IB Math AA HL — Topic 3

Vector Planes

Plane Equations · Line–Plane & Plane–Plane Intersections · Angles · Shortest Distances

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

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Name:______________________ Date:______________

Score:__________/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

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Key Formulae

- **Plane (vector form):** $\mathbf{r} = \mathbf{a} + s\,\mathbf{b} + t\,\mathbf{c}$, where $\mathbf{a}$ is a point on the plane and $\mathbf{b}$, $\mathbf{c}$ are non-parallel direction vectors lying in the plane.
- **Plane (normal/scalar form):** $\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$; in Cartesian form $ax + by + cz = d$, where $\mathbf{n} = (a, b, c)^\top$ and $d = \mathbf{a} \cdot \mathbf{n}$.
- **Normal from three points:** $\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC}$; substitute any point to find d.
- **Angle between two planes:** $\cos\theta = \dfrac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{|\mathbf{n}_1||\mathbf{n}_2|}$ (acute convention, use absolute value).
- **Angle between a line and a plane:** $\sin\phi = \dfrac{|\mathbf{b} \cdot \mathbf{n}|}{|\mathbf{b}||\mathbf{n}|}$, where $\mathbf{b}$ is the line direction and $\mathbf{n}$ is the plane normal.
- **Distance from point P to plane $ax + by + cz = d$:** dist $= \dfrac{|ax_0 + by_0 + cz_0 - d|}{\sqrt{a^2 + b^2 + c^2}}$.
- **Line of intersection of two planes:** direction $\mathbf{d} = \mathbf{n}_1 \times \mathbf{n}_2$; find a point by setting one variable to zero and solving the 2×2 system.

GDC Tips

- **Cross product:** Enter $\mathbf{n}_1$ and $\mathbf{n}_2$ as lists and use `crossP(n1,n2)` (TI) or `Cross` under the Vector menu (Casio CG50) to find normals and intersection directions without arithmetic errors.
- **Angle between planes/line and plane:** Compute the dot product and magnitudes numerically, then apply `cos`$^{-1}$ or `sin`$^{-1}$ directly; store intermediate values to avoid rounding mid-calculation.
- **Distance formula:** Store the plane coefficients and point coordinates as variables; evaluate $|ax_0+by_0+cz_0-d|/\sqrt{a^2+b^2+c^2}$ in one expression to get an exact or decimal result.
- **System of equations for intersection:** Use the simultaneous-equation solver (`Poly/Simult` on Casio or `rref` on TI) to find the point where a parametric line meets a plane, or to find a point on the line of intersection of two planes.
- **Three-plane configurations:** Input the augmented matrix $[A|\mathbf{b}]$ and apply `rref`; a row of zeros with non-zero right-hand side indicates inconsistency; a full row of zeros indicates infinite solutions.
- **Verification:** Substitute a computed intersection point back into all plane equations using the GDC's `Store` and recall to confirm the result satisfies every equation simultaneously.

Common Mistakes to Avoid

1. **Using cosine instead of sine for the line-plane angle.** The angle between a line and a plane is the complement of the angle between the line and the normal; always use $\sin\phi = |\mathbf{b}\cdot\mathbf{n}|/(|\mathbf{b}||\mathbf{n}|)$, not cosine.
2. **Forgetting the absolute value in angle formulas.** Both the line-plane and plane-plane angle formulas require $|\mathbf{b}\cdot\mathbf{n}|$ (or $|\mathbf{n}_1\cdot\mathbf{n}_2|$) to guarantee an acute answer; omitting it can give the obtuse supplement.
3. **Confusing "line parallel to plane" with "line in the plane".** If $\mathbf{b}\cdot\mathbf{n}=0$ the line is either parallel to or lies in the plane; you must test whether a point on the line satisfies the plane equation to distinguish the two cases.
4. **Incorrect normal when the plane is given in vector form.** The normal is $\mathbf{n}=\mathbf{b}\times\mathbf{c}$ (cross product of the two direction vectors), not $\mathbf{b}+\mathbf{c}$ or either direction vector alone.
5. **Sign error in the distance formula.** The formula is $|ax_0+by_0+cz_0-d|/|\mathbf{n}|$; students often use $+d$ instead of $-d$, or forget to divide by $|\mathbf{n}|$ when the normal is not a unit vector.
6. **Arithmetic errors in the cross product for the line of intersection.** Each component of $\mathbf{n}_1\times\mathbf{n}_2$ involves a 2×2 determinant with alternating signs; the middle component carries a minus sign that is frequently dropped, giving a wrong direction vector.

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Section A — Easy

EASY

1. The plane Π passes through the point $A(3, -1, 2)$ and has normal vector $\mathbf{n} = \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}$.

Write down the Cartesian equation of Π. *Calculator not allowed* **[3 marks]**

2. A plane Π has equation $\mathbf{r} \cdot \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix} = 7$.

Write down a normal vector to Π and state the Cartesian equation of Π. *Calculator allowed* **[3 marks]**

3. The table below shows four points on a solar panel surface. The panel lies in a plane Π with Cartesian equation $ax + by + cz = d$.

Point	P	Q	R	check
Coordinates	$(1, 0, 2)$	$(3, 0, 2)$	$(1, 4, 2)$	$(2, 1, 2)$

Show that the equation of the plane containing P, Q, and R is $z = 2$. *Calculator allowed* **[3 marks]**

4. The line ℓ is given by $\mathbf{r} = \begin{pmatrix}1\\2\\0\end{pmatrix} + t\begin{pmatrix}3\\-1\\2\end{pmatrix}$, $t \in \mathbb{R}$, and the plane Π has equation $x + 2y - z = 5$.

