IB Math AA HL — Topic 4

Normal Distribution

The Normal Curve · Normal Calculations · Standardisation & z-Values · Unknown Parameters

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Scan me for help

Name:________________ Date:________

Score:________/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

Scan me for help

Key Formulae

- **Normal distribution notation:** $X \sim N(\mu, \sigma^2)$, where μ is the mean and σ^2 is the variance.
- **Standardisation (z-score):** $z = \dfrac{x-\mu}{\sigma}$, so that $Z \sim N(0,1)$.
- **Symmetry of the standard normal:** $P(Z < -z) = P(Z > z) = 1 - P(Z < z)$.
- **Complement rule:** $P(X > a) = 1 - P(X < a)$; $\quad P(a < X < b) = P(X < b) - P(X < a)$.
- **Symmetry about the mean:** $P(X < \mu) = P(X > \mu) = 0.5$.
- **Two-parameter z-equations:** given $P(X < a) = p_1$ and $P(X < b) = p_2$, form $\dfrac{a-\mu}{\sigma} = z_1$ and $\dfrac{b-\mu}{\sigma} = z_2$, then solve simultaneously for μ and σ.
- **Expected count:** if $P(X > k) = p$ and a sample of size n is taken, the expected number above k is np.

GDC Tips

- **Normal CDF** (`normalcdf`): computes $P(a < X < b)$ for $X \sim N(\mu, \sigma^2)$. Enter lower bound, upper bound, μ, σ. Use -10^{99} as $-\infty$ and 10^{99} as $+\infty$.
- **Inverse Normal** (`invNorm`): finds the value a such that $P(X < a) = p$. Enter the *left-tail* probability p, then μ, then σ. Always specify the tail direction (left).
- **One unknown parameter**: standardise to get z via `invNorm` on the standard normal ($\mu = 0$, $\sigma = 1$), then solve the equation $\frac{a-\mu}{\sigma} = z$ for the unknown.
- **Two unknown parameters**: use `invNorm` twice (once for each probability condition) to obtain z_1 and z_2; write two equations in μ and σ and solve simultaneously by hand or using the equation solver.
- **Normal-then-Binomial chain**: compute $p = P(X > k)$ using `normalcdf`, then enter p into `binomcdf` or `binompdf` as the success probability for $Y \sim B(n, p)$.

Common Mistakes to Avoid

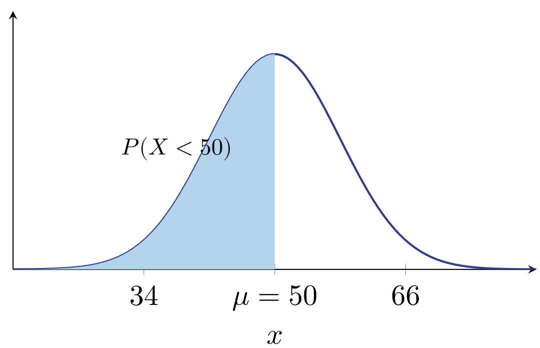
1. **Wrong sign in the z-equation.** When a condition lies *below* the mean, the z-value is negative. Students frequently write a positive z for a lower-tail condition, flipping the equation and giving an incorrect value of μ or σ.
2. **Confusing σ and σ^2 in GDC input.** The `normalcdf` and `invNorm` commands require the standard deviation σ, not the variance σ^2. Entering the variance produces a completely wrong answer with no error message.
3. **Using the right-tail probability in `invNorm`.** The GDC's `invNorm` expects a *left*-tail (cumulative) probability. If the question states $P(X > a) = 0.1$, students must convert to $P(X < a) = 0.9$ before entering the value.
4. **Symmetry errors when the mean is not zero.** Students apply $P(X < \mu - k) = P(X > \mu + k)$ correctly but then forget that this reflection is about μ, not about zero, leading to incorrect boundary values in non-standard distributions.
5. **Rounding z-values too early.** Rounding a z-value to 2 significant figures before substituting it into the simultaneous equations introduces large errors in μ and σ. Retain at least 4 significant figures in intermediate z-values throughout.
6. **Incorrect expected count formula.** Students write the expected count as $n \times P(X = k)$ (a single point probability, which is zero for a continuous distribution) rather than $n \times P(X > k)$ or $n \times P(a < X < b)$ as required by the context.

