IB Math AA HL — Topic 3

Geometry Toolkit

Coordinate Geometry · Radian Measure · Arcs & Sectors

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:_____ Date:_____

Score:_____/120

Scan me for help

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

Key Formulae

- **Distance:** $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **Midpoint:** $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- **Degree ↔ Radian:** $\theta_{\text{rad}} = \theta_{\text{deg}} \times \frac{\pi}{180}$ and $\theta_{\text{deg}} = \theta_{\text{rad}} \times \frac{180}{\pi}$
- **Arc length (radians):** $l = r\theta$ **Arc length (degrees):** $l = \frac{\theta}{360} \times 2\pi r$
- **Sector area (radians):** $A = \frac{1}{2}r^2\theta$ **Sector area (degrees):** $A = \frac{\theta}{360} \times \pi r^2$
- **Perimeter of a sector:** $P = 2r + l = 2r + r\theta$ (two radii plus the arc)
- **Segment area:** $A_{\text{seg}} = \frac{1}{2}r^2(\theta - \sin\theta)$ (θ in radians; equals sector area minus triangle area)
- **Triangle area (used in segments):** $A_{\triangle} = \frac{1}{2}r^2 \sin\theta$

GDC Tips

- **Mode check first:** before any trig calculation, confirm the GDC is set to Radians or Degrees as required by the question; find this in Settings → Angle.
- **Solving for r or θ:** use Menu → Algebra → Solve (CAS) or the Equation Solver (non-CAS) when the unknown appears inside $l = r\theta$ or $A = \frac{1}{2}r^2\theta$ and arithmetic is awkward.
- **Composite areas:** store intermediate sector and triangle areas in GDC variables (A, B, ...) to avoid rounding errors before the final subtraction or addition.
- **Degree→Radian conversion:** type 45° in radian mode and the GDC returns the exact decimal; alternatively use Ans$\times\pi \div 180$ to convert a stored degree value.
- **Segment areas with non-standard angles:** evaluate $\frac{1}{2}r^2(\theta - \sin\theta)$ directly on the home screen in radian mode; the GDC handles $\sin\theta$ without extra steps.
- **Graphing to check arc-length equations:** plot $Y_1 = r \cdot X$ and $Y_2 = l$ (the given arc length) then use Intersection to find θ numerically as a sanity check.

Common Mistakes to Avoid

1. **Forgetting the two radii in sector perimeter.** Students write $P = l = r\theta$ and ignore the two straight sides; the correct perimeter is always $P = 2r + r\theta$.
2. **Using degrees directly in radian formulae.** Substituting $\theta = 60$ (degrees) into $A = \frac{1}{2}r^2\theta$ gives an answer 60 times too large; convert to $\theta = \frac{\pi}{3}$ radians first and show the conversion line.
3. **Segment area taken as sector area alone.** A segment is the region between a chord and its arc; its area is sector minus triangle, i.e. $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$, not just $\frac{1}{2}r^2\theta$.
4. **Mixing degree and radian sector-area formulae.** Using $A = \frac{\theta}{360}\pi r^2$ with θ already in radians (or vice versa) gives a completely wrong answer; identify which formula applies and keep units consistent throughout.
5. **Rounding intermediate values and losing accuracy.** Storing a rounded value of θ or r mid-solution then using it in a second formula produces accumulated error; keep full precision until the final step and round only the stated answer to 3 s.f. (or as required).
6. **Confusing arc length with sector area.** Both involve r and θ but $l = r\theta$ is length (one power of r) while $A = \frac{1}{2}r^2\theta$ is area (two powers of r); checking units (cm vs cm^2) immediately reveals this error.

Section A — Easy EASY (10 questions × 3 marks = 30 marks)

Scan me for help

1. EASY Calculator [3 marks]

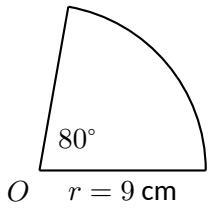
Points $A(-3, 2)$ and $B(5, 8)$ are in the coordinate plane. Find the distance AB.

2. EASY No Calculator [3 marks]

Convert $\frac{5\pi}{6}$ radians to degrees.

3. EASY Calculator [3 marks]

A sector has radius $r = 9$ cm and central angle $\theta = 80°$. Find the arc length l of the sector, giving your answer to 3 significant figures.

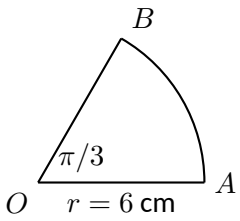


4. **EASY** **Calculator** **[3 marks]**

Find the midpoint M of the segment joining $P(1, -5)$ and $Q(7, 3)$.

5. **EASY** **No Calculator** **[3 marks]**

A sector OAB has radius $r = 6$ cm and central angle $\theta = \dfrac{\pi}{3}$ radians. Find the exact area of the sector.



