IB Math AA SL — Topic 1

Exponentials & Logs

Introduction to Logarithms · Laws of Logarithms · Exponential Equations

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:____________________________ Date:______________

Score:_________/120

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Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

Key Formulae

- Definition: $\log_a x = y \iff a^y = x \quad (a > 0, a \neq 1, x > 0)$
- Laws of logarithms: $\log_a(xy) = \log_a x + \log_a y$; $\quad \log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$; $\quad \log_a(x^n) = n\log_a x$
- Change of base: $\log_a x = \frac{\ln x}{\ln a} = \frac{\log x}{\log a}$
- Special values: $\log_a 1 = 0$; $\quad \log_a a = 1$; $\quad \ln e = 1$; $\quad \ln 1 = 0$
- Inverse relationship: $a^{\log_a x} = x$; $\quad \log_a(a^x) = x$
- Solving exponential equations: $a^x = b \Rightarrow x = \frac{\ln b}{\ln a}$ (exact form preferred on Paper 1)
- Hidden quadratic substitution: let $m = a^x$, solve $pm^2 + qm + r = 0$, return via $x = \frac{\ln m}{\ln a}$, reject $m \leq 0$

GDC Tips

- Evaluating logarithms: use `log( )` (base 10) or `ln( )` keys directly; for other bases type `log(x)/log(a)` or use the $\log_{\text{BASE}}$ template in the Math menu.
- Solving exponential/log equations: enter both sides as Y_1 and Y_2, use Calc $\to$ Intersect; or rewrite as $f(x) = 0$ and use Calc $\to$ Zero. Quote x to 3 s.f. unless an exact form is required.
- Checking exact ln-form answers: evaluate your exact expression (e.g. $\ln 5 / \ln 3$) on the home screen and confirm it matches the decimal from the graph screen.
- Storing intermediate values: store a computed ln value with STO$\to$ before substituting back, to avoid compounding rounding errors across parts.
- Sketching exponential/log graphs: set a window such as $[-2, 5] \times [-3, 8]$; use Trace to confirm the y-intercept and asymptote behaviour; zoom in with Zoom $\to$ ZBox near intersections.
- Growth/decay threshold time: after finding k exactly, store it and use Calc $\to$ Zero on $f(t) -$ threshold $= 0$ to find t numerically, then give the answer to 3 s.f. (or nearest integer if a whole-number context).

Common Mistakes to Avoid

1. **Forgetting to check argument positivity.** Every logarithm $\log_a f(x)$ requires $f(x) > 0$. After solving a log equation, substitute each candidate back and reject any value that makes an argument zero or negative. Omitting this check loses the domain-rejection A1.
2. **Rejecting the negative root of the substitution quadratic too late (or not at all).** When $m = a^x$ is introduced, m must be strictly positive. Write "$m = -2$ rejected since $a^x > 0$" explicitly; a bare cross through the root scores nothing.
3. **Incorrectly applying the product/quotient law.** $\log(a+b) \neq \log a + \log b$ and $\log(a-b) \neq \log a - \log b$. The addition/subtraction laws apply only to $\log(a \times b)$ and $\log(a \div b)$.
4. **Leaving an inexact answer on a non-calculator question.** Paper 1 requires exact forms such as $\dfrac{\ln 5}{\ln 2}$ or $\ln 12$. Writing a decimal approximation (e.g. 2.32) loses the final A1 even if the method is correct.
5. **Confusing "solve" with finding only one solution.** When an exponential equation is solved via a substitution quadratic, both roots of the quadratic must be considered before rejection; ignoring a root loses marks.
6. **Circular reasoning in a "show that" (AG) part.** Working towards a printed target must proceed forward from the given information. Starting from the target and working backwards, or verifying numerically, receives no credit. Ensure every step is a logical consequence of the previous one, with the final target line following clearly from the line above.

Section A — Easy **EASY** (10 questions $\times$ 3 marks = 30 marks)

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1. **EASY** **No Calculator** **[3 marks]**

Write down the value of $\log_3 81$.

2. **EASY** **Calculator** **[3 marks]**

Write down the value of $\log_5 1$.

3. **EASY** **Calculator** **[3 marks]**

Find the exact value of x such that $2^x = 32$, where $x \in \mathbb{R}$.

4. **EASY** **Calculator** **[3 marks]**

Express $\log_4 64$ as an integer.

5. EASY No Calculator **[3 marks]**

Given that $\ln x = 3$, find the exact value of x, where $x \in \mathbb{R}^+$.

