IB Math AA HL — Topic 3
Trigonometry

Right-Angled Trigonometry · Sine & Cosine Rule · Elevation & Depression · Bearings

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:_____ Date:_____

Score:_____/120

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Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

Key Formulae

- **Pythagoras' Theorem:** $a^2 + b^2 = c^2$ where c is the hypotenuse of a right-angled triangle.
- **Right-Angle Trigonometry:** $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$, $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$, $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$
- **Sine Rule:** $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ (use when a side–angle pair is known)
- **Cosine Rule (side):** $a^2 = b^2 + c^2 - 2bc\cos A$ (use when two sides and the included angle, or all three sides, are known)
- **Cosine Rule (angle):** $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
- **Area of a Triangle:** Area $= \dfrac{1}{2}ab\sin C$ where C is the included angle between sides a and b.
- **Bearings:** Three-figure clockwise angles measured from North; the alternate interior angle between two North lines equals the difference in the given bearings.

GDC Tips

- **Degree/Radian mode:** Before any triangle calculation, confirm the GDC is in Degree mode (MODE → DEGREE on TI-84; Settings → Angle: Degrees on Casio CG-50).
- **Inverse trig for angles:** Use $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ (SHIFT + trig key on Casio) to find angles; store the unrounded result with STO before rounding.
- **Ambiguous-case check:** After finding $\sin B = k$ with $k < 1$, evaluate both $B_1 = \sin^{-1}(k)$ and $B_2 = 180^\circ - B_1$ on the GDC, then test $A + B_2 < 180^\circ$ to determine validity.
- **Cosine rule from three sides:** Enter $\cos^{-1}\left(\dfrac{b^2 + c^2 - a^2}{2bc}\right)$ as a single expression to avoid rounding errors in intermediate steps.
- **Area and bearing problems:** Use the Pol/Rec conversion functions (Casio) or component arithmetic (TI) to resolve bearing legs into North and East components when summing a multi-leg journey.
- **Storing exact surds:** For no-calculator checks, verify decimal answers by storing $\sqrt{3}/2$, $1/\sqrt{2}$ etc. via the GDC's exact or fraction mode where available.

Common Mistakes to Avoid

1. **Ambiguous case ignored.** When using the sine rule to find an angle, students find only the acute solution and never test 180° minus that value. Always check whether the obtuse angle also satisfies the triangle's angle sum.
2. **Sine rule applied without a complete side–angle pair.** The sine rule requires at least one known side opposite a known angle. Choosing it when only two sides and their included angle are given (SAS) forces the cosine rule instead; state the reason for your rule choice.
3. **Premature rounding in multi-step problems.** Rounding a side or angle to 3 s.f. mid-solution and using that rounded value in the next step accumulates error. Carry full GDC precision (or exact surds) until the final answer.
4. **Bearing angle derived incorrectly.** Students subtract bearings without drawing the north lines at both points, missing the correct alternate or co-interior angle. Always sketch both north arrows and mark the included angle between the two legs explicitly.
5. **Area formula with the wrong angle.** Area $= \frac{1}{2}ab\sin C$ requires C to be the angle between sides a and b. Using a non-included angle gives an incorrect area; label sides and the included angle on a diagram before substituting.
6. **Degree/radian mode mismatch.** Evaluating $\cos(115)$ while the GDC is in radian mode returns the wrong sign and magnitude. Confirm the mode at the start of every trigonometry question and write "degrees" or "radians" explicitly in working.

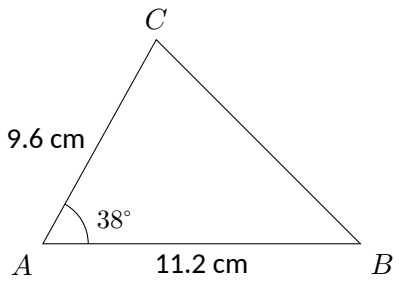
Section A — Easy EASY (10 questions × 3 marks = 30 marks)

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1. EASY Calculator **[3 marks]**

In triangle ABC, angle $A = 38°$, $AB = 11.2$ cm and $AC = 9.6$ cm. The diagram shows the triangle (not to scale).

Find the length of BC, giving your answer to 3 significant figures. Calculator allowed



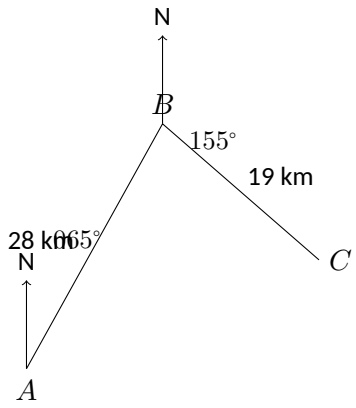
2. EASY No Calculator **[3 marks]**

In right-angled triangle PQR, angle $Q = 90°$, $PQ = 5$ cm and $QR = 12$ cm.

