IB Math AA HL — Topic 4

Binomial Distribution

The Binomial Distribution · Calculating Binomial Probabilities

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:______________________________ Date:_______________

Score:__________/120

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Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

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Key Formulae

- **Binomial probability (exact):** $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 0, 1, \ldots, n$
- **Complement rule:** $P(X \geq k) = 1 - P(X \leq k-1); \quad P(X > k) = 1 - P(X \leq k)$
- **Expected value:** $E(X) = np$
- **Variance and standard deviation:** $\text{Var}(X) = np(1-p); \quad \sigma = \sqrt{np(1-p)}$
- **At-least-one complement:** $P(X \geq 1) = 1 - P(X = 0) = 1 - (1-p)^n$
- **Conditional probability:** $P(X = a \mid X \geq 1) = \dfrac{P(X = a)}{1 - P(X = 0)}, \quad a \geq 1$
- **Four conditions for $X \sim B(n, p)$:** fixed number of trials n; each trial has exactly two outcomes (success/failure); constant probability p of success; trials are independent of each other.

GDC Tips

- **Exact probability `binompdf`:** `binompdf(n, p, k)` gives $P(X = k)$. Enter as `binompdf(20, 0.3, 5)` for $n = 20$, $p = 0.3$, $k = 5$.
- **Cumulative probability `binomcdf`:** `binomcdf(n, p, k)` gives $P(X \leq k)$. Always write the boundary conversion first, e.g. $P(X \geq 6) = 1 -$ `binomcdf(n, p, 5)` before pressing enter.
- **Upper-tail shortcut:** For $P(X > k)$, use $1 -$ `binomcdf(n, p, k)`; for $P(X \geq k)$, use $1 -$ `binomcdf(n, p, k-1)`. Write both conversions in working.
- **Smallest n search:** Store the expression $1 - (1 - p)^n$ or use `binomcdf` in a table; increase n by 1 until the target probability is first exceeded, then state the direction of rounding explicitly.
- **Conditional binomial on GDC:** Compute numerator with `binompdf` and denominator with $1 -$ `binomcdf(n, p, 0)`; divide the stored values to avoid rounding errors mid-calculation.
- **Checking $E(X)$ and Var(X):** Use np and $np(1 - p)$ directly; no GDC list needed. If n or p is unknown, set up the simultaneous equations algebraically before using the GDC to solve.

Common Mistakes to Avoid

1. **Off-by-one boundary error.** Students write `binomcdf(n, p, k)` for $P(X \geq k)$ instead of $1 -$ `binomcdf(n, p, k-1)`. Always convert the inequality *before* entering the GDC: $P(X \geq k) = 1 - P(X \leq k - 1)$, so the upper limit entered is $k - 1$, not k.
2. **Confusing $P(X > k)$ with $P(X \geq k)$.** These differ by exactly $P(X = k)$. Write the strict or non-strict inequality explicitly in every line of working, and convert: $P(X > k) = 1 - P(X \leq k)$ while $P(X \geq k) = 1 - P(X \leq k - 1)$.
3. **Using variance instead of standard deviation (or vice versa).** Var$(X) = np(1 - p)$ is not the standard deviation. If a question asks for σ, take the square root: $\sigma = \sqrt{np(1 - p)}$. Mixing these gives answers that differ by a square or square root.
4. **Incomplete justification of the binomial model.** Stating only "there are two outcomes" earns no marks. All four conditions must be addressed in context: fixed n, two outcomes, constant p, and independence of trials. Identify any condition that may be questionable (e.g. sampling without replacement violates independence).
5. **Rounding too early in multi-step calculations.** Storing a rounded intermediate probability (e.g. $P(X = 3) \approx 0.234$ instead of the full display) and multiplying introduces cumulative error. Keep all decimal places in the GDC memory and round only the *final* answer to 3 significant figures.
6. **Wrong direction when rounding n in smallest-n problems.** After solving $1 - (1 - p)^n > c$ algebraically or by table, students round n *down* to the nearest integer. Because the inequality must be *exceeded*, always round *up*, and verify by substituting both n and $n - 1$ back into the probability to confirm the threshold is first crossed at the chosen n.