Scan me for help

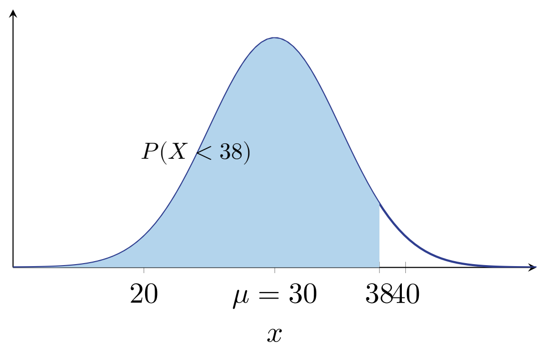
Section A — Easy

EASY

1. The random variable X is normally distributed with mean $\mu = 50$ and standard deviation $\sigma = 8$. Write down $P(X < 50)$. *Calculator not allowed* **[3 marks]**



2. The random variable $X \sim N(30, 5^2)$.
Find $P(X < 38)$. *Calculator allowed* **[3 marks]**



3. The masses, M kg, of bags of flour produced in a factory are normally distributed with mean 2.0 kg and standard deviation 0.04 kg.
Find $P(M > 2.05)$. *Calculator allowed* **[3 marks]**

4. The random variable $X \sim N(15, \sigma^2)$. It is given that $P(X < 18) = 0.9332$.
Find the value of σ. *Calculator allowed* **[3 marks]**

5. The times, T seconds, taken by students to solve a puzzle are normally distributed with mean μ and standard deviation 12. It is given that $P(T < 80) = 0.7734$.
Find the value of μ. *Calculator allowed* **[3 marks]**

6. The random variable $X \sim N(40, 6^2)$.
Find the value of a such that $P(X < a) = 0.9$. *Calculator allowed* **[3 marks]**

7. The random variable $X \sim N(100, 15^2)$.
A score of x is standardised using $z = \dfrac{x - \mu}{\sigma}$.
Write down the value of z when $x = 127.5$. *Calculator not allowed* **[3 marks]**

8. The random variable $X \sim N(20, 4^2)$.
Find $P(16 < X < 24)$. *Calculator allowed* **[3 marks]**

9. The heights, H cm, of plants in a greenhouse are normally distributed with mean 35 cm and standard deviation 5 cm. There are 200 plants in the greenhouse.
Find the expected number of plants with height greater than 42 cm. *Calculator allowed* **[3 marks]**

10. The random variable $X \sim N(\mu, \sigma^2)$. The following information is known:

- $P(X < 25) = 0.5$
- $P(X > 25) = 0.5$

State the value of μ and explain your reasoning. *Calculator not allowed* **[3 marks]**

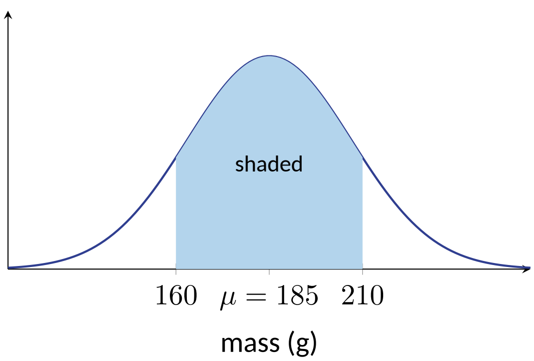
Scan me for help

Section B — Medium

MEDIUM

11. The masses of apples harvested from an orchard are normally distributed with mean $\mu = 185$ g and standard deviation $\sigma = 22$ g. An apple is selected at random. Find the probability that its mass is between 160 g and 210 g.

