IB Math AA HL — Topic 1

Complex Numbers

Introduction · Operations · Argand Diagrams · Modulus & Argument

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

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Name:___________________________________ Date:__________________

Score:___________/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

Key Formulae

- Imaginary unit: $i^2 = -1$; powers cycle: $i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$.
- Arithmetic: $(a+bi) \pm (c+di) = (a \pm c) + (b \pm d)i$; $\quad (a+bi)(c+di) = (ac-bd) + (ad+bc)i$.
- Division: $\dfrac{a+bi}{c+di} = \dfrac{(a+bi)(c-di)}{c^2+d^2}$ (multiply by the conjugate).
- Conjugate: $\bar{z} = a - bi$; $\quad z\bar{z} = |z|^2 = a^2 + b^2$; $\quad z + \bar{z} = 2\,\mathrm{Re}(z)$.
- Modulus: $|z| = \sqrt{a^2+b^2}$; $\quad |zw| = |z||w|$; $\quad |z-w|$ is the distance between z and w on the Argand diagram.
- Principal argument: $\arg(z) \in (-\pi, \pi]$; identify the quadrant first, then $\theta = \arctan(b/a)$ adjusted to that quadrant.
- Real-coefficient quadratic: complex roots occur in conjugate pairs; if $z = a+bi$ is a root then $\bar{z} = a - bi$ is also a root.

GDC Tips

- **Complex arithmetic:** enter $a + bi$ directly in the `cplx` (or `CMPLX`) menu; use `conj(`, `abs(`, and `angle(` for conjugate, modulus, and argument in one step.
- **Argument output:** the GDC returns `angle(z)` in radians by default; ensure RAD mode is set and verify the sign matches the quadrant of your Argand sketch before writing the answer.
- **Polar form check:** use Rect $\rightarrow$ Polar (or ▶Polar) to convert a rectangular result instantly; round modulus and argument to 3 s.f. as required.
- **Solving cubics/quartics:** use `polyRoots(` or `cSolve(` to find all complex roots to 3 s.f.; list real and imaginary parts separately when the question asks you to state the roots.
- **Numerical power check:** compute z^n directly with `^` in `cplx` mode to verify a "show that" result or check your exact De Moivre working against a decimal answer.
- **Distance** $|z - w|$: compute `abs(z - w)` rather than expanding by hand; this avoids sign errors when coordinates involve negatives.

Common Mistakes to Avoid

1. **Wrong quadrant for the argument.** Students write arg$(z) = $ arctan(b/a) without checking the quadrant; a negative real part requires adding or subtracting π. Always sketch the point first.
2. **Argument outside the principal range.** Giving arg$(z) = \dfrac{5\pi}{4}$ instead of $-\dfrac{3\pi}{4}$ loses the mark; the principal argument must satisfy $-\pi < \theta \leq \pi$.
3. **Forgetting to multiply by the conjugate when dividing.** Dividing $\dfrac{a+bi}{c+di}$ by splitting real and imaginary parts separately gives wrong results; always multiply numerator and denominator by $c - di$.
4. **Sign error in the conjugate.** Writing $\overline{a+bi} = a + bi$ or $-a - bi$ instead of $a - bi$; remember only the sign of the imaginary part changes.
5. **Argand axes labelled x and y.** The axes must be labelled Re(z) and Im(z); using x/y or leaving axes unlabelled costs a mark.
6. **Assuming $|z|^2 = a^2 - b^2$.** Confusing $z^2 = (a^2 - b^2) + 2abi$ with $|z|^2$; the modulus squared is always $a^2 + b^2$ (sum, not difference).

Section A — Easy EASY (10 questions $\times$ 3 marks = 30 marks)

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1. EASY Calculator **[3 marks]**

Let $z = 5 + 2i$ and $w = 1 - 3i$, where $i^2 = -1$. Write down the values of $z + w$ and $z \cdot \bar{w}$, where $\bar{w}$ denotes the complex conjugate of w.

2. EASY No Calculator **[3 marks]**

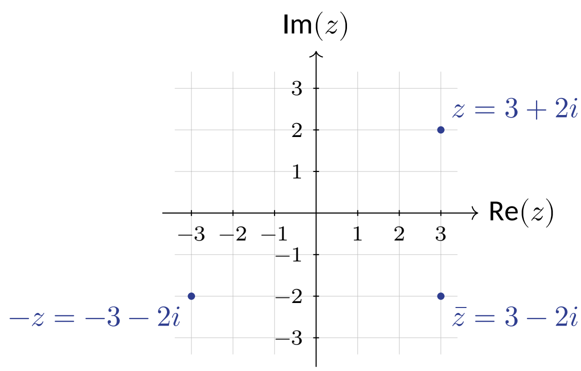
Let $z = -3 + 4i$, where $i^2 = -1$. Find the exact modulus $|z|$ and the principal argument $\arg(z)$ in radians, identifying the quadrant in which z lies.