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Section A — Easy

EASY

1. A fair six-sided die is rolled 8 times. Let X be the number of times a six is obtained. Write down the distribution of X, including the values of all parameters. *Calculator not allowed* **[3 marks]**

2. A factory produces light bulbs, of which 5% are defective. A quality inspector selects 20 bulbs at random. The number of defective bulbs is modelled by $X \sim B(20,\ 0.05)$.
Find $P(X = 2)$. *Calculator not allowed* **[3 marks]**

3. A basketball player has a probability of 0.7 of scoring on any free throw. She takes 15 free throws. The number of successful free throws is modelled by $X \sim B(15,\ 0.7)$.
Find $P(X \leq 10)$. *Calculator allowed* **[3 marks]**

4. Seeds are planted in individual pots. Each seed has a probability of 0.85 of germinating, independently of all other seeds. A tray holds 12 seeds.
The number of seeds that germinate, X, is modelled by a binomial distribution. The table below shows part of the probability distribution.

x	10	11	12
$P(X = x)$	0.2924	0.3012	k

Find $P(X \geq 10)$. *Calculator allowed* **[3 marks]**

5. A multiple-choice quiz has 10 questions, each with 4 options. A student guesses every answer randomly. Let X be the number of correct answers.
State the distribution of X and find $\mathrm{E}(X)$. *Calculator not allowed* **[3 marks]**

6. A spinner is divided into 5 equal sectors numbered 1 to 5. The spinner is spun 30 times. Let X be the number of times the spinner lands on an odd number. $X \sim B(30,\, 0.6)$.
Find $P(X \geq 15)$. *Calculator allowed* **[3 marks]**

7. A survey finds that 40% of people in a town regularly cycle to work. A random sample of 18 residents is taken. Let X be the number of residents who regularly cycle to work.

Parameter	n	p	E(X)	Var(X)
Value	18	0.4		

Find E(X) and Var(X). *Calculator allowed* **[3 marks]**

8. A biased coin has probability 0.3 of landing heads on any toss. The coin is tossed 10 times. Let X be the number of heads obtained. $X \sim B(10,\, 0.3)$.
Find $P(3 \leq X \leq 6)$. *Calculator allowed* **[3 marks]**

9. A box of chocolates contains 25 chocolates, of which 8 are dark chocolate. Gabriella selects chocolates one at a time, replacing each chocolate before the next selection. The number of dark chocolates she selects in 10 trials is modelled by $X \sim B\left(10,\, \frac{8}{25}\right)$.
Find $P(X > 3)$. *Calculator allowed* **[3 marks]**

10. A random variable $X \sim B(n,\, 0.25)$ has $\mathrm{E}(X) = 6$.

Given	p	$\mathrm{E}(X)$
Value	0.25	6

Find the value of n and hence find $\mathrm{Var}(X)$. *Calculator not allowed* **[3 marks]**

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Section B — Medium

MEDIUM

11. A factory produces light bulbs. Each bulb is independently tested and has a probability of 0.08 of being defective. A quality inspector selects a random sample of 20 bulbs.
Let X be the number of defective bulbs in the sample.

(a) Write down the distribution of X, stating the values of any parameters. *Calculator not allowed* **[2 marks]**

(b) Find $\mathrm{P}(X \geq 3)$. *Calculator allowed* **[2 marks]**

12. A biased coin is flipped n times. The probability of obtaining a head on any flip is 0.35. Let X be the number of heads obtained. It is given that $\mathrm{E}(X) = 7$.

(a) Find the value of n. *Calculator not allowed* **[2 marks]**

(b) Find $\mathrm{Var}(X)$. *Calculator not allowed* **[2 marks]**

13. A market gardener reports that 15% of tomato seeds fail to germinate. A tray contains 18 seeds, each germinating independently.
Let X be the number of seeds that fail to germinate.
Find P$(2 \leq X \leq 5)$. *Calculator allowed* **[4 marks]**

14. A basketball player has a probability of 0.72 of scoring a free throw. She takes 12 free throws, each independently.
Let X be the number of successful free throws.
The table below shows some values of the probability distribution of X.

x	8	9	10	11
P$(X = x)$	0.1745	0.2367	p	0.1954

(a) Find the value of p. *Calculator allowed* **[2 marks]**

(b) Find P$(X > 9)$. *Calculator allowed* **[2 marks]**

15. A multiple-choice quiz has 10 questions. Each question has 5 options, of which exactly one is correct. A student guesses every answer independently and at random.
Let X be the number of correct answers.

(a) Find P$(X = 3)$, showing your working clearly by using the binomial probability formula. *Calculator not allowed* **[3 marks]**

(b) Write down E(X). *Calculator not allowed* **[1 marks]**

16. At a call centre, each incoming call independently has a probability of 0.3 of requiring a supervisor. During a busy hour, 15 calls are received.
Let X be the number of calls requiring a supervisor.