Write down the length of PR. Calculator not allowed

3. **EASY** **Calculator** **[3 marks]**

A ship travels from port A on a bearing of $065°$ for 28 km to reach port B. It then travels on a bearing of $155°$ for 19 km to reach port C.
Find the distance AC, giving your answer to 3 significant figures. Calculator allowed



4. **EASY** **Calculator** **[3 marks]**

In triangle DEF, $DE = 9.0$ cm, $EF = 6.5$ cm and angle $DEF = 54°$.
Find the area of triangle DEF, giving your answer to 3 significant figures. Calculator allowed

5. **EASY** **Calculator** **[3 marks]**

In triangle KLM, $KL = 8$ cm, $LM = 14$ cm and $KM = 11$ cm.
Find the size of angle KLM, giving your answer to the nearest degree. Calculator allowed

6. EASY No Calculator [3 marks]

In right-angled triangle XYZ, angle $Y = 90°$ and angle $X = 30°$. The hypotenuse $XZ = 10$ cm.

(a) Write down the exact length of YZ. Calculator not allowed [1]

(b) Write down the exact length of XY. Calculator not allowed [2]

7. EASY Calculator [3 marks]

A ladder leans against a vertical wall. The foot of the ladder is 1.8 m from the base of the wall on horizontal ground. The ladder makes an angle of $72°$ with the ground.
Find the length of the ladder, giving your answer to 3 significant figures. Calculator allowed

8. EASY Calculator [3 marks]

In triangle ABC, $AB = 15$ cm, angle $BAC = 42°$ and angle $ABC = 63°$.
Find the length of BC, giving your answer to 3 significant figures. Calculator allowed

9. EASY Calculator [3 marks]

An observer stands at point P on horizontal ground. The angle of elevation of the top T of a vertical tower QT is $34°$. The observer is 45 m from the base Q of the tower, measured along the ground.
Find the height QT of the tower, giving your answer to 3 significant figures. Calculator allowed

10. **EASY** **Calculator** **[3 marks]**

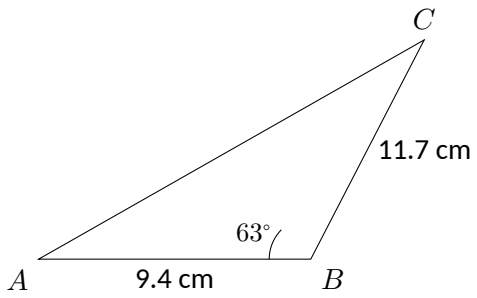
In triangle RST, $RS = 7$ cm, $ST = 10$ cm and angle $RST = 120°$.
Find the length of RT, giving your answer to 3 significant figures. Calculator allowed

Section B — Medium MEDIUM (10 questions × 4 marks = 40 marks)

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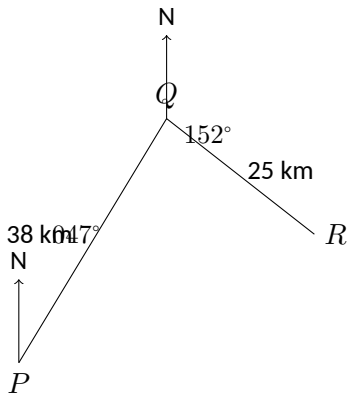
11. MEDIUM Calculator **[4 marks]**

Triangle ABC has $AB = 9.4$ cm, $BC = 11.7$ cm, and angle $ABC = 63°$. Find the area of triangle ABC. Give your answer in cm^2, correct to three significant figures. Calculator allowed



12. **MEDIUM** **Calculator** **[4 marks]**

A ship sails from port P on a bearing of 047° for 38 km to reach point Q. It then sails on a bearing of 152° for 25 km to reach point R. Find the distance PR, in km, correct to three significant figures. The diagram shows the path of the ship. Calculator allowed



13. **MEDIUM** **Calculator** **[4 marks]**

In triangle PQR, $PQ = 7$ cm, $QR = 5$ cm, and $PR = 9$ cm. Find the size of angle QPR, in degrees, correct to one decimal place. Calculator allowed

14. MEDIUM Calculator **[4 marks]**

In triangle KLM, $KL = 12$ cm, angle $KLM = 48°$, and $KM = 10$ cm. Find all possible values of angle KML, giving your answers in degrees correct to one decimal place. (Diagram not to scale.) Calculator allowed

15. MEDIUM Calculator **[4 marks]**

A vertical tower CD stands on horizontal ground. From a point A on the ground, the angle of elevation of the top D is $32°$. From a point B, which is 15 m closer to the base C of the tower along the same straight line, the angle of elevation of D is $47°$. Find the height of the tower CD, in metres, correct to three significant figures. Calculator allowed

16. MEDIUM Calculator **[4 marks]**

A triangular garden ABC has $AB = 8.5$ m, $BC = 6.2$ m, and angle $BAC = 40°$. Find all possible values of angle ABC, giving your answers in degrees correct to one decimal place, justifying whether the situation is ambiguous. Calculator allowed

17. MEDIUM No Calculator **[4 marks]**

Triangle XYZ has $XY = 6\sqrt{3}$ cm, $YZ = 6$ cm, and angle $XYZ = 30°$. Without using a calculator, find the exact area of triangle XYZ in cm^2. Calculator not allowed

18. MEDIUM No Calculator **[4 marks]**

In triangle DEF, $DE = 5$ cm, $EF = 5\sqrt{2}$ cm, and angle $DEF = 45°$. Without using a calculator, find the exact length of DF in the form $a\sqrt{b}$ cm, where $a, b \in \mathbb{Z}^+$. Calculator not allowed

19. MEDIUM Calculator **[4 marks]**

A quadrilateral $ABCD$ has $AB = 14.3$ m, $BC = 9.8$ m, $CD = 11.5$ m, $DA = 10.2$ m, and diagonal $AC = 16.7$ m. Find the total area of quadrilateral $ABCD$, in m^2, correct to three significant figures. Calculator allowed