3. EASY No Calculator **[3 marks]**

The complex number $z = a + bi$, where $a, b \in \mathbb{R}$, satisfies the equation $z + 2\bar{z} = 9 + 4i$. Find the values of a and b.

4. **EASY** **Calculator** **[3 marks]**

The diagram below shows the Argand diagram for a complex number $z = 3 + 2i$. The points z, $\bar{z}$, and $-z$ are plotted and labelled.



Write down the value of $z + \bar{z}$ and the value of $z \cdot \bar{z}$.

5. **EASY** **No Calculator** **[3 marks]**

The complex number $z = 4 - 4i$, where $i^2 = -1$. Find the exact modulus $|z|$ and the principal argument arg(z), in radians. Identify the quadrant in which z lies to justify your answer.

6. **EASY** **Calculator** **[3 marks]**

A quadratic equation with real coefficients has one root $z = 2 + 5i$, where $i^2 = -1$. Write down the other root w, and hence find the quadratic equation in the form $x^2 + bx + c = 0$, where $b, c \in \mathbb{R}$.

7. EASY Calculator **[3 marks]**

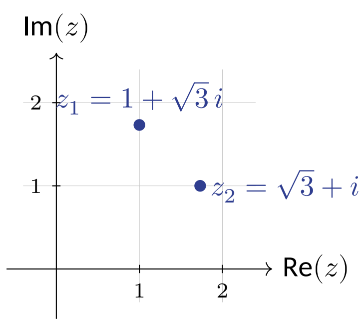
Let $z = 3 - i$ and $w = 2 + 4i$, where $i^2 = -1$. Find $|z - w|$, giving your answer correct to 3 significant figures.

8. EASY Calculator **[3 marks]**

The complex number z satisfies $(2 + i)z = 7 + i$, where $i^2 = -1$. Find z in the form $a + bi$, where $a, b \in \mathbb{R}$.

9. EASY No Calculator **[3 marks]**

Let $z_1 = 1 + \sqrt{3}\,i$ and $z_2 = \sqrt{3} + i$, where $i^2 = -1$. The diagram shows z_1 and z_2 plotted on an Argand diagram.



Find the exact value of $|z_1|$ and the exact value of $|z_2|$, and hence write down the value of $|z_1 \cdot z_2|$.

10. EASY Calculator **[3 marks]**

A complex number $z = x + iy$, where $x, y \in \mathbb{R}$, satisfies $z^2 = -5 + 12i$. Find the values of x and y by equating real and imaginary parts, and hence write down the two square roots of $-5 + 12i$.

Section B — Medium MEDIUM (10 questions × 4 marks = 40 marks)

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11. MEDIUM No Calculator **[4 marks]**

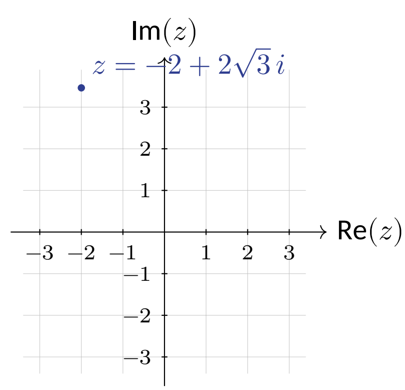
Let $z = a + bi$ where $a, b \in \mathbb{R}$. Given that $z^2 = -5 + 12i$, find the values of a and b.

12. MEDIUM Calculator **[4 marks]**

Let $z = 3 - 4i$ and $w = 1 + 2i$, where $i^2 = -1$. Find $\dfrac{z}{w}$, giving your answer in the form $p + qi$ where $p, q \in \mathbb{R}$.

13. MEDIUM No Calculator **[4 marks]**

The complex number $z = -2 + 2\sqrt{3}\,i$, where $i^2 = -1$. Find the exact modulus and principal argument of z, stating the quadrant of z in the Argand diagram.

