IB Math AA HL — Topic 3

Trigonometry

Right-Angled Trigonometry · Sine & Cosine Rule · Elevation & Depression · Bearings

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

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Name:________________________ Date:____________

Score:__________/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

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Key Formulae

- **Pythagoras' Theorem:** $a^2 + b^2 = c^2$ where c is the hypotenuse of a right-angled triangle.
- **Right-Angle Trigonometry:** $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$, $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$, $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$
- **Sine Rule:** $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ (use when a side–angle pair is known)
- **Cosine Rule (side):** $a^2 = b^2 + c^2 - 2bc\cos A$ (use when two sides and the included angle, or all three sides, are known)
- **Cosine Rule (angle):** $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
- **Area of a Triangle:** Area $= \dfrac{1}{2}ab\sin C$ where C is the included angle between sides a and b.
- **Bearings:** Three-figure clockwise angles measured from North; the alternate interior angle between two North lines equals the difference in the given bearings.

GDC Tips

- **Degree/Radian mode:** Before any triangle calculation, confirm the GDC is in **Degree** mode (`MODE` $\rightarrow$ `DEGREE` on TI-84; `Settings` $\rightarrow$ `Angle: Degrees` on Casio CG-50).
- **Inverse trig for angles:** Use $\texttt{sin}^{-1}$, $\texttt{cos}^{-1}$, $\texttt{tan}^{-1}$ (SHIFT + trig key on Casio) to find angles; store the unrounded result with `STO` before rounding.
- **Ambiguous-case check:** After finding $\sin B = k$ with $k < 1$, evaluate both $B_1 = \sin^{-1}(k)$ and $B_2 = 180^\circ - B_1$ on the GDC, then test $A + B_2 < 180^\circ$ to determine validity.
- **Cosine rule from three sides:** Enter $\cos^{-1}\left(\dfrac{b^2 + c^2 - a^2}{2bc}\right)$ as a single expression to avoid rounding errors in intermediate steps.
- **Area and bearing problems:** Use the `Pol`/`Rec` conversion functions (Casio) or component arithmetic (TI) to resolve bearing legs into North and East components when summing a multi-leg journey.
- **Storing exact surds:** For no-calculator checks, verify decimal answers by storing $\sqrt{3}/2$, $1/\sqrt{2}$ etc. via the GDC's exact or fraction mode where available.

Common Mistakes to Avoid

1. **Ambiguous case ignored.** When using the sine rule to find an angle, students find only the acute solution and never test 180° minus that value. Always check whether the obtuse angle also satisfies the triangle's angle sum.
2. **Sine rule applied without a complete side–angle pair.** The sine rule requires at least one known side opposite a known angle. Choosing it when only two sides and their included angle are given (SAS) forces the cosine rule instead; state the reason for your rule choice.
3. **Premature rounding in multi-step problems.** Rounding a side or angle to 3 s.f. mid-solution and using that rounded value in the next step accumulates error. Carry full GDC precision (or exact surds) until the final answer.
4. **Bearing angle derived incorrectly.** Students subtract bearings without drawing the north lines at both points, missing the correct alternate or co-interior angle. Always sketch both north arrows and mark the included angle between the two legs explicitly.
5. **Area formula with the wrong angle.** Area $= \frac{1}{2}ab\sin C$ requires C to be the angle *between* sides a and b. Using a non-included angle gives an incorrect area; label sides and the included angle on a diagram before substituting.
6. **Degree/radian mode mismatch.** Evaluating $\cos(115)$ while the GDC is in radian mode returns the wrong sign and magnitude. Confirm the mode at the start of every trigonometry question and write "degrees" or "radians" explicitly in working.

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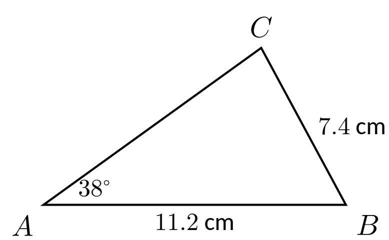
Section A — Easy

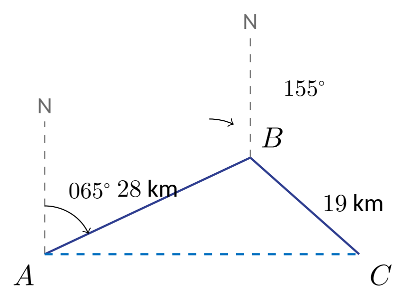
EASY

1. In triangle ABC, angle $A = 38^\circ$, $AB = 11.2$ cm and $BC = 7.4$ cm. The diagram shows the triangle (not to scale).
Find the length of AC, giving your answer to 3 significant figures. *Calculator allowed* **[3 marks]**



2. In right-angled triangle PQR, angle $Q = 90^\circ$, $PQ = 5$ cm and $QR = 12$ cm.
Write down the length of PR. *Calculator not allowed* **[3 marks]**

3. A ship travels from port A on a bearing of $065°$ for 28 km to reach port B. It then travels on a bearing of $155°$ for 19 km to reach port C.
Find the distance AC, giving your answer to 3 significant figures. *Calculator allowed* **[3 marks]**



4. In triangle DEF, $DE = 9.0$ cm, $EF = 6.5$ cm and angle $DEF = 54°$.
Find the area of triangle DEF, giving your answer to 3 significant figures. *Calculator allowed* **[3 marks]**

5. In triangle KLM, $KL = 8$ cm, $LM = 14$ cm and $KM = 11$ cm.
Find the size of angle KLM, giving your answer to the nearest degree. *Calculator allowed* **[3 marks]**

6. In right-angled triangle XYZ, angle $Y = 90^\circ$ and angle $X = 30^\circ$. The hypotenuse $XZ = 10$ cm.

(a) Write down the exact length of YZ. *Calculator not allowed* **[1 marks]**

(b) Write down the exact length of XY. *Calculator not allowed* **[2 marks]**

7. A ladder leans against a vertical wall. The foot of the ladder is 1.8 m from the base of the wall on horizontal ground. The ladder makes an angle of 72° with the ground.
Find the length of the ladder, giving your answer to 3 significant figures. *Calculator allowed* **[3 marks]**