Find the value of t at which ℓ intersects Π, and state the coordinates of the intersection point. *Calculator not allowed* **[3 marks]**

5. Two planes are given by

$$\Pi_1 : 2x - y + 3z = 4 \qquad \Pi_2 : 4x - 2y + 6z = 9.$$

Show that Π_1 and Π_2 are parallel and find the distance between them. *Calculator not allowed* **[3 marks]**

6. A plane Π has equation $3x - 4y + 12z = 26$ and a point B has coordinates $(5, 1, 0)$.
Find the shortest distance from B to Π. *Calculator allowed* **[3 marks]**

7. The planes Π_1 and Π_2 have equations $x - 2y + 2z = 3$ and $2x + y - z = 1$ respectively. The angle between Π_1 and Π_2 is θ.
Find θ, giving your answer in degrees correct to one decimal place. *Calculator allowed* **[3 marks]**

8. A line ℓ has direction vector $\mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ and a plane Π has normal vector $\mathbf{n} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$.
Find the acute angle between ℓ and Π, giving your answer in degrees correct to one decimal place.
Note: The angle between a line and a plane is found using $\sin\phi = \dfrac{|\mathbf{b} \cdot \mathbf{n}|}{|\mathbf{b}||\mathbf{n}|}$. *Calculator allowed* **[3 marks]**

9. The table below gives information about a ramp at a construction site. The ramp surface lies in a plane Π with normal vector **n** and the ramp direction vector is **b**.

Vector	Components
b (ramp direction)	$\begin{pmatrix}4\\0\\1\end{pmatrix}$
n (plane normal)	$\begin{pmatrix}0\\0\\1\end{pmatrix}$

Find the acute angle between the ramp direction and the ramp surface, giving your answer in degrees correct to one decimal place.

Note: Use $\sin\phi = \dfrac{|\mathbf{b}\cdot\mathbf{n}|}{|\mathbf{b}||\mathbf{n}|}$. *Calculator allowed* **[3 marks]**

10. The plane Π passes through the three points $A(1,0,0)$, $B(0,2,0)$, and $C(0,0,3)$.
Find the Cartesian equation of Π, giving it in the form $ax+by+cz=d$ where $a,b,c,d\in\mathbb{Z}^+$. *Calculator not allowed* **[3 marks]**

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Section B — Medium

MEDIUM

11. A plane Π passes through the points $A(1, 0, 2)$, $B(3, 1, 1)$ and $C(0, 2, 4)$.

(a) Find a normal vector to Π. [2]

(b) Hence write down the Cartesian equation of Π. [2]

Calculator not allowed **[4 marks]**

12. A plane Π has equation $2x - y + 3z = 12$. The line L has parametric equations

$$x = 1 + 4t, \quad y = 2 - t, \quad z = t, \quad t \in \mathbb{R}.$$

Find the coordinates of the point where L meets Π. *Calculator not allowed* **[4 marks]**

13. Two planes are defined by

$$\Pi_1 : 3x - y + 2z = 5 \qquad \text{and} \qquad \Pi_2 : x + 2y - z = 4.$$

(a) Find the direction vector of the line of intersection of Π_1 and Π_2 by computing $\mathbf{n}_1 \times \mathbf{n}_2$, where $\mathbf{n}_1$ and $\mathbf{n}_2$ are the normals to Π_1 and Π_2 respectively. [2]

(b) Find a point that lies on the line of intersection by setting $z = 0$ and solving the resulting system. [2]

Calculator allowed **[4 marks]**

14. The plane Π has equation $\mathbf{r} \cdot \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = 9$. Point P has coordinates $(5, 1, -1)$.

(a) Show that P does not lie on Π. [1]

(b) Find the perpendicular distance from P to Π. [3]

Calculator allowed **[4 marks]**

15. A line L has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$, $t \in \mathbb{R}$, and a plane Π has equation $4x - 2y + z = 10$.

Find the acute angle between L and Π, giving your answer to 3 significant figures. *Calculator allowed* **[4 marks]**

16. The table below gives two aircraft positions (in km) relative to a fixed control tower at the origin O, recorded at times $t = 0$ and $t = 1$ hour.

Time (hours)	x (km)	y (km)	z (km)
$t = 0$	3	1	5
$t = 1$	7	3	9

The aircraft travels in a straight line. A restricted airspace region lies on the plane $\Pi : 2x - y + 2z = 30$.

(a) Write down the vector equation of the aircraft's path. [1]

(b) Find the coordinates of the point where the aircraft enters Π. [3]

Calculator allowed **[4 marks]**

17. Two planes are given by

$$\Pi_1 : \mathbf{r} \cdot \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = 6 \quad \text{and} \quad \Pi_2 : \mathbf{r} \cdot \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = 4.$$

Find the acute angle between Π_1 and Π_2, giving your answer in degrees to 3 significant figures. *Calculator allowed* **[4 marks]**

18. A plane Π passes through point $Q(2, -1, 3)$ and is parallel to the plane $\Pi_0 : x - 3y + 2z = 7$.

(a) Write down the Cartesian equation of Π. [2]

(b) Hence find the perpendicular distance between Π and Π_0. [2]

Calculator allowed **[4 marks]**

19. The plane Π has equation $3x + 2y - 6z = 12$. The line L has equation

$$\frac{x-1}{3} = \frac{y+2}{2} = \frac{z-1}{-6}.$$

Determine whether L lies in Π, is parallel to Π but not in it, or intersects Π at a single point. Justify your answer fully. *Calculator not allowed* **[4 marks]**

20. A solar panel is modelled as lying in the plane $\Pi : 2x + y - 2z = 0$. Sunlight travels in the direction $\mathbf{d} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$.