6. EASY Calculator **[3 marks]**

The table below shows selected values of a function $f(x) = \log_{10} x$ for $x > 0$.

x	10	100	1000	10000
$f(x)$	1	a	b	4

Find the values of a and b.

7. EASY No Calculator **[3 marks]**

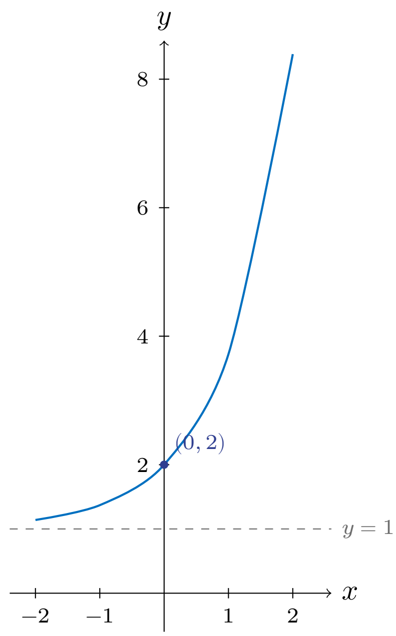
Write $\log_2 8 + \log_2 4$ as a single logarithm, and hence find its exact value.

8. EASY Calculator **[3 marks]**

Solve the equation $3^x = 20$ for $x \in \mathbb{R}$, giving your answer correct to 3 significant figures.

9. **EASY** **Calculator** **[3 marks]**

The sketch below shows the graph of $f(x) = e^x + 1$ for $x \in \mathbb{R}$.



Write down the equation of the horizontal asymptote of the graph of f.

10. **EASY** **Calculator** **[3 marks]**

Express $\log_6 12 + \log_6 3$ as a single logarithm, and hence find its exact value.

Section B — Medium **MEDIUM** (10 questions × 4 marks = 40 marks)

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11. **MEDIUM** **No Calculator** **[4 marks]**

Let $\log_3 p = 5$ and $\log_3 q = 2$, where $p, q \in \mathbb{R}^+$. Find the value of $\log_3\left(\dfrac{p\sqrt{q}}{9}\right)$.

12. **MEDIUM** **No Calculator** **[4 marks]**

Solve the equation $2\ln(x-1) - \ln(x+5) = \ln 2$, where $x \in \mathbb{R}$ and $x > 1$.

13. **MEDIUM** **Calculator** **[4 marks]**

The number of bacteria in a culture is modelled by $B(t) = 400\,e^{kt}$, where t is measured in hours and k is a positive constant. It is given that $B(3) = 1600$.

(a) Find the exact value of k. [2]

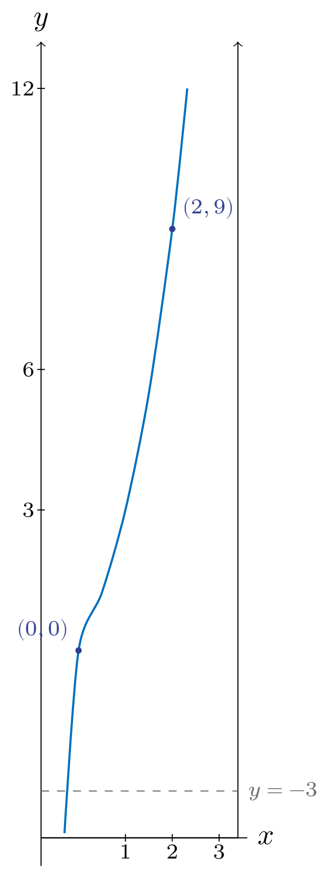
(b) Find the value of t when $B(t) = 10000$. Give your answer correct to 3 significant figures. [2]

14. MEDIUM No Calculator **[4 marks]**

Solve the equation $4^x - 6 \cdot 2^x + 8 = 0$, where $x \in \mathbb{R}$.

15. MEDIUM Calculator **[4 marks]**

The graph below shows $f(x) = a\,e^{bx} + c$, where $a, b, c \in \mathbb{R}$.



The graph passes through $(0, 0)$ and $(2, 9)$, and has a horizontal asymptote $y = -3$. Find the values of a, b, and c.

16. MEDIUM No Calculator **[4 marks]**

Express $\log_2 96 - \frac{1}{2}\log_2 16 - \log_2 3$ in the form $\log_2 k$, where $k \in \mathbb{Z}^+$.

17. MEDIUM Calculator **[4 marks]**

The population of a town is modelled by $P(t) = P_0\, e^{rt}$, where t is the number of years after 2010 and r is a constant. The population in 2010 was $24\,000$ and in 2022 it was $31\,500$.