6. **EASY** **No Calculator** **[3 marks]**

Convert $135°$ to radians, giving your answer as an exact multiple of π.

7. **EASY** **Calculator** **[3 marks]**

A sector has radius $r = 12$ cm and arc length $l = 9$ cm. Find the central angle θ in radians.

8. EASY Calculator **[3 marks]**

Find the gradient of the straight line passing through the points $C(-2, 5)$ and $D(4, -1)$.

9. EASY Calculator **[3 marks]**

A sector has radius $r = 7$ cm and central angle $\theta = 1.2$ radians. Find the perimeter of the sector, giving your answer to 3 significant figures.

10. EASY Calculator **[3 marks]**

A sector has radius $r = 10$ cm and central angle $\theta = 140°$. Find the area of the sector, giving your answer to 3 significant figures.

Section B — Medium MEDIUM (10 questions × 4 marks = 40 marks)

Scan me for help

11. MEDIUM No Calculator **[4 marks]**

The points $A(1, 4)$ and $B(7, -2)$ are given. The point $P(x, y)$ lies on the perpendicular bisector of AB.

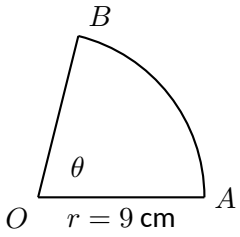
(a) Find the coordinates of the midpoint M of AB. [1]

(b) Find the gradient of the perpendicular bisector of AB. [1]

(c) Hence find the equation of the perpendicular bisector of AB, giving your answer in the form $y = mx + c$. [2]

12. **MEDIUM** **No Calculator** **[4 marks]**

A sector OAB has radius $r = 9$ cm and arc length $l = 12$ cm, where the angle θ is measured in radians.



(a) Find the angle θ of the sector. [1]

(b) Find the area of the sector. [1]

(c) Find the perimeter of the sector. [2]

13. **MEDIUM** **Calculator** **[4 marks]**

A windscreen wiper of length 42 cm sweeps through an angle of $120°$. The pivot is at one end of the wiper.

(a) Convert $120°$ to radians, giving your answer as an exact multiple of π. [1]

(b) Find the area swept by the wiper. Give your answer in cm^2, correct to 3 significant figures. [2]

(c) Find the length of the arc traced by the tip of the wiper, correct to 3 significant figures. [1]

14. MEDIUM Calculator **[4 marks]**

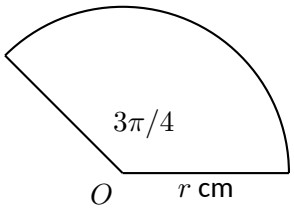
Points $P(-3, 1)$, $Q(5, 5)$ and $R(k, -2)$ are given, where $k \in \mathbb{R}$.

(a) Find the distance PQ. [2]

(b) Given that the gradient of QR is $-\frac{7}{2}$, find the value of k. [2]

15. MEDIUM No Calculator **[4 marks]**

A sector of a circle has radius r cm and angle $\frac{3\pi}{4}$ radians. The area of the sector is 54π cm^2.



(a) Show that $r = 12$ cm. [2]

(b) Find the exact perimeter of the sector. [2]

16. MEDIUM Calculator **[4 marks]**

An irrigation system rotates about a fixed point O, spraying water over a sector of angle $200°$ and radius 18.5 m.

(a) Convert $200°$ to radians, correct to 4 significant figures. [1]

(b) Find the area of the sector, correct to 3 significant figures. [1]

(c) A circular path of radius 5.0 m is centred at O inside the sector. Find the area of the region that lies within the sector but outside the circular path, correct to 3 significant figures. [2]

17. MEDIUM Calculator **[4 marks]**

A running track has a straight section and a semicircular end. The semicircular end has diameter d m, where $d \in \mathbb{R}^+$. An athlete runs along the arc of the semicircle and then 85.0 m along the straight section. The total distance run is 220 m.

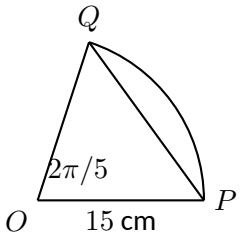
(a) Write down an equation for d in terms of the total distance. [1]

(b) Find the value of d, correct to 3 significant figures. [2]

(c) Find the area enclosed by the semicircle, correct to 3 significant figures. [1]

18. MEDIUM Calculator [4 marks]

The diagram shows a sector OPQ with radius 15 cm and angle $\theta = \dfrac{2\pi}{5}$ radians. The area of the segment (shaded region between the arc PQ and the chord PQ) is to be found.



(a) Find the area of the sector OPQ. [1]

(b) Find the area of triangle OPQ. [1]

(c) Hence find the area of the segment, correct to 3 significant figures. [2]

19. MEDIUM Calculator [4 marks]

A pizza slice is modelled as a sector with radius 14 cm and angle $\dfrac{\pi}{4}$ radians. A second, larger pizza has radius 18 cm and the same arc length as the first.