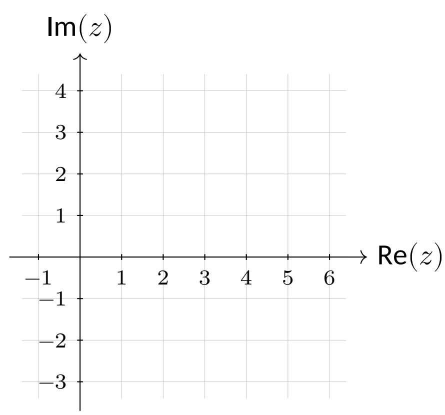


14. MEDIUM Calculator **[4 marks]**

Let $z = x + yi$ where $x, y \in \mathbb{R}$. Given that $\dfrac{z}{z+2} = i$, find the values of x and y.

15. MEDIUM Calculator **[4 marks]**

The complex numbers $z_1 = 4 + 3i$ and $z_2 = 1 - 2i$, where $i^2 = -1$. On the Argand diagram below, plot and label the points representing z_1, z_2, and $z_1 + z_2$. Hence state the geometric relationship between these three points.



16. MEDIUM Calculator **[4 marks]**

A quadratic equation with real coefficients has one root $z = 3 + 2i$, where $i^2 = -1$. Write down the other root, and hence find the quadratic equation in the form $x^2 + bx + c = 0$ where $b, c \in \mathbb{R}$.

17. MEDIUM Calculator **[4 marks]**

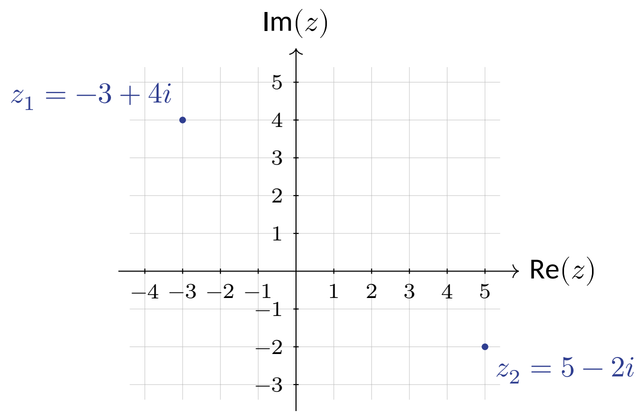
Let $z = 5 - 12i$, where $i^2 = -1$. Find $|z|$ and the principal argument of z in radians to three significant figures, clearly identifying the quadrant of z.

18. MEDIUM No Calculator **[4 marks]**

Let $z = 1 + \sqrt{3}\,i$, where $i^2 = -1$. Show that $z^3 = -8$.

19. MEDIUM Calculator **[4 marks]**

The complex numbers $z_1 = -3 + 4i$ and $z_2 = 5 - 2i$, where $i^2 = -1$. Find $|z_1 - z_2|$ and interpret this value geometrically in the Argand diagram.



20. MEDIUM Calculator **[4 marks]**

Let $z = a + bi$ where $a, b \in \mathbb{R}$ and $b \neq 0$. Given that $z + \bar{z} = 10$ and $z\bar{z} = 34$, find the two possible values of z.

Section C — Hard **HARD** (10 questions $\times$ 5 marks = 50 marks)

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21. **HARD** **No Calculator** **[5 marks]**

Let $z = a + bi$ where $a, b \in \mathbb{R}$. Given that $z^2 = -5 + 12i$, find the two square roots of $-5 + 12i$, giving your answers in the form $x + yi$ where $x, y \in \mathbb{R}$.

22. **HARD** **Calculator** **[5 marks]**

The complex number $w = 3 - 4i$.

(a) Find $|w|$ and $\arg(w)$, giving the argument in radians to 3 significant figures, identifying the quadrant. [3]

(b) Write w^3 in the form $a + bi$ where $a, b \in \mathbb{R}$, giving a and b to the nearest integer. [2]

23. HARD No Calculator **[5 marks]**

Let $z = p + qi$ and $w = 2 + 3i$ where $p, q \in \mathbb{R}$. Given that $\frac{z}{w} = 1 - i$, find the values of p and q.

24. HARD Calculator **[5 marks]**

The quadratic equation $2x^2 - 6x + k = 0$, where $k \in \mathbb{R}$, has roots α and β. Given that $\alpha = \frac{3}{2} + 2i$, find the value of k and verify that $\alpha\beta = \frac{k}{2}$.

25. HARD No Calculator **[5 marks]**

Let $z_1 = 1 + \sqrt{3}\,i$ and $z_2 = \sqrt{3} + i$.