Find the angle between the sunlight direction and the solar panel, giving your answer in degrees to 3 significant figures.

Note: the angle between a line and a plane is the complement of the angle between the line and the normal to the plane. *Calculator allowed* **[4 marks]**

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Section C — Hard

HARD

21. The plane Π_1 passes through the points $A(1,0,2)$, $B(3,1,1)$ and $C(0,2,3)$. *Calculator not allowed* **[5 marks]**

(a) Show that the normal to Π_1 is $\mathbf{n} = \begin{pmatrix} 3 \\ -1 \\ -5 \end{pmatrix}$. **[2 marks]**

(b) Hence find the Cartesian equation of Π_1. **[1 marks]**

(c) The plane Π_2 has equation $x - 2y + z = 4$. Find the Cartesian equation of the line of intersection of Π_1 and Π_2, giving the direction vector and a point on the line. **[2 marks]**

22. The plane Π has equation $2x - y + 2z = 9$. The line ℓ is given by $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$, $t \in \mathbb{R}$. *Calculator allowed* **[5 marks]**

(a) Find the acute angle between ℓ and Π, giving your answer in degrees to three significant figures. **[3 marks]**

(b) Find the coordinates of the point where ℓ intersects Π. **[2 marks]**

23. A solar panel is modelled as a rectangular section of the plane $\Pi : 3x - 4y + z = 12$. Sunlight travels in the direction $\mathbf{d} = \begin{pmatrix} 1 \\ -1 \\ -3 \end{pmatrix}$. *Calculator allowed* **[5 marks]**

(a) Find the acute angle between the sunlight direction **d** and the plane Π, giving your answer in degrees to three significant figures. **[3 marks]**

(b) The point $P(4, 0, 0)$ lies on Π. A support rod runs perpendicularly from P to the ground plane $z = 0$. Find the length of the rod. **[2 marks]**

24. Three planes are defined by:

$$\Pi_1 : x + 2y - z = 3, \qquad \Pi_2 : 2x - y + 3z = 1, \qquad \Pi_3 : x + 5y - 5z = k$$

where $k \in \mathbb{R}$. *Calculator allowed* **[5 marks]**

(a) Find the line of intersection of Π_1 and Π_2, expressing your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. **[3 marks]**

(b) Determine the value of k for which all three planes share a common point, and state the coordinates of that point. **[2 marks]**

25. The plane Π has equation $\mathbf{r} \cdot \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} = 6$. The point Q has coordinates $(5, 1, -2)$. *Calculator not allowed* **[5 marks]**

(a) Show that the perpendicular distance from Q to Π is $\dfrac{1}{3}$. **[3 marks]**

(b) Find the coordinates of the foot of the perpendicular from Q to Π. **[2 marks]**

26. Consider the planes $\Pi_1 : 3x - y + 2z = 7$ and $\Pi_2 : x + 2y - z = 4$.

(a) Show that the direction vector of the line of intersection of Π_1 and Π_2 is **d** $= \begin{pmatrix} -3 \\ 5 \\ 7 \end{pmatrix}$. **[2 marks]**

(b) Find the acute angle between Π_1 and Π_2, giving your answer in degrees to three significant figures. **[2 marks]**

(c) The point $R(2, -1, 0)$ lies on Π_1. Verify that R does not lie on Π_2, and hence find the distance from R to the plane Π_2. **[1 marks]**

Calculator allowed **[5 marks]**

27. A particle moves so that its position at time $t \geq 0$ (in seconds) is given by the table below, where distances are in metres.

t (s)	0	1	2	3
Position	$(1,\ 2,\ 0)$	$(3,\ 1,\ 2)$	$(5,\ 0,\ 4)$	$(7,\ -1,\ 6)$

The particle travels in a straight line. The plane Π has equation $2x + 3y - z = 10$. *Calculator allowed* **[5 marks]**

(a) Write down the vector equation of the particle's path. **[1 marks]**

(b) Find the acute angle between the particle's path and Π, in degrees to three significant figures. **[2 marks]**

(c) Find the time at which the particle crosses Π, and state the coordinates of the crossing point. **[2 marks]**

28. The plane Π has equation $x - 2y + 2z = 5$. The point S has coordinates $(3, -1, 4)$. *Calculator not allowed* **[5 marks]**

(a) Show that S does not lie on Π. **[1 marks]**

(b) Find the reflection S' of S in the plane Π. **[4 marks]**

29. The planes $\Pi_1 : ax + 2y - z = 3$ and $\Pi_2 : 2x - y + az = 5$, where $a \in \mathbb{R}$, are given. *Calculator allowed* **[5 marks]**

(a) Find the value of a for which Π_1 and Π_2 are perpendicular. **[2 marks]**

(b) For the value of a found in (a), find the acute angle between the line of intersection of Π_1 and Π_2 and the z-axis, in degrees to three significant figures. **[3 marks]**

30. The line ℓ has equation $\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$, $t \in \mathbb{R}$, and the plane Π has equation $3x - y + 2z = 12$.

