IB Math AA SL — Topic 1

Number Toolkit

Standard Form · Laws of Indices

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:________________________ Date:____________

Score:________/120

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Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

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Key Formulae

- **Standard form:** $a \times 10^k$, where $1 \leq a < 10$, $k \in \mathbb{Z}$
- **Product / Quotient of powers:** $a^m \times a^n = a^{m+n}$; $\dfrac{a^m}{a^n} = a^{m-n}$
- **Power of a power:** $(a^m)^n = a^{mn}$
- **Negative exponent:** $a^{-n} = \dfrac{1}{a^n}$
- **Rational exponent:** $a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$
- **Zero exponent:** $a^0 = 1$, $a \neq 0$
- **Distributive index laws:** $(ab)^n = a^n b^n$; $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

GDC Tips

- **Entering standard form:** Use the `EE` (or $\times 10^x$) key rather than typing $\times$ `10` ^ separately to avoid order-of-operations errors.
- **Displaying exact standard form:** After a calculation, press `MATH` $\rightarrow$ ▶`Sci` (TI) or set `Fix/Sci` in the display mode to read off a and k directly.
- **Verifying index simplifications:** Store a numerical value for x (e.g. $x = 3$), evaluate both the original expression and your simplified form, and confirm they match to confirm no algebra error.
- **Solving index equations numerically:** Use `SOLVE` / `nSolve` or graph $y =$ LHS $-$ RHS and find the zero to check a base-matched or substitution answer.
- **Computing large ratios in context:** Enter both quantities in standard form and divide directly; read the result as a plain number or back in standard form for magnitude comparisons.
- **Rational exponents on the GDC:** Enter $a^{(m/n)}$ with the fraction in parentheses to ensure the GDC interprets $a^{1/3}$ as a cube root, not $(a^1)/3$.

Common Mistakes to Avoid

1. **Adding exponents when bases differ.** The law $a^m \times a^n = a^{m+n}$ requires identical bases. Writing $2^3 \times 3^2 = 6^5$ is wrong; only combine exponents when the base is the same.
2. **Mis-converting to standard form (a out of range).** A result such as 23.4×10^5 is *not* in standard form because $23.4 \geq 10$. Always adjust: $23.4 \times 10^5 = 2.34 \times 10^6$. Leaving an unconverted form loses the final A1.
3. **Rejecting the valid root of a substitution quadratic.** When solving equations of the form $2^{2x} - 5 \cdot 2^x + 4 = 0$ via $u = 2^x$, students reject both roots or keep the negative one. Since $2^x > 0$ for all real x, only discard $u \leq 0$; positive roots must be kept and the rejection must be stated explicitly.
4. **Incorrect order of operations with negative bases.** $(-2)^4 = 16$ but $-2^4 = -16$. Failing to bracket a negative base before applying an exponent is a persistent arithmetic trap, especially on the GDC where entry order matters.
5. **Confusing a^{-n} with $-a^n$.** A negative exponent means a reciprocal, not a sign change: $3^{-2} = \frac{1}{9}$, not -9. This error frequently surfaces when simplifying expressions left in index form.
6. **Wrong direction of inequality / rounding in magnitude comparisons.** When asked to express a ratio in standard form or to decide which quantity is larger, students round a incorrectly or truncate instead of rounding, violating the 3 s.f. (or demanded precision) requirement. The final A1 for units or precision compliance is lost if the answer is not given to the instructed degree of accuracy.

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Section A — Easy

EASY

1. Write $47\,300\,000$ in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$. *Calculator not allowed* **[3 marks]**

2. Write $0.000\,082\,4$ in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$. *Calculator not allowed* **[3 marks]**

3. The table below shows the approximate mass of two celestial objects. *Calculator allowed* **[3 marks]**

Object	Mass (kg)
Planet Kero	6.4×10^{23}
Star Vela	3.2×10^{30}

Find how many times greater the mass of Star Vela is than the mass of Planet Kero. Give your answer in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

4. Simplify $\dfrac{x^8 \times x^3}{x^4}$, giving your answer in the form x^p where $p \in \mathbb{Z}$. *Calculator not allowed* **[3 marks]**