Calculator allowed **[4 marks]**

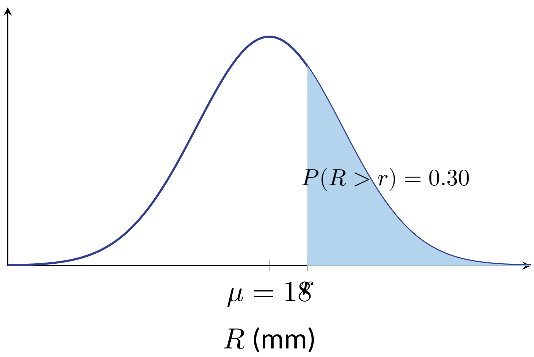


12. The time, T minutes, that customers spend waiting in a queue is normally distributed with mean μ and standard deviation 3.2 min. It is known that $P(T > 12) = 0.0384$. Find the value of μ. *Calculator allowed* **[4 marks]**

13. The heights, H cm, of adult males in a town are normally distributed with $H \sim N(176, \sigma^2)$. Given that $P(H < 160) = 0.0250$, find the value of σ. *Calculator not allowed* **[4 marks]**

14. The scores on a standardised test are normally distributed with mean 62 and standard deviation 11. Candidates from two different schools sit the test. Aiko scores 74 at School A. Bram scores 81 at School B, where the scores at School B are normally distributed with mean 70 and standard deviation 15. By standardising, determine which student performed better relative to their school's distribution, justifying your answer. *Calculator not allowed* **[4 marks]**

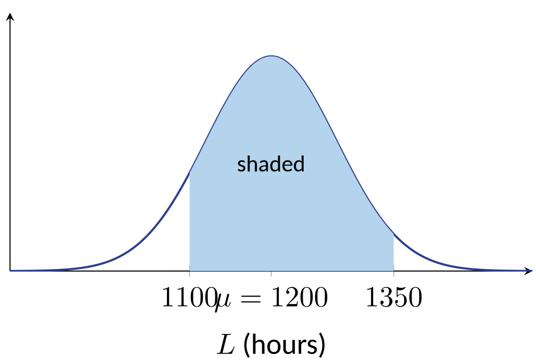
15. The daily rainfall, R mm, in a coastal city during winter is normally distributed with mean 18 mm and standard deviation 5 mm. Find the value of r such that $P(R > r) = 0.30$. *Calculator allowed* **[4 marks]**



16. The lengths of bolts produced by a machine are normally distributed with mean μ mm and standard deviation σ mm. It is given that $P(L < 48) = 0.2119$ and $P(L > 56) = 0.1587$. Write down the two simultaneous equations in μ and σ that arise from standardising these conditions. *Calculator not allowed* **[4 marks]**

17. The volume, V mL, dispensed by a drinks machine is normally distributed with mean 503 mL and standard deviation 4 mL. A cup is rejected if it contains less than 495 mL. In a batch of 500 cups, find the expected number of rejected cups. *Calculator allowed* **[4 marks]**

18. The lifetimes, L hours, of a brand of light bulb are normally distributed with mean 1200 h and standard deviation 80 h. Find $P(1100 < L < 1350)$, giving your answer to three significant figures. *Calculator allowed* **[4 marks]**



19. The mass, M kg, of bags of rice filled by an automated machine is normally distributed with mean 5.05 kg and standard deviation σ kg. The probability that a bag is underweight (i.e. $M < 5.00$ kg) is 0.0668. Find the value of σ, giving your answer to three significant figures. *Calculator allowed* **[4 marks]**

20. The weekly profits, X thousand dollars, of a small business are normally distributed with mean μ and standard deviation 2.4. Given that $P(X < 10) = 0.5$, state the value of μ and hence find $P(X > 13.5)$. *Calculator not allowed* **[4 marks]**

Scan me for help

Section C — Hard

HARD

21. The heights of adult pine trees in a forest are normally distributed with mean μ cm and standard deviation σ cm. It is known that $P(H < 1200) = 0.0668$ and $P(H < 1580) = 0.9772$.

(a) Write down two equations in μ and σ. *Calculator not allowed* **[2 marks]**

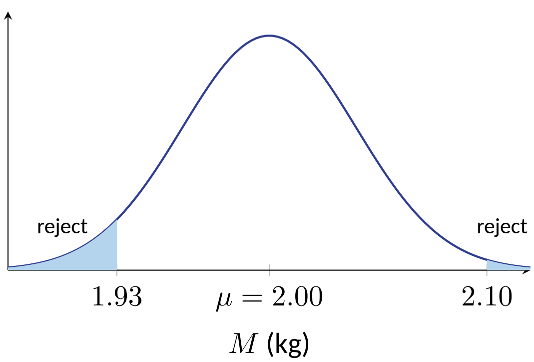
(b) Find the values of μ and σ. **[1 marks]**

(c) Find $P(1350 < H < 1500)$. **[2 marks]**

22. The mass, M kg, of bags of flour filled by a machine is normally distributed with mean $\mu = 2.00$ kg and standard deviation $\sigma = 0.04$ kg. A bag is rejected if its mass is less than 1.93 kg or greater than 2.10 kg.