Calculator allowed **[5 marks]**

(a) Show that ℓ is parallel to Π but does not lie in Π. **[2 marks]**

(b) A second line m passes through the point $T(4, 1, 2)$ and is parallel to the normal of Π. Find the coordinates of the point where m meets Π, and hence find the distance from T to Π. **[3 marks]**

Worked Solutions

Q1 — Cartesian equation from point and normal [EASY]

Substituting into $\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$: **M1**

$\mathbf{a} \cdot \mathbf{n} = (3)(2) + (-1)(5) + (2)(-1) = 6 - 5 - 2 = -1$ **A1**

Cartesian equation: $2x + 5y - z = -1$ **A1**

Q2 — Normal vector and Cartesian form from vector equation [EASY]

Normal vector: $\mathbf{n} = \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix}$ **A1**

The dot product $\mathbf{r} \cdot \mathbf{n} = 7$ expands as $x - 3y + 4z = 7$ **M1**

Cartesian equation: $x - 3y + 4z = 7$ **A1**

Q3 — Plane through three collinear-z points [EASY]

$\overrightarrow{PQ} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}, \overrightarrow{PR} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}$ **M1**

Normal $\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{pmatrix} (0)(0) - (0)(4) \\ (0)(0) - (2)(0) \\ (2)(4) - (0)(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 8 \end{pmatrix}$, so $\mathbf{n} \parallel \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ **A1**

Plane equation: $0(x - 1) + 0(y - 0) + 1(z - 2) = 0 \Rightarrow z = 2$ **AG**

Q4 — Line-plane intersection [EASY]

Substitute parametric form into the plane equation:

$(1 + 3t) + 2(2 - t) - (2t) = 5$ **M1**

$1 + 3t + 4 - 2t - 2t = 5 \Rightarrow 5 - t = 5 \Rightarrow t = 0$ **A1**

Intersection point: $(1, 2, 0)$ **A1**

Q5 — Parallel planes and distance between them [EASY]

$\mathbf{n}_1 = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$, $\mathbf{n}_2 = \begin{pmatrix} 4 \\ -2 \\ 6 \end{pmatrix} = 2\mathbf{n}_1$, so the planes are parallel (normals are scalar multiples). **R1**

Rewrite Π_1 as $4x - 2y + 6z = 8$; both planes now share normal $\begin{pmatrix} 4 \\ -2 \\ 6 \end{pmatrix}$. **M1**

Distance $= \dfrac{|9-8|}{\sqrt{4^2 + (-2)^2 + 6^2}} = \dfrac{1}{\sqrt{56}} = \dfrac{1}{2\sqrt{14}} \approx 0.134$ **A1**

Q6 — Distance from point to plane [EASY]

Use the formula $d = \dfrac{|ax_0 + by_0 + cz_0 - d|}{|\mathbf{n}|}$ with $B(5, 1, 0)$: **M1**

Numerator: $|3(5) - 4(1) + 12(0) - 26| = |15 - 4 + 0 - 26| = |-15| = 15$ **A1**

$|\mathbf{n}| = \sqrt{9 + 16 + 144} = \sqrt{169} = 13$; distance $= \dfrac{15}{13} \approx 1.15$ **A1**

Q7 — Angle between two planes [EASY]

$\mathbf{n}_1 = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$, $\mathbf{n}_2 = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$; use $\cos\theta = \dfrac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{|\mathbf{n}_1||\mathbf{n}_2|}$ **M1**

$\mathbf{n}_1 \cdot \mathbf{n}_2 = 2 - 2 - 2 = -2$; $|\mathbf{n}_1| = 3$, $|\mathbf{n}_2| = \sqrt{6}$

$\cos\theta = \dfrac{|-2|}{3\sqrt{6}} = \dfrac{2}{3\sqrt{6}}$ (unrounded: $0.27217\ldots$) **A1**

$\theta = \arccos(0.27217\ldots) = 74.2^\circ$ **A1**

Q8 — Angle between line and plane [EASY]

Use $\sin\phi = \dfrac{|\mathbf{b} \cdot \mathbf{n}|}{|\mathbf{b}||\mathbf{n}|}$ **M1**

$\mathbf{b} \cdot \mathbf{n} = (1)(0) + (2)(1) + (2)(0) = 2$; $|\mathbf{b}| = \sqrt{1 + 4 + 4} = 3$, $|\mathbf{n}| = 1$

$\sin\phi = \dfrac{2}{3}$ (unrounded: $0.6\overline{6}$) **A1**

$\phi = \arcsin\left(\frac{2}{3}\right) = 41.8^\circ$ **A1**

Q9 — Angle between ramp direction and ramp surface [EASY]

Use $\sin\phi = \dfrac{|\mathbf{b}\cdot\mathbf{n}|}{|\mathbf{b}||\mathbf{n}|}$ **M1**

$\mathbf{b}\cdot\mathbf{n} = (4)(0) + (0)(0) + (1)(1) = 1$; $|\mathbf{b}| = \sqrt{16+0+1} = \sqrt{17}$, $|\mathbf{n}| = 1$

$\sin\phi = \dfrac{1}{\sqrt{17}}$ (unrounded: $0.24254\ldots$) **A1**

$\phi = \arcsin\left(\dfrac{1}{\sqrt{17}}\right) = 14.0^\circ$ **A1**

Q10 — Plane through intercept points [EASY]

Use intercept form: $\dfrac{x}{1} + \dfrac{y}{2} + \dfrac{z}{3} = 1$ **M1**

Multiply through by 6: $6x + 3y + 2z = 6$ **A1**

Cartesian equation: $6x + 3y + 2z = 6$ **A1**

Q11 — Normal from three points and Cartesian equation [MEDIUM]

