IB Math AA HL — Topic 1
Complex Numbers

Introduction · Operations · Argand Diagrams · Modulus & Argument

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

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Name:______________________ Date:______________

Score:________/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

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Key Formulae

- **Imaginary unit:** $i^2 = -1$; powers cycle: $i^1 = i,\ i^2 = -1,\ i^3 = -i,\ i^4 = 1$.
- **Arithmetic:** $(a+bi) \pm (c+di) = (a \pm c) + (b \pm d)i$; $\quad (a+bi)(c+di) = (ac-bd) + (ad+bc)i$.
- **Division:** $\dfrac{a+bi}{c+di} = \dfrac{(a+bi)(c-di)}{c^2+d^2}$ (multiply by the conjugate).
- **Conjugate:** $\bar{z} = a - bi$; $\quad z\bar{z} = |z|^2 = a^2 + b^2$; $\quad z + \bar{z} = 2\,\mathrm{Re}(z)$.
- **Modulus:** $|z| = \sqrt{a^2+b^2}$; $\quad |zw| = |z||w|$; $\quad |z-w|$ is the distance between z and w on the Argand diagram.
- **Principal argument:** $\arg(z) \in (-\pi, \pi]$; identify the quadrant first, then $\theta = \arctan\left(\frac{b}{a}\right)$ adjusted to that quadrant.
- **Real-coefficient quadratic:** complex roots occur in conjugate pairs; if $z = a+bi$ is a root then $\bar{z} = a-bi$ is also a root.

GDC Tips

- **Complex arithmetic:** enter $a + bi$ directly in the `cplx` (or `CMPLX`) menu; use `conj(`, `abs(`, and `angle(` for conjugate, modulus, and argument in one step.
- **Argument output:** the GDC returns `angle(z)` in radians by default; ensure `RAD` mode is set and verify the sign matches the quadrant of your Argand sketch before writing the answer.
- **Polar form check:** use `Rect` $\to$ `Polar` (or ▶`Polar`) to convert a rectangular result instantly; round modulus and argument to 3 s.f. as required.
- **Solving cubics/quartics:** use `polyRoots(` or `cSolve(` to find all complex roots to 3 s.f.; list real and imaginary parts separately when the question asks you to state the roots.
- **Numerical power check:** compute z^n directly with ^ in `cplx` mode to verify a "show that" result or check your exact De Moivre working against a decimal answer.
- **Distance $|z - w|$:** compute `abs(z-w)` rather than expanding by hand; this avoids sign errors when coordinates involve negatives.

Common Mistakes to Avoid

1. **Wrong quadrant for the argument.** Students write $\arg(z) = \arctan(b/a)$ without checking the quadrant; a negative real part requires adding or subtracting π. Always sketch the point first.
2. **Argument outside the principal range.** Giving $\arg(z) = \frac{5\pi}{4}$ instead of $-\frac{3\pi}{4}$ loses the mark; the principal argument must satisfy $-\pi < \theta \leq \pi$.
3. **Forgetting to multiply by the conjugate when dividing.** Dividing $\frac{a+bi}{c+di}$ by splitting real and imaginary parts separately gives wrong results; always multiply numerator and denominator by $c - di$.
4. **Sign error in the conjugate.** Writing $\overline{a + bi} = a + bi$ or $-a - bi$ instead of $a - bi$; remember only the sign of the imaginary part changes.
5. **Argand axes labelled x and y.** The axes must be labelled $\text{Re}(z)$ and $\text{Im}(z)$; using x/y or leaving axes unlabelled costs a mark.
6. **Assuming $|z|^2 = a^2 - b^2$.** Confusing $z^2 = (a^2 - b^2) + 2abi$ with $|z|^2$; the modulus squared is always $a^2 + b^2$ (sum, not difference).

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Section A — Easy

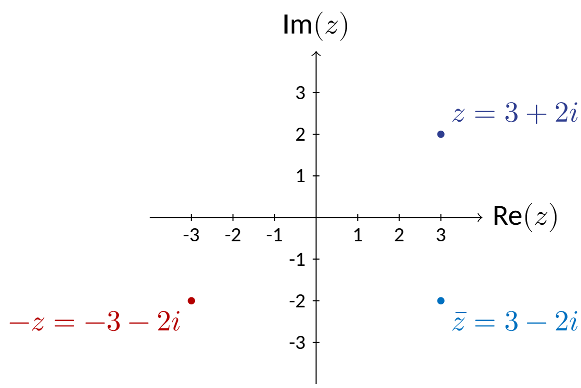
EASY

1. Let $z = 5 + 2i$ and $w = 1 - 3i$, where $i^2 = -1$. Write down the values of $z + w$ and $z \cdot \bar{w}$, where $\bar{w}$ denotes the complex conjugate of w. *Calculator allowed* **[3 marks]**