(a) Find the probability that a randomly selected bag is rejected. *Calculator allowed* **[3 marks]**

(b) In a batch of 500 bags, find the expected number of rejected bags. **[2 marks]**



23. The time, T minutes, that students spend on a mathematics test is normally distributed with mean 85 minutes and standard deviation 12 minutes.

(a) Show that $P(T > 97) \approx 0.159$. *Calculator allowed* **[2 marks]**

(b) Given that $P(T > t_0) = 0.05$, find the value of t_0. **[2 marks]**

(c) Interpret your answer to part (b) in context. **[1 marks]**

24. A student scores 74 in a Physics test and 81 in a Chemistry test. Physics scores are normally distributed with mean 68 and standard deviation 9. Chemistry scores are normally distributed with mean 75 and standard deviation 10.

(a) Find the z-score for each subject. *Calculator not allowed* **[2 marks]**

(b) State, with a reason, in which subject the student performed better relative to the class. **[2 marks]**

(c) Find the probability that a randomly chosen student scores higher than this student in Chemistry. *Calculator allowed* **[1 marks]**

25. The daily rainfall, R mm, in a coastal town is normally distributed. The lower quartile of R is 3.8 mm and the upper quartile is 11.2 mm.

(a) Write down the mean μ of R, justifying your answer. *Calculator not allowed* **[2 marks]**

(b) Find the standard deviation σ of R. *Calculator allowed* **[2 marks]**

(c) Find $P(R > 15)$. **[1 marks]**

26. The lifetimes, L hours, of a brand of light bulbs are normally distributed with mean μ and standard deviation σ. It is known that 10% of bulbs fail before 800 hours and 5% last longer than 1150 hours.

(a) Write down two equations satisfied by μ and σ, giving the z-values correct to 4 significant figures. *Calculator not allowed* **[2 marks]**

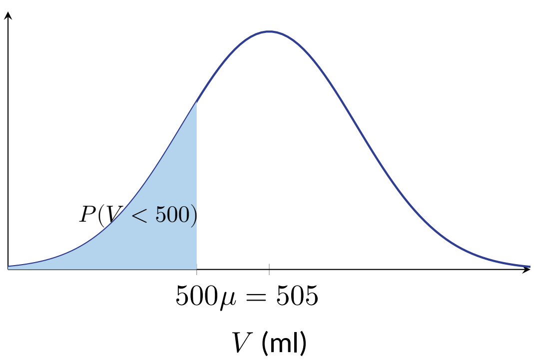
(b) Find μ and σ. *Calculator allowed* **[2 marks]**

(c) Find the probability that a randomly chosen bulb lasts between 900 and 1050 hours. **[1 marks]**

27. The volume, V ml, of juice dispensed by a machine into bottles is normally distributed with mean 505 ml and standard deviation 6 ml. A bottle is labelled as containing 500 ml.

(a) Find the probability that a randomly chosen bottle contains less than the labelled volume. *Calculator allowed* **[2 marks]**

(b) The machine is recalibrated so that only 2% of bottles contain less than 500 ml. The standard deviation remains 6 ml. Find the new mean. **[3 marks]**



28. In a large city, the commute time C minutes of residents is normally distributed with mean μ and standard deviation 15. It is given that $P(C > 60) = 0.2119$.

(a) Find μ. *Calculator allowed* **[2 marks]**

(b) A resident is selected at random. Find the probability that their commute time is within one standard deviation of the mean. **[2 marks]**

(c) A random sample of 8 residents is taken. Find the probability that exactly 3 of them have a commute time within one standard deviation of the mean. **[1 marks]**

29. The exam scores, X, of candidates sitting a standardised test are normally distributed with mean 62 and standard deviation 14.

