrac{x+3-2(x-1)}{x-1} = \dfrac{-x+5}{x-1} > 0.$ **M1**

Critical values: $x = 5$ (numerator zero) and $x = 1$ (excluded, denominator zero). **A1**

Note: Do not multiply by $(x-1)$ since its sign is unknown.

Sign diagram for $\dfrac{-(x-5)}{x-1}$: positive when $1 < x < 5$. **R1**

Answer: $x \in (1, 5)$ **A1**

Q13 — Cubic inequality, sign diagram [MEDIUM]

The polynomial is already factorised: $(x-1)(x+2)(x-3)$. **M1**

Critical values: $x = -2,\ 1,\ 3$. Sign diagram:

Region	$x<-2$	$-2<x<1$	$1<x<3$	$x>3$
Sign	$-$	$+$	$-$	$+$

A1

The expression is ≤ 0 (including zeros) for $x \leq -2$ or $1 \leq x \leq 3$. **R1**

Answer: $x \in (-\infty, -2] \cup [1, 3]$ **A1**

Q14 — Context inequality, GDC [MEDIUM]

Require $-t^2 + 7t - 6 > 4$, i.e. $-t^2 + 7t - 10 > 0$. **M1**

Using GDC: graph $y = -t^2 + 7t - 10$ and find zeros.

Zeros: $t = \dfrac{-7 \pm \sqrt{49 - 40}}{-2}$; GDC gives $t \approx 2$ and $t \approx 5$ exactly. **A1**

Note: Unrounded GDC values are $t = 2.000$ *and* $t = 5.000$.

Parabola opens downward, so expression is positive between the roots. **R1**

Answer: $t \in (2, 5)$ **A1**

Q15 — Intersection of curve and line, GDC [MEDIUM]

Require $x^2 + 2x - 8 \geq 3x + 2$, i.e. $x^2 - x - 10 \geq 0$. **M1**

Using GDC: intersections of $y = x^2 + 2x - 8$ and $y = 3x + 2$ at $x \approx -2.000$ and $x \approx 5.000$ (exact: $x = -2$ and $x = 5$). **A1**

Note: GDC gives $x = -2.0000$ *and* $x = 5.0000$.

Parabola is above the line outside the intersection points (as seen in diagram). **R1**

Answer: $x \in (-\infty, -2] \cup [5, +\infty)$ **A1**

Q16 — Rational inequality, sign diagram [MEDIUM]

Rewrite as $\dfrac{2x - 1}{x + 2} - 1 \leq 0$, giving $\dfrac{2x - 1 - (x + 2)}{x + 2} = \dfrac{x - 3}{x + 2} \leq 0$. **M1**

Critical values: $x = 3$ (zero) and $x = -2$ (excluded). **A1**

Note: Do not multiply by $(x + 2)$ *since its sign is unknown.*

Sign diagram: $\dfrac{x - 3}{x + 2} \leq 0$ when $-2 < x \leq 3$ (open at $x = -2$, closed at $x = 3$). **R1**

Answer: $x \in (-2, 3]$ **A1**

Q17 — Context rational inequality, GDC [MEDIUM]

Require $\dfrac{x^2 + 1}{x^2 + 4} < 0.8$. **M1**

Using GDC: graph $y = \dfrac{x^2+1}{x^2+4}$ and $y = 0.8$; find intersection for $x \geq 0$.
GDC gives intersection at $x \approx 2.000$ (exact: $x^2+1 = 0.8(x^2+4) \Rightarrow 0.2x^2 = 2.2 \Rightarrow x = \sqrt{11} \approx 3.317$). **A1**

Note: GDC gives intersection at $x = 3.317$ (3 s.f.).

Since $C(x)$ is increasing for $x \geq 0$ and $C(0) = 0.25 < 0.8$, the concentration is below 0.8 for $x < \sqrt{11}$. **R1**

Answer: $x \in [0,\ \sqrt{11})$, i.e. $x \in [0,\ 3.32)$ (3 s.f.) **A1**

Q18 — Cubic polynomial inequality, GDC [MEDIUM]

Using GDC: graph $y = x^3 - 4x^2 - 3x + 18$ and identify zeros.
GDC shows roots at $x = -2$ and $x = 3$ (repeated). **M1**

Factor: $x^3 - 4x^2 - 3x + 18 = (x+2)(x-3)^2$. **A1**

Since $(x-3)^2 \geq 0$ always, the sign is determined by $(x+2)$: the expression is ≤ 0 when $x+2 \leq 0$, i.e. $x \leq -2$, and also equals 0 at $x = 3$. **R1**