20. MEDIUM No Calculator **[4 marks]**

An isosceles triangle has two equal sides of length a cm and the angle between them is $\frac{\pi}{3}$ radians. Given that the area of the triangle is $25\sqrt{3}$ cm^2, find the exact value of a. Calculator not allowed

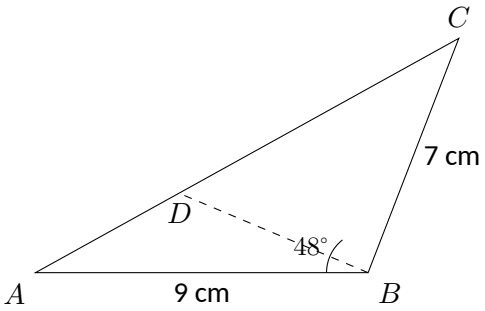
Section C — Hard HARD (10 questions × 5 marks = 50 marks)

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21. HARD Calculator [5 marks]

In triangle ABC, $AB = 9$ cm, $BC = 7$ cm and angle $B = 48°$. A point D lies on AC such that BD bisects angle ABC.

Find the length AD, giving your answer to 3 significant figures. Calculator allowed

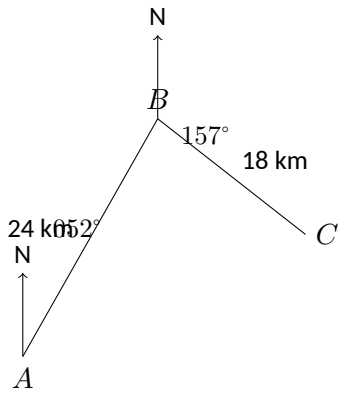


22. HARD No Calculator [5 marks]

In triangle PQR, $PQ = 14$ cm, $QR = 11$ cm, and $PR = 8$ cm. Show that $\cos(\hat{P}) = \frac{139}{224}$, and hence find the exact area of triangle PQR. Calculator not allowed

23. **HARD** **Calculator** **[5 marks]**

A ship sails from port A on a bearing of 052° for 24 km to reach point B. It then sails on a bearing of 157° for 18 km to reach point C. Find the bearing of C from A, giving your answer to the nearest degree. Calculator allowed



24. **HARD** **Calculator** **[5 marks]**

In triangle KLM, $KM = 15$ cm, $LM = 10$ cm and angle $M = 35^\circ$. Find both possible values of angle L, justifying which values are geometrically valid. Calculator allowed

25. **HARD** **Calculator** **[5 marks]**

From point X on horizontal ground, the angle of elevation to the top T of a vertical tower TY is $32°$. From point Z, which is 20 m closer to the base Y along the same straight horizontal line, the angle of elevation to T is $51°$. Find the height of the tower TY, giving your answer to 3 significant figures. Calculator allowed

26. **HARD** **Calculator** **[5 marks]**

A quadrilateral $ABCD$ has $AB = 8$ cm, $BC = 6$ cm, $CD = 9$ cm, $DA = 7$ cm and diagonal $AC = 11$ cm. Find the area of quadrilateral $ABCD$, giving your answer to 3 significant figures. Calculator allowed

27. **HARD** **No Calculator** **[5 marks]**

In triangle RST, $RS = 5$ cm, $ST = 12$ cm and angle $S = 90°$. Find the length RT.
A second triangle RTU shares side RT, with $TU = 9$ cm and angle $RTU = 110°$. Find the area of triangle RTU, giving your answer to 3 significant figures. Calculator not allowed

28. HARD No Calculator [5 marks]

In triangle ABC, $BC = a$, $CA = b$, $AB = c$, and the area of the triangle is denoted Δ. Given that $a = 13$, $b = 9$, $c = 7$ (lengths in cm), use the cosine rule to find $\cos(\widehat{A})$, and hence find the exact value of $\sin(\widehat{A})$ in surd form. State the exact area Δ. Calculator not allowed

29. HARD Calculator [5 marks]

Two observers P and Q stand on horizontal ground. Q is due east of P, and the distance $PQ = 50$ m. Both observe a hot-air balloon B directly above a point on the ground between them. From P, the angle of elevation of B is $54°$; from Q, the angle of elevation of B is $38°$. Find the height of the balloon above the ground, giving your answer to 3 significant figures. Calculator allowed

30. HARD No Calculator [5 marks]

In triangle XYZ, $XY = p$ cm, $YZ = (p + 3)$ cm and angle $Y = 60°$. The area of triangle XYZ is $10\sqrt{3}$ cm^2. Find the value of p and hence find the length XZ, giving your answer in the form $\sqrt{n}$ cm where $n \in \mathbb{Z}^+$. Calculator not allowed

Mark Schemes

Q1 — Cosine Rule for a Side SAS EASY

Two sides (AB, AC) and the included angle A are known, so the cosine rule applies directly (this is genuine SAS – AB and AC both meet at the given angle A). **M1**

$BC^2 = AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos A$

$BC^2 = 11.2^2 + 9.6^2 - 2(11.2)(9.6)\cos 38^\circ = 125.44 + 92.16 - 215.04 \times 0.78801\ldots = 48.146\ldots$ **A1**

$BC = \sqrt{48.146\ldots} = 6.94$ cm **A1**

Note: Award final A1 only for a correctly rounded answer of 6.94 cm. Follow through from their unrounded value.