(a) Find the arc length of the first pizza slice, giving your answer as an exact multiple of π. [1]

(b) Find the angle of the second pizza's sector, giving your answer as an exact fraction of π. [1]

(c) Find the difference in area between the two sectors, giving your answer correct to 3 significant figures. [2]

20. MEDIUM Calculator **[4 marks]**

The points $C(2, -1)$ and $D(8, 3)$ are the endpoints of a diameter of a circle.

(a) Find the centre of the circle. [1]

(b) Find the exact radius of the circle. [1]

(c) Find the equation of the circle in the form $(x-h)^2 + (y-k)^2 = r^2$. [2]

Section C — Hard HARD (10 questions × 5 marks = 50 marks)

Scan me for help

21. HARD No Calculator [5 marks]

A circle has centre O and radius r cm. A chord PQ subtends an angle of $\frac{2\pi}{3}$ radians at O. The area of the minor segment cut off by PQ is $12\pi - 9\sqrt{3}$ cm^2.

(a) Show that $r = 6$ cm. [3 marks]

(b) Find the perimeter of the minor segment, giving your answer in the form $a\pi + b\sqrt{3}$ cm, where $a, b \in \mathbb{Q}$. [2 marks]

22. HARD Calculator [5 marks]

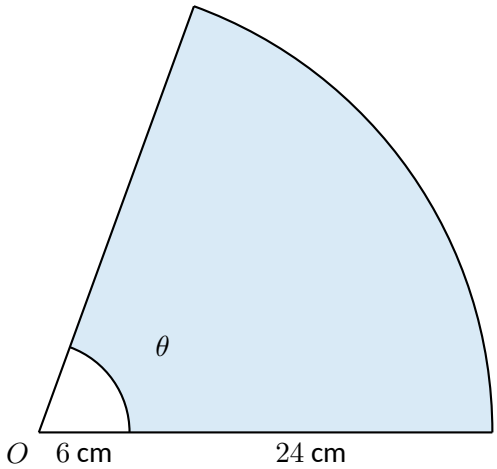
Points $A(-3, 5)$ and $B(7, -1)$ lie in the coordinate plane. The perpendicular bisector of AB meets the y-axis at point C.

(a) Find the coordinates of C. [3 marks]

(b) Hence find the area of triangle OAC, where O is the origin. [2 marks]

23. **HARD** **Calculator** **[5 marks]**

A windscreen wiper has a blade of length 24 cm. The wiper pivots at point O and the near edge of the blade is 6 cm from O. In one sweep, the wiper rotates through an angle of θ radians, cleaning an area of 756 cm^2.



(a) Show that $\theta = 7/4$ radians. [3 marks]

(b) Find the total length of the boundary of the cleaned region. Give your answer correct to 3 significant figures. [2 marks]

24. **HARD** **Calculator** **[5 marks]**

An arc AB of a circle with centre O and radius r cm has length l cm and subtends an angle θ radians at O. It is given that $l = 15$ cm and that the area of sector OAB is 67.5 cm^2.

(a) Find the values of r and θ. [3 marks]

(b) A second sector has the same perimeter as sector OAB but its radius is 12 cm. Find the area of the second sector. [2 marks]

25. **HARD** **Calculator** **[5 marks]**

The points $P(1, -2)$, $Q(5, 4)$ and $R(k, 0)$, where $k \in \mathbb{R}$, are such that PQ is perpendicular to QR.

(a) Find the value of k. [3 marks]

(b) Find the exact length PR. [2 marks]

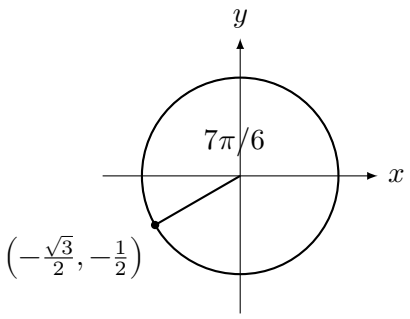
26. **HARD** **Calculator** **[5 marks]**

Two sectors share the same centre O. Sector OAB has radius 9 cm and angle α radians. Sector OCD has radius 15 cm and angle β radians. The arc length of OAB equals the arc length of OCD, and the sum of the two sector areas is 189 cm^2.

Find the values of α and β.

27. **HARD** **No Calculator** **[5 marks]**

The angle $\frac{7\pi}{6}$ radians is marked on a unit circle.



(a) Write down the exact coordinates of the corresponding point on the unit circle. [2 marks]

(b) Hence find the exact area of the minor sector formed by the angle $\frac{\pi}{6}$ and the unit circle, and show that the major sector area is $\frac{11\pi}{12}$ square units. [3 marks]

28. HARD Calculator **[5 marks]**

An irrigation system sprays water over a field. The sprayer is fixed at point O and rotates, covering a sector of radius 45 m. The sprayer covers an angle of $220°$.