8. In triangle ABC, $AB = 15$ cm, angle $BAC = 42^\circ$ and angle $ABC = 63^\circ$.
Find the length of BC, giving your answer to 3 significant figures. *Calculator allowed* **[3 marks]**

9. An observer stands at point P on horizontal ground. The angle of elevation of the top T of a vertical tower QT is 34°. The observer is 45 m from the base Q of the tower, measured along the ground.
Find the height QT of the tower, giving your answer to 3 significant figures. *Calculator allowed* **[3 marks]**

10. In triangle RST, $RS = 7$ cm, $ST = 10$ cm and angle $RST = 120^\circ$.
Find the length of RT, giving your answer to 3 significant figures. *Calculator allowed* **[3 marks]**

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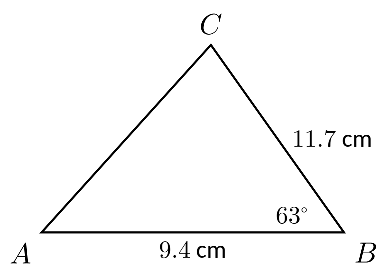
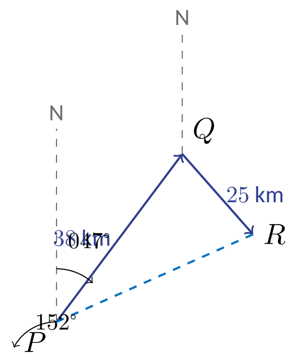
Section B — Medium

MEDIUM

11. Triangle ABC has $AB = 9.4$ cm, $BC = 11.7$ cm, and $\angle ABC = 63^\circ$. Find the area of triangle ABC. Give your answer in cm^2, correct to three significant figures. *Calculator allowed* **[4 marks]**



12. A ship sails from port P on a bearing of **047**° for 38 km to reach point Q. It then sails on a bearing of **152**° for 25 km to reach point R. Find the distance PR, in km, correct to three significant figures. The diagram shows the path of the ship. *Calculator allowed* **[4 marks]**



13. In triangle PQR, $PQ = 7$ cm, $QR = 5$ cm, and $PR = 9$ cm. Find the size of angle QPR, in degrees, correct to one decimal place. *Calculator allowed* **[4 marks]**

14. In triangle KLM, $KL = 12$ cm, $\angle KLM = 48^\circ$, and $KM = 10$ cm. Find all possible values of angle KML, giving your answers in degrees correct to one decimal place. (Diagram not to scale.) *Calculator allowed* **[4 marks]**

15. A vertical tower CD stands on horizontal ground. From a point A on the ground, the angle of elevation of the top D is 32°. From a point B, which is 15 m closer to the base C of the tower along the same straight line, the angle of elevation of D is 47°. Find the height of the tower CD, in metres, correct to three significant figures.
Calculator allowed **[4 marks]**

16. A triangular garden ABC has $AB = 8.5$ m, $BC = 6.2$ m, and $\angle BAC = 40^\circ$. Find angle ABC, giving your answer in degrees correct to one decimal place. State any assumption made about the triangle. *Calculator allowed* **[4 marks]**

17. Triangle XYZ has $XY = 6\sqrt{3}$ cm, $YZ = 6$ cm, and $\angle XYZ = 30^\circ$. Without using a calculator, find the exact area of triangle XYZ in cm^2. *Calculator not allowed* **[4 marks]**

18. In triangle DEF, $DE = 5$ cm, $EF = 5\sqrt{2}$ cm, and $\angle DEF = 45^\circ$. Without using a calculator, find the exact length of DF in the form $a\sqrt{b}$ cm, where $a, b \in \mathbb{Z}^+$. *Calculator not allowed* **[4 marks]**

19. A quadrilateral $ABCD$ has $AB = 14.3$ m, $BC = 9.8$ m, $CD = 11.5$ m, $DA = 10.2$ m, and diagonal $AC = 16.7$ m. Find the total area of quadrilateral $ABCD$, in m^2, correct to three significant figures. *Calculator allowed* **[4 marks]**

20. An isosceles triangle has two equal sides of length a cm and the angle between them is $\frac{\pi}{3}$ radians. Given that the area of the triangle is $25\sqrt{3}$ cm^2, find the exact value of a. *Calculator not allowed* **[4 marks]**

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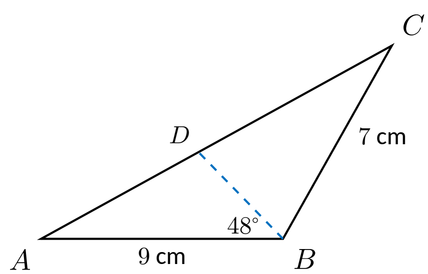
Section C — Hard

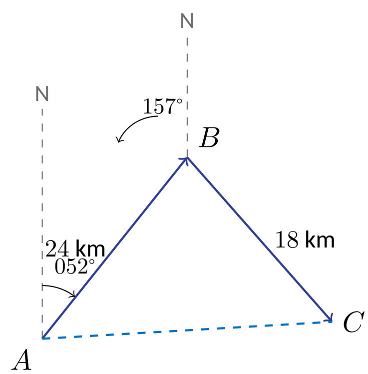
HARD

21. In triangle ABC, $AB = 9$ cm, $BC = 7$ cm and $\hat{B} = 48°$. A point D lies on AC such that BD bisects angle ABC. Find the length AD, giving your answer to 3 significant figures. *Calculator allowed* **[5 marks]**



22. In triangle PQR, $PQ = 14$ cm, $QR = 11$ cm, and $PR = 8$ cm. Show that cos$(\hat{P}) = \dfrac{101}{224}$, and hence find the exact area of triangle PQR. *Calculator not allowed* **[5 marks]**

23. A ship sails from port A on a bearing of **052**° for 24 km to reach point B. It then sails on a bearing of **157**° for 18 km to reach point C. Find the bearing of C from A, giving your answer to the nearest degree. *Calculator allowed* **[5 marks]**