(a) Find P$(X \leq 4)$. *Calculator allowed* **[2 marks]**

(b) Find P$(X = 2 \mid X \leq 4)$. *Calculator allowed* **[2 marks]**

17. Seeds of a particular plant are sold in packets. Each seed independently has a probability of 0.9 of sprouting. A gardener buys a packet of n seeds and wants the probability of at least one seed sprouting to exceed 0.9999. Find the smallest value of n that satisfies this condition. *Calculator allowed* **[4 marks]**

18. A survey finds that 40% of adults in a town own a bicycle. A random sample of 16 adults is chosen. Let X be the number of bicycle owners in the sample.

Event	$X \leq 5$	$X \leq 6$	$X \leq 7$	$X \leq 8$
Probability	0.3294	0.5272	0.7161	0.8577

(a) Using the table, find P$(6 \leq X \leq 8)$. *Calculator allowed* **[2 marks]**

(b) Find P$(X > 6)$, clearly converting the boundary before calculating. *Calculator allowed* **[2 marks]**

19. A random variable $X \sim B(n, p)$ has mean $\mathrm{E}(X) = 6$ and variance $\mathrm{Var}(X) = 4.2$.

(a) Find the values of n and p. *Calculator not allowed* **[3 marks]**

(b) State the mode of X. *Calculator allowed* **[1 marks]**

20. A technician services 14 computers in a day. Each computer independently has a probability of 0.25 of needing a hardware repair.
Let X be the number of computers needing a hardware repair.

(a) Find $\mathrm{P}(X \geq 5)$. *Calculator allowed* **[2 marks]**

(b) Given that at least one computer needs a hardware repair, find the probability that exactly two computers need a hardware repair. *Calculator allowed* **[2 marks]**

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Section C — Hard

HARD

21. A factory produces electronic components. Each component independently has a probability of 0.08 of being defective. A quality inspector selects a random sample of 25 components.
Let X be the number of defective components in the sample.

(a) Show that $\mathrm{E}(X) = 2$ and find $\mathrm{Var}(X)$. [2]

(b) Find the probability that the inspector finds more than 4 defective components. [2]

(c) Given that at least one component is defective, find the probability that exactly two are defective. Give your answer to 3 significant figures. [1]

Calculator allowed **[5 marks]**

22. A biased coin has probability p of landing heads on any toss. The coin is tossed 20 times. Let X denote the number of heads obtained. It is given that Var$(X) = 4.8$.

(a) Show that p satisfies $20p(1-p) = 4.8$, and hence find the two possible values of p. [3]

(b) Given that E$(X) > 10$, write down the value of p and find $P(X \geq 15)$. [2]

Calculator not allowed **[5 marks]**

23. Seeds are sold in packets of n. Each seed germinates independently with probability 0.85. A gardener wants the probability that all n seeds in a packet germinate to be less than 0.05.

(a) Write down an inequality in n satisfied by this condition. [1]

(b) Find the smallest value of n that satisfies this condition. [2]

(c) For this value of n, find the probability that at least $\lfloor 0.8n \rfloor$ seeds germinate. [2]

Calculator allowed **[5 marks]**

24. A student claims that the number of faulty bolts, X, produced per hour in a machine shop can be modelled by $X \sim B(40,\, 0.15)$.

(a) State the four conditions required for a binomial model to be valid, referring to this context in each condition. [4]

(b) Write down one condition that may not hold in practice, and explain why. [1]

Calculator not allowed **[5 marks]**

25. In a large population, 30% of adults wear contact lenses. A random sample of 18 adults is selected. Let W be the number of adults in the sample who wear contact lenses, so $W \sim B(18,\, 0.3)$.
The table below shows selected probabilities for W.

w	3	4	5	6
$P(W = w)$	0.1046	0.1681	0.2017	0.1873

(a) Using only the values in the table, identify the mode of W. Justify your answer by comparing consecutive probabilities. [2]

(b) Find $P(3 \le W \le 6)$. [1]

(c) Find $P(W \le 2 \mid W \le 6)$. [2]

Calculator allowed **[5 marks]**

26. A basketball player makes each free-throw independently with probability 0.72. During a training session, she attempts 15 free-throws.

(a) Find the probability that she makes exactly 12 free-throws. Show your method by writing out the binomial coefficient and relevant powers. [2]

(b) Find the probability that she makes fewer than 10 free-throws. [1]

(c) Find the probability that she makes exactly 12 free-throws given that she makes at least 10. [2]

Calculator not allowed **[5 marks]**

27. A medical screening test gives a positive result independently for each patient tested with probability 0.06. A clinic tests n patients in a day. The clinic manager wants the probability that at least one patient tests positive to exceed 0.99.