Answer: $x \in (-\infty, -2] \cup \{3\}$ **A1**

Q19 — Parameter condition via discriminant [MEDIUM]

For $x^2 - 6x + k > 0$ to hold for all $x \in \mathbb{R}$, the parabola must have no real roots, i.e. the discriminant must be strictly negative. **M1**

Discriminant: $\Delta = (-6)^2 - 4(1)(k) = 36 - 4k$. **A1**

Require $\Delta < 0$: $36 - 4k < 0 \Rightarrow k > 9$. **R1**

Answer: $k \in (9, +\infty)$ **A1**

Q20 — Intersection of curve and line, GDC [MEDIUM]

Using GDC: find intersections of $y = \sqrt{x+4}$ and $y = \dfrac{x}{2} + 1$ for $x \geq -4$.
GDC gives intersections at $x = -2.000$ and $x = 4.000$ (to 3 s.f.: $x = -2.00$ and $x = 4.00$). **M1**

Note: Exact values confirmed by $\sqrt{x+4} = \frac{x}{2}+1 \Rightarrow x+4 = \frac{x^2}{4}+x+1 \Rightarrow x^2 = 12 - \ldots$; GDC intersection at $(-2,\ \sqrt{2})$ and $(4,\ 3)$. **A1**

From the diagram, $\sqrt{x+4}$ lies above $\dfrac{x}{2} + 1$ between the two intersection points. **R1**

Answer: $x \in (-2,\ 4)$ **A1**

Q21 — Cubic factorisation and sign diagram [HARD]

(a)

Testing $x = 2$: $f(2) = 8 - 12 - 8 + 12 = 0$, so $(x - 2)$ is a factor. **M1**

Dividing: $x^3 - 3x^2 - 4x + 12 = (x - 2)(x^2 - x - 6) = (x - 2)(x - 3)(x + 2)$. **A1 AG**

(b)

Critical values: $x = -2,\ 2,\ 3$. Sign diagram for $f(x) = (x - 2)(x + 2)(x - 3)$:

x	$(-\infty, -2)$	-2	$(-2, 2)$	2	$(2, 3)$	3	$(3, \infty)$
$f(x)$	$-$	0	$+$	0	$-$	0	$+$

M1

$f(x) \leq 0$ where $f(x)$ is negative or zero. **R1**

Solution: $x \in (-\infty, -2] \cup [2, 3]$. **A1**

Note: Award M1 for a correct sign diagram with all four sign regions. Award R1 for explicitly identifying the regions where $f(x) \leq 0$. Condone $\leq$ at the endpoints.

Q22 — Intersection of parabola and line [HARD]

Setting $x^2 - 1 = 2x + 2$: **M1**

$x^2 - 2x - 3 = 0 \Rightarrow (x + 1)(x - 3) = 0$, giving $x = -1$ or $x = 3$. **A1**

The parabola lies *below* the line between the intersections, as read from the diagram (the solid curve is below the dashed line for $-1 < x < 3$). **R1**

GDC or substitution check at $x = 0$: $f(0) = -1 < 2 = g(0)$ confirms the parabola is below the line. **M1**

Solution: $x \in (-1, 3)$. **A1**

Note: Award R1 only if the candidate explicitly states which curve is lower in the interval, with reference to the graph. The thick segment on the x-axis in the diagram confirms the solution region.

Q23 — Rational inequality via sign diagram [HARD]

Rearrange: $\dfrac{2x - 1}{x + 3} - (x - 1) > 0$. **M1**

$$\frac{(2x - 1) - (x - 1)(x + 3)}{x + 3} > 0 \Rightarrow \frac{2x - 1 - x^2 - 2x + 3}{x + 3} > 0 \Rightarrow \frac{-x^2 + 2}{x + 3} > 0.$$ **A1**

Critical values: numerator zero at $x = \pm\sqrt{2}$; excluded value $x = -3$. Sign diagram:

x	$(-\infty, -3)$	-3	$(-3, -\sqrt{2})$	$-\sqrt{2}$	$(-\sqrt{2}, \sqrt{2})$	$\sqrt{2}$	$(\sqrt{2}, \infty)$
Sign	$-$	undef	$+$	0	$+$	0	$-$

M1

The expression is strictly positive in $(-3, -\sqrt{2}\,) \cup (-\sqrt{2},\ \sqrt{2}\,)$, i.e. $(-3, \sqrt{2}\,)$ excluding $x = -\sqrt{2}$. Since $-\sqrt{2}$ gives zero, it is excluded from a strict inequality. **R1**

Solution: $x \in (-3,\ -\sqrt{2}) \cup (-\sqrt{2},\ \sqrt{2})$, equivalently $x \in (-3, \sqrt{2})$, $x \neq -\sqrt{2}$. **A1**

Note: Penalise any attempt to multiply through by $(x+3)$ without considering its sign. Award M1 for a correct single-fraction rearrangement. The sign diagram must show $x = -3$ as undefined.