(a) Find the value of k such that $P(62 - k < X < 62 + k) = 0.80$. *Calculator allowed* **[3 marks]**

(b) Hence find $P(X > 62 + k)$. *Calculator not allowed* **[1 marks]**

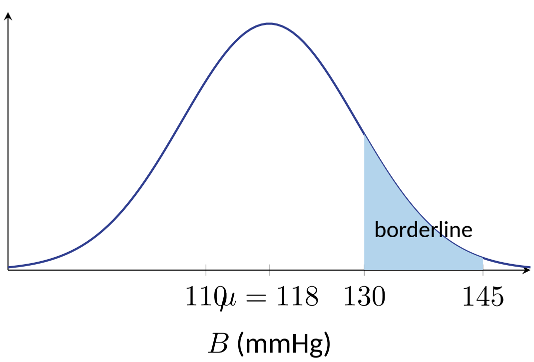
(c) Scores above $62 + k$ are awarded a distinction. In a group of 200 candidates, find the expected number who receive a distinction. **[1 marks]**

30. The blood pressure readings, B mmHg, of patients at a clinic are normally distributed with mean 118 and standard deviation 11.

(a) Find $P(110 < B < 130)$. *Calculator allowed* **[2 marks]**

(b) A patient is classified as "borderline" if their reading is between 130 and 145. Find the probability that a randomly chosen patient is borderline. **[2 marks]**

(c) In a group of 60 patients, find the expected number who are borderline. **[1 marks]**



Worked Solutions

Q1 — Symmetry of normal distribution [EASY]

The normal distribution is symmetric about its mean. **R1**

Since $X \sim N(50, 8^2)$, the mean is $\mu = 50$, so exactly half the distribution lies below the mean. **M1**

$P(X < 50) = 0.5$ **A1**

Q2 — Direct normal probability [EASY]

Using **normalcdf** on GDC with lower $= -\infty$, upper $= 38$, $\mu = 30$, $\sigma = 5$: **M1**

Unrounded value: $0.94520\ldots$ **(A1)**

$P(X < 38) = 0.945$ **A1**

Q3 — Normal tail probability [EASY]

Using **normalcdf** on GDC with lower $= 2.05$, upper $= \infty$, $\mu = 2.0$, $\sigma = 0.04$: **M1**

Unrounded value: $0.10565\ldots$ **(A1)**

$P(M > 2.05) = 0.106$ **A1**

Q4 — Finding standard deviation from probability [EASY]

Using **invNorm** with $p = 0.9332$, $\mu = 0$, $\sigma = 1$: $z = 1.500\ldots$ **M1**

Setting up the standardisation equation: $\dfrac{18 - 15}{\sigma} = 1.500$ **(M1)**

$\sigma = \dfrac{3}{1.500} = 2.00$ **A1**

Q5 — Finding the mean from probability [EASY]

Using **invNorm** with $p = 0.7734$, $\mu = 0$, $\sigma = 1$: $z = 0.7500\ldots$ **M1**

Setting up the standardisation equation: $\dfrac{80 - \mu}{12} = 0.7500$ **(M1)**

$\mu = 80 - 0.7500 \times 12 = 71.0$ **A1**

Q6 — Inverse normal to find a value [EASY]

Using **invNorm** on GDC with $p = 0.9$, $\mu = 40$, $\sigma = 6$: **M1**

Unrounded value: $a = 47.690\ldots$ **(A1)**

$a = 47.7$ **A1**

Q7 — Standardisation calculation [EASY]

Applying the standardisation formula with $x = 127.5$, $\mu = 100$, $\sigma = 15$: **M1**

$z = \dfrac{127.5 - 100}{15}$ **(M1)**

$z = \dfrac{27.5}{15} = 1.83$ **A1**

Q8 — Symmetric interval probability [EASY]

Using **normalcdf** on GDC with lower $= 16$, upper $= 24$, $\mu = 20$, $\sigma = 4$: **M1**

Unrounded value: $0.68269\ldots$ **(A1)**

$P(16 < X < 24) = 0.683$ **A1**

Q9 — Expected count from normal probability [EASY]

Using **normalcdf** on GDC with lower $= 42$, upper $= \infty$, $\mu = 35$, $\sigma = 5$: **M1**