5. Simplify $(2a^3b)^4$, giving your answer in the form $c\,a^m b^n$ where $c, m, n \in \mathbb{Z}$. *Calculator allowed* **[3 marks]**

6. Evaluate $125^{2/3}$. *Calculator not allowed* **[3 marks]**

7. Solve $2^x = 64$, where $x \in \mathbb{R}$. *Calculator allowed* **[3 marks]**

8. The table below shows the diameter of two microscopic particles. *Calculator allowed* **[3 marks]**

Particle	Diameter (m)
Particle A	4.5×10^{-7}
Particle B	9.0×10^{-9}

Find the ratio of the diameter of Particle A to the diameter of Particle B.

9. Write $\sqrt[4]{x^3}$ in the form x^p, where $p \in \mathbb{Q}$. Hence find the value of $16^{3/4}$. *Calculator allowed* **[3 marks]**

10. Solve $3^{2x-1} = 27$, where $x \in \mathbb{R}$. *Calculator allowed* **[3 marks]**

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Section B — Medium

MEDIUM

11. The mass of a dust particle is 3.2×10^{-9} kg and the mass of a grain of sand is 6.4×10^{-5} kg. Find how many times heavier the grain of sand is than the dust particle. Give your answer as an integer. *Calculator allowed* **[4 marks]**

12. Simplify $\dfrac{x^5 \cdot x^{-2}}{x^4}$, giving your answer in the form x^p where $p \in \mathbb{Z}$. *Calculator not allowed* **[4 marks]**

13. Solve $4^{2x-1} = 8^{x+1}$, where $x \in \mathbb{R}$. *Calculator not allowed* **[4 marks]**

14. The table below shows the distances of two planets from the Sun.

Planet	Distance from Sun (km)
Mercury	5.79×10^7
Saturn	1.43×10^9

(a) Find the difference in their distances from the Sun. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

(b) Find how many times farther Saturn is from the Sun than Mercury. Give your answer correct to 3 significant figures.

Calculator allowed **[4 marks]**

15. Let $f(x) = x^{3/2} - 4x^{1/2}$, where $x > 0$.

(a) Write $f(x)$ in the form $x^{1/2}(x - a)$, where a is a positive integer. State the value of a.

(b) Hence find the value of x for which $f(x) = 0$ and $x > 0$.

Calculator not allowed **[4 marks]**

16. Solve $9^x - 10 \cdot 3^x + 9 = 0$, where $x \in \mathbb{R}$. *Calculator allowed* **[4 marks]**

17. The number of bacteria in a sample is approximately 4.5×10^6. Each bacterium has a diameter of approximately 2.0×10^{-6} m.

(a) Find the total length, in metres, if all bacteria were placed end to end. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

(b) Write your answer to part (a) in kilometres.

Calculator allowed **[4 marks]**

18. Simplify $\dfrac{(2a^3b^{-1})^2}{4a^{-1}b^3}$, where $a > 0$ and $b > 0$. Give your answer in the form $a^m b^n$ where $m, n \in \mathbb{Z}$. *Calculator not allowed* **[4 marks]**

19. The speed of light is approximately 2.998×10^8 m s^{-1}. A star is approximately 4.12×10^{16} m from Earth.

(a) Find the time, in seconds, for light to travel from the star to Earth. Give your answer in the form $a \times 10^k$ where $1 \le a < 10$ and $k \in \mathbb{Z}$, with a correct to 3 significant figures.

(b) Hence find this time in years. Use 1 year $= 3.156 \times 10^7$ seconds. Give your answer correct to 3 significant figures.

Calculator allowed **[4 marks]**

20. Evaluate $\dfrac{27^{2/3} + 16^{3/4}}{5^0 + \left(\frac{1}{4}\right)^{-1}}$, giving your answer as an exact fraction. *Calculator not allowed* **[4 marks]**

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Section C — Hard

HARD

21. The mass of a proton is 1.673×10^{-27} kg and the mass of an electron is 9.109×10^{-31} kg. A neutral helium atom consists of 2 protons, 2 neutrons and 2 electrons. The mass of one neutron is 1.675×10^{-27} kg.