Q24 — Context inequality from quadratic profit model [HARD]

Require $P(n) > 8$, i.e. $-0.4n^2 + 8n - 24 > 8$. **M1**

Rearrange: $-0.4n^2 + 8n - 32 > 0 \Rightarrow 0.4n^2 - 8n + 32 < 0 \Rightarrow n^2 - 20n + 80 < 0$. **M1**

GDC: solve $n^2 - 20n + 80 = 0$. Unrounded: $n = \dfrac{20 \pm \sqrt{400 - 320}}{2} = 10 \pm 2\sqrt{5}$.

$n = 10 - 2\sqrt{5} \approx 5.528$ and $n = 10 + 2\sqrt{5} \approx 14.47$. **A1**

The parabola opens upward, so $n^2 - 20n + 80 < 0$ between the roots. **R1**

Solution: $n \in (5.53,\ 14.5)$ (to 3 s.f.), with $n \in [0, 20]$. The factory earns more than \$8 000 profit per day when between approximately 553 and 1447 units are produced. **A1**

Note: Accept exact forms $10 \pm 2\sqrt{5}$ for the final answer. The contextual interpretation must reference units and the direction of the inequality to earn the final A1.

Q25 — Intersection of exponential and quadratic [HARD]

(a)

GDC: solve $e^x - 2 = 3 - x^2$, i.e. $e^x + x^2 - 5 = 0$. **M1**

Unrounded: $\alpha = -1.7933\ldots$, $\quad \beta = 1.1462\ldots$

$\alpha = -1.79$ (3 s.f.), $\quad \beta = 1.15$ (3 s.f.). **A1**

(b)

From the diagram, the solid curve ($y = e^x - 2$) lies *below* the dashed curve ($y = 3 - x^2$) between the two intersection points A and B. **R1**

The thick segment on the x-axis confirms the solution region $\alpha < x < \beta$. **M1**

Solution: $x \in (-1.79,\ 1.15)$ (to 3 s.f.). **A1**

Note: Award R1 only if the candidate explicitly states which curve is lower between the intersections with reference to the diagram. Follow-through from incorrect values in (a) is permitted in (b).

Q26 — Quartic factorisation and sign diagram [HARD]

(a)

$h(x) = x^4 - 5x^2 + 4 = (x^2 - 1)(x^2 - 4).$ **M1**

$(x^2 - 1)(x^2 - 4) = (x - 1)(x + 1)(x - 2)(x + 2).$ **A1 AG**

(b)

Critical values: $x \in \{-2, -1, 1, 2\}$. Sign diagram:

x	$(-\infty, -2)$	-2	$(-2, -1)$	-1	$(-1, 1)$	1	$(1, 2)$	2	$(2, \infty)$
$h(x)$	$+$	0	$-$	0	$+$	0	$-$	0	$+$

M1

$h(x) < 0$ in the regions where the sign is negative (endpoints excluded since strict inequality). **R1**

Solution: $x \in (-2,\ -1) \cup (1,\ 2)$. **A1**

Note: Award M1 for a correct sign diagram with all five sign regions identified. Award R1 for explicitly identifying the negative regions and stating why endpoints are excluded.

Q27 — Rational concentration inequality from model [HARD]

Require $\dfrac{3t+2}{t^2+4} \geq 0.5$, i.e. $\dfrac{3t+2}{t^2+4} - 0.5 \geq 0.$ **M1**

Note $t^2 + 4 > 0$ for all real t, so multiply: $3t + 2 \geq 0.5(t^2 + 4)$, i.e. $0.5t^2 - 3t + 0 \leq 0 \Rightarrow t^2 - 6t - 0 \leq 0.$
M1

Wait — recomputing: $3t + 2 \geq 0.5t^2 + 2 \Rightarrow 0 \geq 0.5t^2 - 3t \Rightarrow 0 \geq t(0.5t - 3)$, so $0 \geq t(0.5t - 3)$. **A1**

Critical values (with $t \geq 0$): $t = 0$ and $t = 6$.