Q2 — Pythagoras Exact Answer EASY

Applying Pythagoras' theorem: $PR^2 = PQ^2 + QR^2$ **M1**

$PR^2 = 5^2 + 12^2 = 25 + 144 = 169$ **A1**

$PR = 13$ cm **A1**

Q3 — Bearings and Cosine Rule EASY

The angle at B between BA and BC: the bearing from A to B is 065°, so the back-bearing is $065^\circ + 180^\circ = 245^\circ$. The bearing from B to C is 155°.

Angle $ABC = 245^\circ - 155^\circ = 90^\circ$ **M1**

$AC^2 = 28^2 + 19^2 = 784 + 361 = 1145$ **A1**

$AC = \sqrt{1145} = 33.8$ km **A1**

Note: Award M1 for correct derivation of the 90° angle at B from the bearings. Final A1 requires answer rounded to 3 s.f.

Q4 — Area of a Triangle EASY

Area $= \frac{1}{2} \times DE \times EF \times \sin(\angle DEF)$ **M1**

Area $= \frac{1}{2} \times 9.0 \times 6.5 \times \sin 54^\circ$ **A1**

Area $= \frac{1}{2} \times 9.0 \times 6.5 \times 0.80902\ldots = 23.663\ldots = 23.7$ cm^2 **A1**

Note: Final A1 for 23.7 cm^2 rounded to 3 s.f.

Q5 — Cosine Rule for an Angle EASY

Three sides are known, so the cosine rule for an angle applies.

$\cos(\angle KLM) = \dfrac{KL^2 + LM^2 - KM^2}{2 \cdot KL \cdot LM}$ **M1**

$\cos(\angle KLM) = \dfrac{64 + 196 - 121}{2(8)(14)} = \dfrac{139}{224} = 0.62054\ldots$ **A1**

$\angle KLM = \cos^{-1}(0.62054\ldots) = 51.6\ldots^\circ \approx 52^\circ$ **A1**

Q6 — Exact 30-60-90 Triangle EASY

(a) $YZ = XZ\sin 30^\circ = 10 \times \frac{1}{2} = 5$ cm **A1**

(b) $XY = XZ\cos 30^\circ = 10 \times \dfrac{\sqrt{3}}{2}$ **M1**

$XY = 5\sqrt{3}$ cm **A1**

Q7 — Right-Triangle Elevation EASY

The angle with the ground is 72° and the adjacent side (ground distance) is 1.8 m.

$\cos 72^\circ = \dfrac{1.8}{\text{ladder length}}$ **M1**

$\text{ladder length} = \dfrac{1.8}{\cos 72^\circ} = \dfrac{1.8}{0.30902\ldots} = 5.824\ldots$ **A1**

$\text{ladder length} = 5.82$ m **A1**

Note: Award final A1 for 5.82 m to 3 s.f.

Q8 — Sine Rule for a Side EASY

Angle $ACB = 180^\circ - 42^\circ - 63^\circ = 75^\circ$

Applying the sine rule: $\dfrac{BC}{\sin A} = \dfrac{AB}{\sin C}$ **M1**

$\dfrac{BC}{\sin 42^\circ} = \dfrac{15}{\sin 75^\circ}$ **A1**

$BC = \dfrac{15\sin 42^\circ}{\sin 75^\circ} = \dfrac{15 \times 0.66913\ldots}{0.96593\ldots} = 10.390\ldots = 10.4$ cm **A1**

Note: M1 for correct statement of sine rule with their angle C.

Q9 — Angle of Elevation EASY

In right-angled triangle PQT, $PQ = 45$ m and angle of elevation $= 34^\circ$.

$\tan 34^\circ = \dfrac{QT}{PQ}$ **M1**

$QT = 45 \times \tan 34^\circ = 45 \times 0.67451\ldots = 30.353\ldots$ **A1**
$QT = 30.4$ m **A1**
Note: Final A1 for 30.4 m to 3 s.f.

Q10 — Cosine Rule Obtuse Angle EASY

Two sides and the included angle are known, so the cosine rule applies.
$RT^2 = RS^2 + ST^2 - 2 \cdot RS \cdot ST \cdot \cos(\angle RST)$ **M1**
$RT^2 = 7^2 + 10^2 - 2(7)(10)\cos 120^\circ = 49 + 100 - 140 \times (-0.5) = 149 + 70 = 219$ **A1**
$RT = \sqrt{219} = 14.799\ldots = 14.8$ cm **A1**

Q11 — Area Using the Sine Formula MEDIUM

Using area $= \frac{1}{2}ab\sin C$ with the included angle at B: **M1**
Area $= \frac{1}{2} \times 9.4 \times 11.7 \times \sin 63^\circ$ **M1**
$= \frac{1}{2} \times 9.4 \times 11.7 \times 0.89101\ldots = 49.02\ldots$ **A1**
Area $= 49.0$ cm^2 **A1**
Note: Award M1M1A1A0 for unrounded answer only. Final A1 requires correct rounding to 3 s.f.