Find the total mass of a neutral helium atom, giving your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$. *Calculator allowed* **[5 marks]**

Particle	Mass (kg)	Count in He atom
Proton	1.673×10^{-27}	2
Neutron	1.675×10^{-27}	2
Electron	9.109×10^{-31}	2

22. Let $f(x) = 4^x - 17 \cdot 2^x + 16$, where $x \in \mathbb{R}$.

Find all values of x for which $f(x) = 0$. *Calculator not allowed* **[5 marks]**

23. Show that $\dfrac{8^{n+1} - 8^n}{7} = 8^n$, where $n \in \mathbb{Z}$. *Calculator not allowed* **[5 marks]**

24. The diameter of a spherical bacterium is 3.8×10^{-6} m. The diameter of a spherical virus is 1.9×10^{-8} m. The volume of a sphere of diameter d is $\frac{\pi d^3}{6}$.

Find the ratio of the volume of the bacterium to the volume of the virus, giving your answer as an integer power of 10 multiplied by a value rounded to 3 significant figures. *Calculator allowed* **[5 marks]**

25. Let $p = 2^a$ and $q = 2^b$, where $a, b \in \mathbb{R}$.

(a) Express $\frac{p^3\sqrt{q}}{(pq)^2}$ in the form $2^{f(a,b)}$, where $f(a,b)$ is a linear expression in a and b. **[3 marks]**

(b) Hence find the value of $\frac{p^3\sqrt{q}}{(pq)^2}$ when $a = 4$ and $b = 6$. **[2 marks]** *Calculator not allowed*

26. The number of bacteria in a culture at time t hours is modelled by $N = N_0 \cdot k^t$, where N_0 and k are positive constants with $k > 1$.

At time $t = 0$, there are 2.50×10^3 bacteria. At time $t = 5$, there are 8.00×10^4 bacteria.

(a) Find the value of N_0 and the value of k, giving k correct to 3 significant figures. **[3 marks]**

(b) Find the time at which the number of bacteria first exceeds 1.00×10^6, giving your answer correct to the nearest minute. **[2 marks]** *Calculator allowed*

27. Solve the equation $3^{2x+1} - 28 \cdot 3^x + 9 = 0$, where $x \in \mathbb{R}$. *Calculator not allowed* **[5 marks]**

28. The distance from Earth to a distant star is $d_1 = 4.22 \times 10^{16}$ m. The distance from Earth to a nearby star is $d_2 = 3.86 \times 10^{13}$ m.

(a) Find $\dfrac{d_1}{d_2}$, giving your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$. **[2 marks]**

(b) Light travels at 3.00×10^8 m s^{-1}. Find the difference in light-travel times from Earth to each star, giving your answer in years, correct to 3 significant figures. Use 1 year $= 3.156 \times 10^7$ s. **[3 marks]** *Calculator allowed*

29. Find all values of $x \in \mathbb{R}$ such that $\left(\dfrac{1}{4}\right)^{x^2-3} = 32^x$. *Calculator not allowed* **[5 marks]**

30. The expression $\dfrac{\sqrt[3]{x^5} \cdot x^{-\frac{1}{2}}}{\left(x^{\frac{2}{3}}\right)^4}$ can be written in the form x^p for $x > 0$.

(a) Find the value of p. **[3 marks]**

(b) Hence determine all values of $x > 0$ for which $\dfrac{\sqrt[3]{x^5} \cdot x^{-\frac{1}{2}}}{\left(x^{\frac{2}{3}}\right)^4} < x^{-1}$, justifying whether each solution found is valid. **[2 marks]** *Calculator not allowed*

Worked Solutions

Q1 — Standard form: large number [EASY]

Moving the decimal point 7 places to the left:

$$47\,300\,000 = 4.73 \times 10^7$$

M1

$a = 4.73$ identified correctly **A1**

$k = 7$ **A1**

Note: Award M1 for evidence of moving the decimal point; award both A1s only if the answer is in the demanded form $a \times 10^k$ with $1 \leq a < 10$.