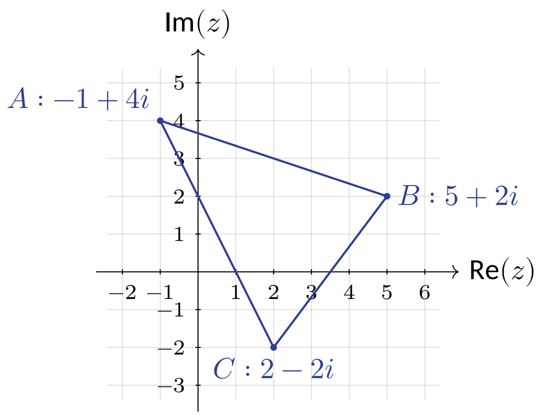
(a) Show that $|z_1| = |z_2| = 2$. [1]

(b) Find arg(z_1) and arg(z_2), identifying each quadrant. [2]

(c) Hence find the exact modulus and argument of $\frac{z_1}{z_2}$. [2]

26. HARD Calculator **[5 marks]**

The points A, B, and C on an Argand diagram represent the complex numbers $z_1 = -1 + 4i$, $z_2 = 5 + 2i$, and $z_3 = 2 - 2i$ respectively. Find $|z_1 - z_3|$ and $|z_2 - z_3|$, and hence determine whether triangle ABC is isosceles, scalene, or right-angled, justifying your answer.



27. HARD No Calculator **[5 marks]**

Let $z = r(\cos\theta + i\sin\theta)$ where $r > 0$ and $\theta \in (-\pi, \pi]$. Given that $z + \bar{z} = \sqrt{6}$ and $z\bar{z} = 3$, find the exact value of r and all possible values of θ, identifying each quadrant.

28. HARD Calculator **[5 marks]**

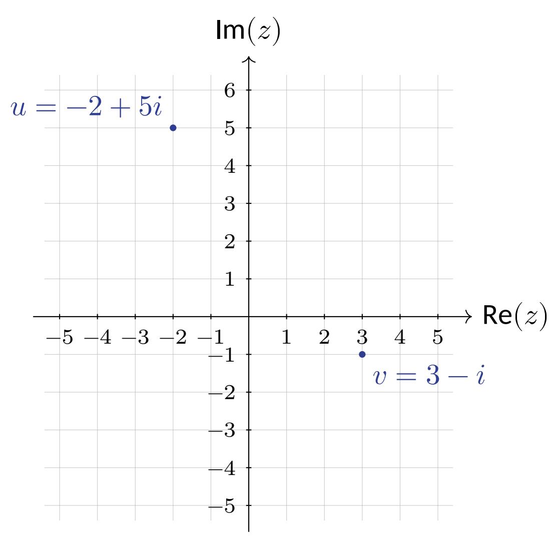
An electrical circuit has impedance $Z_1 = 4.7 + 3.2i$ ohms and $Z_2 = 2.1 - 5.8i$ ohms connected in parallel, so the combined impedance satisfies $\dfrac{1}{Z} = \dfrac{1}{Z_1} + \dfrac{1}{Z_2}$. Find $|Z|$ to 3 significant figures.

29. HARD Calculator **[5 marks]**

Let $z = x + yi$ where $x, y \in \mathbb{R}$. Given that $z^2 + 2\bar{z} = 3 + 4i$, find all complex numbers z that satisfy this equation.

30. HARD Calculator **[5 marks]**

The complex numbers $u = -2 + 5i$ and $v = 3 - i$ are shown on the Argand diagram below. On the same diagram, plot and label the points representing $u + v$, $u - v$, and $\bar{u}$. Find the exact distance between $u + v$ and $u - v$.



Mark Schemes

Q1 — Sum and Product with Conjugate EASY

$\bar{w} = 1 + 3i$ **(A1)**

$z + w = (5 + 1) + (2 - 3)i = 6 - i$ **A1**

$z \cdot \bar{w} = (5 + 2i)(1 + 3i) = 5 + 15i + 2i + 6i^2 = 5 + 17i - 6 = -1 + 17i$ **A1**

Q2 — Exact Modulus and Argument EASY

$|z| = \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$ **A1**

z lies in the second quadrant (negative real part, positive imaginary part). **R1**

$\arg(z) = \pi - \arctan(4/3)$ **A1**

Note: Award A0 if the quadrant reasoning is absent.