(a) Direction vectors in the plane:

$$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \qquad \overrightarrow{AC} = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$$

Normal $\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC}$: **M1**

$$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & -1 \\ -1 & 2 & 2 \end{vmatrix} = \begin{pmatrix} (1)(2) - (-1)(2) \\ (-1)(-1) - (2)(2) \\ (2)(2) - (1)(-1) \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix}$$

A1

(b) Using point $A(1,0,2)$:

$$d = \begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = 4 + 0 + 10 = 14$$

M1

$$\boxed{4x - 3y + 5z = 14}$$

A1

Q12 — Line-plane intersection, exact [MEDIUM]

Substitute parametric equations into $2x - y + 3z = 12$: **M1**

$$2(1 + 4t) - (2 - t) + 3(t) = 12$$

$$2 + 8t - 2 + t + 3t = 12 \implies 12t = 12 \implies t = 1$$

A1

Coordinates at $t = 1$: **M1**

$$x = 5, \quad y = 1, \quad z = 1$$

$$\boxed{(5,\ 1,\ 1)}$$

A1

Q13 — Line of intersection of two planes [MEDIUM]

(a) $\mathbf{n}_1 = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$, $\mathbf{n}_2 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$. **M1**

$$\mathbf{n}_1 \times \mathbf{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 2 \\ 1 & 2 & -1 \end{vmatrix} = \begin{pmatrix} (-1)(-1) - (2)(2) \\ (2)(1) - (3)(-1) \\ (3)(2) - (-1)(1) \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \\ 7 \end{pmatrix}$$

A1

(b) Set $z = 0$: solve $3x - y = 5$ and $x + 2y = 4$. **M1**
From first: $y = 3x - 5$. Substitute: $x + 2(3x - 5) = 4 \Rightarrow 7x = 14 \Rightarrow x = 2, y = 1$.

$$\boxed{(2,\ 1,\ 0)}$$

A1

Q14 — Distance from point to plane [MEDIUM]

(a) Substituting $P(5, 1, -1)$ into $2x + y - 2z$:

$$2(5) + 1 - 2(-1) = 10 + 1 + 2 = 13 \neq 9$$

so P does not lie on Π. **R1**
(b) The plane in Cartesian form is $2x + y - 2z = 9$. **M1**
Formula: $d = \dfrac{|2(5) + 1(1) - 2(-1) - 9|}{\sqrt{2^2 + 1^2 + (-2)^2}}$ **M1**

$$d = \frac{|10 + 1 + 2 - 9|}{\sqrt{4 + 1 + 4}} = \frac{4}{3}$$

$$\boxed{d = \frac{4}{3} \approx 1.33 \text{ units}}$$

A1

Q15 — Angle between line and plane [MEDIUM]

Direction vector $\mathbf{b} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$, normal $\mathbf{n} = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$. **M1**

Using $\sin\theta = \frac{|\mathbf{b}\cdot\mathbf{n}|}{|\mathbf{b}||\mathbf{n}|}$ (angle between line and plane, not the complement): **M1**

$$\mathbf{b}\cdot\mathbf{n} = 8 + 2 + 2 = 12, \quad |\mathbf{b}| = \sqrt{4+1+4} = 3, \quad |\mathbf{n}| = \sqrt{16+4+1} = \sqrt{21}$$

$$\sin\theta = \frac{12}{3\sqrt{21}} = \frac{4}{\sqrt{21}}$$

A1

$$\theta = \arcsin\left(\frac{4}{\sqrt{21}}\right) \approx 60.7^\circ \text{ (GDC)}$$

$$\boxed{\theta \approx 60.7^\circ}$$

A1

Note: using cos instead of sin gives the angle with the normal (the complement); this is a common error.

Q16 — Aircraft path meets a restricted plane [MEDIUM]

(a) Direction: $\begin{pmatrix} 7 \\ 3 \\ 9 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix}$.

$$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix} + t\begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix}, \quad t \in \mathbb{R}$$

A1

(b) Substitute into $2x - y + 2z = 30$ (GDC or by hand): **M1**

$$2(3+4t) - (1+2t) + 2(5+4t) = 30$$

$$6 + 8t - 1 - 2t + 10 + 8t = 30 \implies 14t = 15 \implies t = \frac{15}{14}$$

A1

$$x = 3 + \frac{60}{14} = \frac{81}{14}, \quad y = 1 + \frac{30}{14} = \frac{22}{14} = \frac{11}{7}, \quad z = 5 + \frac{60}{14} = \frac{130}{14} = \frac{65}{7}$$

$$\boxed{\left(\frac{81}{14},\ \frac{11}{7},\ \frac{65}{7}\right) \approx (5.79,\ 1.57,\ 9.29)\text{ km}}$$

A1

Q17 — Angle between two planes [MEDIUM]

$\mathbf{n}_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$, $\mathbf{n}_2 = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$. **M1**

$$\mathbf{n}_1 \cdot \mathbf{n}_2 = 2 - 2 - 3 = -3$$

$$|\mathbf{n}_1| = \sqrt{1+4+1} = \sqrt{6}, \quad |\mathbf{n}_2| = \sqrt{4+1+9} = \sqrt{14}$$

M1

$$\cos\phi = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{|\mathbf{n}_1||\mathbf{n}_2|} = \frac{3}{\sqrt{6}\cdot\sqrt{14}} = \frac{3}{\sqrt{84}} \approx 0.32733\ldots$$

A1

$$\phi = \arccos(0.32733\ldots) \approx 70.9^\circ \text{ (GDC)}$$

$$\boxed{\phi \approx 70.9^\circ}$$

A1

Note: the absolute value of the dot product gives the acute angle between the planes.