Q2 — Standard form: small number [EASY]

Moving the decimal point 5 places to the right:

$$0.000\,082\,4 = 8.24 \times 10^{-5}$$

M1

$a = 8.24$ identified correctly **A1**

$k = -5$ **A1**

Note: Award M1 for evidence of locating the first significant figure; accept $k = -5$ written separately. Both A1s require the demanded form.

Q3 — Standard form ratio in context [EASY]

METHOD 1

Attempt to divide the two quantities: $\dfrac{3.2 \times 10^{30}}{6.4 \times 10^{23}}$ **(M1)**

$$= \frac{3.2}{6.4} \times 10^{30-23} = 0.5 \times 10^7$$

(A1)

$$= 5.0 \times 10^6$$

A1

Note: The final A1 requires the answer expressed in the demanded form $a \times 10^k$ with $1 \leq a < 10$; award (A1) for 0.5×10^7 or 5×10^6 seen before converting.

Q4 — Laws of indices: simplify [EASY]

Attempt to apply multiplication and division laws of indices: x^{8+3-4} **M1**

$= x^{11-4}$ **A1**

$= x^7$, so $p = 7$ **A1**

Note: Award M1 for a correct intermediate step showing addition or subtraction of at least two indices; final A1 requires the answer in the demanded form x^p.

Q5 — Laws of indices: power of a product [EASY]

Attempt to raise each factor to the power 4: $2^4 \cdot a^{3\times4} \cdot b^{1\times4}$ **M1**

$2^4 = 16$ **A1**

$16\,a^{12}b^4$ **A1**

Note: Award M1 for distributing the exponent 4 over at least two factors; both A1s require the fully simplified form.

Q6 — Rational exponent: exact evaluation [EASY]

Write $125^{2/3} = \left(125^{1/3}\right)^2$ OR $\left(\sqrt[3]{125}\right)^2$ **M1**

$125^{1/3} = 5$ **A1**

$5^2 = 25$ **A1**

Note: Accept the equivalent rewriting $125^{2/3} = \left(125^2\right)^{1/3}$ for M1 provided a correct cube root is subsequently evaluated; award A1 A1 for the same final answer via this route.

Q7 — Index equation: matching bases [EASY]

Write 64 as a power of 2: $64 = 2^6$ **M1**

So $2^x = 2^6$ **(A1)**

$x = 6$ **A1**

Note: Award M1 for any evidence that 64 is expressed as a power of 2; the implied A1 may be awarded for the equation $2^x = 2^6$ seen without further working.

Q8 — Standard form: ratio of measurements [EASY]

METHOD 1

Attempt to divide: $\dfrac{4.5\times10^{-7}}{9.0\times10^{-9}}$ **(M1)**

$$= \frac{4.5}{9.0} \times 10^{-7-(-9)} = 0.5 \times 10^{2}$$

(A1)

Ratio $= 50 : 1$ (Particle A is 50 times wider than Particle B) **A1**

Note: Accept 50 as the final answer; do not penalise if the ratio is written as 50 without the $:1$ notation provided the comparison is clear.

Q9 — Surd-index interplay and rational exponent [EASY]

$\sqrt[4]{x^3} = x^{3/4}$, so $p = \dfrac{3}{4}$ **A1**

Hence $16^{3/4} = \left(16^{1/4}\right)^3$; attempt to evaluate $16^{1/4}$ **M1**

$16^{1/4} = 2$, so $16^{3/4} = 2^3 = 8$ **A1**

Note: The first A1 is for the correct index $p = \frac{3}{4}$ in any equivalent form; M1 requires use of the result from the first part ("hence" lock applies). Do not award the second A1 for a decimal approximation.

Q10 — Index equation: matching bases [EASY]

Write $27 = 3^3$, so $3^{2x-1} = 3^3$ **M1**

Equate exponents: $2x - 1 = 3$ **A1**

$2x = 4 \implies x = 2$ **A1**

Note: Award M1 for expressing both sides as powers of 3; award the second A1 only for the correct value $x = 2$.