Q12 — Bearings Journey Cosine Rule MEDIUM

The angle at Q between QP (back-bearing 227°) and QR (bearing 152°):
Angle $PQR = 227^\circ - 152^\circ = 75^\circ$ **M1**
Note: M1 awarded for correctly deriving the included angle 75° from the bearings. The back-bearing from Q to P is $047^\circ + 180^\circ = 227^\circ$.
Applying the cosine rule: $PR^2 = PQ^2 + QR^2 - 2 \cdot PQ \cdot QR\cos(\angle PQR)$ **M1**
$PR^2 = 38^2 + 25^2 - 2(38)(25)\cos 75^\circ = 1444 + 625 - 1900 \times 0.25882\ldots = 1577.4\ldots$ **A1**
$PR = 39.7$ km **A1**

Q13 — Cosine Rule to Find an Angle MEDIUM

We have all three sides, so apply the cosine rule to find angle QPR. **M1**
$$\cos(\angle QPR) = \frac{PQ^2 + PR^2 - QR^2}{2 \cdot PQ \cdot PR} = \frac{49 + 81 - 25}{2 \times 7 \times 9} = \frac{105}{126}$$ **M1**
$\cos(\angle QPR) = 0.83333\ldots$ **A1**
$\angle QPR = \cos^{-1}(0.83333\ldots) = 33.6^\circ$ **A1**

Q14 — Ambiguous Case of the Sine Rule MEDIUM

Using the sine rule: $\dfrac{\sin(\angle KML)}{KL} = \dfrac{\sin(\angle KLM)}{KM}$ **M1**

$\sin(\angle KML) = \dfrac{12\sin 48^\circ}{10} = \dfrac{12 \times 0.74314\ldots}{10} = 0.89177\ldots$ **A1**

First solution: $\angle KML = \arcsin(0.89177\ldots) = 63.1^\circ$

Check: $48^\circ + 63.1^\circ = 111.1^\circ < 180^\circ$ ✓ valid **A1**

Second solution: $\angle KML = 180^\circ - 63.1^\circ = 116.9^\circ$

Check: $48^\circ + 116.9^\circ = 164.9^\circ < 180^\circ$ ✓ valid

Both values are valid: $\angle KML = 63.1^\circ$ or $\angle KML = 116.9^\circ$ **A1**

Note: Final A1 requires both values stated with validity check for each. Full precision: $\arcsin(0.89177\ldots) = 63.099\ldots^\circ$, *which rounds to* 63.1° *(3 s.f.).*

Q15 — Two-Triangle Elevation MEDIUM

Let $CD = h$ m and $BC = x$ m, so $AC = (x + 15)$ m.

From B: $\tan 47^\circ = \dfrac{h}{x}$, so $h = x\tan 47^\circ$ **M1**

From A: $\tan 32^\circ = \dfrac{h}{x+15}$, so $h = (x+15)\tan 32^\circ$ **M1**

Setting equal: $x\tan 47^\circ = (x+15)\tan 32^\circ$

$x(1.07237\ldots - 0.62487\ldots) = 15 \times 0.62487\ldots$

$x = \dfrac{9.3730\ldots}{0.44749\ldots} = 20.944\ldots$ m **A1**

$h = 20.944\ldots \times \tan 47^\circ = 20.944\ldots \times 1.07237\ldots = 22.5$ m **A1**

Note: Award final A1 only for answer in range 22.4–22.5 m. Unrounded intermediate values must be carried through.

Q16 — Sine Rule Ambiguous Case MEDIUM

Two sides (AB, BC) and a non-included angle (A) are known – this is SSA, so check for the ambiguous case. BC is opposite the known angle A; AB is opposite the unknown angle ACB.

By the sine rule: $\dfrac{\sin(\angle ACB)}{AB} = \dfrac{\sin A}{BC}$ **M1**

$\sin(\angle ACB) = \dfrac{AB\sin A}{BC} = \dfrac{8.5 \times \sin 40^\circ}{6.2} = \dfrac{8.5 \times 0.64279\ldots}{6.2} = 0.88124\ldots$ **A1**

$\angle ACB = 61.8^\circ$ or $180^\circ - 61.8^\circ = 118.2^\circ$.

Ambiguous-case test: $BC < AB$ and $AB\sin A = 5.464\ldots < BC = 6.2$, so *both* branches are geometrically valid. **M1**

Hence angle $ABC = 180^\circ - A - \angle ACB$ for each branch:

Branch 1: $\angle ABC = 180^\circ - 40^\circ - 61.8^\circ = 78.2^\circ$

Branch 2: $\angle ABC = 180^\circ - 40^\circ - 118.2^\circ = 21.8^\circ$

Both possible values: $\angle ABC = 78.2^\circ$ or $\angle ABC = 21.8^\circ$ **A1**

Note: $\angle ACB$ (not $\angle ABC$) is the angle directly opposite the known side AB – label the sine-rule output correctly before using the angle sum to reach $\angle ABC$.