Q18 — Parallel planes and distance between them [MEDIUM]

(a) Since $\Pi \parallel \Pi_0$, its normal is $\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$. Using $Q(2,-1,3)$: **M1**

$$d = 1(2) - 3(-1) + 2(3) = 2 + 3 + 6 = 11$$

$$\boxed{\Pi : x - 3y + 2z = 11}$$

A1

(b) Distance between parallel planes $ax+by+cz=d_1$ and $ax+by+cz=d_2$: **M1**

$$D = \frac{|d_1 - d_2|}{|\mathbf{n}|} = \frac{|11-7|}{\sqrt{1+9+4}} = \frac{4}{\sqrt{14}} \approx 1.07 \text{ units (GDC)}$$

$$\boxed{D = \frac{4}{\sqrt{14}} \approx 1.07 \text{ units}}$$

A1

Q19 — Classify line relative to plane [MEDIUM]

Direction vector of L: $\mathbf{b} = \begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix}$. Normal to Π: $\mathbf{n} = \begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix}$. **M1**

$$\mathbf{b} \cdot \mathbf{n} = 9 + 4 + 36 = 49 \neq 0$$

Since **b** is parallel to **n** (in fact $\mathbf{b} = \mathbf{n}$), we have $\mathbf{b} \cdot \mathbf{n} \neq 0$, so L is **not** parallel to Π. **R1**
*Note: we must check whether **b** is perpendicular to **n** (line parallel to plane) or not.*
Test the point on L at $t = 0$: $(1, -2, 1)$. Check in Π: **M1**

$$3(1) + 2(-2) - 6(1) = 3 - 4 - 6 = -7 \neq 12$$

Since $\mathbf{b} \cdot \mathbf{n} \neq 0$, the line **intersects** Π at a single point. **A1**

Q20 — Angle between sunlight direction and solar panel [MEDIUM]

$\mathbf{d} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$, $\mathbf{n} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ (normal to Π). **M1**
First find angle α between **d** and **n**: **M1**

$$\cos\alpha = \frac{|\mathbf{d} \cdot \mathbf{n}|}{|\mathbf{d}||\mathbf{n}|} = \frac{|2 + 2 + 4|}{\sqrt{1 + 4 + 4} \cdot \sqrt{4 + 1 + 4}} = \frac{8}{3 \cdot 3} = \frac{8}{9}$$

A1

Angle between line and plane $= 90^\circ - \alpha$:

$$\alpha = \arccos\left(\tfrac{8}{9}\right) \approx 27.266\ldots^\circ, \qquad \theta = 90^\circ - 27.266\ldots^\circ \approx 62.7^\circ \text{ (GDC)}$$

$$\boxed{\theta \approx 62.7^\circ}$$

A1

Note: the angle between a line and a plane uses $\sin\theta = |\mathbf{d} \cdot \mathbf{n}|/(|\mathbf{d}||\mathbf{n}|)$ equivalently; using cos directly gives the complement (the angle with the normal), which is a common error.

Q21 — Plane through three points; line of intersection [HARD]

(a) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$ **M1**

$$\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix} = \begin{pmatrix} (1)(1) - (-1)(2) \\ (-1)(-1) - (2)(1) \\ (2)(2) - (1)(-1) \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ -5 \end{pmatrix}$$ **A1 AG**

(b) Using point $A(1,0,2)$: $3(1)+(-1)(0)+(-5)(2)=3-10=-7$, so $\Pi_1 : 3x-y-5z=-7$ **A1**

(c) Direction of intersection: $\mathbf{d}=\mathbf{n}_1\times\mathbf{n}_2=\begin{pmatrix}3\\-1\\-5\end{pmatrix}\times\begin{pmatrix}1\\-2\\1\end{pmatrix}=\begin{pmatrix}(-1)(1)-(-5)(-2)\\(-5)(1)-(3)(1)\\(3)(-2)-(-1)(1)\end{pmatrix}=\begin{pmatrix}-11\\-8\\-5\end{pmatrix}$

M1

Set $z=0$: solve $3x-y=-7$ and $x-2y=4$; from second $x=4+2y$; substitute: $12+6y-y=-7\Rightarrow y=-\frac{19}{5}$, $x=4-\frac{38}{5}=-\frac{18}{5}$.

Point: $\left(-\frac{18}{5},-\frac{19}{5},0\right)$; direction: $\begin{pmatrix}-11\\-8\\-5\end{pmatrix}$ (or scalar multiple). **A1**

Note: Accept any scalar multiple of the direction vector; accept any valid point on both planes.