Q11 — Ratio of masses in standard form [MEDIUM]

Dividing the two masses:

$$\frac{6.4 \times 10^{-5}}{3.2 \times 10^{-9}}$$ **M1**

$$= \frac{6.4}{3.2} \times 10^{-5-(-9)} = 2 \times 10^{4}$$ **(A1)**

$$= 20\,000$$ **A1**

The grain of sand is $20\,000$ times heavier. **A1**

Note: Award final A1 only if answer is given as an integer, as required by the stem. Accept 2×10^4 for the third mark with the integer conversion earning the fourth.

Q12 — Index law simplification [MEDIUM]

Applying laws of indices:

Numerator: $x^5 \cdot x^{-2} = x^{5+(-2)} = x^3$ **M1**

$\dfrac{x^3}{x^4} = x^{3-4}$ **M1**

$= x^{-1}$ so $p = -1$ **A2**

Note: Award A2 for the correct final answer x^{-1}. Award A1 only if $p = -1$ is clearly stated but the demanded form is not written. Both M1 marks may be awarded together if the candidate correctly combines all three powers in one step.

Q13 — Solving index equation by matching bases [MEDIUM]

Expressing both sides as powers of 2:

$4^{2x-1} = (2^2)^{2x-1} = 2^{4x-2}$ and $8^{x+1} = (2^3)^{x+1} = 2^{3x+3}$ **M1**

Setting exponents equal: $4x - 2 = 3x + 3$ **M1**

$x = 5$ **A2**

Note: Award M1M1A2 as above. Award M1M1A1 if $x = 5$ is obtained but with an arithmetic slip in the rearrangement that is then corrected inconsistently. Accept equivalent base-conversion using base 4 or base 8 for full marks via METHOD 2.

METHOD 2

$4^{2x-1} = 8^{x+1} \Rightarrow (4)^{2x-1} = (4^{3/2})^{x+1} \Rightarrow 2x - 1 = \frac{3}{2}(x+1)$ **M1M1**

$4x - 2 = 3x + 3 \Rightarrow x = 5$ **A2**

Q14 — Planetary distances in standard form [MEDIUM]

(a) Difference $= 1.43 \times 10^9 - 5.79 \times 10^7$ **M1**

$= 1.43 \times 10^9 - 0.0579 \times 10^9 = 1.3721 \times 10^9$ km **A1**

(b) $\dfrac{1.43 \times 10^9}{5.79 \times 10^7}$ **M1**

$= 24.6976\ldots \approx 24.7$ (3 s.f.) **A1**

Note: In part (a) accept 1.37×10^9 if correctly rounded. In part (b) the answer must be given to 3 s.f. for the final A1.

Q15 — Factoring a surd-index expression [MEDIUM]

(a) $f(x) = x^{3/2} - 4x^{1/2} = x^{1/2}(x-4)$ **A1**

$a = 4$ **A1**

(b) Setting $f(x) = 0$: $x^{1/2}(x-4) = 0$ **M1**

Since $x > 0$, $x^{1/2} \neq 0$, so $x = 4$ **A1**

Note: In part (b) award M1 for equating their factored form to zero and considering both factors. The root $x = 0$ must be rejected because the domain is $x > 0$; candidates who state only $x = 4$ without acknowledging the rejection of $x = 0$ may be awarded A1 if the domain restriction makes the rejection implicit.

Q16 — Quadratic substitution in index equation [MEDIUM]

Substitution: Let $u = 3^x$, so $9^x = (3^2)^x = u^2$ **M1**

$u^2 - 10u + 9 = 0$ **A1**

$(u-1)(u-9) = 0 \Rightarrow u = 1$ or $u = 9$ **M1**

$3^x = 1 \Rightarrow x = 0$ and $3^x = 9 = 3^2 \Rightarrow x = 2$ **A1**

Note: Both values of x required for the final A1. Award M1 for a valid attempt to factorise or use the quadratic formula on their substituted equation. No root rejection is required here as both values of u are positive.

Q17 — Total length of bacteria in standard form [MEDIUM]

(a) Total length $= 4.5 \times 10^6 \times 2.0 \times 10^{-6}$ **M1**

$= (4.5 \times 2.0) \times 10^{6+(-6)} = 9.0 \times 10^0 = 9.0$ m **A1**

In the form $a \times 10^k$: 9.0×10^0 m

(b) 9.0 m $= 9.0 \times 10^{-3}$ km **M1**

$= 9.0 \times 10^{-3}$ km **A1**

Note: In part (a) accept 9 m or 9.0×10^0 m. In part (b) the conversion factor of 10^{-3} must be applied; accept 0.009 km but not without a unit.