(a) Show that this condition requires $0.94^n < 0.01$. [2]

(b) Hence find the minimum value of n, showing all working. [2]

(c) Interpret your answer to part (b) in context. [1]

Calculator allowed **[5 marks]**

28. A call centre receives complaints in batches of 12. Each complaint independently requires escalation with probability p. It is known that the probability of exactly one complaint requiring escalation is 0.3 times the probability of exactly two complaints requiring escalation.

(a) Show that $\dfrac{1}{p} - 1 = \dfrac{11}{6}$. [3]

(b) Hence find p, and write down E(X) where X is the number of escalated complaints per batch. [2]

Calculator not allowed **[5 marks]**

29. A food manufacturer inspects cans leaving a production line. Each can independently has probability 0.04 of containing a labelling error. Cans are inspected in boxes of 30.

Number of errors k	0	1	2	3	≥ 4
$P(X = k)$	a	b	0.1191	0.0168	c

where $X \sim B(30,\, 0.04)$.

(a) Find the values of a and b. [2]

(b) Find c and verify that the probabilities sum to 1 (to 4 decimal places). [2]

(c) A box is selected at random. Given that $X \geq 1$, find the probability that $X \geq 3$. [1]

Calculator allowed **[5 marks]**

30. A game show contestant spins a wheel 10 times. On each spin the probability of winning a prize is 0.35, independently of all other spins. Let Y denote the total number of prizes won.

(a) Find $P(Y = 4)$. [2]

(b) Find the value of k such that $P(Y \leq k) \geq 0.75$ for the first time as k increases from 0. [2]

(c) The contestant wins a bonus if $Y > \mathsf{E}(Y)$. Find the probability of winning the bonus and interpret this result in context. [1]

Calculator allowed **[5 marks]**

Worked Solutions

Q1 — Identifying binomial parameters [EASY]

A six appears with probability $p = \dfrac{1}{6}$ on each roll, there are $n = 8$ independent rolls, and each roll has exactly two outcomes (six or not six). **R1**

Therefore $X \sim B\left(8, \dfrac{1}{6}\right)$, with $n = 8$ and $p = \dfrac{1}{6}$. **A1A1**

Note: Award A1 for correct $n = 8$ and A1 for correct $p = \frac{1}{6}$ (or equivalent decimal 0.167). Award R1 for identifying fixed n, two outcomes, constant p, and independence (at least two conditions mentioned).

Q2 — Exact binomial probability by hand [EASY]

Using the binomial formula with $n = 20$, $p = 0.05$, $k = 2$:

$$P(X = 2) = \binom{20}{2}(0.05)^2(0.95)^{18}$$

M1

$\binom{20}{2} = 190$, so $P(X = 2) = 190 \times 0.0025 \times (0.95)^{18}$ **(A1)**

$P(X = 2) = 190 \times 0.0025 \times 0.39740\ldots = 0.18868\ldots \approx 0.189$ **A1**

Note: Award M1 for writing the correct binomial formula with $\binom{20}{2}$ shown explicitly. Award final A1 for answer rounding correctly to 3 s.f.

Q3 — Binomial cdf [EASY]

Using GDC: $\texttt{binomcdf}(15,\ 0.7,\ 10)$ **M1**

$P(X \leq 10) = 0.48490\ldots$ **(A1)**

$P(X \leq 10) \approx 0.485$ **A1**

Note: Award M1 for correct identification of $P(X \leq 10)$ and use of binomcdf with parameters $(15, 0.7, 10)$.

Q4 — Binomial upper tail probability [EASY]

$P(X = 12) = k = (0.85)^{12} = 0.14224\ldots \approx 0.142$ **(A1)**

$P(X \geq 10) = P(X = 10) + P(X = 11) + P(X = 12)$ **M1**

$P(X \geq 10) = 0.2924 + 0.3012 + 0.1422 = 0.7358\ldots \approx 0.736$ **A1**

Note: Award M1 for summing the three terms. Accept GDC: $\mathit{binomcdf}(12, 0.85, 12)$ −

$binomcdf(12, 0.85, 9)$ giving the same result.

Q5 — Binomial distribution and expected value [EASY]

Each question has probability $p = \frac{1}{4} = 0.25$ of being correct; there are $n = 10$ independent questions. **A1**

$X \sim B(10,\ 0.25)$ **A1**

$\mathrm{E}(X) = np = 10 \times 0.25 = 2.5$ **A1**

Note: Award first A1 for stating the distribution with correct parameters. Award second A1 for $\mathrm{E}(X) = np$ applied correctly.