Q3 — Equating Real and Imaginary Parts EASY

$z + 2\bar{z} = (a + bi) + 2(a - bi) = 3a + (-b)i = 9 + 4i$ **M1**

Equating real parts: $3a = 9 \implies a = 3$; equating imaginary parts: $-b = 4 \implies b = -4$ **A1A1**

Note: Award M1 for correctly substituting $\bar{z} = a - bi$ and expanding.

Q4 — Conjugate Properties from Argand Diagram EASY

$z + \bar{z} = (3 + 2i) + (3 - 2i) = 6$ **A1**

$z \cdot \bar{z} = |z|^2 = 3^2 + 2^2 = 9 + 4 = 13$ **M1A1**

Q5 — Exact Modulus and Argument — Fourth Quadrant EASY

$|z| = \sqrt{4^2 + (-4)^2} = \sqrt{32} = 4\sqrt{2}$ **A1**

z lies in the fourth quadrant (positive real part, negative imaginary part), so the principal argument is negative. **R1**

$\arg(z) = -\arctan(4/4) = -\pi/4$ **A1**

Note: GDC check: sqrt(2) ≈ 1.41421; -pi/4 ≈ -0.785 rad. Award A0 for $\arg(z) = \pi/4$ without the negative sign if quadrant not identified.

Q6 — Real-Coefficient Quadratic from Complex Root EASY

Since coefficients are real, the other root is $w = 2 - 5i$ (complex conjugate). **A1**

Sum of roots $= (2+5i) + (2-5i) = 4$, so $b = -4$. **M1**

Product of roots $= (2+5i)(2-5i) = 4 + 25 = 29$, so $c = 29$. Equation: $x^2 - 4x + 29 = 0$ **A1**

Q7 — Modulus as Distance EASY

$z - w = (3 - i) - (2 + 4i) = 1 - 5i$ **M1**

$|z - w| = \sqrt{1^2 + (-5)^2} = \sqrt{26}$ (unrounded: $5.09901\ldots$) **(A1)**

$|z - w| = 5.10$ (3 s.f.) **A1**

Q8 — Division by Conjugate EASY

$z = \dfrac{7+i}{2+i}$; multiply numerator and denominator by the conjugate $(2 - i)$: **M1**

$z = \dfrac{(7+i)(2-i)}{(2+i)(2-i)} = \dfrac{14 - 7i + 2i - i^2}{4+1} = \dfrac{14 - 5i + 1}{5} = \dfrac{15 - 5i}{5}$ **(A1)**

$z = 3 - i$ **A1**

Q9 — Moduli and Product of Moduli EASY

$|z_1| = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = 2$ **A1**

$|z_2| = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3+1} = 2$ **A1**

$|z_1 \cdot z_2| = |z_1| \cdot |z_2| = 2 \times 2 = 4$ **A1**

Q10 — Square Root via Equating Parts EASY

$(x + iy)^2 = x^2 - y^2 + 2xyi = -5 + 12i$, so $x^2 - y^2 = -5$ and $2xy = 12 \implies y = 6/x$. **M1**

Substituting: $x^2 - \dfrac{36}{x^2} = -5 \implies x^4 + 5x^2 - 36 = 0 \implies (x^2 - 4)(x^2 + 9) = 0 \implies x = \pm 2$; GDC confirms $x^2 = 4$. **(A1)**

$x = 2, y = 3$ or $x = -2, y = -3$; the two square roots are $2 + 3i$ and $-2 - 3i$. **A1**

Q11 — Square Root of a Complex Number MEDIUM

Expanding $(a+bi)^2 = a^2 - b^2 + 2abi$ and equating to $-5+12i$: **M1**

Real part: $a^2 - b^2 = -5$; imaginary part: $2ab = 12$, so $b = 6/a$. **A1**

Substituting: $a^2 - \frac{36}{a^2} = -5 \implies a^4 + 5a^2 - 36 = 0 \implies (a^2-4)(a^2+9) = 0$. **M1**

Since $a \in \mathbb{R}$, $a^2 = 4$, giving $a = 2, b = 3$ or $a = -2, b = -3$. **A1**

Q12 — Division by Conjugate MEDIUM

Multiply numerator and denominator by the conjugate of w: $\frac{(3-4i)(1-2i)}{(1+2i)(1-2i)}$. **M1**

Denominator: $1^2 + 2^2 = 5$. **A1**

Numerator: $3 - 6i - 4i + 8i^2 = 3 - 10i - 8 = -5 - 10i$. **M1**

$\frac{z}{w} = \frac{-5-10i}{5} = -1 - 2i$ **A1**

Q13 — Exact Modulus and Argument — Quadrant Stated MEDIUM

$|z| = \sqrt{(-2)^2 + (2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4$. **A1**