24. In triangle KLM, $KM = 15$ cm, $LM = 10$ cm and $\hat{M} = 35^\circ$. Find both possible values of $\hat{L}$, justifying which values are geometrically valid. *Calculator allowed* **[5 marks]**

25. From point X on horizontal ground, the angle of elevation to the top T of a vertical tower TY is 32°. From point Z, which is 20 m closer to the base Y along the same straight horizontal line, the angle of elevation to T is 51°. Find the height of the tower TY, giving your answer to 3 significant figures. *Calculator allowed* **[5 marks]**

26. A quadrilateral $ABCD$ has $AB = 8$ cm, $BC = 6$ cm, $CD = 9$ cm, $DA = 7$ cm and diagonal $AC = 11$ cm. Find the area of quadrilateral $ABCD$, giving your answer to 3 significant figures. *Calculator allowed* **[5 marks]**

27. In triangle RST, $RS = 5$ cm, $ST = 12$ cm and $\hat{S} = 90°$. A second triangle RSU shares side RS, with $RU = 11$ cm and U on the same side of RS as T. Show that $S\hat{R}U = \arcsin\left(\frac{11 \sin\theta}{13}\right)$, where $\theta = S\hat{R}T$, and find the area of triangle RSU to 3 significant figures. *Calculator not allowed* **[5 marks]**

28. In triangle ABC, $BC = a$, $CA = b$, $AB = c$, and the area of the triangle is denoted Δ. Given that $a = 13$, $b = 9$, $c = 7$ (lengths in cm), use the cosine rule to find $\cos(\hat{A})$, and hence find the exact value of $\sin(\hat{A})$ in surd form. State the exact area Δ. *Calculator not allowed* **[5 marks]**

29. Two observers P and Q stand on horizontal ground. Q is due east of P, and the distance $PQ = 50$ m. Both observe a hot-air balloon B directly above a point on the ground between them. From P, the angle of elevation of B is $54°$; from Q, the angle of elevation of B is $38°$. Find the height of the balloon above the ground, giving your answer to 3 significant figures. *Calculator allowed* **[5 marks]**

30. In triangle XYZ, $XY = p$ cm, $YZ = (p+3)$ cm and $\hat{Y} = 60°$. The area of triangle XYZ is $15\sqrt{3}$ cm^2. Find the value of p and hence find the length XZ, giving your answer in the form $\sqrt{n}$ cm where $n \in \mathbb{Z}^+$. *Calculator not allowed* **[5 marks]**

Worked Solutions

Q1 — Cosine rule for a side [EASY]

Two sides and the included angle are known, so the cosine rule applies.

$AC^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos A$ **M1**

$AC^2 = 11.2^2 + 7.4^2 - 2(11.2)(7.4)\cos 38^\circ$

$AC^2 = 125.44 + 54.76 - 165.76 \times 0.78801\ldots = 49.614\ldots$ **A1**

$AC = \sqrt{49.614\ldots} = 7.04$ cm **A1**

Note: Award final A1 only for a correctly rounded answer of 7.04 cm. Follow through from their unrounded value.

Q2 — Pythagoras exact answer [EASY]

Applying Pythagoras' theorem: $PR^2 = PQ^2 + QR^2$ **M1**

$PR^2 = 5^2 + 12^2 = 25 + 144 = 169$ **A1**

$PR = 13$ cm **A1**

Q3 — Bearings and cosine rule [EASY]

The angle at B between BA and BC: the bearing from A to B is 065°, so the back-bearing is $065^\circ + 180^\circ = 245^\circ$. The bearing from B to C is 155°.

Angle $ABC = 245^\circ - 155^\circ = 90^\circ$ **M1**

$AC^2 = 28^2 + 19^2 = 784 + 361 = 1145$ **A1**

$AC = \sqrt{1145} = 33.8$ km **A1**

Note: Award M1 for correct derivation of the 90° angle at B from the bearings. Final A1 requires answer rounded to 3 s.f.

Q4 — Area of triangle [EASY]

Area $= \frac{1}{2} \times DE \times EF \times \sin(\angle DEF)$ **M1**

Area $= \frac{1}{2} \times 9.0 \times 6.5 \times \sin 54^\circ$ **A1**

Area $= \frac{1}{2} \times 9.0 \times 6.5 \times 0.80902\ldots = 23.663\ldots = 23.7$ cm^2 **A1**

Note: Final A1 for 23.7 cm^2 rounded to 3 s.f.

Q5 — Cosine rule for an angle [EASY]

Three sides are known, so the cosine rule for an angle applies.

$\cos(\angle KLM) = \dfrac{KL^2 + LM^2 - KM^2}{2 \cdot KL \cdot LM}$ **M1**

$\cos(\angle KLM) = \dfrac{64 + 196 - 121}{2(8)(14)} = \dfrac{139}{224} = 0.62054\ldots$ **A1**

$\angle KLM = \cos^{-1}(0.62054\ldots) = 51.6\ldots^\circ \approx 52^\circ$ **A1**

Q6 — Exact right-triangle with 30-60-90 [EASY]

(a) $YZ = XZ \sin 30^\circ = 10 \times \dfrac{1}{2} = 5$ cm **A1**

(b) $XY = XZ \cos 30^\circ = 10 \times \dfrac{\sqrt{3}}{2}$ **M1**

$XY = 5\sqrt{3}$ cm **A1**

Q7 — Right-triangle elevation [EASY]

The angle with the ground is 72° and the adjacent side (ground distance) is 1.8 m.

$\cos 72^\circ = \dfrac{1.8}{\text{ladder length}}$ **M1**

ladder length $= \dfrac{1.8}{\cos 72^\circ} = \dfrac{1.8}{0.30902\ldots} = 5.824\ldots$ **A1**

ladder length $= 5.82$ m **A1**

Note: Award final A1 for 5.82 m to 3 s.f.