Q6 — Binomial upper tail via complement [EASY]

$P(X \geq 15) = 1 - P(X \leq 14)$ **M1**

Using GDC: $\texttt{binomcdf}(30,\ 0.6,\ 14) = 0.17465\ldots$ **(A1)**

$P(X \geq 15) = 1 - 0.17465\ldots = 0.82534\ldots \approx 0.825$ **A1**

Note: Award M1 for writing the correct boundary conversion $P(X \geq 15) = 1 - P(X \leq 14)$ before the GDC call.

Q7 — E(X) and Var(X) from parameters [EASY]

$\mathrm{E}(X) = np = 18 \times 0.4 = 7.2$ **A1**

$\mathrm{Var}(X) = np(1-p) = 18 \times 0.4 \times 0.6$ **M1**

$\mathrm{Var}(X) = 4.32$ **A1**

Note: Award M1 for correct formula $np(1-p)$ with values substituted. Both answers must be stated for full marks.

Q8 — Binomial probability over a range [EASY]

$P(3 \leq X \leq 6) = P(X \leq 6) - P(X \leq 2)$ **M1**

Using GDC: $\texttt{binomcdf}(10,\ 0.3,\ 6) - \texttt{binomcdf}(10,\ 0.3,\ 2)$ **(A1)**

$= 0.98941\ldots - 0.38278\ldots = 0.60663\ldots \approx 0.607$ **A1**

Note: Award M1 for writing $P(X \leq 6) - P(X \leq 2)$ with correct boundary conversion shown explicitly before GDC use.

Q9 — Binomial complement for strict inequality [EASY]

$P(X > 3) = 1 - P(X \leq 3)$ **M1**

Using GDC: binomcdf$(10,\ 0.32,\ 3) = 0.57921\ldots$ **(A1)**

$P(X > 3) = 1 - 0.57921\ldots = 0.42078\ldots \approx 0.421$ **A1**

Note: $p = \frac{8}{25} = 0.32$. Award M1 for correct boundary conversion $P(X > 3) = 1 - P(X \leq 3)$ written before GDC call.

Q10 — Reverse E(X) to find n, then Var(X) [EASY]

$\mathrm{E}(X) = np \Rightarrow 6 = n \times 0.25$ **M1**

$n = \dfrac{6}{0.25} = 24$ **A1**

$\mathrm{Var}(X) = np(1-p) = 24 \times 0.25 \times 0.75 = 4.5$ **A1**

Note: Award M1 for setting up $np = 6$ correctly. Follow through their n into the variance formula if n is incorrect but method is clear.

Q11 — Binomial distribution setup and upper-tail probability [MEDIUM]

(a) $X \sim B(20,\ 0.08)$ **A1A1**

Award A1 for $n = 20$, A1 for $p = 0.08$.

(b) $\mathrm{P}(X \geq 3) = 1 - \mathrm{P}(X \leq 2)$ **M1**

Using GDC: `binomcdf`$(20,\ 0.08,\ 2) = 0.79736\ldots$

$\mathrm{P}(X \geq 3) = 1 - 0.79736\ldots = 0.203$ **A1**

Q12 — Finding n and variance from E(X) [MEDIUM]

(a) $\mathrm{E}(X) = np \Rightarrow 0.35n = 7$ **M1**

$n = 20$ **A1**

(b) $\mathrm{Var}(X) = np(1-p) = 20 \times 0.35 \times 0.65$ **M1**

$\mathrm{Var}(X) = 4.55$ **A1**

Q13 — Binomial interval probability [MEDIUM]

$X \sim B(18,\ 0.15)$ **(A1)**

$\mathrm{P}(2 \leq X \leq 5) = \mathrm{P}(X \leq 5) - \mathrm{P}(X \leq 1)$ **M1**

Using GDC: `binomcdf`$(18,\ 0.15,\ 5)$ − `binomcdf`$(18,\ 0.15,\ 1)$

$= 0.93890\ldots - 0.27434\ldots$ **(A1)**

$= 0.665$ **A1**

Q14 — Reading table and upper-tail probability [MEDIUM]

(a) $X \sim B(12,\ 0.72)$ **(M1)**

Using GDC: $\texttt{binompdf}(12,\ 0.72,\ 10) = 0.21565\ldots$

$p = 0.216$ **A1**

(b) $\mathsf{P}(X > 9) = 1 - \mathsf{P}(X \leq 9)$ **M1**

Using GDC: $\texttt{binomcdf}(12,\ 0.72,\ 9) = 0.43063\ldots$

$\mathsf{P}(X > 9) = 1 - 0.43063\ldots = 0.569$ **A1**

Q15 — Exact binomial pdf by hand and E(X) [MEDIUM]