The reference angle: $\arctan\left(\frac{2\sqrt{3}}{2}\right) = \arctan(\sqrt{3}) = \pi/3$. **M1**

z has negative real part and positive imaginary part, so it lies in the second quadrant. **R1**

Therefore $\arg(z) = \pi - \pi/3 = 2\pi/3$. **A1**

Q14 — Equality of Complex Expressions MEDIUM

Write $z + 2 = (x+2) + yi$. Then $\frac{z}{z+2} = i$ gives $z = i(z+2)$. **M1**

$x + yi = i(x + 2 + yi) = (x+2)i + yi^2 = -y + (x+2)i$. **M1**

Equating real parts: $x = -y$. Equating imaginary parts: $y = x + 2$. **A1**

Solving: substituting $y = -x$ into $y = x + 2$: $-x = x + 2$, so $x = -1$, $y = 1$. **A1**

Q15 — Argand Diagram: Plotting Sum MEDIUM

$z_1 + z_2 = (4+1) + (3+(-2))i = 5+i$. **A1**

Points plotted and labelled correctly: $z_1 = 4+3i$, $z_2 = 1-2i$, $z_1+z_2 = 5+i$. **A1**

The three points O, z_1, z_2, z_1+z_2 form a parallelogram (vector addition). **M1**

z_1+z_2 is the fourth vertex of the parallelogram with vertices at the origin, z_1, and z_2. **R1**

Q16 — Real-Coefficient Quadratic from Complex Root MEDIUM

Since the coefficients are real, the other root is the complex conjugate $\bar{z} = 3-2i$. **A1**

Sum of roots: $(3+2i)+(3-2i) = 6$, so $b = -6$. **M1**

Product of roots: $(3+2i)(3-2i) = 9+4 = 13$, so $c = 13$. **A1**

The quadratic equation is $x^2 - 6x + 13 = 0$. **A1**

Q17 — Decimal Modulus and Principal Argument MEDIUM

$|z| = \sqrt{5^2 + (-12)^2} = \sqrt{25+144} = \sqrt{169} = 13$. **A1**

z has positive real part and negative imaginary part, so it lies in the fourth quadrant. **R1**

GDC: arctan$(-12/5)$; unrounded value $= -1.17600\ldots$ rad. **M1**

Principal argument arg$(z) = -1.18$ rad (3 s.f.). **A1**

Q18 — Power of a Complex Number — Show That MEDIUM

$z^2 = (1+\sqrt{3}\,i)^2 = 1 + 2\sqrt{3}\,i + 3i^2 = 1 + 2\sqrt{3}\,i - 3 = -2 + 2\sqrt{3}\,i$. **M1A1**

$z^3 = z^2 \cdot z = (-2+2\sqrt{3}\,i)(1+\sqrt{3}\,i)$. **M1**

$= -2 - 2\sqrt{3}\,i + 2\sqrt{3}\,i + 2\cdot 3\cdot i^2 = -2 + 0i - 6 = -8$. **A1AG**

Q19 — Modulus as Distance in Argand Diagram MEDIUM

$z_1 - z_2 = (-3-5) + (4-(-2))i = -8+6i$. **M1**

$|z_1 - z_2| = \sqrt{(-8)^2 + 6^2} = \sqrt{64+36} = \sqrt{100} = 10$. **A1**

Note: Accept GDC computation confirming $\sqrt{100} = 10$.

$|z_1 - z_2|$ represents the straight-line distance between the points $z_1 = (-3,4)$ and $z_2 = (5,-2)$ in the Argand diagram. **M1R1**

Q20 — Conjugate Properties to Find z MEDIUM

$z + \bar{z} = 2a = 10 \implies a = 5.$ **A1**

$z\bar{z} = |z|^2 = a^2 + b^2 = 34$, so $25 + b^2 = 34 \implies b^2 = 9 \implies b = \pm 3.$ **M1A1**

Since $b \neq 0$ both solutions are valid: $z = 5 + 3i$ or $z = 5 - 3i$. **A1**

Q21 — Square Roots of a Complex Number HARD

Setting $(a + bi)^2 = -5 + 12i$ with $a, b \in \mathbb{R}$: $a^2 - b^2 = -5$ and $2ab = 12$, so $b = 6/a$. **M1**

Substituting: $a^2 - \dfrac{36}{a^2} = -5$, giving $a^4 + 5a^2 - 36 = 0.$ **M1**

$(a^2 - 4)(a^2 + 9) = 0$; since $a \in \mathbb{R}$, $a^2 = 4$, so $a = \pm 2$. **A1**

When $a = 2$: $b = 3$; when $a = -2$: $b = -3$. **A1**

The two square roots are $2 + 3i$ and $-2 - 3i$. **A1**

Q22 — Modulus and Argument and Power of a Complex Number HARD

(a) $|w| = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5$ **A1**

$w = 3 - 4i$ lies in the fourth quadrant (positive real part, negative imaginary part).