Q18 — Multi-law index simplification [MEDIUM]

Expanding the numerator:

$(2a^3b^{-1})^2 = 4a^6b^{-2}$ **M1**

$\dfrac{4a^6b^{-2}}{4a^{-1}b^3} = a^{6-(-1)} \cdot b^{-2-3}$ **M1**

$= a^7b^{-5}$ so $m = 7, n = -5$ **A2**

Note: Award A2 for the correct final expression a^7b^{-5} in the demanded form. Deduct one mark (award A1) if the demanded form is correct but one of m or n carries an arithmetic error. The coefficient of 1 need *not be written explicitly.*

Q19 — Time for light to reach Earth [MEDIUM]

(a) Time $= \dfrac{4.12 \times 10^{16}}{2.998 \times 10^{8}}$ **M1**

$= \dfrac{4.12}{2.998} \times 10^{16-8} = 1.37424\ldots \times 10^{8}$ s **A1**

$\approx 1.37 \times 10^{8}$ s (3 s.f.)

(b) Time in years $= \dfrac{1.37424\ldots \times 10^{8}}{3.156 \times 10^{7}}$ **M1**

$= 4.3543\ldots \approx 4.35$ years (3 s.f.) **A1**

Note: In part (b) candidates must use their unrounded answer from part (a) (or full-precision value) to avoid a premature rounding error; award A1ft for a correctly rounded answer consistent with their part (a). The unit "years" is required for the final A1.

Q20 — Evaluating rational exponents exactly [MEDIUM]

Numerator:

$27^{2/3} = (27^{1/3})^{2} = 3^{2} = 9$ **M1**

$16^{3/4} = (16^{1/4})^{3} = 2^{3} = 8$ **A1**

Denominator:

$5^{0} = 1$, $\left(\frac{1}{4}\right)^{-1} = 4$, so denominator $= 1 + 4 = 5$ **M1**

$\dfrac{9+8}{5} = \dfrac{17}{5}$ **A1**

Note: Award M1 for a correct method to evaluate either $27^{2/3}$ or $16^{3/4}$. Award A1 for both numerator terms correct (9 and 8). Award the second M1 for correct evaluation of the denominator as 5. Final A1 for the exact fraction $\dfrac{17}{5}$; do not accept 3.4 as it is not in exact form.

Q21 — Standard form: total mass of helium atom [HARD]

Mass of 2 protons:
$2 \times 1.673 \times 10^{-27} = 3.346 \times 10^{-27}$ kg **(A1)**

Mass of 2 neutrons:
$2 \times 1.675 \times 10^{-27} = 3.350 \times 10^{-27}$ kg **(A1)**

Mass of 2 electrons:
$2 \times 9.109 \times 10^{-31} = 1.8218 \times 10^{-30} = 0.0018218 \times 10^{-27}$ kg **M1**

Attempt to add all three contributions (same power of 10):

$(3.346 + 3.350 + 0.0018218) \times 10^{-27}$ **M1**

$= 6.6978218 \times 10^{-27}$

$= 6.70 \times 10^{-27}$ kg **A1**

Note: Accept 6.698×10^{-27} *or any correctly rounded value to 3 or 4 s.f. Award final A1 only if answer is in correct demanded form* $a \times 10^k$, $1 \le a < 10$.

Q22 — Index equation via substitution quadratic [HARD]

Substitution: Let $u = 2^x$, so $4^x = (2^x)^2 = u^2$ **M1**

Equation becomes $u^2 - 17u + 16 = 0$ **A1**

Factorising: $(u - 1)(u - 16) = 0$ **M1**

$u = 1$ or $u = 16$

Since $u = 2^x > 0$, both values are valid. **R1**

$2^x = 1 \Rightarrow x = 0; \quad 2^x = 16 = 2^4 \Rightarrow x = 4$ **A1**

Note: Award R1 for explicitly stating both positive values are acceptable (trap: no root to reject here, but the justification is required). Full A1 requires both values of x.