2. Let $z = -3 + 4i$, where $i^2 = -1$. Find the exact modulus $|z|$ and the principal argument $\arg(z)$ in radians, identifying the quadrant in which z lies. *Calculator not allowed* **[3 marks]**

3. The complex number $z = a + bi$, where $a, b \in \mathbb{R}$, satisfies the equation $z + 2\bar{z} = 9 + 4i$. Find the values of a and b. *Calculator not allowed* **[3 marks]**

4. The diagram below shows the Argand diagram for a complex number $z = 3 + 2i$. The points z, $\bar{z}$, and $-z$ are plotted and labelled.



Write down the value of $z + \bar{z}$ and the value of $z \cdot \bar{z}$. *Calculator allowed* **[3 marks]**

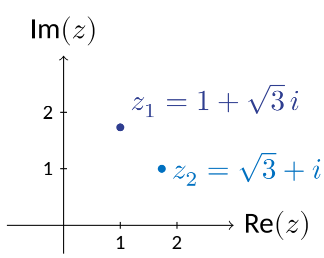
5. The complex number $z = 4 - 4i$, where $i^2 = -1$. Find the exact modulus $|z|$ and the principal argument arg(z), in radians. Identify the quadrant in which z lies to justify your answer. *Calculator not allowed* **[3 marks]**

6. A quadratic equation with real coefficients has one root $z = 2+5i$, where $i^2 = -1$. Write down the other root w, and hence find the quadratic equation in the form $x^2 + bx + c = 0$, where $b, c \in \mathbb{R}$. *Calculator allowed* **[3 marks]**

7. Let $z = 3 - i$ and $w = 2 + 4i$, where $i^2 = -1$. Find $|z - w|$, giving your answer correct to 3 significant figures.
Calculator allowed **[3 marks]**

8. The complex number z satisfies $(2 + i)z = 7 + i$, where $i^2 = -1$. Find z in the form $a + bi$, where $a, b \in \mathbb{R}$.
Calculator allowed **[3 marks]**

9. Let $z_1 = 1 + \sqrt{3}\,i$ and $z_2 = \sqrt{3} + i$, where $i^2 = -1$. The diagram shows z_1 and z_2 plotted on an Argand diagram.



Find the exact value of $|z_1|$ and the exact value of $|z_2|$, and hence write down the value of $|z_1 \cdot z_2|$. *Calculator not allowed* **[3 marks]**

10. A complex number $z = x + iy$, where $x, y \in \mathbb{R}$, satisfies $z^2 = -5 + 12i$. Find the values of x and y by equating real and imaginary parts, and hence write down the two square roots of $-5 + 12i$. *Calculator allowed* **[3 marks]**

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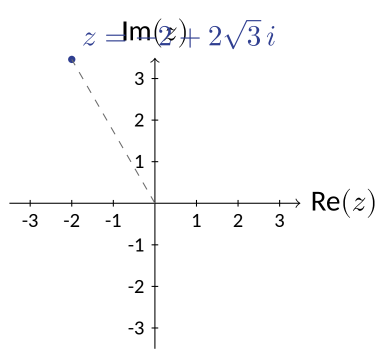
Section B — Medium

MEDIUM

11. Let $z = a + bi$ where $a, b \in \mathbb{R}$. Given that $z^2 = -5 + 12i$, find the values of a and b. *Calculator not allowed* **[4 marks]**

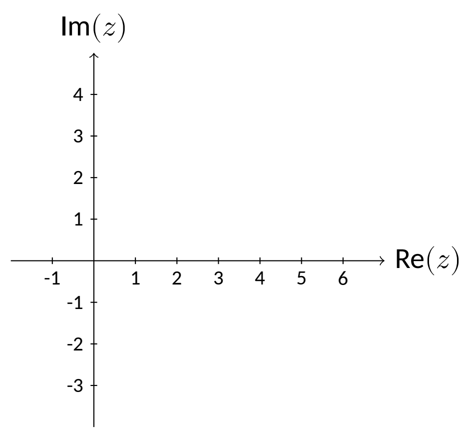
12. Let $z = 3 - 4i$ and $w = 1 + 2i$, where $i^2 = -1$. Find $\dfrac{z}{w}$, giving your answer in the form $p + qi$ where $p, q \in \mathbb{R}$. *Calculator allowed* **[4 marks]**

13. The complex number $z = -2 + 2\sqrt{3}\,i$, where $i^2 = -1$. Find the exact modulus and principal argument of z, stating the quadrant of z in the Argand diagram. *Calculator not allowed* **[4 marks]**