(a) $X \sim B\left(10,\ \frac{1}{5}\right)$ **(A1)**

$\mathsf{P}(X = 3) = \begin{pmatrix} 10 \\ 3 \end{pmatrix} \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right)^7$ **M1**

$= 120 \times \dfrac{1}{125} \times \dfrac{16384}{78125} = \dfrac{1966080}{9765625}$

$= 0.201$ **A1**

(b) $\mathsf{E}(X) = np = 10 \times 0.2 = 2$ **A1**

Q16 — Binomial cdf and conditional probability [MEDIUM]

(a) $X \sim B(15,\ 0.3)$ **(A1)**

Using GDC: $\texttt{binomcdf}(15,\ 0.3,\ 4) = 0.51549\ldots$

$\mathsf{P}(X \leq 4) = 0.515$ **A1**

(b) $\mathsf{P}(X = 2) = \texttt{binompdf}(15,\ 0.3,\ 2) = 0.09156\ldots$ **M1**

$\mathsf{P}(X = 2 \mid X \leq 4) = \dfrac{0.09156\ldots}{0.51549\ldots} = 0.178$ **A1**

Q17 — Smallest n for at-least-one complement [MEDIUM]

$X \sim B(n,\ 0.9)$; require $\mathsf{P}(X \geq 1) > 0.9999$ **M1**

$1 - \mathsf{P}(X = 0) > 0.9999$ **M1**

$\mathsf{P}(X = 0) = (0.1)^n < 0.0001$ **(A1)**

Using GDC (table search or $\texttt{binomcdf}$): test $n = 4$: $(0.1)^4 = 0.0001$ (not strictly less than); $n = 5$:

$(0.1)^5 = 0.00001 < 0.0001$ ✓

Smallest $n = 5$ **A1**

Note: n must be rounded up; $n = 4$ gives equality, not strict inequality.

Q18 — Interval probability from cumulative table [MEDIUM]

(a) $\text{P}(6 \le X \le 8) = \text{P}(X \le 8) - \text{P}(X \le 5)$ **M1**

$= 0.8577 - 0.3294 = 0.528$ **A1**

(b) $\text{P}(X > 6) = 1 - \text{P}(X \le 6)$ **M1**

$= 1 - 0.5272 = 0.473$ **A1**

Note: M1 requires the boundary conversion $\text{P}(X > 6) = 1 - \text{P}(X \le 6)$ to be written explicitly.

Q19 — Finding n and p from mean and variance [MEDIUM]

(a) $np = 6$ and $np(1-p) = 4.2$ **M1**

Dividing: $1 - p = \dfrac{4.2}{6} = 0.7 \Rightarrow p = 0.3$ **A1**

$n = \dfrac{6}{0.3} = 20$ **A1**

(b) Mode $= 6$ **A1**

Note: For $X \sim B(20, 0.3)$, $(n+1)p = 6.3$, so the mode is 6.

Q20 — Upper-tail probability and conditional binomial [MEDIUM]

(a) $X \sim B(14,\ 0.25)$ **(A1)**

$\text{P}(X \ge 5) = 1 - \text{P}(X \le 4)$ **M1**

Using GDC: $\texttt{binomcdf}(14,\ 0.25,\ 4) = 0.72056\ldots$

$\text{P}(X \ge 5) = 1 - 0.72056\ldots = 0.279$ **A1**

(b) $\text{P}(X = 2 \mid X \ge 1) = \dfrac{\text{P}(X = 2)}{1 - \text{P}(X = 0)}$ **M1**

$\texttt{binompdf}(14,\ 0.25,\ 2) = 0.15577\ldots$; $\text{P}(X = 0) = (0.75)^{14} = 0.01782\ldots$

$= \dfrac{0.15577\ldots}{1 - 0.01782\ldots} = \dfrac{0.15577\ldots}{0.98218\ldots} = 0.159$ **A1**

Q21 — Defective components, conditional binomial [HARD]

(a) $\text{E}(X) = np = 25 \times 0.08 = 2$ **AG**

$\text{Var}(X) = np(1-p) = 25 \times 0.08 \times 0.92 = 1.84$ **A1**

(b) $P(X > 4) = 1 - P(X \leq 4)$ **M1**

Using $\texttt{binomcdf}(25,\ 0.08,\ 4)$: $P(X \leq 4) = 0.95881\ldots$

$P(X > 4) = 1 - 0.95881\ldots = 0.0412$ (3 s.f.) **A1**

(c) $P(X = 2 \mid X \geq 1) = \dfrac{P(X = 2)}{1 - P(X = 0)}$

$\texttt{binompdf}(25, 0.08, 2) = 0.26553\ldots; \quad P(X = 0) = 0.92^{25} = 0.12398\ldots$

$= \dfrac{0.26553\ldots}{1 - 0.12398\ldots} = \dfrac{0.26553\ldots}{0.87602\ldots} = 0.303$ (3 s.f.) **A1**

Note: Accept answers rounding to 0.303.