(a) Convert $220°$ to radians, giving your answer correct to 4 significant figures. [1 marks]

(b) Find the area watered by the sprayer. Give your answer correct to 3 significant figures. [2 marks]

(c) A fence runs along the straight boundary of the sector. A second fence runs along the curved boundary. Find the total length of fencing required. Give your answer correct to 3 significant figures. [2 marks]

29. HARD Calculator **[5 marks]**

Points $A(2, 6)$, $B(8, 4)$ and C lie in the coordinate plane. C lies on the perpendicular bisector of AB and $|AC| = |BC| = \sqrt{53}$.

(a) Find the coordinates of C. [3 marks]

(b) State, with a reason, whether C lies inside or outside the circle with diameter AB. [2 marks]

30. HARD No Calculator **[5 marks]**

A sector of radius r cm and angle θ radians has perimeter P cm and area A cm^2. It is given that $P = 24$ and $A = 27$.

(a) Show that r satisfies the equation $r^2 - 12r + 27 = 0$. [3 marks]

(b) Hence find all possible values of r and the corresponding values of θ. [2 marks]

Worked Solutions

Q1 — Distance Between Two Points EASY

Using the distance formula with $A(-3, 2)$ and $B(5, 8)$: **M1**

$AB = \sqrt{(5-(-3))^2 + (8-2)^2} = \sqrt{8^2 + 6^2} = \sqrt{64+36} = \sqrt{100}$ **(A1)**

$AB = 10$ units **A1**

Q2 — Radian to Degree Conversion EASY

Multiplying by $\frac{180^\circ}{\pi}$: **M1**

$\frac{5\pi}{6} \times \frac{180^\circ}{\pi} = \frac{5 \times 180^\circ}{6}$ **(A1)**

$= 150^\circ$ **A1**

Q3 — Arc Length from a Degree Angle EASY

Converting 80° to radians: $\theta = 80 \times \frac{\pi}{180} = \frac{4\pi}{9}$ rad **M1**

Applying $l = r\theta$: $l = 9 \times \frac{4\pi}{9} = 4\pi = 12.566\ldots$ **(A1)**

$l = 12.6$ cm **A1**

Note: Award M1 for correct substitution into $l = r\theta$ with θ in radians. Accept 12.6 cm only after correct conversion.

Q4 — Midpoint of a Segment EASY

Using the midpoint formula: $M = \left(\frac{x_P + x_Q}{2}, \frac{y_P + y_Q}{2}\right)$ **M1**

$= \left(\frac{1+7}{2}, \frac{-5+3}{2}\right)$ **(A1)**

$M = (4, -1)$ **A1**

Q5 — Exact Sector Area in Radians EASY

Using $A = \frac{1}{2}r^2\theta$ with $r = 6$ cm and $\theta = \frac{\pi}{3}$: **M1**

$A = \frac{1}{2} \times 6^2 \times \frac{\pi}{3} = \frac{1}{2} \times 36 \times \frac{\pi}{3} = \frac{36\pi}{6}$ **(A1)**

$A = 6\pi$ cm^2 **A1**

Q6 — Degree to Radian Conversion EASY

Multiplying by $\frac{\pi}{180^\circ}$: $135^\circ \times \frac{\pi}{180^\circ}$ **M1**

$= \frac{135\pi}{180} = \frac{3\pi}{4}$ **(A1)**

$135^\circ = \frac{3\pi}{4}$ radians **A1**

Q7 — Central Angle from Arc Length EASY

Using $l = r\theta$ with $l = 9$ cm and $r = 12$ cm: $9 = 12\theta$ **M1**

$\theta = \frac{9}{12}$ **(A1)**

$\theta = 0.75$ radians **A1**

Q8 — Gradient Between Two Points EASY

Using the gradient formula with $C(-2, 5)$ and $D(4, -1)$: $m = \frac{y_D - y_C}{x_D - x_C} = \frac{-1 - 5}{4 - (-2)}$ **M1**

$= \frac{-6}{6}$ **(A1)**

$m = -1$ **A1**

Q9 — Perimeter of a Sector EASY

The perimeter of a sector consists of two radii and the arc: $P = 2r + l = 2r + r\theta$ **M1**

Substituting $r = 7$ cm and $\theta = 1.2$ rad: $P = 2(7) + 7(1.2) = 14 + 8.4$ **(A1)**

$P = 22.4$ cm **A1**

Note: Award M0 if only the arc length $l = 8.40$ cm is given as the perimeter (arc-only error).

Q10 — Sector Area from a Degree Angle EASY

Converting 140° to radians: $\theta = 140 \times \dfrac{\pi}{180} = \dfrac{7\pi}{9} = 2.4435\ldots$ rad **M1**

Applying $A = \dfrac{1}{2}r^2\theta$ with $r = 10$ cm: $A = \dfrac{1}{2} \times 100 \times \dfrac{7\pi}{9} = \dfrac{350\pi}{9} = 122.17\ldots$ **(A1)**

$A = 122$ cm^2 **A1**

Note: Unrounded value $122.17\ldots$ must be seen before the rounded answer.