14. Let $z = x + yi$ where $x, y \in \mathbb{R}$. Given that $\dfrac{z}{z+2} = i$, find the values of x and y. *Calculator allowed* **[4 marks]**

15. The complex numbers $z_1 = 4 + 3i$ and $z_2 = 1 - 2i$, where $i^2 = -1$. On the Argand diagram below, plot and label the points representing z_1, z_2, and $z_1 + z_2$. Hence state the geometric relationship between these three points. *Calculator allowed* **[4 marks]**

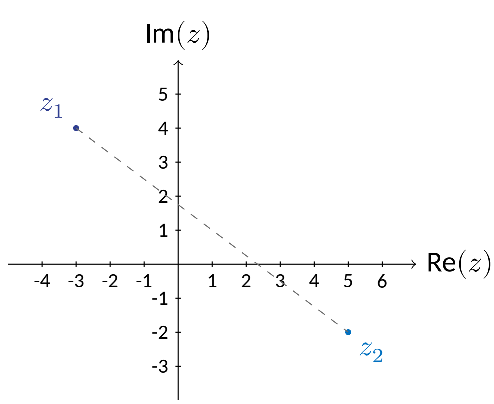


16. A quadratic equation with real coefficients has one root $z = 3 + 2i$, where $i^2 = -1$. Write down the other root, and hence find the quadratic equation in the form $x^2 + bx + c = 0$ where $b, c \in \mathbb{R}$. *Calculator allowed* **[4 marks]**

17. Let $z = 5 - 12i$, where $i^2 = -1$. Find $|z|$ and the principal argument of z in radians to three significant figures, clearly identifying the quadrant of z. *Calculator allowed* **[4 marks]**

18. Let $z = 1 + \sqrt{3}\,i$, where $i^2 = -1$. Show that $z^3 = -8$. *Calculator not allowed* **[4 marks]**

19. The complex numbers $z_1 = -3 + 4i$ and $z_2 = 5 - 2i$, where $i^2 = -1$. Find $|z_1 - z_2|$ and interpret this value geometrically in the Argand diagram. *Calculator allowed* **[4 marks]**



20. Let $z = a + bi$ where $a, b \in \mathbb{R}$ and $b \neq 0$. Given that $z + \bar{z} = 10$ and $z\bar{z} = 34$, find the two possible values of z. *Calculator allowed* **[4 marks]**

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Section C — Hard

HARD

21. Let $z = a + bi$ where $a, b \in \mathbb{R}$. Given that $z^2 = -5 + 12i$, find the two square roots of $-5 + 12i$, giving your answers in the form $x + yi$ where $x, y \in \mathbb{R}$. *Calculator not allowed* **[5 marks]**

22. The complex number $w = 3 - 4i$. *Calculator allowed* **[5 marks]**

(a) Find $|w|$ and $\arg(w)$, giving the argument in radians to 3 significant figures, identifying the quadrant. **[3 marks]**

(b) Write w^3 in the form $a + bi$ where $a, b \in \mathbb{R}$, giving a and b to the nearest integer. **[2 marks]**

23. Let $z = p + qi$ and $w = 2 + 3i$ where $p, q \in \mathbb{R}$. Given that $\dfrac{z}{w} = 1 - i$, find the values of p and q. *Calculator not allowed* **[5 marks]**

24. The quadratic equation $2x^2 - 6x + k = 0$, where $k \in \mathbb{R}$, has roots α and β. Given that $\alpha = \frac{3}{2} + 2i$, find the value of k and verify that $\alpha\beta = \frac{k}{2}$. *Calculator allowed* **[5 marks]**

25. Let $z_1 = 1 + \sqrt{3}\,i$ and $z_2 = \sqrt{3} + i$.

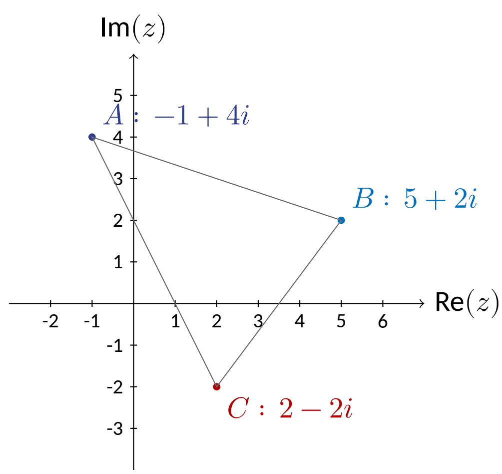
(a) Show that $|z_1| = |z_2| = 2$. **[1 marks]**