Q22 — Variance reverse to find p [HARD]

(a) $\text{Var}(X) = np(1-p)$, so $20p(1-p) = 4.8$ **AG**

$20p - 20p^2 = 4.8 \implies 20p^2 - 20p + 4.8 = 0 \implies 25p^2 - 25p + 6 = 0$ **M1**

$(5p - 2)(5p - 3) = 0$ **M1**

$p = 0.4$ or $p = 0.6$ **A1**

(b) $\text{E}(X) = 20p > 10 \implies p > 0.5$, so $p = 0.6$ **A1**

$P(X \geq 15) = 1 - P(X \leq 14) = 1 - \text{binomcdf}(20, 0.6, 14)$

$= 1 - 0.87441\ldots = 0.126$ (3 s.f.) **A1**

Note: No GDC allowed; $P(X \geq 15)$ evaluated by sum is acceptable.

Q23 — Smallest n for germination threshold [HARD]

(a) $P(\text{all germinate}) = 0.85^n < 0.05$ **A1**

(b) Taking logarithms: $n \ln(0.85) < \ln(0.05)$ **M1**

Since $\ln(0.85) < 0$, the inequality reverses: $n > \dfrac{\ln 0.05}{\ln 0.85} = \dfrac{-2.9957\ldots}{-0.16252\ldots} = 18.43\ldots$

Smallest integer $n = 19$ **A1**

(c) $\lfloor 0.8 \times 19 \rfloor = \lfloor 15.2 \rfloor = 15$

$P(X \geq 15) = 1 - P(X \leq 14)$ where $X \sim B(19, 0.85)$ **M1**

$= 1 - \text{binomcdf}(19,\ 0.85,\ 14) = 1 - 0.26364\ldots = 0.736$ (3 s.f.) **A1**

Note: Boundary $P(X \geq 15) = 1 - P(X \leq 14)$ must be stated.

Q24 — Binomial model validity, faulty bolts [HARD]

(a)

Fixed n: There are a fixed number of bolts produced per hour, stated as 40. **A1**

Two outcomes: Each bolt is either faulty or not faulty — only two possible outcomes. **A1**

Constant p: The probability of any bolt being faulty is constant at 0.15 throughout the hour. **A1**

Independence: Whether one bolt is faulty does not affect whether any other bolt is faulty. **A1**

(b) The independence condition may not hold in practice: if the machine develops a fault part-way through the hour, then bolts produced after that point are more likely to be faulty, so consecutive bolts are not independent. **R1**

Q25 — Mode identification and conditional probability [HARD]

(a) From the table: $P(W = 5) = 0.2017 > P(W = 4) = 0.1681$ and $P(W = 5) = 0.2017 > P(W = 6) = 0.1873$, so $W = 5$ is the mode. **M1A1**

(b) $P(3 \leq W \leq 6) = 0.1046 + 0.1681 + 0.2017 + 0.1873 = 0.6617$ **A1**

(c) $P(W \leq 2 \mid W \leq 6) = \dfrac{P(W \leq 2)}{P(W \leq 6)}$ **M1**

$P(W \leq 2) = 1 - P(W \geq 3)$; using GDC: binomcdf$(18, 0.3, 2) = 0.09884\ldots$

$P(W \leq 6) =$ binomcdf$(18, 0.3, 6) = 0.70796\ldots$

$P(W \leq 2 \mid W \leq 6) = \dfrac{0.09884\ldots}{0.70796\ldots} = 0.140$ (3 s.f.) **A1**

Q26 — Exact binomial by hand, conditional [HARD]

(a) $P(X = 12) = \dbinom{15}{12}(0.72)^{12}(0.28)^3$ **M1**

$= 455 \times (0.72)^{12} \times (0.28)^3$

$(0.72)^{12} = 0.023298\ldots; \quad (0.28)^3 = 0.021952$

$P(X = 12) = 455 \times 0.023298\ldots \times 0.021952 = 0.2326\ldots \approx 0.233$ (3 s.f.) **A1**

(b) $P(X < 10) = P(X \leq 9) =$ binomcdf$(15,\ 0.72,\ 9) = 0.1484\ldots \approx 0.148$ (3 s.f.) **A1**