Q22 — Angle between line and plane; intersection point [HARD]

(a) Direction of ℓ: $\mathbf{b}=\begin{pmatrix}1\\-1\\2\end{pmatrix}$; normal of Π: $\mathbf{n}=\begin{pmatrix}2\\-1\\2\end{pmatrix}$. **M1**

$$\sin\theta=\frac{|\mathbf{b}\cdot\mathbf{n}|}{|\mathbf{b}||\mathbf{n}|}=\frac{|2+1+4|}{\sqrt{6}\cdot 3}=\frac{7}{3\sqrt{6}}$$

GDC: $\theta=\arcsin\left(\frac{7}{3\sqrt{6}}\right)=\arcsin(0.95346\ldots)=72.4^\circ$ (3 s.f.) **A1A1**

*Note: Award A1 for correct sine formula setup, A1 for final answer. Complement error (*cos *instead of* sin*) loses second A1.*

(b) Substitute **r** into plane: $2(1+t)-(3-t)+2(2t)=9$ **M1**

$2+2t-3+t+4t=9\Rightarrow 7t=10\Rightarrow t=\frac{10}{7}$

Point: $\left(1+\frac{10}{7},\,3-\frac{10}{7},\,\frac{20}{7}\right)=\left(\frac{17}{7},\,\frac{11}{7},\,\frac{20}{7}\right)$ **A1**

Q23 — Sunlight angle on a plane; perpendicular rod length [HARD]

(a) $\mathbf{d}=\begin{pmatrix}1\\-1\\-3\end{pmatrix}$; $\mathbf{n}=\begin{pmatrix}3\\-4\\1\end{pmatrix}$. **M1**

$$\sin\theta=\frac{|\mathbf{d}\cdot\mathbf{n}|}{|\mathbf{d}||\mathbf{n}|}=\frac{|3+4-3|}{\sqrt{11}\cdot\sqrt{26}}=\frac{4}{\sqrt{286}}$$

GDC: $\theta=\arcsin\left(\frac{4}{\sqrt{286}}\right)=\arcsin(0.23645\ldots)=13.7^\circ$ (3 s.f.) **A1A1**

Note: The angle between a line and a plane uses $\sin\theta=\frac{|\mathbf{d}\cdot\mathbf{n}|}{|\mathbf{d}||\mathbf{n}|}$*; using* cos *gives the complement and is penalised.*

(b) $P = (4, 0, 0)$ lies on Π since $3(4) - 4(0) + 0 = 12$. **(M1)**

The support rod runs from $P(4, 0, 0)$ perpendicular to $z = 0$; it is vertical, so length $=$ z-coordinate of $P = 0$ m.

Since P already has $z = 0$, the rod has zero length; that is, P lies on the ground plane. **A1**

Note: Award A1 for correctly identifying length = 0 with justification.

Q24 — Three planes; common point via line of intersection [HARD]

(a) Direction: $\mathbf{d} = \mathbf{n}_1 \times \mathbf{n}_2 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \times \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} (2)(3) - (-1)(-1) \\ (-1)(2) - (1)(3) \\ (1)(-1) - (2)(2) \end{pmatrix} = \begin{pmatrix} 5 \\ -5 \\ -5 \end{pmatrix}$ **M1**

Simplify: $\mathbf{d} = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$. Set $z = 0$: $x + 2y = 3$, $2x - y = 1$; solve: $x = 1, y = 1$. **(A1)**

$\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ **A1**

(b) Substitute into Π_3: $(1 + t) + 5(1 - t) - 5(0 - t) = k \Rightarrow 1 + t + 5 - 5t + 5t = k \Rightarrow t + 6 = k$ **M1**

GDC check: for a unique common point, any t gives a unique k; with $t = 0$: $k = 6$. Point: $(1, 1, 0)$. **A1**

Note: $k = 6$; point $(1, 1, 0)$. Accept equivalent parametric substitution.

Q25 — Distance from point to plane; foot of perpendicular [HARD]

(a) Plane in Cartesian form: $2x - y + 2z = 6$; $\mathbf{n} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$, $|\mathbf{n}| = 3$. **M1**

Distance $= \dfrac{|2(5) - 1(1) + 2(-2) - 6|}{3} = \dfrac{|10 - 1 - 4 - 6|}{3} = \dfrac{|-1|}{3} = \dfrac{1}{3}$ **M1A1 AG**

Note: Both method marks required for AG: one for formula, one for correct substitution.

(b) The perpendicular from $Q(5, 1, -2)$ has parametric form: $\begin{pmatrix} 5 + 2s \\ 1 - s \\ -2 + 2s \end{pmatrix}$ **M1**

Substitute into plane: $2(5 + 2s) - (1 - s) + 2(-2 + 2s) = 6 \Rightarrow 10 + 4s - 1 + s - 4 + 4s = 6 \Rightarrow 9s = 1 \Rightarrow s = \frac{1}{9}$

Foot: $\left(5 + \frac{2}{9}, 1 - \frac{1}{9}, -2 + \frac{2}{9}\right) = \left(\frac{47}{9}, \frac{8}{9}, -\frac{16}{9}\right)$ **A1**

Q26 — Intersection direction; angle between planes; distance [HARD]

(a) $\mathbf{n}_1 = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$, $\mathbf{n}_2 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$. **M1**

$\mathbf{d} = \mathbf{n}_1 \times \mathbf{n}_2 = \begin{pmatrix} (-1)(-1)-(2)(2) \\ (2)(1)-(3)(-1) \\ (3)(2)-(-1)(1) \end{pmatrix} = \begin{pmatrix} 1-4 \\ 2+3 \\ 6+1 \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \\ 7 \end{pmatrix}$ **A1 AG**

(b) $\cos\alpha = \dfrac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{|\mathbf{n}_1||\mathbf{n}_2|} = \dfrac{|3-2-2|}{\sqrt{14}\cdot\sqrt{6}} = \dfrac{1}{\sqrt{84}}$; GDC: $\alpha = \arccos\left(\frac{1}{\sqrt{84}}\right) = 83.7^\circ$ (3 s.f.) **M1A1**

(c) Test $R(2,-1,0)$ in Π_2: $2-2(-1)-0=4$, and the plane requires 4, so R *does* lie on Π_2.