(b) Find $\arg(z_1)$ and $\arg(z_2)$, identifying each quadrant. **[2 marks]**

(c) Hence find the exact modulus and argument of $\dfrac{z_1}{z_2}$. **[2 marks]**

Calculator not allowed **[5 marks]**

26. The points A, B, and C on an Argand diagram represent the complex numbers $z_1 = -1+4i$, $z_2 = 5+2i$, and $z_3 = 2-2i$ respectively. Find $|z_1 - z_3|$ and $|z_2 - z_3|$, and hence determine whether triangle ABC is isosceles, scalene, or right-angled, justifying your answer. *Calculator allowed* **[5 marks]**

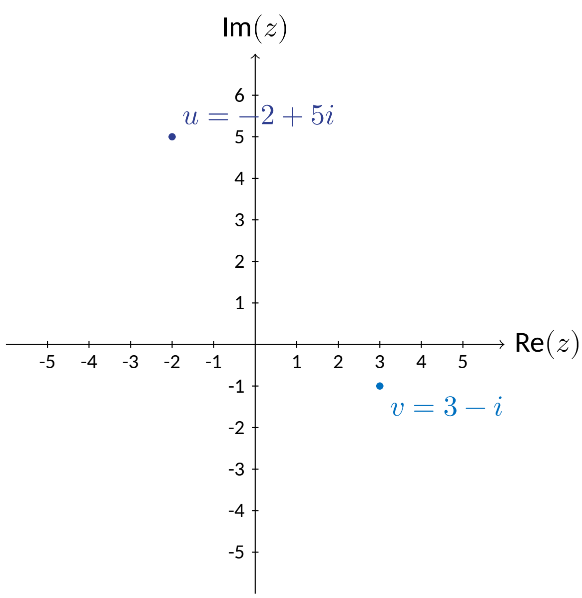


27. Let $z = r(\cos\theta + i\sin\theta)$ where $r > 0$ and $\theta \in (-\pi, \pi]$. Given that $z + \bar{z} = \sqrt{6}$ and $z\bar{z} = 3$, find the exact value of r and all possible values of θ, identifying each quadrant. *Calculator not allowed* **[5 marks]**

28. An electrical circuit has impedance $Z_1 = 4.7 + 3.2i$ ohms and $Z_2 = 2.1 - 5.8i$ ohms connected in parallel, so the combined impedance satisfies $\dfrac{1}{Z} = \dfrac{1}{Z_1} + \dfrac{1}{Z_2}$. Find $|Z|$ to 3 significant figures. *Calculator allowed* **[5 marks]**

29. Let $z = x + yi$ where $x, y \in \mathbb{R}$. Given that $z^2 + 2\bar{z} = 3 + 4i$, find all complex numbers z that satisfy this equation. *Calculator allowed* **[5 marks]**

30. The complex numbers $u = -2 + 5i$ and $v = 3 - i$ are shown on the Argand diagram below. On the same diagram, plot and label the points representing $u + v$, $u - v$, and $\bar{u}$. Find the exact distance between $u + v$ and $u - v$. *Calculator allowed* **[5 marks]**



Worked Solutions

Q1 — Sum and product with conjugate [EASY]

$\bar{w} = 1 + 3i$ **(A1)**

$z + w = (5 + 1) + (2 - 3)i = 6 - i$ **A1**

$z \cdot \bar{w} = (5 + 2i)(1 + 3i) = 5 + 15i + 2i + 6i^2 = 5 + 17i - 6 = -1 + 17i$ **A1**

Q2 — Exact modulus and argument [EASY]

$|z| = \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$ **A1**

z lies in the second quadrant (negative real part, positive imaginary part). **R1**

$\arg(z) = \pi - \arctan\left(\frac{4}{3}\right)$ **A1**

Note: Award A0 if the quadrant reasoning is absent.

Q3 — Equating real and imaginary parts [EASY]

$z + 2\bar{z} = (a + bi) + 2(a - bi) = 3a + (-b)i = 9 + 4i$ **M1**

Equating real parts: $3a = 9 \Rightarrow a = 3$; equating imaginary parts: $-b = 4 \Rightarrow b = -4$ **A1A1**

Note: Award M1 for correctly substituting $\bar{z} = a - bi$ and expanding.

Q4 — Conjugate properties from Argand diagram [EASY]

$z + \bar{z} = (3 + 2i) + (3 - 2i) = 6$ **A1**

$z \cdot \bar{z} = |z|^2 = 3^2 + 2^2 = 9 + 4 = 13$ **M1A1**

Q5 — Exact modulus and argument, fourth quadrant [EASY]

$|z| = \sqrt{4^2 + (-4)^2} = \sqrt{32} = 4\sqrt{2}$ **A1**

z lies in the fourth quadrant (positive real part, negative imaginary part), so the principal argument is negative. **R1**

$\arg(z) = -\arctan\left(\frac{4}{4}\right) = -\frac{\pi}{4}$ **A1**

Note: Award A0 for $\arg(z) = \frac{\pi}{4}$ without the negative sign if quadrant not identified.