$P(H > 42) = 0.08076\ldots$ **(A1)**

Expected number $= 200 \times 0.08076\ldots = 16.2$ (rounded to 16 plants) **A1**

Q10 — Mean from symmetry reasoning [EASY]

Since $P(X < 25) = 0.5$ and $P(X > 25) = 0.5$, the value 25 divides the distribution into two equal halves. **R1**

For a normal distribution, the point of symmetry where $P(X < c) = 0.5$ is exactly the mean. **M1**

Therefore $\mu = 25$. **A1**

Q11 — Normal probability between two values [MEDIUM]

Using GDC: `normalcdf`(lower$= 160$, upper$= 210$, $\mu = 185$, $\sigma = 22$) **M1A1**

Unrounded value: $0.74210\ldots$ **(A1)**

$P(160 < X < 210) = 0.742$ **A1**

Q12 — Find unknown mean from one probability condition [MEDIUM]

$P(T > 12) = 0.0384$, so $P(T < 12) = 0.9616$ **M1**

Using GDC: `invNorm`$(0.9616, \mu = 0, \sigma = 1) = 1.7701\ldots$ **(A1)**

Standardising: $\dfrac{12-\mu}{3.2} = 1.7701\ldots$ **M1**

$\mu = 12 - 3.2 \times 1.7701\ldots = 6.34$ min **A1**

Q13 — Find unknown standard deviation from one probability [MEDIUM]

$P(H < 160) = 0.0250$; since 0.0250 corresponds to a standard normal value, recognise $z = -1.96$ (standard result) **M1**

Standardising: $\dfrac{160-176}{\sigma} = -1.96$ **A1**

$\sigma = \dfrac{-16}{-1.96}$ **M1**

$\sigma = 8.16$ cm **A1**

Q14 — Compare performance via z-scores [MEDIUM]

Aiko's z-score: $z_A = \dfrac{74-62}{11} = \dfrac{12}{11} = 1.0\overline{09}$ **M1A1**

Bram's z-score: $z_B = \dfrac{81-70}{15} = \dfrac{11}{15} = 0.7\overline{3}$ **A1**

Since $z_A > z_B$, Aiko performed better relative to her school's distribution. **R1**

Q15 — Inverse normal, upper tail [MEDIUM]

$P(R > r) = 0.30 \Rightarrow P(R < r) = 0.70$ **M1**

Using GDC: `invNorm`$(0.70, \mu = 18, \sigma = 5)$ **M1**

Unrounded value: $r = 20.621\ldots$ **(A1)**

$r = 20.6$ mm **A1**

Q16 — Set up two simultaneous z-equations [MEDIUM]

Standardise the first condition: $P(L < 48) = 0.2119 \Rightarrow z_1 = -0.800$ (standard result) **M1**

$\dfrac{48-\mu}{\sigma} = -0.800$ **A1**

Standardise the second condition: $P(L > 56) = 0.1587 \Rightarrow z_2 = 1.00$ (standard result) **M1**

$\dfrac{56-\mu}{\sigma} = 1.00$ **A1**

Note: Award A1A1 only if both signs are correct.

Q17 — Expected count from normal probability [MEDIUM]

$P(V < 495)$: using GDC `normalcdf`(lower$= -\infty$, upper$= 495$, $\mu = 503$, $\sigma = 4$) **M1**

Unrounded: $P(V < 495) = 0.022750\ldots$ **A1**

Expected number $= 500 \times 0.022750\ldots$ **M1**

$= 11.375\ldots \approx 11$ cups **A1**

Note: Accept 11.4 *or* 11.

Q18 — Normal probability, two-sided interval [MEDIUM]

Using GDC: `normalcdf`(lower$= 1100$, upper$= 1350$, $\mu = 1200$, $\sigma = 80$) **M1A1**

Unrounded value: $0.88924\ldots$ **(A1)**

$P(1100 < L < 1350) = 0.889$ **A1**

Q19 — Find unknown standard deviation via inverse normal [MEDIUM]

$P(M < 5.00) = 0.0668$; using GDC `invNorm`$(0.0668, 0, 1) = -1.5003\ldots$ **M1A1**