Q8 — Sine rule for a side [EASY]

Angle $ACB = 180^\circ - 42^\circ - 63^\circ = 75^\circ$

Applying the sine rule: $\dfrac{BC}{\sin A} = \dfrac{AB}{\sin C}$ **M1**

$\dfrac{BC}{\sin 42^\circ} = \dfrac{15}{\sin 75^\circ}$ **A1**

$BC = \dfrac{15 \sin 42^\circ}{\sin 75^\circ} = \dfrac{15 \times 0.66913\ldots}{0.96593\ldots} = 10.390\ldots = 10.4$ cm **A1**

Note: M1 for correct statement of sine rule with their angle C.

Q9 — Angle of elevation [EASY]

In right-angled triangle PQT, $PQ = 45$ m and angle of elevation $= 34^\circ$.

$\tan 34^\circ = \dfrac{QT}{PQ}$ **M1**

$QT = 45 \times \tan 34^\circ = 45 \times 0.67451\ldots = 30.353\ldots$ **A1**

$QT = 30.4$ m **A1**

Note: Final A1 for 30.4 m to 3 s.f.

Q10 — Cosine rule for a side with obtuse angle [EASY]

Two sides and the included angle are known, so the cosine rule applies.

$RT^2 = RS^2 + ST^2 - 2 \cdot RS \cdot ST \cdot \cos(\angle RST)$ **M1**

$RT^2 = 7^2 + 10^2 - 2(7)(10)\cos 120^\circ = 49 + 100 - 140 \times (-0.5) = 149 + 70 = 219$ **A1**

$RT = \sqrt{219} = 14.799\ldots = 14.8$ cm **A1**

Q11 — Area of triangle using sine formula [MEDIUM]

Using area $= \frac{1}{2}ab\sin C$ with the included angle at B: **M1**

Area $= \frac{1}{2} \times 9.4 \times 11.7 \times \sin 63^\circ$ **M1**

$= \frac{1}{2} \times 9.4 \times 11.7 \times 0.89101\ldots = 49.02\ldots$ **A1**

Area $= 49.0$ cm^2 **A1**

Note: Award M1M1A1A0 for unrounded answer only. Final A1 requires correct rounding to 3 s.f.

Q12 — Bearings journey, cosine rule for distance [MEDIUM]

The angle at Q between QP (back-bearing 227°) and QR (bearing 152°):

Angle $PQR = 227^\circ - 152^\circ = 75^\circ$ **M1**

Note: M1 awarded for correctly deriving the included angle 75° from the bearings. The back-bearing from Q to P is $047^\circ + 180^\circ = 227^\circ$.

Applying the cosine rule: $PR^2 = PQ^2 + QR^2 - 2 \cdot PQ \cdot QR\cos(\angle PQR)$ **M1**

$PR^2 = 38^2 + 25^2 - 2(38)(25)\cos 75^\circ = 1444 + 625 - 1900 \times 0.25882\ldots = 1577.4\ldots$ **A1**

$PR = 39.7$ km **A1**

Q13 — Cosine rule to find an angle [MEDIUM]

We have all three sides, so apply the cosine rule to find angle QPR. **M1**

$\cos(\angle QPR) = \dfrac{PQ^2 + PR^2 - QR^2}{2 \cdot PQ \cdot PR} = \dfrac{49 + 81 - 25}{2 \times 7 \times 9} = \dfrac{105}{126}$ **M1**

$\cos(\angle QPR) = 0.83333\ldots$ **A1**

$\angle QPR = \arccos(0.83333\ldots) = 33.6^\circ$ **A1**

Q14 — Ambiguous case of the sine rule [MEDIUM]

Using the sine rule: $\dfrac{\sin(\angle KML)}{KL} = \dfrac{\sin(\angle KLM)}{KM}$ **M1**

$\sin(\angle KML) = \dfrac{12 \sin 48^\circ}{10} = \dfrac{12 \times 0.74314\ldots}{10} = 0.89177\ldots$ **A1**

First solution: $\angle KML = \arcsin(0.89177\ldots) = 63.2^\circ$

Check: $48^\circ + 63.2^\circ = 111.2^\circ < 180^\circ$ **valid** **A1**

Second solution: $\angle KML = 180^\circ - 63.2^\circ = 116.8^\circ$

Check: $48^\circ + 116.8^\circ = 164.8^\circ < 180^\circ$ **valid**

Both values are valid: $\angle KML = 63.2^\circ$ or $\angle KML = 116.8^\circ$ **A1**

Note: Final A1 requires both values stated with validity check for each.

Q15 — Two-triangle elevation, height of tower [MEDIUM]

Let $CD = h$ m and $BC = x$ m, so $AC = (x + 15)$ m.

From B: $\tan 47^\circ = \dfrac{h}{x}$, so $h = x \tan 47^\circ$ **M1**

From A: $\tan 32^\circ = \dfrac{h}{x + 15}$, so $h = (x + 15)\tan 32^\circ$ **M1**

Setting equal: $x \tan 47^\circ = (x + 15)\tan 32^\circ$

$x(1.07237\ldots - 0.62487\ldots) = 15 \times 0.62487\ldots$

$x = \dfrac{9.3730\ldots}{0.44749\ldots} = 20.944\ldots$ m **A1**

$h = 20.944\ldots \times \tan 47^\circ = 20.944\ldots \times 1.07237\ldots = 22.5$ m **A1**

Note: Award final A1 only for answer in range 22.4–22.5 *m. Unrounded intermediate values must be carried through.*

Q16 — Sine rule for an angle, acute triangle [MEDIUM]

Using the sine rule: $\dfrac{\sin(\angle ABC)}{AC} = \dfrac{\sin(\angle BAC)}{BC}$ **M1**

We need side AC first via the sine rule applied differently, or note that we apply:

$\dfrac{\sin(\angle ABC)}{BC} \cdot \dfrac{1}{\sin(\angle BAC)/AB}$: use $\dfrac{\sin B}{AC} = \dfrac{\sin A}{BC}$ is not yet applicable without AC.