Q11 — Perpendicular Bisector of a Segment MEDIUM

(a) $M = \left(\dfrac{1+7}{2}, \dfrac{4+(-2)}{2}\right) = (4, 1)$ **A1**

(b) Gradient of AB: $m_{AB} = \dfrac{-2-4}{7-1} = \dfrac{-6}{6} = -1$ **(M1)**

Gradient of perpendicular bisector: $m_\perp = 1$ **A1**

(c) Using point $M(4, 1)$: $y - 1 = 1(x - 4)$ **(M1)**

$y = x - 3$ **A1**

Note: Award final A1 only if the equation is in the form $y = mx + c$.

Q12 — Arc Angle Area and Perimeter of a Sector MEDIUM

(a) Using $l = r\theta$: $\theta = \dfrac{l}{r} = \dfrac{12}{9} = 4/3$ rad **A1**

(b) $A = \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(9)^2\left(\dfrac{4}{3}\right) = \dfrac{1}{2}(81)\left(\dfrac{4}{3}\right) = 54$ cm^2 **A1**

(c) Perimeter $= 2r + l$ **M1**

$= 2(9) + 12 = 30$ cm **A1**

Note: Award M1 only if both radii and the arc are included. Do not award if arc length alone is given.

Q13 — Windscreen Wiper Arc and Sector Area MEDIUM

(a) $120^\circ \times \dfrac{\pi}{180} = \dfrac{2\pi}{3}$ radians **A1**

(b) $A = \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(42)^2\left(\dfrac{2\pi}{3}\right)$ **M1**

$= \dfrac{1}{2}(1764)\left(\dfrac{2\pi}{3}\right) = 588\pi \approx 1850$ cm^2 **A1**

(c) $l = r\theta = 42 \times \dfrac{2\pi}{3} = 28\pi \approx 87.96\ldots \approx 88.0$ cm **A1**

Note: Accept $1847.9\ldots$ cm^2 before rounding. Unrounded value must be seen.

Q14 — Distance and Gradient Condition MEDIUM

(a) $PQ = \sqrt{(5-(-3))^2 + (5-1)^2}$ **M1**

$= \sqrt{64+16} = \sqrt{80} = 4\sqrt{5} \approx 8.94$ **A1**

(b) Gradient of QR: $\dfrac{-2-5}{k-5} = -\dfrac{7}{2}$ **M1**

$-14 = -7(k-5) \Rightarrow k-5 = 2 \Rightarrow k = 7$ **A1**

Q15 — Sector Area to Find Radius and Exact Perimeter MEDIUM

(a) Area of sector: $\dfrac{1}{2}r^2\theta = 54\pi$ **M1**

$\dfrac{1}{2}r^2 \cdot \dfrac{3\pi}{4} = 54\pi \Rightarrow \dfrac{3\pi r^2}{8} = 54\pi \Rightarrow r^2 = \dfrac{54\pi \times 8}{3\pi} = 144$ **A1**

$r = 12$ cm **AG**

(b) Arc length: $l = r\theta = 12 \times \dfrac{3\pi}{4} = 9\pi$ cm **M1**

Perimeter $= 2r + l = 2(12) + 9\pi = (24 + 9\pi)$ cm **A1**

Note: M1 for perimeter $= 2r + l$; do not award if only arc length given.

Q16 — Irrigation Sector and Composite Region MEDIUM

(a) $200^\circ \times \dfrac{\pi}{180} = 3.4906\ldots$ rad ≈ 3.491 rad **A1**

(b) $A_{\text{sector}} = \dfrac{1}{2}(18.5)^2(3.4906\ldots) = \dfrac{1}{2}(342.25)(3.4906\ldots) = 597.33\ldots$ **(M1)**

≈ 597 m^2 **A1**

(c) Area of inner circular sector with same angle: $A_{\text{inner}} = \dfrac{1}{2}(5.0)^2(3.4906\ldots) = 43.63\ldots$ m^2 **M1**

Required area $= 597.33\ldots - 43.63\ldots = 553.70\ldots \approx 554$ m^2 **A1**

Note: The inner region is a sector (not a full circle) since it lies inside the 200° sector.