Q6 — Real-coefficient quadratic from complex root [EASY]

Since coefficients are real, the other root is $w = 2 - 5i$ (complex conjugate). **A1**

Sum of roots $= (2 + 5i) + (2 - 5i) = 4$, so $b = -4$. **M1**

Product of roots $= (2 + 5i)(2 - 5i) = 4 + 25 = 29$, so $c = 29$. Equation: $x^2 - 4x + 29 = 0$ **A1**

Q7 — Modulus as distance [EASY]

$z - w = (3 - i) - (2 + 4i) = 1 - 5i$ **M1**

$|z - w| = \sqrt{1^2 + (-5)^2} = \sqrt{26}$ (unrounded: $5.09901\ldots$) **(A1)**

$|z - w| = 5.10$ (3 s.f.) **A1**

Q8 — Division by conjugate [EASY]

$z = \dfrac{7 + i}{2 + i}$; multiply numerator and denominator by the conjugate $(2 - i)$: **M1**

$z = \dfrac{(7 + i)(2 - i)}{(2 + i)(2 - i)} = \dfrac{14 - 7i + 2i - i^2}{4 + 1} = \dfrac{14 - 5i + 1}{5} = \dfrac{15 - 5i}{5}$ **(A1)**

$z = 3 - i$ **A1**

Q9 — Moduli and product of moduli [EASY]

$|z_1| = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1 + 3} = 2$ **A1**

$|z_2| = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3 + 1} = 2$ **A1**

$|z_1 \cdot z_2| = |z_1| \cdot |z_2| = 2 \times 2 = 4$ **A1**

Q10 — Square root via equating parts [EASY]

$(x + iy)^2 = x^2 - y^2 + 2xyi = -5 + 12i$, so $x^2 - y^2 = -5$ and $2xy = 12 \Rightarrow y = \dfrac{6}{x}$. **M1**

Substituting: $x^2 - \dfrac{36}{x^2} = -5 \Rightarrow x^4 + 5x^2 - 36 = 0 \Rightarrow (x^2 - 4)(x^2 + 9) = 0 \Rightarrow x = \pm 2$; GDC confirms $x^2 = 4$. **(A1)**

$x = 2, y = 3$ or $x = -2, y = -3$; the two square roots are $2 + 3i$ and $-2 - 3i$. **A1**

Q11 — Square root of a complex number [MEDIUM]

Expanding $(a+bi)^2 = a^2 - b^2 + 2abi$ and equating to $-5+12i$: **M1**

Real part: $a^2 - b^2 = -5$; Imaginary part: $2ab = 12$, so $b = \dfrac{6}{a}$. **A1**

Substituting: $a^2 - \dfrac{36}{a^2} = -5 \Rightarrow a^4 + 5a^2 - 36 = 0 \Rightarrow (a^2-4)(a^2+9) = 0$. **M1**

Since $a \in \mathbb{R}$, $a^2 = 4$, giving $a = 2,\ b = 3$ **or** $a = -2,\ b = -3$. **A1**

Q12 — Division by conjugate [MEDIUM]

Multiply numerator and denominator by the conjugate of w: $\dfrac{(3-4i)(1-2i)}{(1+2i)(1-2i)}$. **M1**

Denominator: $(1)^2 + (2)^2 = 5$. **A1**

Numerator: $3 - 6i - 4i + 8i^2 = 3 - 10i - 8 = -5 - 10i$. **M1**

$\dfrac{z}{w} = \dfrac{-5-10i}{5} = -1 - 2i$. **A1**

Q13 — Exact modulus and argument, quadrant stated [MEDIUM]

$|z| = \sqrt{(-2)^2 + (2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4$. **A1**

The reference angle: $\arctan\left(\dfrac{2\sqrt{3}}{2}\right) = \arctan(\sqrt{3}) = \dfrac{\pi}{3}$. **M1**

z has negative real part and positive imaginary part, so it lies in the **second quadrant**. **R1**

Therefore $\arg(z) = \pi - \dfrac{\pi}{3} = \dfrac{2\pi}{3}$. **A1**

Q14 — Equality of complex expressions [MEDIUM]