Standardising: $\dfrac{5.00 - 5.05}{\sigma} = -1.5003\ldots$ **M1**

$\sigma = \dfrac{0.05}{1.5003\ldots} = 0.0333$ kg **A1**

Q20 — Symmetry then upper tail probability [MEDIUM]

Since $P(X < 10) = 0.5$, the mean equals 10, so $\mu = 10$. **R1A1**

Standardise: $z = \dfrac{13.5 - 10}{2.4} = \dfrac{3.5}{2.4} = 1.458\overline{3}$ **M1**

$P(X > 13.5) = P(Z > 1.4583\ldots) = 1 - \Phi(1.4583\ldots) = 0.0724$ **A1**

Note: Award M1 for correct standardisation using their $\mu = 10$. *Final answer must be obtained without GDC.*

Q21 — Two unknown parameters, pine tree heights [HARD]

(a) Standardising both conditions:

$\dfrac{1200 - \mu}{\sigma} = -1.5$ (since $P(Z < -1.5) \approx 0.0668$) **A1**

$\dfrac{1580-\mu}{\sigma}=2$ (since $P(Z<2)\approx 0.9772$) **A1**

Note: Accept z-values stated as -1.500 and 2.000. Signs must be correct.

(b) Subtracting the first equation from the second:

$\dfrac{380}{\sigma}=3.5 \Rightarrow \sigma=\dfrac{380}{3.5}=\dfrac{760}{7}\approx 108.571\ldots\approx 109$ **(A1)**

Substituting: $\mu=1580-2(108.571\ldots)\approx 1363$ **A1**

So $\mu\approx 1363$ cm, $\sigma\approx 109$ cm.

Note: Accept $\sigma=108$ or 109 and corresponding μ from their equations.

(c) Using GDC: normalcdf$(1350,\ 1500,\ 1362.86,\ 108.571)$

$=0.48895\ldots\approx 0.489$ **A1**

Q22 — Rejection rate, flour bags [HARD]

(a) $P(M<1.93)+P(M>2.10)$ **M1**

Using GDC: normalcdf$(-\infty,\ 1.93,\ 2.00,\ 0.04)$

$=0.04006\ldots$

normalcdf$(2.10,\ \infty,\ 2.00,\ 0.04)=0.00621\ldots$ **(A1)**

$P(\text{rejected})=0.04006+0.00621=0.04627\ldots\approx 0.0463$ **A1**

(b) Expected number $=500\times 0.04627\ldots$ **M1**

$=23.135\ldots\approx 23$ bags **A1**

Q23 — Test time, inverse normal and interpretation [HARD]

(a) Using GDC: normalcdf$(97,\ \infty,\ 85,\ 12)$

$=0.15866\ldots$ **M1**

≈ 0.159 **AG**

Note: Must show the unrounded value. Answer given — no mark for the rounded value alone.

(b) Using GDC: invNorm$(0.95,\ 85,\ 12)$ (upper tail 5% means left-tail probability 0.95) **M1**

$=104.74\ldots\approx 105$ minutes **A1**

(c) 95% of students finish within approximately 105 minutes, so only 5% of students take longer than $t_0\approx 105$ minutes to complete the test. **R1**

Q24 — Comparing z-scores across subjects [HARD]

(a) Physics: $z = \dfrac{74 - 68}{9} = \dfrac{6}{9} = 0.667$ (3 s.f.) **A1**

Chemistry: $z = \dfrac{81 - 75}{10} = \dfrac{6}{10} = 0.600$ **A1**

(b) The Physics z-score (0.667) is greater than the Chemistry z-score (0.600), so the student performed better relative to the class in Physics. **R1A1**

Note: R1 for the comparison of z-scores stated explicitly; A1 for the correct conclusion.