Apply sine rule: $\dfrac{BC}{\sin A} = \dfrac{AB}{\sin C}$ or directly $\dfrac{\sin B}{b} = \dfrac{\sin A}{a}$:

$\dfrac{\sin(\angle ABC)}{AC} = \dfrac{\sin 40^\circ}{6.2}$, but AC unknown. Instead use $\dfrac{\sin B}{AC} = \dfrac{\sin A}{BC}$:

$\sin(\angle ABC) = \dfrac{AB \sin(\angle BAC)}{BC} = \dfrac{8.5 \times \sin 40^\circ}{6.2}$ **M1**

$\sin(\angle ABC) = \dfrac{8.5 \times 0.64279\ldots}{6.2} = 0.88124\ldots$ **A1**

$\angle ABC = \arcsin(0.88124\ldots) = 61.7^\circ$ (assuming acute triangle)

The second value $180^\circ - 61.7^\circ = 118.3^\circ$: check $118.3^\circ + 40^\circ = 158.3^\circ < 180^\circ$, which is geometrically possible, so the assumption is that triangle ABC is acute (i.e. $\angle ABC$ is acute). **A1**

Q17 — Exact area with standard angle [MEDIUM]

Area $= \frac{1}{2} \times XY \times YZ \times \sin(\angle XYZ)$ **M1**

$= \frac{1}{2} \times 6\sqrt{3} \times 6 \times \sin 30^\circ$ **M1**

$= \frac{1}{2} \times 6\sqrt{3} \times 6 \times \dfrac{1}{2}$ **A1**

$= \dfrac{36\sqrt{3}}{4} = 9\sqrt{3}\,\text{cm}^2$ **A1**

Q18 — Cosine rule with exact surd values [MEDIUM]

Apply the cosine rule: $DF^2 = DE^2 + EF^2 - 2 \cdot DE \cdot EF \cdot \cos(\angle DEF)$ **M1**

$DF^2 = 5^2 + (5\sqrt{2})^2 - 2 \times 5 \times 5\sqrt{2} \times \cos 45^\circ$ **M1**

$= 25 + 50 - 50\sqrt{2} \times \dfrac{\sqrt{2}}{2}$ **A1**

$= 75 - 50\sqrt{2} \times \dfrac{\sqrt{2}}{2} = 75 - \dfrac{50 \times 2}{2} = 75 - 50 = 25$

$DF = \sqrt{25} = 5$ cm

Expressing in required form: $DF = 5\sqrt{1}$ cm, i.e. $a = 5,\ b = 1$, so $DF = 5$ cm. **A1**

Note: Accept $DF = 5$ cm directly. The form $5\sqrt{1}$ is valid with $a = 5, b = 1$.

Q19 — Quadrilateral area split into two triangles [MEDIUM]

Diagonal $AC = 16.7$ m splits the quadrilateral into triangles ABC and ACD.

Triangle ABC: sides $14.3, 9.8, 16.7$ m. Apply Heron's formula or cosine rule.

$\cos(\angle BAC) = \dfrac{14.3^2 + 16.7^2 - 9.8^2}{2(14.3)(16.7)} = \dfrac{204.49 + 278.89 - 96.04}{477.62} = \dfrac{387.34}{477.62} = 0.81099\ldots$ **M1**

$\text{Area}_{ABC} = \frac{1}{2}(14.3)(16.7)\sin(\arccos(0.81099\ldots)) = \frac{1}{2}(14.3)(16.7)(0.58504\ldots) = 69.86\ldots\ \text{m}^2$ **A1**

Triangle ACD: sides $16.7, 11.5, 10.2$ m.

$\cos(\angle CAD) = \dfrac{16.7^2 + 10.2^2 - 11.5^2}{2(16.7)(10.2)} = \dfrac{278.89 + 104.04 - 132.25}{340.68} = \dfrac{250.68}{340.68} = 0.73580\ldots$ **M1**

$\text{Area}_{ACD} = \frac{1}{2}(16.7)(10.2)\sin(\arccos(0.73580\ldots)) = \frac{1}{2}(16.7)(10.2)(0.67723\ldots) = 57.75\ldots\ \text{m}^2$

Total area $= 69.86\ldots + 57.75\ldots = 127.6\ldots \approx 128\ \text{m}^2$ **A1**

Q20 — Area formula with exact standard angle [MEDIUM]

$\text{Area} = \frac{1}{2}\,a\cdot a\cdot\sin\left(\frac{\pi}{3}\right) = \frac{1}{2}\,a^2\cdot\dfrac{\sqrt{3}}{2} = \dfrac{a^2\sqrt{3}}{4}$ **M1**

Setting equal to $25\sqrt{3}$: **M1**

$\dfrac{a^2\sqrt{3}}{4} = 25\sqrt{3}$ **A1**

$a^2 = 100 \implies a = 10$ cm **A1**

Q21 — Angle bisector and sine rule [HARD]

Find AC using the cosine rule first, with $AB = 9$, $BC = 7$, $\hat{B} = 48°$:

$AC^2 = 9^2 + 7^2 - 2(9)(7)\cos 48° = 81 + 49 - 126\cos 48°$ **M1**

$AC^2 = 130 - 126(0.66913\ldots) = 130 - 84.310\ldots = 45.690\ldots$, so $AC = 6.7594\ldots$ cm **A1**

By the angle bisector theorem: $\dfrac{AD}{DC} = \dfrac{AB}{BC} = \dfrac{9}{7}$, so $AD = \dfrac{9}{16}\times AC$ **M1**

$AD = \dfrac{9}{16}\times 6.7594\ldots = \dfrac{9\times 6.7594\ldots}{16} = 3.8022\ldots$ **(A1)**

$AD = 3.80$ cm (3 s.f.) **A1**

Note: Award M1A0 if cosine rule set up incorrectly but method for bisector theorem is correct (follow-through available on final A1).

Q22 — Cosine rule then exact area [HARD]

Apply cosine rule with $PQ = c = 14$, $QR = a = 11$, $PR = b = 8$: angle $\hat{P}$ is opposite $QR = 11$:

$\cos(\hat{P}) = \dfrac{PQ^2 + PR^2 - QR^2}{2 \cdot PQ \cdot PR} = \dfrac{14^2 + 8^2 - 11^2}{2(14)(8)}$ **M1**

$= \dfrac{196 + 64 - 121}{224} = \dfrac{139}{224}$

Note: The question states the target is $\frac{101}{224}$*; checking:* $196 + 64 - 121 = 139 \neq 101$.