Q17 — Running Track Semicircle and Diameter MEDIUM

(a) Arc of semicircle $= \dfrac{\pi d}{2}$; total distance: $\dfrac{\pi d}{2} + 85.0 = 220$ **A1**

(b) $\dfrac{\pi d}{2} = 135 \Rightarrow d = \dfrac{270}{\pi}$ **M1**

$d = 85.94\ldots \approx 85.9$ m **A1**

(c) Radius $= \dfrac{d}{2} = \dfrac{135}{\pi}$; area of semicircle $= \dfrac{1}{2}\pi r^2 = \dfrac{1}{2}\pi\left(\dfrac{135}{\pi}\right)^2 = \dfrac{9112.5}{\pi} = 2900.88\ldots \approx 2900$ m^2

A1

Q18 — Segment Area via Sector Minus Triangle MEDIUM

(a) $A_{\text{sector}} = \frac{1}{2}r^2\theta = \frac{1}{2}(15)^2\left(\frac{2\pi}{5}\right) = \frac{1}{2}(225)\left(\frac{2\pi}{5}\right) = 45\pi \approx 141.37\ldots \text{ cm}^2$ **A1**

(b) $A_{\triangle} = \frac{1}{2}r^2\sin\theta = \frac{1}{2}(225)\sin\left(\frac{2\pi}{5}\right) = 112.5 \times 0.9511\ldots = 107.0\ldots \text{ cm}^2$ **A1**

(c) $A_{\text{segment}} = A_{\text{sector}} - A_{\triangle}$ **M1**

$= 141.37\ldots - 107.0\ldots = 34.37\ldots \approx 34.4 \text{ cm}^2$ **A1**

Note: Award M1 for the correct subtraction formula. Both unrounded values must be seen before rounding.

Q19 — Two Sectors with Equal Arc Lengths MEDIUM

(a) $l_1 = r_1\theta_1 = 14 \times \frac{\pi}{4} = \frac{7\pi}{2}$ cm **A1**

(b) $\frac{7\pi}{2} = 18\theta_2 \Rightarrow \theta_2 = \frac{7\pi}{36}$ radians **A1**

(c) $A_1 = \frac{1}{2}(14)^2\left(\frac{\pi}{4}\right) = \frac{1}{2}(196)\left(\frac{\pi}{4}\right) = \frac{49\pi}{2} = 76.969\ldots \text{ cm}^2$ **M1**

$A_2 = \frac{1}{2}(18)^2\left(\frac{7\pi}{36}\right) = \frac{1}{2}(324)\left(\frac{7\pi}{36}\right) = \frac{63\pi}{2} = 98.960\ldots \text{ cm}^2$

Difference $= 98.960\ldots - 76.969\ldots = 21.99\ldots \approx 22.0 \text{ cm}^2$ **A1**

Q20 — Circle from Diameter Endpoints MEDIUM

(a) Centre $= \left(\frac{2+8}{2}, \frac{-1+3}{2}\right) = (5, 1)$ **A1**

(b) $r = \frac{1}{2}\sqrt{(8-2)^2 + (3-(-1))^2} = \frac{1}{2}\sqrt{36+16} = \frac{1}{2}\sqrt{52} = \sqrt{13}$ **M1A1**

(c) $(x-5)^2 + (y-1)^2 = 13$ **A1**

Note: Award final A1 only if centre and radius from (a) and (b) are correctly substituted.

Q21 — Segment Area and Perimeter HARD

(a) Area of minor segment = area of sector − area of triangle.

Area of sector $= \frac{1}{2}r^2 \cdot \frac{2\pi}{3} = \frac{\pi r^2}{3}$ **M1**

Area of triangle $= \frac{1}{2}r^2\sin\left(\frac{2\pi}{3}\right) = \frac{1}{2}r^2 \cdot \frac{\sqrt{3}}{2} = \frac{r^2\sqrt{3}}{4}$

Segment area: $\frac{\pi r^2}{3} - \frac{r^2\sqrt{3}}{4} = 12\pi - 9\sqrt{3}$ **M1**

Comparing coefficients: $\frac{r^2}{3} = 12 \Rightarrow r^2 = 36 \Rightarrow r = 6$ **AG**

Note: Check $\frac{r^2\sqrt{3}}{4} = \frac{36\sqrt{3}}{4} = 9\sqrt{3}$ ✓

(b) Perimeter = arc + chord.

Arc $= r\theta = 6 \cdot \frac{2\pi}{3} = 4\pi$ cm

Chord $= 2r\sin\left(\frac{\theta}{2}\right) = 2(6)\sin\left(\frac{\pi}{3}\right) = 12 \cdot \frac{\sqrt{3}}{2} = 6\sqrt{3}$ cm **M1**

Perimeter $= 4\pi + 6\sqrt{3}$ cm, so $a = 4, b = 6$ **A1**

Q22 — Perpendicular Bisector Meets the y-axis HARD

(a) Midpoint of AB: $M = \left(\frac{-3+7}{2}, \frac{5+(-1)}{2}\right) = (2, 2)$ **M1**

Gradient of AB: $m_{AB} = \frac{-1-5}{7-(-3)} = \frac{-6}{10} = -\frac{3}{5}$

Gradient of perpendicular bisector: $m_\perp = \frac{5}{3}$

Equation: $y - 2 = \frac{5}{3}(x-2) \Rightarrow y = \frac{5}{3}x - \frac{4}{3}$ **A1**

At $x = 0$: $y = -\frac{4}{3}$, so $C = (0, -4/3)$ **A1**

(b) Triangle OAC has vertices $O(0,0)$, $A(-3,5)$, $C(0,-4/3)$.