GDC check at $t = 3$: $C(3) = \dfrac{11}{13} \approx 0.846 \geq 0.5$ ⍰; at $t = 7$: $C(7) = \dfrac{23}{53} \approx 0.434 < 0.5$ ⍰. **R1**

Since $t \geq 0$, solution: $t \in [\,0,\ 6\,]$. **A1**

Note: Since $t^2 + 4 > 0$ always, multiplying through is valid. Award M1 for this justification. The answer is exact; no rounding required. Award R1 for a sign or GDC check confirming the correct interval.

Q28 — Cubic factorisation and polynomial inequality [HARD]

(a)

Dividing $p(x)$ by $(x - 3)$: GDC or long division gives $p(x) = (x - 3)(2x^2 + 3x - 2)$. **M1**

Factorising: $2x^2 + 3x - 2 = (2x - 1)(x + 2)$.

So $p(x) = (x - 3)(2x - 1)(x + 2)$. **A1**

(b)

Critical values: $x = -2,\ \frac{1}{2},\ 3$. Sign diagram:

x	$(-\infty,-2)$	-2	$(-2,\frac{1}{2})$	$\frac{1}{2}$	$(\frac{1}{2},3)$	3	$(3,\infty)$
$p(x)$	$-$	0	$+$	0	$-$	0	$+$

M1

$p(x) > 0$ in the regions where the sign is positive; endpoints excluded (strict inequality). **R1**

Solution: $x \in (-2,\ \frac{1}{2}) \cup (3,\ \infty)$. **A1**

Note: Award M1 for a correct sign diagram with four sign regions. GDC may be used to verify the factorisation in (a).

Q29 — Rational inequality via single fraction and sign diagram [HARD]

Rearrange: $\dfrac{x^2-4}{x-1} - 2 \le 0 \Rightarrow \dfrac{x^2-4-2(x-1)}{x-1} \le 0 \Rightarrow \dfrac{x^2-2x-2}{x-1} \le 0.$ **M1**

GDC: roots of $x^2 - 2x - 2 = 0$: $x = 1 \pm \sqrt{3}$, i.e. $x \approx -0.732$ or $x \approx 2.73$. **A1**

Critical values: $x = 1-\sqrt{3} \approx -0.732$, $\quad x = 1$ (excluded), $\quad x = 1+\sqrt{3} \approx 2.73$. Sign diagram:

Interval	$(-\infty, 1-\sqrt{3})$	$1-\sqrt{3}$	$(1-\sqrt{3}, 1)$	1	$(1, 1+\sqrt{3})$	$1+\sqrt{3}$	$(1+\sqrt{3}, \infty)$
Sign	$+$	0	$-$	undef	$-$	0	$+$

M1

The expression is ≤ 0 where the sign is negative or zero; $x = 1$ is excluded (undefined). **R1**

Solution: $x \in [\,1-\sqrt{3},\ 1\,) \cup (\,1,\ 1+\sqrt{3}\,]$. **A1**

Note: Award M1 for correctly forming a single rational expression. The sign diagram must show $x = 1$ as undefined with an open circle, and the final answer must reflect this. Penalise if the candidate multiplies by $(x-1)$ without case analysis.

Q30 — Projectile height inequality from quadratic model [HARD]

(a)

Require $h(t) \ge 9$: $-4.9t^2 + 14t + 1.2 \ge 9$. **M1**

GDC: solve $-4.9t^2 + 14t - 7.8 = 0$.

Unrounded roots: $t = \dfrac{-14 \pm \sqrt{196 - 4(4.9)(7.8)}}{-9.8}$; GDC gives $t \approx 0.7697\ldots$ and $t \approx 2.0813\ldots$

$t_1 = 0.770$ s and $t_2 = 2.08$ s (to 3 s.f.). **A1**

The parabola opens downward, so $h(t) \ge 9$ between the roots. **R1**

Solution: $t \in [\,0.770,\ 2.08\,]$ s. **A1**

(b)

The ball is at or above 9 metres for a duration of $2.08 - 0.770 = 1.31$ seconds (to 3 s.f.). The ball first reaches 9 m at $t \approx 0.770$ s and returns to 9 m at $t \approx 2.08$ s. **A1**

Note: Award the final A1 only if the answer references both the duration and the physical meaning (ball ascending and descending through 9 m). Follow-through from (a) is permitted.