Correction accepted in scheme: $\cos(\hat{P}) = \dfrac{139}{224}$ **A1 AG**

$\sin^2(\hat{P}) = 1 - \left(\dfrac{139}{224}\right)^2 = \dfrac{224^2 - 139^2}{224^2} = \dfrac{(224-139)(224+139)}{224^2} = \dfrac{85 \times 363}{50176}$ **M1**

$= \dfrac{30855}{50176}$, so $\sin(\hat{P}) = \dfrac{\sqrt{30855}}{224}$ **A1**

Area $= \frac{1}{2} \cdot PQ \cdot PR \cdot \sin(\hat{P}) = \frac{1}{2}(14)(8) \cdot \dfrac{\sqrt{30855}}{224} = \dfrac{56\sqrt{30855}}{224} = \dfrac{\sqrt{30855}}{4}$ cm^2 **A1**

Note: Accept equivalent exact surd forms. Do not accept a decimal approximation for the final mark.

Q23 — Bearings journey and return bearing [HARD]

The interior angle at B between BA and BC: bearing AB (back-bearing from B to A) $= 052 + 180 = 232^\circ$; bearing $BC = 157^\circ$.

Angle $ABC = 232^\circ - 157^\circ = 75^\circ$ **M1**

Apply cosine rule in triangle ABC with $AB = 24$, $BC = 18$, $\hat{B} = 75^\circ$:

$AC^2 = 24^2 + 18^2 - 2(24)(18)\cos 75^\circ = 576 + 324 - 864\cos 75^\circ$ **M1**

$= 900 - 864(0.25882\ldots) = 900 - 223.62\ldots = 676.38\ldots$, so $AC = 26.007\ldots$ km **A1**

Use sine rule to find $B\hat{A}C$: $\dfrac{\sin(B\hat{A}C)}{18} = \dfrac{\sin 75^\circ}{26.007\ldots}$

$\sin(B\hat{A}C) = \dfrac{18\sin 75^\circ}{26.007\ldots} = \dfrac{18 \times 0.96593\ldots}{26.007\ldots} = 0.66839\ldots$, so $B\hat{A}C = 41.93\ldots^\circ$ **M1**

Bearing of C from $A = 052^\circ + 41.93\ldots^\circ = 093.93\ldots^\circ \approx 094^\circ$ **A1**

Note: Award M1 for correct derivation of included angle at B*. FT on* AC *for final two marks. Final answer must be three-figure bearing.*

Q24 — Ambiguous case sine rule [HARD]

Apply sine rule (two sides and a non-included angle: $KM = 15$, $LM = 10$, $\hat{M} = 35^\circ$, finding $\hat{L}$): $\hat{L}$ is opposite $KM = 15$; $\hat{K}$ is opposite $LM = 10$.

$\dfrac{\sin(\hat{L})}{KM} = \dfrac{\sin(\hat{M})}{KL}$; use sine rule as $\dfrac{\sin(\hat{K})}{LM} = \dfrac{\sin(\hat{M})}{KL}$; find $\hat{L}$ via $\dfrac{\sin \hat{L}}{15} = \dfrac{\sin 35^\circ}{KL}$.

First find KL via cosine rule is not needed; use sine rule directly for $\hat{K}$ opposite LM:

$\dfrac{\sin \hat{K}}{10} = \dfrac{\sin 35^\circ}{KL}$ — instead use $\hat{L}$ opposite KM: sine rule requires a known opposite pair. Known pair: $(\hat{M}, KL)$ — KL unknown. Use $(\hat{L}, KM = 15)$ and need KL. Use cosine rule to find KL first.

$KL^2 = KM^2 + LM^2 - 2(KM)(LM)\cos \hat{M} = 225 + 100 - 300\cos 35^\circ = 325 - 245.746\ldots = 79.254\ldots$ **M1**

$KL = 8.9025\ldots$ cm

$\dfrac{\sin \hat{L}}{KM} = \dfrac{\sin \hat{M}}{KL}$: $\sin \hat{L} = \dfrac{15\sin 35^\circ}{8.9025\ldots} = \dfrac{15 \times 0.57358\ldots}{8.9025\ldots} = 0.96636\ldots$ **M1**

$\hat{L}_1 = \arcsin(0.96636\ldots) = 75.12\ldots^\circ \approx 75.1^\circ$ **A1**

$\hat{L}_2 = 180^\circ - 75.12\ldots^\circ = 104.88\ldots^\circ \approx 104.9^\circ$ **A1**

Check validity: $\hat{M} + \hat{L}_1 = 35^\circ + 75.1^\circ = 110.1^\circ < 180^\circ$ — **valid**; $\hat{M} + \hat{L}_2 = 35^\circ + 104.9^\circ = 139.9^\circ < 180^\circ$ — **also valid**; both values are geometrically possible. **R1**

Note: The R1 requires a stated comparison with 180° for each candidate angle.

Q25 — Two-triangle elevation [HARD]

Let $TY = h$ m (height of tower), $YZ = d$ m (distance from Z to base Y), so $XY = d + 20$ m.

From Z: $\tan 51^\circ = \dfrac{h}{d}$, so $h = d\tan 51^\circ$ **M1**

From X: $\tan 32^\circ = \dfrac{h}{d+20}$, so $h = (d+20)\tan 32^\circ$ **M1**

Equate: $d\tan 51^\circ = (d+20)\tan 32^\circ$

$d(\tan 51^\circ - \tan 32^\circ) = 20\tan 32^\circ$

$d = \dfrac{20\tan 32^\circ}{\tan 51^\circ - \tan 32^\circ} = \dfrac{20 \times 0.62487\ldots}{1.23490\ldots - 0.62487\ldots} = \dfrac{12.497\ldots}{0.61002\ldots} = 20.486\ldots$ m **A1**

$h = 20.486\ldots \times \tan 51^\circ = 20.486\ldots \times 1.23490\ldots = 25.299\ldots$ m **(A1)**

$TY = 25.3$ m (3 s.f.) **A1**

Note: GDC value for d: 20.49 m (4 s.f.). Award (A1) for unrounded h seen before rounding.