Area $= \frac{1}{2}\left|x_O(y_A - y_C) + x_A(y_C - y_O) + x_C(y_O - y_A)\right|$ **M1**

$= \frac{1}{2}\left|0\left(5+\frac{4}{3}\right) + (-3)\left(-\frac{4}{3}-0\right) + 0(0-5)\right| = \frac{1}{2}\left|(-3)\left(-\frac{4}{3}\right)\right| = \frac{1}{2}(4) = 2$, so the area is 2 square units. **A1**

Q23 — Windscreen Wiper Annular Sector HARD

(a) The cleaned region is an annular sector (large sector minus small sector).

Area $= \frac{1}{2}\theta(R^2 - r^2)$, where $R = 6 + 24 = 30$ cm, $r = 6$ cm **M1**

$\frac{1}{2}\theta(30^2 - 6^2) = 756$

$\frac{1}{2}\theta(900 - 36) = 756 \Rightarrow \frac{1}{2}\theta(864) = 756$ **M1**

$432\theta = 756 \Rightarrow \theta = \dfrac{756}{432} = 7/4$ **A1**

Since this equals the printed target, the result is established. **AG**

(b) Boundary = outer arc + inner arc + 2 straight edges.

Outer arc $= R\theta = 30 \times \dfrac{7}{4} = \dfrac{105}{2} = 52.5$ cm

Inner arc $= r\theta = 6 \times \dfrac{7}{4} = \dfrac{21}{2} = 10.5$ cm

Two straight edges $= 2 \times (30 - 6) = 2 \times 24 = 48$ cm **M1**

Total $= 52.5 + 10.5 + 48 = 111$ cm **A1**

Note: GDC gives 111.000... cm; answer is exact.

Q24 — Sector from Arc Length and Area HARD

(a) Using $l = r\theta$ and $A = \dfrac{1}{2}rl$: $A = \dfrac{1}{2}rl \Rightarrow 67.5 = \dfrac{1}{2} \times r \times 15 \Rightarrow r = \dfrac{2 \times 67.5}{15} = 9$ cm **M1A1**

$\theta = \dfrac{l}{r} = \dfrac{15}{9} = 5/3$ radians **A1**

(b) Perimeter of first sector $= 2r + l = 2(9) + 15 = 33$ cm **M1**

Second sector: $2(12) + l_2 = 33 \Rightarrow l_2 = 9$ cm

Area of second sector $= \dfrac{1}{2}r_2 l_2 = \dfrac{1}{2}(12)(9) = 54$ cm^2 **A1**

Q25 — Perpendicular Lines and Exact Distance HARD

(a) Gradient of PQ: $m_{PQ} = \dfrac{4 - (-2)}{5 - 1} = \dfrac{6}{4} = \dfrac{3}{2}$ **M1**

Since $PQ \perp QR$: $m_{QR} = -\dfrac{2}{3}$

Gradient of QR: $m_{QR} = \dfrac{0 - 4}{k - 5} = -\dfrac{2}{3}$ **M1**

$\dfrac{-4}{k-5} = -\dfrac{2}{3} \Rightarrow -12 = -2(k-5) \Rightarrow 12 = 2k - 10 \Rightarrow k = 11$ **A1**

(b) $P = (1, -2)$, $R = (11, 0)$

$|PR| = \sqrt{(11-1)^2 + (0-(-2))^2} = \sqrt{100 + 4} = \sqrt{104} = 2\sqrt{26}$ **M1A1**

Q26 — Two Sectors with Equal Arc Lengths and Combined Area HARD

Arc of $OAB = 9\alpha$; arc of $OCD = 15\beta$.

Equal arcs: $9\alpha = 15\beta \Rightarrow \alpha = \dfrac{5\beta}{3}$ **M1**

Sum of areas: $\dfrac{1}{2}(9)^2\alpha + \dfrac{1}{2}(15)^2\beta = 189$ **M1**

$\dfrac{81\alpha}{2} + \dfrac{225\beta}{2} = 189 \Rightarrow 81\alpha + 225\beta = 378$ **A1**

Substituting $\alpha = \dfrac{5\beta}{3}$: $81 \cdot \dfrac{5\beta}{3} + 225\beta = 378 \Rightarrow 135\beta + 225\beta = 378 \Rightarrow 360\beta = 378$ **M1**

$\beta = \dfrac{378}{360} = 1.05$ rad; $\alpha = \dfrac{5}{3}(1.05) = 1.75$ rad **A1**

Note: GDC solve gives $\beta = 21/20$ and $\alpha = 7/4$ exactly.