Q17 — Exact Area Standard Angle MEDIUM

Area $= \frac{1}{2} \times XY \times YZ \times \sin(\angle XYZ)$ **M1**

$= \frac{1}{2} \times 6\sqrt{3} \times 6 \times \sin 30^\circ$ **M1**

$= \frac{1}{2} \times 6\sqrt{3} \times 6 \times \dfrac{1}{2}$ **A1**

$= \dfrac{36\sqrt{3}}{4} = 9\sqrt{3}\ \text{cm}^2$ **A1**

Q18 — Cosine Rule with Exact Surds MEDIUM

Apply the cosine rule: $DF^2 = DE^2 + EF^2 - 2 \cdot DE \cdot EF \cdot \cos(\angle DEF)$ **M1**

$DF^2 = 5^2 + (5\sqrt{2})^2 - 2 \times 5 \times 5\sqrt{2} \times \cos 45^\circ$ **M1**

$= 25 + 50 - 50\sqrt{2} \times \dfrac{\sqrt{2}}{2}$ **A1**

$= 75 - \dfrac{50 \times 2}{2} = 75 - 50 = 25$

$DF = \sqrt{25} = 5\sqrt{1}$ cm, i.e. $a = 5$, $b = 1$, so $DF = 5$ cm. **A1**

Note: Accept $DF = 5$ cm directly. The form $5\sqrt{1}$ is valid with $a = 5$, $b = 1$.

Q19 — Quadrilateral Area Split into Two Triangles MEDIUM

Diagonal $AC = 16.7$ m splits the quadrilateral into triangles ABC and ACD.

Triangle ABC: sides 14.3, 9.8, 16.7 m.

$\cos(\angle BAC) = \dfrac{14.3^2 + 16.7^2 - 9.8^2}{2(14.3)(16.7)} = \dfrac{204.49 + 278.89 - 96.04}{477.62} = \dfrac{387.34}{477.62} = 0.81099\ldots$ **M1**

$\text{Area}_{ABC} = \frac{1}{2}(14.3)(16.7)\sin(\arccos(0.81099\ldots)) = \frac{1}{2}(14.3)(16.7)(0.58504\ldots) = 69.86\ldots\ \text{m}^2$ **A1**

Triangle ACD: sides 16.7, 11.5, 10.2 m.

$\cos(\angle CAD) = \dfrac{16.7^2 + 10.2^2 - 11.5^2}{2(16.7)(10.2)} = \dfrac{278.89 + 104.04 - 132.25}{340.68} = \dfrac{250.68}{340.68} = 0.73580\ldots$ **M1**

$\text{Area}_{ACD} = \frac{1}{2}(16.7)(10.2)\sin(\arccos(0.73580\ldots)) = \frac{1}{2}(16.7)(10.2)(0.67723\ldots) = 57.67\ldots\ \text{m}^2$

Total area $= 69.86\ldots + 57.67\ldots = 127.5\ldots \approx 128\ \text{m}^2$ **A1**

Q20 — Area Formula Exact Standard Angle MEDIUM

Area $= \frac{1}{2}a \cdot a \cdot \sin\left(\frac{\pi}{3}\right) = \frac{1}{2}a^2 \cdot \frac{\sqrt{3}}{2} = \frac{a^2\sqrt{3}}{4}$ **M1**

Setting equal to $25\sqrt{3}$: **M1**

$\frac{a^2\sqrt{3}}{4} = 25\sqrt{3}$ **A1**

$a^2 = 100 \implies a = 10$ cm **A1**

Q21 — Angle Bisector and Sine Rule HARD

Find AC using the cosine rule first, with $AB = 9$, $BC = 7$, angle $B = 48°$:

$AC^2 = 9^2 + 7^2 - 2(9)(7)\cos 48° = 81 + 49 - 126\cos 48°$ **M1**

$AC^2 = 130 - 126(0.66913\ldots) = 130 - 84.310\ldots = 45.690\ldots$, so $AC = 6.7594\ldots$ cm **A1**

By the angle bisector theorem: $\frac{AD}{DC} = \frac{AB}{BC} = \frac{9}{7}$, so $AD = \frac{9}{16} \times AC$ **M1**

$AD = \frac{9}{16} \times 6.7594\ldots = \frac{9 \times 6.7594\ldots}{16} = 3.8022\ldots$ **(A1)**

$AD = 3.80$ cm (3 s.f.) **A1**

Note: Award M1A0 if cosine rule set up incorrectly but method for bisector theorem is correct (follow-through available on final A1).

Q22 — Cosine Rule Then Exact Area HARD

Apply cosine rule with $PQ = c = 14$, $QR = a = 11$, $PR = b = 8$: angle $\hat{P}$ is opposite $QR = 11$:

$\cos(\hat{P}) = \frac{PQ^2 + PR^2 - QR^2}{2 \cdot PQ \cdot PR} = \frac{14^2 + 8^2 - 11^2}{2(14)(8)}$ **M1**

$= \frac{196 + 64 - 121}{224} = \frac{139}{224}$ **A1 AG**

$\sin^2(\hat{P}) = 1 - \left(\frac{139}{224}\right)^2 = \frac{224^2 - 139^2}{224^2} = \frac{(224-139)(224+139)}{224^2} = \frac{85 \times 363}{50176}$ **M1**

$= \frac{30855}{50176}$, so $\sin(\hat{P}) = \frac{\sqrt{30855}}{224}$ **A1**

Area $= \frac{1}{2} \cdot PQ \cdot PR \cdot \sin(\hat{P}) = \frac{1}{2}(14)(8) \cdot \frac{\sqrt{30855}}{224} = \frac{56\sqrt{30855}}{224} = \frac{\sqrt{30855}}{4}$ cm^2 **A1**

Note: Accept equivalent exact surd forms, e.g. $\frac{11\sqrt{255}}{4}$. Do not accept a decimal approximation for the final mark.