(c) Using GDC: normalcdf$(81,\ \infty,\ 75,\ 10)$

$= 0.27425\ldots \approx 0.274$ **A1**

Q25 — Quartiles determine parameters, rainfall [HARD]

(a) Since the normal distribution is symmetric about its mean, and the quartiles are equidistant from the mean:

$\mu = \dfrac{3.8 + 11.2}{2} = \dfrac{15.0}{2} = 7.5$ mm **A1**

Justified by symmetry of the normal distribution about μ. **R1**

(b) $P(R < 3.8) = 0.25$, so using GDC: invNorm$(0.25,\ 0,\ 1) = -0.6745\ldots$ **M1**

$\dfrac{3.8 - 7.5}{\sigma} = -0.6745 \Rightarrow \sigma = \dfrac{3.7}{0.6745} = 5.4855\ldots \approx 5.49$ mm **A1**

(c) Using GDC: normalcdf$(15,\ \infty,\ 7.5,\ 5.4855)$

$= 0.08520\ldots \approx 0.0852$ **A1**

Q26 — Two unknown parameters, light bulb lifetimes [HARD]

(a) $P(L < 800) = 0.10$, so $\dfrac{800 - \mu}{\sigma} = -1.282$ **A1**

$P(L > 1150) = 0.05$, so $\dfrac{1150 - \mu}{\sigma} = 1.645$ **A1**

Note: z-values must be -1.282 and $+1.645$ with correct signs. Both required for 2 marks.

(b) From the two equations: $\mu - 1.282\sigma = 800$ and $\mu + 1.645\sigma = 1150$ **(M1)**

Subtracting: $2.927\sigma = 350 \Rightarrow \sigma = 119.6\ldots \approx 120$ hours

$\mu = 800 + 1.282 \times 119.6 = 953.3\ldots \approx 953$ hours **A1**

(c) Using GDC: normalcdf$(900,\ 1050,\ 953.3,\ 119.6)$

$= 0.5223\ldots \approx 0.522$ **A1**

Q27 — Recalibrating machine mean, juice volume [HARD]

(a) Using GDC: normalcdf$(-\infty,\ 500,\ 505,\ 6)$ **M1**

$= 0.20233\ldots \approx 0.202$ **A1**

(b) Require $P(V < 500) = 0.02$ with $\sigma = 6$.

Using GDC: invNorm$(0.02,\ 0,\ 1) = -2.0537\ldots$ **M1**

Setting up: $\dfrac{500-\mu}{6} = -2.0537$ **M1**

$\mu = 500 + 2.0537 \times 6 = 500 + 12.322\ldots = 512.322\ldots \approx 512$ ml **A1**

Q28 — Find mean then normal-binomial chain, commute time [HARD]

(a) $P(C > 60) = 0.2119 \Rightarrow P(C < 60) = 0.7881$ **M1**

Using GDC: invNorm$(0.7881,\ \mu,\ 15)$; equivalently invNorm$(0.7881,\ 0,\ 1) = 0.8000\ldots$

$\dfrac{60-\mu}{15} = 0.800 \Rightarrow \mu = 60 - 12 = 48$ minutes **A1**

(b) $P(\mu - 15 < C < \mu + 15) = P(33 < C < 63)$ **M1**

Using GDC: normalcdf$(33,\ 63,\ 48,\ 15) = 0.68269\ldots \approx 0.683$ **A1**

(c) Let $Y \sim B(8,\ 0.683)$.

$P(Y = 3) =$ binomPdf$(8,\ 0.683,\ 3) = 0.04880\ldots \approx 0.0488$ **A1**

Q29 — Symmetric interval, inverse normal and expected count [HARD]

(a) $P(62 - k < X < 62 + k) = 0.80$, so by symmetry $P(X < 62 + k) = 0.90$ **M1**

Using GDC: invNorm$(0.90,\ 62,\ 14)$ **M1**

$= 79.938\ldots$ so $62 + k = 79.938\ldots \Rightarrow k = 17.938\ldots \approx 17.9$ **A1**

(b) By symmetry of the normal distribution:

$P(X > 62 + k) = \dfrac{1 - 0.80}{2} = 0.10$ **A1**

Note: No GDC permitted. Must use symmetry argument explicitly.

(c) Expected number $= 200 \times 0.10 = 20$ candidates **A1**

Q30 — Blood pressure, borderline classification [HARD]

(a) Using GDC: normalcdf$(110,\ 130,\ 118,\ 11)$ **M1**

$= 0.72792\ldots \approx 0.728$ **A1**

(b) Using GDC: normalcdf$(130,\ 145,\ 118,\ 11)$ **M1**

$= 0.11967\ldots \approx 0.120$ **A1**

(c) Expected number $= 60 \times 0.11967\ldots = 7.180\ldots \approx 7$ patients **A1**