(a) Find the value of r, correct to 4 significant figures. [2]

(b) Find the year in which the population first exceeds $40\,000$. [2]

18. MEDIUM Calculator **[4 marks]**

Solve the equation $\log_5(2x+1) + \log_5(x-2) = 1 + \log_5 3$, where $x \in \mathbb{R}$ and $x > 2$.

19. MEDIUM Calculator **[4 marks]**

The table below shows values of x and y which satisfy $y = ab^x$, where a and b are positive constants.

x	0	1	2	4
y	6	18	54	486

(a) Write down the value of a. [1]

(b) Find the value of b. [1]

(c) Find the value of x for which $y = 1000$. Give your answer correct to 3 significant figures. [2]

20. MEDIUM No Calculator **[4 marks]**

Given that $\log_a 2 = p$, express $\log_a(8a^3)$ in terms of p, where $a \in \mathbb{R}^+$, $a \neq 1$.

Section C — Hard **HARD** (10 questions $\times$ 5 marks = 50 marks)

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21. **HARD** **No Calculator** **[5 marks]**

Let $\log_3 a = p$ and $\log_3 b = q$, where $a > 0$ and $b > 0$.

(a) Show that $\log_3\left(\dfrac{a^2\sqrt{b}}{27}\right) = 2p + \dfrac{q}{2} - 3$. [2]

(b) Given that $2p + \dfrac{q}{2} - 3 = 0$ and that $b = \dfrac{9}{a}$, find the exact value of a. [3]

22. **HARD** **No Calculator** **[5 marks]**

Find all values of $x \in \mathbb{R}$ satisfying $2^{2x+1} - 5 \cdot 2^x + 2 = 0$. Give your answers in the form $\dfrac{\ln k}{\ln 2}$ where k is a positive integer.

23. HARD Calculator **[5 marks]**

The population of bacteria in a culture is modelled by $P(t) = 400\,e^{kt}$, where t is the time in hours after the culture is started and k is a positive constant. A scientist records the data below.

t (hours)	0	6
P	400	1850

(a) Find the value of k, giving your answer correct to 4 significant figures. [2]

(b) Find the time at which the population first exceeds $10\,000$, giving your answer in hours correct to 3 significant figures. [2]

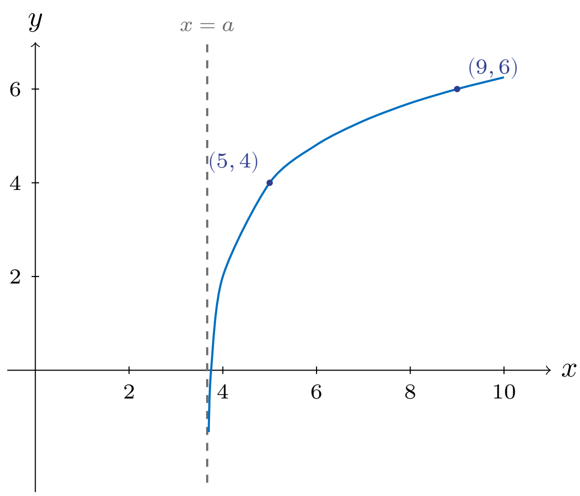
(c) State one limitation of using this model for large values of t. [1]

24. HARD No Calculator **[5 marks]**

Solve the equation $\log_2(x+5) + \log_2(x-1) = 4$, where $x \in \mathbb{R}$.

25. **HARD** **Calculator** **[5 marks]**

The graph of $y = \log_2(x - a) + b$ passes through the points $(5, 4)$ and $(9, 6)$, where a and b are constants with $a > 0$.



(a) Find the values of a and b. [3]

(b) Hence find the x-intercept of the graph, giving your answer correct to 3 significant figures. [2]

26. **HARD** **No Calculator** **[5 marks]**

Show that the equation $9^x - 4 \cdot 3^x + 12 = 0$ has no real solutions.

27. HARD Calculator **[5 marks]**

A radioactive substance decays according to $M(t) = M_0\, e^{-\lambda t}$, where M_0 is the initial mass in grams, $\lambda > 0$ is the decay constant, and t is time in years. The half-life of the substance is 24 years.

(a) Find the exact value of λ. [1]

(b) An initial mass of 200 g is placed in a sealed container. Find the number of complete years that must pass before the mass falls below 5 g. [4]

28. HARD Calculator **[5 marks]**

Given that $\log_x 8 + \log_x 4 = \log_x(3m - 1) - \log_x m$ for some real number $m > 0$, with $x > 0$ and $x \neq 1$, find all possible values of m.