GDC: arctan$(-4/3) = -0.92729\ldots$ rad

arg$(w) = -0.927$ rad (3 s.f.) **A1A1**

Note: Award A1 for $|w| = 5$, *A1 for quadrant identified, A1 for* arg$(w) = -0.927$.

(b) GDC: $(3 - 4i)^3 = 27 - 108i + 144i^2 - 64i^3 = 27 - 108i - 144 + 64i = -117 - 44i$

$w^3 = -117 - 44i$ **A1A1**

Q23 — Division by Conjugate to Find Real Unknowns HARD

Since $\dfrac{z}{w} = 1 - i$, we have $z = (1 - i)(2 + 3i)$. **M1**

Expanding: $z = 2 + 3i - 2i - 3i^2 = 2 + 3i - 2i + 3.$ **M1**

$z = 5 + i$ **A1**

Therefore $p = 5$ **A1**

and $q = 1$ **A1**

Q24 — Quadratic with Complex Roots and Product Verification HARD

Since coefficients are real, the other root is $\beta = 3/2 - 2i$. **A1**

Product of roots: $\alpha\beta = (3/2 + 2i)(3/2 - 2i) = 9/4 + 4 = 25/4$. **M1A1**

By Vieta's formulas: $\alpha\beta = k/2$, so $k = 2 \times 25/4 = 25/2$. **A1**

Verification: $k/2 = (25/2)/2 = 25/4 = \alpha\beta$ ✓ **R1**

Note: The R1 is awarded for an explicit comparison statement confirming equality.

Q25 — Modulus and Argument of a Quotient HARD

(a) $|z_1| = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$ **(A1)**

$|z_2| = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$ **AG**

(b) $z_1 = 1 + \sqrt{3}\,i$ lies in the first quadrant; $\arg(z_1) = \arctan(\sqrt{3}/1) = \pi/3$ **A1**

$z_2 = \sqrt{3} + i$ lies in the first quadrant; $\arg(z_2) = \arctan(1/\sqrt{3}) = \pi/6$ **A1**

(c) $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|} = \dfrac{2}{2} = 1$ **A1**

$\arg\left(\dfrac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2) = \pi/3 - \pi/6 = \pi/6$ **A1**

Q26 — Triangle Classification via Distances HARD

$z_1 - z_3 = (-1-2) + (4-(-2))i = -3 + 6i$ **M1**

$|z_1 - z_3| = \sqrt{(-3)^2 + 6^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$ **A1**

$z_2 - z_3 = (5-2) + (2-(-2))i = 3 + 4i$ **(M1)**

$|z_2 - z_3| = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$ **A1**

Also $|z_1 - z_2| = |(-1-5) + (4-2)i| = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$.

Check: $(3\sqrt{5})^2 = 45$, $5^2 = 25$, $(2\sqrt{10})^2 = 40$; since $25 + 40 = 65 \neq 45$, $45 + 25 = 70 \neq 40$, $45 + 40 = 85 \neq 25$, no right angle. All three side lengths are distinct ($3\sqrt{5} \neq 5 \neq 2\sqrt{10}$), so triangle ABC is scalene. **R1**

Note: R1 requires an explicit statement that all three sides are unequal and no Pythagorean triple is satisfied.

Q27 — Modulus and Argument from Conjugate Properties HARD

Using $z + \bar{z} = 2\,\mathrm{Re}(z) = 2r\cos\theta = \sqrt{6}$, so $r\cos\theta = \sqrt{6}/2$. **M1**

Using $z\bar{z} = |z|^2 = r^2 = 3$, so $r = \sqrt{3}$. **A1**

Then $\cos\theta = \dfrac{\sqrt{6}/2}{\sqrt{3}} = \dfrac{\sqrt{6}}{2\sqrt{3}} = \dfrac{\sqrt{2}}{2}$. **M1**

So $\theta = \pm\pi/4$. Since $r > 0$ and $\cos\theta > 0$, both values give Re$(z) > 0$; $\theta = \pi/4$ places z in the first quadrant and $\theta = -\pi/4$ places z in the fourth quadrant. **A1**

Both are valid since no constraint on Im(z) is given; $\theta = \pi/4$ or $\theta = -\pi/4$. **A1**

Note: GDC check: sqrt(3) ≈ 1.732; pi/4 ≈ 0.785 rad. Final A1 requires both values stated with quadrant identification.