Note: Upon testing, R satisfies Π_2, so the distance is 0. Award A1 for correct verification and conclusion. **A1**

Note: The question asks to verify R does NOT lie on Π_2; since R satisfies Π_2, the distance is 0 m. Full mark for correct arithmetic and honest conclusion.

Q27 — Particle line; angle with plane; crossing time [HARD]

(a) Direction vector from table: $\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ (change per second).

$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ **A1**

(b) $\mathbf{b} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$, $\mathbf{n} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$, $|\mathbf{b}| = 3$, $|\mathbf{n}| = \sqrt{14}$. **M1**

$\sin\theta = \dfrac{|\mathbf{b}\cdot\mathbf{n}|}{|\mathbf{b}||\mathbf{n}|} = \dfrac{|4-3-2|}{3\sqrt{14}} = \dfrac{1}{3\sqrt{14}}$; GDC: $\theta = \arcsin(0.08909\ldots) = 5.11^\circ$ (3 s.f.) **A1**

Note: Use of cos for line-plane angle is the flagged complement error; penalise with M1 A0.

(c) Substitute into Π: $2(1+2t)+3(2-t)-(2t)=10$ **M1**

$2+4t+6-3t-2t=10 \Rightarrow -t+8=10 \Rightarrow t=-2$.

GDC confirms $t=-2$; point: $(1-4,\ 2+2,\ 0-4)=(-3,\ 4,\ -4)$. Time: $t=-2$ s. **A1**

Note: A negative time is valid; award full marks. Accept unrounded intermediate value shown.

Q28 — Reflection of a point in a plane [HARD]

(a) Substitute $S(3,-1,4)$ into LHS of Π: $3-2(-1)+2(4)=3+2+8=13\neq 5$, so $S\notin\Pi$. **A1**

(b) The line through S perpendicular to Π: $\mathbf{r} = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} + s\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$. **M1**

Foot F: substitute into Π: $(3+s) - 2(-1-2s) + 2(4+2s) = 5$ **M1**

$3 + s + 2 + 4s + 8 + 4s = 5 \Rightarrow 9s + 13 = 5 \Rightarrow s = -\frac{8}{9}$

$F = \left(3 - \frac{8}{9},\ -1 + \frac{16}{9},\ 4 - \frac{16}{9}\right) = \left(\frac{19}{9},\ \frac{7}{9},\ \frac{20}{9}\right)$ **A1**

Reflection: $S' = 2F - S = \left(\frac{38}{9} - 3,\ \frac{14}{9} + 1,\ \frac{40}{9} - 4\right) = \left(\frac{11}{9},\ \frac{23}{9},\ \frac{4}{9}\right)$ **A1**

Q29 — Perpendicular planes; angle between line of intersection and z-axis [HARD]

(a) $\Pi_1 \perp \Pi_2 \Leftrightarrow \mathbf{n}_1 \cdot \mathbf{n}_2 = 0$: $\begin{pmatrix} a \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ a \end{pmatrix} = 2a - 2 - a = a - 2 = 0$ **M1**

$a = 2$ **A1**

(b) With $a = 2$: $\Pi_1 : 2x + 2y - z = 3$, $\Pi_2 : 2x - y + 2z = 5$. **M1**

Direction of intersection: $\mathbf{d} = \mathbf{n}_1 \times \mathbf{n}_2 = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \times \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} (2)(2) - (-1)(-1) \\ (-1)(2) - (2)(2) \\ (2)(-1) - (2)(2) \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \\ -6 \end{pmatrix}$

z-axis direction: $\mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

$\sin\phi = \dfrac{|\mathbf{d} \cdot \mathbf{k}|}{|\mathbf{d}||\mathbf{k}|} = \dfrac{6}{\sqrt{81}} = \dfrac{6}{9} = \dfrac{2}{3}$; GDC: $\phi = \arcsin\left(\frac{2}{3}\right) = 41.8^\circ$ (3 s.f.) **A1A1**

Note: Award first A1 for correct **d***; second A1 for final angle. Complement error penalised with A0 on final mark.*

Q30 — Line parallel to plane; distance from point to plane [HARD]

(a) Direction of ℓ: $\mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$; normal of Π: $\mathbf{n} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$. **M1**

$\mathbf{b} \cdot \mathbf{n} = 3 - 2 - 2 = -1 \neq 0$.

Note: Since $\mathbf{b} \cdot \mathbf{n} \neq 0$*, the line is NOT parallel to* Π*; it intersects* Π*. The question as set contains an inconsistency; award M1 for correct dot product and A1 for honest conclusion.* **A1**

Corrected conclusion: ℓ *is NOT parallel to* Π*; it intersects it.*

(b) Line m through $T(4, 1, 2)$ parallel to **n**: $\mathbf{r} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} + s\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$. **M1**

Substitute into Π: $3(4+3s)-(1-s)+2(2+2s)=12 \Rightarrow 12+9s-1+s+4+4s=12 \Rightarrow 14s = -3 \Rightarrow s=-\frac{3}{14}$

Meeting point: $\left(4-\frac{9}{14},\, 1+\frac{3}{14},\, 2-\frac{6}{14}\right)=\left(\frac{47}{14},\, \frac{17}{14},\, \frac{22}{14}\right)$. **A1**

Distance $= |s|\cdot|\mathbf{n}| = \frac{3}{14}\cdot\sqrt{14} = \dfrac{3}{\sqrt{14}} = \dfrac{3\sqrt{14}}{14} \approx 0.802$ m (GDC). **A1**