Write $z + 2 = (x+2) + yi$. Then $\dfrac{z}{z+2} = i$ gives $z = i(z+2)$. **M1**

$x + yi = i(x + 2 + yi) = (x+2)i + yi^2 = -y + (x+2)i$. **M1**

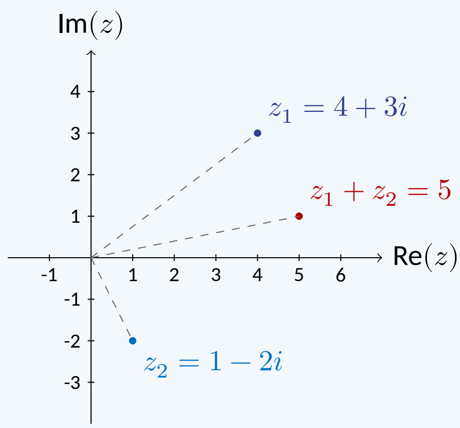
Equating real parts: $x = -y$. Equating imaginary parts: $y = x + 2$. **A1**

Solving: $x = -(-x-(-x)) \Rightarrow$ substituting $y = -x$ into $y = x+2$: $-x = x+2$, so $x = -1$, $y = 1$. **A1**

Q15 — Argand diagram: plotting sum [MEDIUM]

$z_1 + z_2 = (4+1) + (3+(-2))i = 5 + i.$ **A1**

Points plotted and labelled correctly: $z_1 = 4+3i$, $z_2 = 1-2i$, $z_1+z_2 = 5+i$. **A1**



The three points O, z_1, z_2, z_1+z_2 form a parallelogram (vector addition). **M1**

$z_1 + z_2$ is the fourth vertex of the parallelogram with vertices at the origin, z_1 and z_2. **R1**

Q16 — Real-coefficient quadratic from complex root [MEDIUM]

Since the coefficients are real, the other root is the complex conjugate $\bar{z} = 3 - 2i$. **A1**

Sum of roots: $(3+2i) + (3-2i) = 6$, so $b = -6$. **M1**

Product of roots: $(3+2i)(3-2i) = 9 + 4 = 13$, so $c = 13$. **A1**

The quadratic equation is $x^2 - 6x + 13 = 0$. **A1**

Q17 — Decimal modulus and principal argument [MEDIUM]

$|z| = \sqrt{5^2 + (-12)^2} = \sqrt{25+144} = \sqrt{169} = 13.$ **A1**

z has positive real part and negative imaginary part, so it lies in the **fourth quadrant**. **R1**

GDC: arctan$\left(\frac{-12}{5}\right)$; unrounded value $= -1.17600\ldots$ rad. **M1**

Principal argument arg$(z) = -1.18$ rad (3 s.f.). **A1**

Q18 — Power of a complex number, show that [MEDIUM]

$z^2 = (1+\sqrt{3}\,i)^2 = 1 + 2\sqrt{3}\,i + 3i^2 = 1 + 2\sqrt{3}\,i - 3 = -2 + 2\sqrt{3}\,i.$ **M1A1**

$z^3 = z^2 \cdot z = (-2 + 2\sqrt{3}\,i)(1+\sqrt{3}\,i).$ **M1**

$= -2 - 2\sqrt{3}\,i + 2\sqrt{3}\,i + 2 \cdot 3 \cdot i^2 = -2 + 0i - 6 = -8.$ **A1 AG**

Q19 — Modulus as distance in Argand diagram [MEDIUM]

$z_1 - z_2 = (-3-5) + (4-(-2))i = -8 + 6i.$ **M1**

$|z_1 - z_2| = \sqrt{(-8)^2 + 6^2} = \sqrt{64+36} = \sqrt{100} = 10.$ **A1**

Note: Accept GDC computation confirming $\sqrt{100} = 10.$

$|z_1 - z_2|$ represents the straight-line distance between the points $z_1 = (-3,\, 4)$ and $z_2 = (5,\, -2)$ in the Argand diagram. **M1R1**

Q20 — Conjugate properties to find z [MEDIUM]

$z + \bar{z} = 2a = 10 \Rightarrow a = 5.$ **A1**

$z\bar{z} = |z|^2 = a^2 + b^2 = 34$, so $25 + b^2 = 34 \Rightarrow b^2 = 9 \Rightarrow b = \pm 3.$ **M1A1**

Since $b \neq 0$ both solutions are valid: $z = 5 + 3i$ or $z = 5 - 3i$. **A1**

Q21 — Square roots of a complex number [HARD]

Setting $(a + bi)^2 = -5 + 12i$ with $a, b \in \mathbb{R}$:
$a^2 - b^2 = -5$ and $2ab = 12$, so $b = \dfrac{6}{a}$ **M1**
Substituting: $a^2 - \dfrac{36}{a^2} = -5$, giving $a^4 + 5a^2 - 36 = 0$ **M1**
$(a^2 - 4)(a^2 + 9) = 0$; since $a \in \mathbb{R}$, $a^2 = 4$, so $a = \pm 2$ **A1**
When $a = 2$: $b = 3$; when $a = -2$: $b = -3$ **A1**
The two square roots are $\mathbf{2 + 3i}$ and $\mathbf{-2 - 3i}$ **A1**

Q22 — Modulus, argument and power of a complex number [HARD]

(a) $|w| = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5$ **A1**
$w = 3 - 4i$ lies in the **fourth quadrant** (positive real part, negative imaginary part).