Q23 — Show that: index law manipulation [HARD]

Attempt to factor numerator: **M1**

$8^{n+1} - 8^n = 8^n \cdot 8^1 - 8^n \cdot 1 = 8^n(8 - 1)$ **A1**

$= 8^n \cdot 7$ **A1**

Dividing both sides by 7:

$\dfrac{8^n \cdot 7}{7} = 8^n$ **A1**

$\therefore \dfrac{8^{n+1} - 8^n}{7} = 8^n$ **AG**

Note: M1 requires the candidate to write $8^{n+1} = 8^n \cdot 8$ *or equivalent; award A1 for correct factored form; final line carries no mark (AG firewall). Do not award full marks for purely numerical verification.*

[4 marks shown; 1 mark is M1 for method]

Additional M1: Explicit statement that $8^{n+1} - 8^n = 8^n(8 - 1)$ and $8 - 1 = 7$ so the 7s cancel. **M1**

Total: M1 M1 A1 A1 = 4; AG carries 0. Correct forward chain to printed result earns all 4 available marks.

Q24 — Volume ratio in standard form [HARD]

Ratio of volumes $= \left(\dfrac{d_1}{d_2}\right)^3$ (since $\frac{\pi d^3/6}{\pi d^3/6}$ simplifies) **M1**

$\dfrac{d_1}{d_2} = \dfrac{3.8 \times 10^{-6}}{1.9 \times 10^{-8}} = \dfrac{3.8}{1.9} \times 10^{(-6)-(-8)} = 2 \times 10^2 = 200$ **A1**

Cubing: $(200)^3 = (2 \times 10^2)^3 = 8 \times 10^6$ **M1**

Ratio $= 8.00 \times 10^6$ **A1**

The volume of the bacterium is 8.00×10^6 times larger than the volume of the virus. **A1**

Note: M1 for recognising volume ratio equals (diameter ratio)^3; second M1 for correct cubing; final A1 for correct demanded form to 3 s.f. Penalise if ratio is inverted and not corrected.

Q25 — Index law simplification with substitution [HARD]

(a)

Write $p = 2^a$, $q = 2^b$; substitute: **M1**

$\dfrac{(2^a)^3 \cdot (2^b)^{1/2}}{(2^a \cdot 2^b)^2} = \dfrac{2^{3a} \cdot 2^{b/2}}{2^{2(a+b)}} = \dfrac{2^{3a+b/2}}{2^{2a+2b}}$ **A1**

$= 2^{(3a+b/2)-(2a+2b)} = 2^{a-\frac{3b}{2}}$ **A1**

So $f(a,b) = a - \dfrac{3b}{2}$.

(b) Substitute $a = 4$, $b = 6$: **M1**

$2^{4-\frac{3(6)}{2}} = 2^{4-9} = 2^{-5} = \dfrac{1}{32}$ **A1**

Note: In (a), award first A1 for correct numerator/denominator indices before subtraction; second A1 for correct final exponent. FT their $f(a,b)$ in (b).

Q26 — Exponential model: find k and time [HARD]

(a)

At $t = 0$: $N_0 = 2.50 \times 10^3$ **A1**

At $t = 5$: $2.50 \times 10^3 \cdot k^5 = 8.00 \times 10^4$ **M1**

$k^5 = \dfrac{8.00 \times 10^4}{2.50 \times 10^3} = 32$

$k = 32^{1/5} = 2.00$ **A1**

(b) Solve $2.50 \times 10^3 \cdot 2^t = 1.00 \times 10^6$: **(M1)**

$2^t = \dfrac{1.00 \times 10^6}{2.50 \times 10^3} = 400$

$t = \log_2(400) = 8.64385\ldots$ hours $= 518.631\ldots$ minutes

$t = 519$ minutes (to the nearest minute) **(A1)**

Note: Full precision shown; GDC solve acceptable. Award (A1) only for 519 (rounding direction trap: must round up as first exceedance is required).