(c) $P(X = 12 \mid X \geq 10) = \dfrac{P(X = 12)}{P(X \geq 10)}$ **M1**

$P(X \geq 10) = 1 - P(X \leq 9) = 1 - 0.1484\ldots = 0.8516\ldots$

$= \dfrac{0.2326\ldots}{0.8516\ldots} = 0.273$ (3 s.f.) **A1**

Q27 — Smallest n for at-least-one exceeds 0.99 [HARD]

(a) Let $X \sim B(n, 0.06)$. $P(\text{at least one positive}) = 1 - P(X = 0)$ **M1**

$P(X = 0) = (1 - 0.06)^n = 0.94^n$

Condition: $1 - 0.94^n > 0.99 \implies 0.94^n < 0.01$ **AG** **A1**

(b) Taking logarithms: $n \ln(0.94) < \ln(0.01)$ **M1**

$n > \dfrac{\ln(0.01)}{\ln(0.94)} = \dfrac{-4.6052\ldots}{-0.061875\ldots} = 74.43\ldots$

Minimum integer $n = 75$ **A1**

(c) The clinic must test at least 75 patients in a day to be more than 99% certain that at least one patient tests positive. **R1**

Q28 — Ratio condition to find p algebraically [HARD]

(a) $X \sim B(12, p)$.

$P(X = 1) = \dbinom{12}{1} p(1-p)^{11} = 12p(1-p)^{11}$

$P(X = 2) = \dbinom{12}{2} p^2(1-p)^{10} = 66p^2(1-p)^{10}$ **M1**

Setting $P(X = 1) = 0.3 \times P(X = 2)$:

$12p(1-p)^{11} = 0.3 \times 66p^2(1-p)^{10}$ **M1**

Dividing both sides by $p(1-p)^{10}$: $12(1-p) = 19.8p$

$12 - 12p = 19.8p \implies 12 = 31.8p$

Rearranging: $\dfrac{12}{p} = 31.8 \implies \dfrac{1}{p} - 1 = \dfrac{31.8 - 12}{12} = \dfrac{19.8}{12} = \dfrac{11}{6}$ **AG** **A1**

(b) $\dfrac{1}{p} = 1 + \dfrac{11}{6} = \dfrac{17}{6} \implies p = \dfrac{6}{17}$ **A1**

$\mathrm{E}(X) = np = 12 \times \dfrac{6}{17} = \dfrac{72}{17} \approx 4.24$ **A1**

Q29 — Probability table verification and conditional [HARD]

(a) $X \sim B(30, 0.04)$

$a = P(X = 0) =$ binompdf$(30,\ 0.04,\ 0) = 0.96^{30} = 0.2939\ldots \approx 0.2939$ **A1**

$b = P(X = 1) =$ binompdf$(30,\ 0.04,\ 1) = 30 \times 0.04 \times 0.96^{29} = 0.3673\ldots \approx 0.3673$ **A1**

(b) $c = 1 - (a + b + 0.1191 + 0.0168)$ **M1**

$c = 1 - (0.2939 + 0.3673 + 0.1191 + 0.0168) = 1 - 0.7971 = 0.0029$

Sum $= 0.2939 + 0.3673 + 0.1191 + 0.0168 + 0.0029 = 1.0000$ ✓ **A1**

(c) $P(X \geq 3 \mid X \geq 1) = \dfrac{P(X \geq 3)}{P(X \geq 1)} = \dfrac{0.0168 + 0.0029}{1 - 0.2939} = \dfrac{0.0197}{0.7061} = 0.0279$ (3 s.f.) **A1**

Q30 — Game show wheel, median and interpretation [HARD]

(a) $Y \sim B(10,\ 0.35)$

$P(Y = 4) = \dbinom{10}{4}(0.35)^4(0.65)^6$ **M1**

Using `binompdf`$(10,\ 0.35,\ 4) = 0.23768\ldots \approx 0.238$ (3 s.f.) **A1**

(b) Compute $P(Y \leq k)$ via `binomcdf`$(10,\ 0.35,\ k)$ for increasing k: **M1**

$k = 2$: $0.2616\ldots$; $\quad k = 3$: $0.5138\ldots$; $\quad k = 4$: $0.7515\ldots > 0.75$

First k for which $P(Y \leq k) \geq 0.75$ is $k = 4$ **A1**

(c) $\mathrm{E}(Y) = 10 \times 0.35 = 3.5$, so bonus requires $Y > 3.5$, i.e. $Y \geq 4$

$P(Y \geq 4) = 1 - P(Y \leq 3) = 1 - 0.5138\ldots = 0.486$ (3 s.f.)

The contestant has approximately a 48.6% chance of winning the bonus, meaning it is slightly less likely than not that they win the bonus in any single game. **R1**