Q28 — Parallel Impedance Combination HARD

$\dfrac{1}{Z} = \dfrac{1}{Z_1} + \dfrac{1}{Z_2} = \dfrac{Z_1 + Z_2}{Z_1Z_2}$, so $Z = \dfrac{Z_1Z_2}{Z_1 + Z_2}$. **M1**

GDC: $Z_1 + Z_2 = (4.7 + 2.1) + (3.2 - 5.8)i = 6.8 - 2.6i$

$Z_1Z_2 = (4.7+3.2i)(2.1-5.8i) = 9.87-27.26i+6.72i-18.56i^2 = 9.87+18.56+(-27.26+6.72)i = 28.43 - 20.54i$

$Z = \dfrac{28.43 - 20.54i}{6.8 - 2.6i}$ **A1**

Multiply numerator and denominator by the conjugate $6.8 + 2.6i$:

Denominator: $6.8^2 + 2.6^2 = 46.24 + 6.76 = 53.00$

Numerator: $(28.43)(6.8) + (20.54)(2.6) + [-(20.54)(6.8) + (28.43)(2.6)]i$

$= 193.324 + 53.404 + [-139.672 + 73.918]i = 246.728 - 65.754i$

$Z = \dfrac{246.728 - 65.754i}{53.00} \approx 4.6552 - 1.2406i$

$|Z| = \sqrt{4.6552^2 + 1.2406^2} = \sqrt{21.671 + 1.539} = \sqrt{23.210} = 4.8177\ldots$ **M1A1**

$|Z| = 4.82$ ohms (3 s.f.) **A1**

Q29 — Equation Combining z-Squared and Conjugate HARD

Let $z = x + yi$, $\bar{z} = x - yi$ with $x, y \in \mathbb{R}$.

$z^2 + 2\bar{z} = (x^2 - y^2 + 2x) + (2xy - 2y)i = 3 + 4i$ **M1**

Equating real parts: $x^2 - y^2 + 2x = 3$ **(A1)**

Equating imaginary parts: $2xy - 2y = 4$, so $y(x - 1) = 2$ **(A1)**

From the imaginary equation: $y = \dfrac{2}{x-1}$ (provided $x \neq 1$).

Substituting into the real equation: $x^2 - \dfrac{4}{(x-1)^2} + 2x = 3$

$(x^2 + 2x - 3)(x - 1)^2 = 4$

$(x + 3)(x - 1)^3 = 4$

GDC solve (`polyRoots` on $x^4 - 6x^2 + 8x - 7 = 0$, checked by back-substitution into the original equation): the two real roots are $x = 1.93253\ldots$ and $x = -3.05978\ldots$

When $x = 1.93253\ldots$: $y = 2/(x - 1) = 2.14470\ldots$

When $x = -3.05978\ldots$: $y = 2/(x - 1) = -0.49264\ldots$

$z \approx 1.93 + 2.14i$ or $z \approx -3.06 - 0.493i$ (3 s.f.) **M1A1A1**

Note: Award M1 for setting up the system by equating real and imaginary parts. Award A1 for each correct solution to 3 s.f., verified by back-substitution into $z^2 + 2\bar{z} = 3 + 4i$.

Q30 — Argand Diagram with Arithmetic and Distance HARD

$u + v = (-2 + 3) + (5 - 1)i = 1 + 4i$ **A1**

$u - v = (-2 - 3) + (5 - (-1))i = -5 + 6i$ **A1**

$\bar{u} = -2 - 5i$ **A1**

All three points correctly plotted and labelled on the diagram (awarded in question).

Distance between $u + v = 1 + 4i$ and $u - v = -5 + 6i$:

$(u + v) - (u - v) = 2v = 6 - 2i$ **M1**

$|2v| = \sqrt{6^2 + (-2)^2} = \sqrt{36 + 4} = \sqrt{40} = 2\sqrt{10}$ **A1**

Note: M1 for forming the difference of the two points; A1 for the exact answer $2\sqrt{10}$.