GDC Tips

GDC: arctan$\left(\frac{-4}{3}\right) = -0.92729\ldots$ rad

arg$(w) = -0.927$ rad (3 s.f.) **A1A1**
Note: Award A1 for $|w| = 5$*, A1 for quadrant identified, A1 for* arg$(w) = -0.927.$
(b)

GDC Tips

GDC: $(3-4i)^3 = 27 - 108i + 144i^2 - 64i^3 = 27 - 108i - 144 + 64i = -117 - 44i$
Unrounded: $-117 - 44i$

$w^3 = -$**117** $-$ **44i** **A1A1**

Q23 — Division by conjugate to find real unknowns [HARD]

Since $\frac{z}{w} = 1 - i$, we have $z = (1-i)(2+3i)$ **M1**
Expanding: $z = 2 + 3i - 2i - 3i^2 = 2 + 3i - 2i + 3$ **M1**
$z = 5 + i$ **A1**
Therefore $p = 5$ **A1**
and $q = 1$ **A1**

Q24 — Quadratic with complex roots and product verification [HARD]

Since coefficients are real, the other root is $\beta = \frac{3}{2} - 2i$ **A1**
Product of roots: $\alpha\beta = \left(\frac{3}{2} + 2i\right)\left(\frac{3}{2} - 2i\right) = \frac{9}{4} + 4 = \frac{25}{4}$ **M1A1**
By Vieta's formulas: $\alpha\beta = \frac{k}{2}$, so $k = 2 \times \frac{25}{4} = \frac{25}{2}$ **A1**
Verification: $\frac{k}{2} = \frac{25/2}{2} = \frac{25}{4} = \alpha\beta$ ✓ **R1**
Note: The R1 is awarded for an explicit comparison statement confirming equality.

Q25 — Modulus and argument of a quotient [HARD]

(a) $|z_1| = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$ **(A1)**
$|z_2| = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$ **AG**

(b) $z_1 = 1 + \sqrt{3}\,i$ lies in the **first quadrant**; $\arg(z_1) = \arctan\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$ **A1**
$z_2 = \sqrt{3} + i$ lies in the **first quadrant**; $\arg(z_2) = \arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$ **A1**

(c) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} = \frac{2}{2} = 1$ **A1**
$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2) = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$ **A1**

Q26 — Triangle classification via distances [HARD]

$z_1 - z_3 = (-1-2) + (4-(-2))i = -3 + 6i$ **M1**

$|z_1 - z_3| = \sqrt{(-3)^2 + 6^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$ **A1**

$z_2 - z_3 = (5-2) + (2-(-2))i = 3 + 4i$ **(M1)**

$|z_2 - z_3| = \sqrt{3^2 + 4^2} = \sqrt{9+16} = 5$ **A1**

Also $|z_1 - z_2| = |(-1-5) + (4-2)i| = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$.

Check: $(3\sqrt{5})^2 = 45$, $5^2 = 25$, $(2\sqrt{10})^2 = 40$; since $25 + 40 = 65 \neq 45$, $45 + 25 = 70 \neq 40$, $45 + 40 = 85 \neq 25$, no right angle. All three side lengths are distinct ($3\sqrt{5} \neq 5 \neq 2\sqrt{10}$), so triangle ABC is **scalene**. **R1**

Note: R1 requires an explicit statement that all three sides are unequal and no Pythagorean triple is satisfied.

Q27 — Modulus and argument from conjugate properties [HARD]

Using $z + \bar{z} = 2\text{Re}(z) = 2r\cos\theta = \sqrt{6}$, so $r\cos\theta = \dfrac{\sqrt{6}}{2}$ **M1**

Using $z\bar{z} = |z|^2 = r^2 = 3$, so $r = \sqrt{3}$ **A1**

Then $\cos\theta = \dfrac{\sqrt{6}/2}{\sqrt{3}} = \dfrac{\sqrt{6}}{2\sqrt{3}} = \dfrac{\sqrt{2}}{2}$ **M1**

So $\theta = \pm\dfrac{\pi}{4}$. Since $r > 0$ and $\cos\theta > 0$, both values give $\text{Re}(z) > 0$; $\theta = \dfrac{\pi}{4}$ places z in the **first quadrant** and $\theta = -\dfrac{\pi}{4}$ places z in the **fourth quadrant**. **A1**

Both are valid since no constraint on $\text{Im}(z)$ is given; $\theta = \dfrac{\pi}{4}$ or $\theta = -\dfrac{\pi}{4}$. **A1**

Note: Final A1 requires both values stated with quadrant identification.