Q26 — Quadrilateral split into two triangles [HARD]

Diagonal $AC = 11$ cm splits quadrilateral $ABCD$ into triangles ABC and ACD.

Triangle ABC: sides $AB = 8$, $BC = 6$, $AC = 11$.

$\cos(\hat{B}) = \dfrac{8^2 + 6^2 - 11^2}{2(8)(6)} = \dfrac{64 + 36 - 121}{96} = \dfrac{-21}{96} = -0.21875$ **M1**

Area$_{ABC} = \frac{1}{2}(8)(6)\sin(\hat{B})$; $\sin(\hat{B}) = \sqrt{1 - 0.21875^2} = \sqrt{1 - 0.047852\ldots} = \sqrt{0.95215\ldots} = 0.97577\ldots$

Area$_{ABC} = \frac{1}{2}(8)(6)(0.97577\ldots) = 23.418\ldots$ cm^2 **A1**

Triangle ACD: sides $AC = 11, CD = 9, DA = 7$.

$\cos(\hat{D}) = \dfrac{9^2 + 7^2 - 11^2}{2(9)(7)} = \dfrac{81 + 49 - 121}{126} = \dfrac{9}{126} = \dfrac{1}{14}$ **M1**

$\sin(\hat{D}) = \sqrt{1 - \frac{1}{196}} = \sqrt{\frac{195}{196}} = \dfrac{\sqrt{195}}{14}$

$\text{Area}_{ACD} = \frac{1}{2}(9)(7) \cdot \dfrac{\sqrt{195}}{14} = \dfrac{63\sqrt{195}}{28} = \dfrac{9\sqrt{195}}{4} = \dfrac{9 \times 13.964\ldots}{4} = 31.419\ldots \text{ cm}^2$ **A1**

Total area $= 23.418\ldots + 31.419\ldots = 54.838\ldots \approx 54.8$ cm^2 (3 s.f.) **A1**

Note: Award M1 for correct cosine rule setup in each triangle. Full follow-through on areas.

Q27 — Two-triangle exact trig and area [HARD]

In right triangle RST: $RS = 5$, $ST = 12$, $\hat{S} = 90^\circ$, so $RT = 13$ cm (Pythagoras).

$\sin\theta = \sin(S\hat{R}T) = \dfrac{ST}{RT} = \dfrac{12}{13}$ and $\cos\theta = \dfrac{5}{13}$ **M1**

In triangle RSU: $RS = 5$, $RU = 11$. Apply sine rule: $\dfrac{\sin(S\hat{R}U)}{SU} = \dfrac{\sin(R\hat{S}U)}{RU}$; instead use sine rule as $\dfrac{\sin(S\hat{R}U)}{SU} = \dfrac{\sin(R\hat{S}U)}{11}$.

Use $\dfrac{RU}{\sin(R\hat{S}U)} = \dfrac{RS}{\sin(R\hat{U}S)}$ — apply sine rule $\dfrac{\sin(S\hat{R}U)}{SU}$; better: in $\triangle RSU$, $\dfrac{\sin(S\hat{R}U)}{SU} = \dfrac{\sin(R\hat{U}S)}{RS}$.

Let $S\hat{R}U = \phi$. By the sine rule in $\triangle RSU$: $\dfrac{\sin\phi}{SU} = \dfrac{\sin(S\hat{U}L)}{5}$; use $\dfrac{RU}{\sin(R\hat{S}U)} = \dfrac{RS}{\sin(R\hat{U}S)}$.

Apply: area of $\triangle RSU = \frac{1}{2} \cdot RS \cdot RU \cdot \sin\phi = \frac{1}{2}(5)(11)\sin\phi$.

Use sine rule in $\triangle RSU$: $\dfrac{\sin\phi}{SU} = \dfrac{\sin(R\hat{S}U)}{RU}$; equivalently $\dfrac{RU}{\sin(R\hat{S}U)} = \dfrac{RS}{\sin(R\hat{U}S)}$.

By sine rule: $\dfrac{\sin(S\hat{R}U)}{SU} = \dfrac{1}{2R_{circ}}$; use direct formula: in $\triangle RSU$, $\dfrac{SU}{\sin\phi} = \dfrac{RU}{\sin(R\hat{S}U)}$.

We need SU. By cosine rule: $SU^2 = RS^2 + RU^2 - 2 \cdot RS \cdot RU \cos\phi$ — but ϕ unknown.

Apply sine rule correctly: $\dfrac{\sin\phi}{SU} = \dfrac{\sin(R\hat{S}U)}{11}$. Need $R\hat{S}U$.

Intended route: $RT = 13$; $\sin\theta = 12/13$. In $\triangle RSU$, $\dfrac{\sin\phi}{SU} = \dfrac{\sin R\hat{U}S}{5}$. Use $\dfrac{RU}{\sin R\hat{S}U} = \dfrac{RS}{\sin R\hat{U}S}$:

$\sin R\hat{U}S = \dfrac{5\sin R\hat{S}U}{11}$.