Q27 — Unit Circle Exact Coordinates and Sector Areas HARD

(a) The angle $\dfrac{7\pi}{6}$ lies in the third quadrant.

Exact coordinates: $(-\sqrt{3}/2,\ -1/2)$ **A1A1**

(b) Minor sector has angle $\dfrac{\pi}{6}$ and radius 1: $A_{\text{minor}} = \dfrac{1}{2}(1)^2 \cdot \dfrac{\pi}{6} = \pi/12$ square units **A1**

Total circle area $= \pi(1)^2 = \pi$

Major sector angle $= 2\pi - \dfrac{\pi}{6} = \dfrac{11\pi}{6}$

Major sector area $= \dfrac{1}{2}(1)^2 \cdot \dfrac{11\pi}{6} = 11\pi/12$ square units **M1**

Or: Major area $= \pi - \dfrac{\pi}{12} = \dfrac{12\pi - \pi}{12} = \dfrac{11\pi}{12}$ **AG**

Note: Both methods valid; must show subtraction or explicit angle calculation.

Q28 — Irrigation Sector in Degrees Converted to Radians HARD

(a) $220^\circ \times \dfrac{\pi}{180} = \dfrac{11\pi}{9}$ rad $= 3.840$ rad (4 s.f.) **A1**

(b) Area $= \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(45)^2 \times 3.8397\ldots$ **M1**

$= \dfrac{1}{2}(2025)(3.8397\ldots) = 3887.4\ldots \approx 3890$ m^2 (3 s.f.) **A1**

Note: GDC: $0.5 \times 45^2 \times (220\pi/180) = 3887.4\ldots$ m^2.

(c) Fencing $= 2r +$ arc $= 2(45) + 45 \times 3.8397\ldots$ **M1**

$= 90 + 172.79\ldots = 262.79\ldots \approx 263$ m (3 s.f.) **A1**

Note: GDC: $2(45) + 45(220\pi/180) = 262.79\ldots$ m. Must include both straight edges.

Q29 — Perpendicular Bisector and Point Inside or Outside a Circle HARD

(a) Midpoint of AB: $M = \left(\frac{2+8}{2}, \frac{6+4}{2}\right) = (5, 5)$ **M1**

Gradient of AB: $m_{AB} = \frac{4-6}{8-2} = -\frac{1}{3}$; perpendicular gradient $= 3$

Perpendicular bisector: $y - 5 = 3(x - 5) \Rightarrow y = 3x - 10$

C lies on the perpendicular bisector; $|AC|^2 = 53$
$(x-2)^2 + (3x - 10 - 6)^2 = 53 \Rightarrow (x-2)^2 + (3x-16)^2 = 53$ **M1**

Writing $x = 5+t$, $y = 3(5+t) - 10 = 5 + 3t$: $|AC|^2 = (3+t)^2 + (3t-1)^2 = 10t^2 + 10 = 53 \Rightarrow t^2 = 4.3$

$C = \left(5 + \sqrt{4.3},\ 5 + 3\sqrt{4.3}\right)$ or $C = \left(5 - \sqrt{4.3},\ 5 - 3\sqrt{4.3}\right)$

GDC gives $C \approx (7.07, 11.22)$ or $C \approx (2.93, -1.22)$ **A1**

(b) The circle with diameter AB has centre $M(5, 5)$ and radius $= \frac{|AB|}{2} = \frac{\sqrt{36+4}}{2} = \frac{\sqrt{40}}{2} = \sqrt{10}$ **M1**

$|MC|^2 = 10 \times 4.3 = 43 > 10$, so $|MC| > \sqrt{10}$

C lies **outside** the circle, since its distance from the centre exceeds the radius. **R1**

Q30 — Sector Perimeter and Area System HARD

(a) Perimeter of sector: $2r + r\theta = P \Rightarrow 2r + r\theta = 24 \Rightarrow r\theta = 24 - 2r$ **M1**

Area of sector: $A = \frac{1}{2}r^2\theta = 27 \Rightarrow r^2\theta = 54 \Rightarrow \theta = \frac{54}{r^2}$

Substituting into $r\theta = 24 - 2r$: $r \cdot \frac{54}{r^2} = 24 - 2r \Rightarrow \frac{54}{r} = 24 - 2r$ **M1**

$54 = 24r - 2r^2 \Rightarrow 2r^2 - 24r + 54 = 0 \Rightarrow r^2 - 12r + 27 = 0$ **AG**

(b) $(r-3)(r-9) = 0 \Rightarrow r = 3$ or $r = 9$ **M1**

When $r = 3$: $\theta = \frac{54}{9} = 6$ rad; when $r = 9$: $\theta = \frac{54}{81} = 2/3$ rad **A1**

Note: Both solutions are valid since $\theta < 2\pi$ for $r = 3$ ($\theta = 6 < 2\pi$) and $r = 9$.