Q23 — Bearings Journey and Return Bearing HARD

The interior angle at B between BA and BC: bearing AB (back-bearing from B to A) $= 052 + 180 = 232°$; bearing $BC = 157°$.

Angle $ABC = 232° - 157° = 75°$ **M1**

Apply cosine rule in triangle ABC with $AB = 24$, $BC = 18$, angle $B = 75°$:

$AC^2 = 24^2 + 18^2 - 2(24)(18)\cos 75° = 576 + 324 - 864\cos 75°$ **M1**

$= 900 - 864(0.25882\ldots) = 900 - 223.62\ldots = 676.38\ldots$, so $AC = 26.007\ldots$ km **A1**

Use sine rule to find angle BAC: $\dfrac{\sin(\angle BAC)}{18} = \dfrac{\sin 75°}{26.007\ldots}$

$\sin(\angle BAC) = \dfrac{18\sin 75°}{26.007\ldots} = \dfrac{18 \times 0.96593\ldots}{26.007\ldots} = 0.66839\ldots$, so $\angle BAC = 41.93\ldots°$ **M1**

Bearing of C from $A = 052° + 41.93\ldots° = 093.93\ldots° \approx 094°$ **A1**

Note: Award M1 for correct derivation of included angle at B. FT on AC for final two marks. Final answer must be a three-figure bearing.

Q24 — Ambiguous Case Sine Rule SAS Check HARD

$KM = 15$ and $LM = 10$ both meet at the given angle $M = 35°$ – this is SAS, so the triangle is uniquely determined (not ambiguous). First find KL via the cosine rule:

$KL^2 = KM^2 + LM^2 - 2(KM)(LM)\cos M = 225 + 100 - 300\cos 35° = 325 - 245.746\ldots = 79.254\ldots$ **M1**

$KL = 8.9025\ldots$ cm **A1**

Using the sine rule to find angle L: $\dfrac{\sin L}{KM} = \dfrac{\sin M}{KL}$, giving $\sin L = \dfrac{15\sin 35°}{8.9025\ldots} = 0.96636\ldots$

$\hat{L}_1 = \arcsin(0.96636\ldots) = 75.1°$ and $\hat{L}_2 = 180° - 75.1° = 104.9°$ **M1**

Since the triangle is SAS (already uniquely fixed by the cosine-rule side KL), only *one* of these two arcsin branches can be correct – the sine rule's inverse function cannot itself distinguish them. Disambiguate using the side-angle inequality: $KM = 15$ cm is the largest given side and lies opposite L, so L must be the *largest* angle in the triangle. Since $\hat{L}_2 = 104.9°$ is obtuse and $\hat{L}_1 = 75.1°$ is not the largest angle once $K = 180 - 35 - 104.9 = 40.1°$ is compared, $\hat{L}_2$ is the valid angle. **A1**

Confirming directly with the cosine rule using all three known sides: $\cos L = \dfrac{KL^2 + LM^2 - KM^2}{2 \cdot KL \cdot LM} \Longrightarrow L = 104.9°$, matching $\hat{L}_2$ exactly and rejecting $\hat{L}_1 = 75.1°$ as a spurious root of the sine rule. **R1**

Note: The R1 requires an explicit justification (side-angle inequality or direct cosine-rule check) for rejecting the acute branch. Only $L = 104.9°$ is a valid answer.

Q25 — Two-Triangle Elevation HARD

Let $TY = h$ m (height of tower), $YZ = d$ m (distance from Z to base Y), so $XY = d + 20$ m.

From Z: $\tan 51° = \dfrac{h}{d}$, so $h = d\tan 51°$ **M1**

From X: $\tan 32° = \dfrac{h}{d+20}$, so $h = (d+20)\tan 32°$ **M1**

Equate: $d\tan 51^\circ = (d+20)\tan 32^\circ$
$d(\tan 51^\circ - \tan 32^\circ) = 20\tan 32^\circ$
$d = \dfrac{20 \times 0.62487\ldots}{1.23490\ldots - 0.62487\ldots} = \dfrac{12.497\ldots}{0.61002\ldots} = 20.486\ldots$ m **A1**
$h = 20.486\ldots \times \tan 51^\circ = 20.486\ldots \times 1.23490\ldots = 25.299\ldots$ m **(A1)**
$TY = 25.3$ m (3 s.f.) **A1**
Note: GDC value for d: 20.49 m (4 s.f.). Award (A1) for unrounded h seen before rounding.