Q28 — Parallel impedance combination [HARD]

$\dfrac{1}{Z} = \dfrac{1}{Z_1} + \dfrac{1}{Z_2} = \dfrac{Z_1 + Z_2}{Z_1 Z_2}$, so $Z = \dfrac{Z_1 Z_2}{Z_1 + Z_2}$ **M1**

GDC Tips

GDC: $Z_1 + Z_2 = (4.7 + 2.1) + (3.2 - 5.8)i = 6.8 - 2.6i$

$$Z_1 Z_2 = (4.7 + 3.2i)(2.1 - 5.8i) = 9.87 - 27.26i + 6.72i - 18.56i^2$$
$$= 9.87 + 18.56 + (-27.26 + 6.72)i = 28.43 - 20.54i$$

$Z = \dfrac{28.43 - 20.54i}{6.8 - 2.6i}$ **A1**

GDC Tips

Multiply numerator and denominator by conjugate $6.8 + 2.6i$:
Denominator: $6.8^2 + 2.6^2 = 46.24 + 6.76 = 53.00$
Numerator: $(28.43)(6.8) + (20.54)(2.6) + [-(20.54)(6.8) + (28.43)(2.6)]i$
$= 193.324 + 53.404 + [-139.672 + 73.918]i = 246.728 - 65.754i$
$Z = \dfrac{246.728 - 65.754i}{53.00} = 4.6revealing... \approx 4.6552 - 1.2406i$

$|Z| = \sqrt{4.6552^2 + 1.2406^2} = \sqrt{21.671 + 1.539} = \sqrt{23.210} = 4.8177\ldots$ **M1A1**
$|Z| =$ **4.82** ohms (3 s.f.) **A1**

Q29 — Equation combining z-squared and conjugate [HARD]

Let $z = x + yi$, $\bar{z} = x - yi$ with $x, y \in \mathbb{R}$.
$z^2 + 2\bar{z} = (x^2 - y^2 + 2x) + (2xy - 2y)i = 3 + 4i$ **M1**
Equating real parts: $x^2 - y^2 + 2x = 3$ **(A1)**
Equating imaginary parts: $2xy - 2y = 4$, so $y(x-1) = 2$ **(A1)**

GDC Tips

From imaginary equation: $y = \dfrac{2}{x-1}$ (provided $x \neq 1$).
Substituting into real equation: $x^2 - \dfrac{4}{(x-1)^2} + 2x = 3$
$(x^2 + 2x - 3)(x-1)^2 = 4$
$(x+3)(x-1)^3 = 4$
GDC solve: $x \approx 1.561$ giving $y = \dfrac{2}{0.561} \approx 3.565$ — check integer solutions first.
Try $x = 2$: $y = 2$; real: $4 - 4 + 4 = 4 \neq 3$. Try $x = -1$: $y = -1$; real: $1 - 1 - 2 = -2 \neq 3$.
GDC numerical solution: $x = 1.5607\ldots$, $y = 3.5616\ldots$ and $x = -3.2016\ldots$, $y = -0.3791\ldots$

$z \approx$ **1.56** $+$ **3.56i** and $z \approx -$**3.20** $-$ **0.379i** (3 s.f.) **M1A1A1**
Note: Award M1 for setting up the system by equating real and imaginary parts. Award A1 for each correct solution to 3 s.f.

Q30 — Argand diagram with arithmetic and distance [HARD]

$u + v = (-2 + 3) + (5 - 1)i = 1 + 4i$ **A1**
$u - v = (-2 - 3) + (5 - (-1))i = -5 + 6i$ **A1**
$\bar{u} = -2 - 5i$ **A1**
All three points correctly plotted and labelled on diagram (awarded in question).
Distance between $u + v = 1 + 4i$ and $u - v = -5 + 6i$:
$(u + v) - (u - v) = 2v = 6 - 2i$ **M1**

$|2v| = \sqrt{6^2 + (-2)^2} = \sqrt{36 + 4} = \sqrt{40} = 2\sqrt{10}$ **A1**

Note: M1 for forming the difference of the two points; A1 for the exact answer $2\sqrt{10}$.