Q27 — Index equation: substitution quadratic [HARD]

Write $3^{2x+1} = 3 \cdot (3^x)^2$; let $u = 3^x > 0$: **M1**

$3u^2 - 28u + 9 = 0$ **A1**

Solve quadratic: $(3u-1)(u-9) = 0$ **M1**

$u = \dfrac{1}{3}$ or $u = 9$

Since $u = 3^x > 0$, both roots are valid; reject neither. **R1**

$3^x = \dfrac{1}{3} = 3^{-1} \Rightarrow x = -1; \quad 3^x = 9 = 3^2 \Rightarrow x = 2$ **A1**

Note: Award M1 for correctly substituting $u = 3^x$ and forming a three-term quadratic; R1 requires explicit statement that both positive values are admissible. Final A1 requires both $x = -1$ and $x = 2$.

Q28 — Standard form ratio and light-travel time [HARD]

(a)

$\dfrac{d_1}{d_2} = \dfrac{4.22 \times 10^{16}}{3.86 \times 10^{13}}$ **M1**

$= \dfrac{4.22}{3.86} \times 10^{16-13} = 1.0933\ldots \times 10^3$

$= 1.09 \times 10^3$ (3 s.f.) **A1**

(b) Time to distant star: $t_1 = \dfrac{4.22 \times 10^{16}}{3.00 \times 10^8} = 1.4\overline{06} \times 10^8$ s **M1**

Time to nearby star: $t_2 = \dfrac{3.86 \times 10^{13}}{3.00 \times 10^8} = 1.2\overline{8} \times 10^5$ s

Difference: $\Delta t = (1.40\overline{6} \times 10^8 - 1.28\overline{6} \times 10^5)$ s $\approx 1.40538 \times 10^8$ s **A1**

In years: $\dfrac{1.40538 \times 10^8}{3.156 \times 10^7} = 4.4531\ldots \approx 4.45$ years **A1**

Note: Award M1 for dividing each distance by speed of light to obtain times; accept methods that compute difference in metres first then divide. Final A1 requires answer in years to 3 s.f.

Q29 — Index equation with quadratic exponent [HARD]

Express with base 2:

$$\left(\frac{1}{4}\right)^{x^2-3} = (2^{-2})^{x^2-3} = 2^{-2(x^2-3)} = 2^{-2x^2+6}$$ **M1**

$32^x = (2^5)^x = 2^{5x}$ **A1**

Equate exponents:

$-2x^2 + 6 = 5x$ **M1**

$2x^2 + 5x - 6 = 0$

Solve: $x = \dfrac{-5 \pm \sqrt{25+48}}{4} = \dfrac{-5 \pm \sqrt{73}}{4}$ **A1**

Both roots are real; no domain restriction eliminates either. Both values valid: **R1**

$x = \dfrac{-5+\sqrt{73}}{4}$ or $x = \dfrac{-5-\sqrt{73}}{4}$ **(included in A1)**

Note: Award M1 for correct rewriting of both sides as powers of 2; second M1 for equating exponents to form a quadratic; A1 for correct exact surd answers; R1 for explicit statement that both roots satisfy the original equation (no restriction on x).

Q30 — Index simplification and inequality [HARD]

(a)

$$\frac{x^{5/3} \cdot x^{-1/2}}{x^{8/3}}$$ **M1**

Numerator index: $\dfrac{5}{3} - \dfrac{1}{2} = \dfrac{10}{6} - \dfrac{3}{6} = \dfrac{7}{6}$ **A1**

$x^{7/6-8/3} = x^{7/6-16/6} = x^{-9/6} = x^{-3/2}$ **A1**

So $p = -\dfrac{3}{2}$.

(b) The inequality becomes $x^{-3/2} < x^{-1}$ for $x > 0$. **M1**

Since $x > 0$, multiply both sides by $x^{3/2}$:
$1 < x^{-1+3/2} = x^{1/2}$, so $x^{1/2} > 1$, giving $x > 1$. **A1**

Validity: $x > 1 > 0$, so all such x lie within the stated domain $x > 0$; solution is valid. **(R1 absorbed into A1)**

Note: Award M1 for substituting their p and -1 correctly into an inequality; A1 for $x > 1$ with a valid algebraic justification. FT their p from (a) provided the inequality is not trivially impossible.