Q26 — Quadrilateral Split into Two Triangles HARD

Diagonal $AC = 11$ cm splits quadrilateral $ABCD$ into triangles ABC and ACD.
Triangle ABC: sides $AB = 8$, $BC = 6$, $AC = 11$.
$\cos(\hat{B}) = \dfrac{8^2 + 6^2 - 11^2}{2(8)(6)} = \dfrac{64 + 36 - 121}{96} = \dfrac{-21}{96} = -0.21875$ **M1**
$\text{Area}_{ABC} = \frac{1}{2}(8)(6)\sin(\hat{B})$; $\sin(\hat{B}) = \sqrt{1 - 0.21875^2} = 0.97577\ldots$
$\text{Area}_{ABC} = \frac{1}{2}(8)(6)(0.97577\ldots) = 23.418\ldots$ cm^2 **A1**
Triangle ACD: sides $AC = 11$, $CD = 9$, $DA = 7$.
$\cos(\hat{D}) = \dfrac{9^2 + 7^2 - 11^2}{2(9)(7)} = \dfrac{81 + 49 - 121}{126} = \dfrac{9}{126} = \dfrac{1}{14}$ **M1**
$\sin(\hat{D}) = \sqrt{1 - \frac{1}{196}} = \dfrac{\sqrt{195}}{14}$
$\text{Area}_{ACD} = \frac{1}{2}(9)(7) \cdot \dfrac{\sqrt{195}}{14} = \dfrac{63\sqrt{195}}{28} = \dfrac{9\sqrt{195}}{4} = \dfrac{9 \times 13.964\ldots}{4} = 31.419\ldots$ cm^2 **A1**
Total area $= 23.418\ldots + 31.419\ldots = 54.838\ldots \approx 54.8$ cm^2 (3 s.f.) **A1**
Note: Award M1 for correct cosine rule setup in each triangle. Full follow-through on areas.

Q27 — Two-Triangle Right-Angle and SAS Area HARD

In right triangle RST: $RS = 5$, $ST = 12$, angle $S = 90^\circ$, so by Pythagoras: **M1**
$RT^2 = 5^2 + 12^2 = 25 + 144 = 169 \implies RT = 13$ cm
Triangle RTU shares side $RT = 13$ cm, with $TU = 9$ cm and included angle $RTU = 110^\circ$ – this is SAS, so the triangle is uniquely determined. **M1**
$\text{Area} = \frac{1}{2} \cdot RT \cdot TU \cdot \sin(\angle RTU) = \frac{1}{2}(13)(9)\sin 110^\circ$ **A1**
$= \frac{1}{2}(13)(9)(0.93969\ldots) = 54.972\ldots$ **A1**
Area $= 55.0$ cm^2 (3 s.f.) **A1**
Note: Award full follow-through if $RT = 13$ is correctly carried into the area formula even if a numerical slip occurred in finding it.

Q28 — Cosine Rule Exact Surd and Area HARD

Cosine rule for $\hat{A}$ (opposite $a = BC = 13$, with $b = CA = 9$, $c = AB = 7$):

$\cos\hat{A} = \dfrac{b^2 + c^2 - a^2}{2bc} = \dfrac{81 + 49 - 169}{2(9)(7)} = \dfrac{-39}{126} = -\dfrac{13}{42}$ **M1A1**

$\sin^2\hat{A} = 1 - \dfrac{169}{1764} = \dfrac{1764 - 169}{1764} = \dfrac{1595}{1764}$

$\sin\hat{A} = \dfrac{\sqrt{1595}}{42}$ **M1A1**

$\Delta = \frac{1}{2}bc\sin\hat{A} = \frac{1}{2}(9)(7)\cdot\dfrac{\sqrt{1595}}{42} = \dfrac{63\sqrt{1595}}{84} = \dfrac{3\sqrt{1595}}{4}$ cm^2 **A1**

Note: 1595 *does not simplify further* ($1595 = 5 \times 11 \times 29$). *Accept* $\dfrac{3\sqrt{1595}}{4}$ *only; decimal not accepted for final mark.*

Q29 — Two-Observer Balloon Height HARD

Let F be the point on the ground directly below balloon B, with $PF = x$ m, so $QF = (50 - x)$ m, and $BF = h$ m.

From P: $\tan 54^\circ = \dfrac{h}{x}$, so $x = \dfrac{h}{\tan 54^\circ}$ **M1**

From Q: $\tan 38^\circ = \dfrac{h}{50 - x}$, so $50 - x = \dfrac{h}{\tan 38^\circ}$ **M1**

Adding: $50 = h\left(\dfrac{1}{\tan 54^\circ} + \dfrac{1}{\tan 38^\circ}\right) = h(\cot 54^\circ + \cot 38^\circ)$ **M1**

$h = \dfrac{50}{\cot 54^\circ + \cot 38^\circ} = \dfrac{50}{0.72654\ldots + 1.27994\ldots} = \dfrac{50}{2.00649\ldots} = 24.919\ldots$ m **A1**

$h = 24.9$ m (3 s.f.) **A1**

Note: GDC: $\cot 54^\circ = 0.72654$, $\cot 38^\circ = 1.27994$. *Award M1 for each correct equation; third M1 for combining to eliminate* x.

Q30 — Area Equation Exact Cosine Rule HARD

Area $= \frac{1}{2}\cdot XY\cdot YZ\cdot\sin\hat{Y} = \frac{1}{2}\cdot p(p+3)\sin 60^\circ = \dfrac{\sqrt{3}}{4}p(p+3)$ **M1**

Set equal to $10\sqrt{3}$: $\dfrac{\sqrt{3}}{4}p(p+3) = 10\sqrt{3}$ **M1**

$p(p+3) = 40 \implies p^2 + 3p - 40 = 0 \implies (p+8)(p-5) = 0$

Taking the positive root: $p = 5$ **A1**

With $p = 5$: $XY = 5$ cm, $YZ = 8$ cm.

Cosine rule: $XZ^2 = XY^2 + YZ^2 - 2\cdot XY\cdot YZ\cos 60^\circ = 25 + 64 - 2(5)(8)\cdot\frac{1}{2} = 89 - 40 = 49$ **M1**

$XZ = \sqrt{49} = 7$ cm, so $n = 49$. **A1**