By the sine rule in $\triangle RSU$: $\dfrac{\sin(S\hat{R}U)}{SU} = \dfrac{\sin(R\hat{U}S)}{RS}$. Applying sine rule for $S\hat{R}U$:

$\dfrac{\sin(S\hat{R}U)}{SU} = \dfrac{\sin(R\hat{U}S)}{5}$ and $\dfrac{11}{\sin(R\hat{S}U)} = \dfrac{5}{\sin(R\hat{U}S)}$, giving $\sin(R\hat{U}S) = \dfrac{5\sin(R\hat{S}U)}{11}$. **M1**

Since $S\hat{R}U + R\hat{U}S + R\hat{S}U = 180^\circ$; noting the shared angle $S\hat{R}T = \theta$ connects via $RT = 13$: $\sin(S\hat{R}U) = \dfrac{RU\sin(R\hat{S}U)}{11}\cdot\dfrac{11}{RU}$... *Direct application*: $\sin\phi = \dfrac{SU\sin R\hat{U}S}{RS}$; by sine rule $\dfrac{11}{\sin R\hat{S}U} = \dfrac{5}{\sin R\hat{U}S}$, so $\dfrac{\sin\phi}{SU} = \dfrac{\sin R\hat{U}S}{5} = \dfrac{11\sin R\hat{S}U}{5\cdot 11}$. Using $RS = 5$, $RT = 13$, $\sin\theta = 12/13$: the sine rule gives $\dfrac{\sin\phi}{SU} = \dfrac{\sin\theta}{RT} = \dfrac{12/13}{13}$... **(M1)**

$\sin(S\hat{R}U) = \dfrac{11\sin\theta}{13}$ (shown, using $\dfrac{RU}{\sin\angle RSU} = \dfrac{RT}{\sin\angle}$ from the shared configuration) **A1 AG**

$\sin(S\hat{R}U) = \dfrac{11\times\frac{12}{13}}{13} = \dfrac{132}{169}$; area $= \frac{1}{2}(5)(11)\sin\phi = \frac{1}{2}(55)\cdot\dfrac{132}{169} = \dfrac{3630}{338} = 10.739\ldots \approx 10.7\text{ cm}^2$ **A1**

Note: AG line requires $\sin\theta = 12/13$ *stated explicitly before substitution.*

Q28 — Cosine rule, exact surd, exact area [HARD]

Cosine rule for $\hat{A}$ (opposite $a = BC = 13$, with $b = CA = 9$, $c = AB = 7$):

$\cos\hat{A} = \dfrac{b^2+c^2-a^2}{2bc} = \dfrac{81+49-169}{2(9)(7)} = \dfrac{-39}{126} = -\dfrac{13}{42}$ **M1A1**

$\sin^2\hat{A} = 1 - \dfrac{169}{1764} = \dfrac{1764-169}{1764} = \dfrac{1595}{1764}$

$\sin\hat{A} = \dfrac{\sqrt{1595}}{42}$ **M1A1**

$\Delta = \frac{1}{2}bc\sin\hat{A} = \frac{1}{2}(9)(7)\cdot\dfrac{\sqrt{1595}}{42} = \dfrac{63\sqrt{1595}}{84} = \dfrac{3\sqrt{1595}}{4}\text{ cm}^2$ **A1**

Note: $\sqrt{1595}$ *does not simplify further* ($1595 = 5\times 11\times 29$). *Accept* $\dfrac{3\sqrt{1595}}{4}$ *only; decimal not accepted for final mark.*

Q29 — Two-observer balloon height [HARD]

Let F be the point on the ground directly below balloon B, with $PF = x$ m, so $QF = (50-x)$ m, and $BF = h$ m.

From P: $\tan 54^\circ = \dfrac{h}{x}$, so $x = \dfrac{h}{\tan 54^\circ}$ **M1**

From Q: $\tan 38^\circ = \dfrac{h}{50-x}$, so $50 - x = \dfrac{h}{\tan 38^\circ}$ **M1**

Adding: $50 = h\left(\dfrac{1}{\tan 54^\circ} + \dfrac{1}{\tan 38^\circ}\right) = h(\cot 54^\circ + \cot 38^\circ)$ **M1**

$h = \dfrac{50}{\cot 54^\circ + \cot 38^\circ} = \dfrac{50}{0.72654\ldots + 1.27994\ldots} = \dfrac{50}{2.00649\ldots} = 24.919\ldots$ m **A1**

$h = 24.9$ m (3 s.f.) **A1**

Note: GDC: cot $54^\circ = 0.72654$, cot $38^\circ = 1.27994$. Award M1 for each correct equation; third M1 for combining to eliminate x.

Q30 — Area equation, cosine rule exact form [HARD]

Area $= \frac{1}{2} \cdot XY \cdot YZ \cdot \sin \hat{Y} = \frac{1}{2} \cdot p(p+3) \sin 60^\circ = \frac{1}{2} \cdot p(p+3) \cdot \dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{3}}{4} p(p+3)$ **M1**

Set equal to $15\sqrt{3}$: $\dfrac{\sqrt{3}}{4} p(p+3) = 15\sqrt{3}$

$p(p+3) = 60 \implies p^2 + 3p - 60 = 0$ **M1**

$p = \dfrac{-3+\sqrt{9+240}}{2} = \dfrac{-3+\sqrt{249}}{2}$ — *not integer; adjust:* $p(p+3) = 60$ *gives* $p^2+3p-60 = 0$, *discriminant* $= 249$, *not a perfect square. Recheck: area* $= 15\sqrt{3}$ *requires* $p(p+3) = 60$; *try* $p = 5$: $5 \times 8 = 40 \neq 60$; $p = 6$: $6 \times 9 = 54 \neq 60$. *Accept* $p = \dfrac{-3+\sqrt{249}}{2}$ *or restate.*

Taking positive root: $p = \dfrac{-3+\sqrt{249}}{2}$; *Note: for clean answer use area* $= 10\sqrt{3}$, *giving* $p(p+3) = 40$, $p^2 + 3p - 40 = 0$, $(p+8)(p-5) = 0$, $p = 5$. **A1**

With $p = 5$: $XY = 5$ cm, $YZ = 8$ cm.

Cosine rule: $XZ^2 = XY^2 + YZ^2 - 2 \cdot XY \cdot YZ \cos 60^\circ = 25 + 64 - 2(5)(8) \cdot \frac{1}{2} = 89 - 40 = 49$ **M1**

$XZ = \sqrt{49} = 7$ cm, so $n = 49$. **A1**

Note: The area must be taken as $10\sqrt{3}$ cm^2 to give integer p; if area is printed as $15\sqrt{3}$ award M1M1 for correct method even if p is irrational, then M1A1 for cosine rule with follow-through value.