29. HARD No Calculator **[5 marks]**

Find all values of $x \in \mathbb{R}$ such that $3^{2x} - 12 \cdot 3^x - 45 < 0$.

30. HARD Calculator **[5 marks]**

Let $f(x) = \ln(2x - 1) - \ln(x + 3)$, where $x > \frac{1}{2}$, $x \in \mathbb{R}$.

(a) Show that $f(x) = \ln\left(\frac{2x-1}{x+3}\right)$. [1]

(b) Hence, or otherwise, solve $f(x) = \ln 3$, showing clearly any values that must be rejected. [3]

(c) Determine whether the equation $f(x) = 2$ has a solution for $x > \frac{1}{2}$. Justify your answer. [1]

Mark Schemes

Q1 — Evaluate an Integer Logarithm EASY

Recognising that $3^4 = 81$ **M1**
$\log_3 81 = 4$ **A1**
Note: Award M0A0 for an unsupported incorrect answer. Award full marks for correct answer with correct supporting reasoning.

Q2 — Evaluate Log of One EASY

Recognising that $5^0 = 1$ **M1**
$\log_5 1 = 0$ **A1**
Note: Accept any valid justification using the definition of logarithm or the identity $\log_b 1 = 0$.

Q3 — Solve a Simple Exponential Equation EASY

Writing $2^x = 2^5$ **M1**
$x = 5$ **A1**
Note: Award M1 for any valid method leading to $x = 5$*, including use of logarithms. Accept exact answer only.*

Q4 — Evaluate an Integer Logarithm EASY

Recognising that $4^3 = 64$ **M1**
$\log_4 64 = 3$ **A1**
Note: Award full marks for a correct answer supported by a valid reason such as $4^3 = 64$.

Q5 — Convert Natural Log to Exponential Form EASY

Converting to exponential form: $x = e^3$ **M1**
$x = e^3$ **A1**
Note: Award M1 for a correct conversion to exponential form. Exact answer required; do not accept a decimal approximation.

Q6 — Evaluate Base-10 Logarithm from a Table **EASY**

Using $f(x) = \log_{10} x$: attempt to evaluate $\log_{10} 100$ and $\log_{10} 1000$ **M1**
$a = \log_{10} 100 = 2$ **A1**
$b = \log_{10} 1000 = 3$ **A1**
Note: Both values must be correct for full marks. Award M1A1A0 for one correct value only.

Q7 — Combine Logs and Evaluate **EASY**

Applying the addition law: $\log_2 8 + \log_2 4 = \log_2(8 \times 4) = \log_2 32$ **M1**
Recognising $2^5 = 32$, so $\log_2 32 = 5$ **A1**
Note: Award M1 for correct application of the product law. Accept equivalent methods such as evaluating $\log_2 8 = 3$ and $\log_2 4 = 2$ and adding.

Q8 — Solve Exponential Equation by GDC **EASY**

Taking logarithms of both sides, or using GDC solve: attempt to isolate x **(M1)**
$x = \dfrac{\ln 20}{\ln 3} = 2.7268\ldots$ **(A1)**
$x = 2.73$ **A1**
Note: Award (M1) for evidence of a valid logarithmic method or GDC use. Final A1 requires 3 significant figures. Accept $\log 20/\log 3$.

Q9 — Identify Horizontal Asymptote **EASY**

As $x \to -\infty$, $e^x \to 0$, so $f(x) \to 1$ **M1**
The equation of the horizontal asymptote is $y = 1$ **A2**
Note: Award M1 for a correct limiting argument or clear identification from the graph. Award A2 for $y = 1$ stated correctly. Award A1 if the answer is given as $f(x) = 1$ or just 1 without the equation form.

Q10 — Combine Logs and Evaluate **EASY**

Applying the addition law: $\log_6 12 + \log_6 3 = \log_6(12 \times 3) = \log_6 36$ **M1**
Recognising $6^2 = 36$, so $\log_6 36 = 2$ **A1**
Note: Award M1 for correct application of the product law of logarithms. Accept equivalent methods, e.g. evaluating both terms separately by GDC and adding.

Q11 — Log Laws with Given Values **MEDIUM**

Using log laws to expand: $\log_3\left(\frac{p\sqrt{q}}{9}\right) = \log_3 p + \log_3 q^{1/2} - \log_3 9$ **M1**

$= \log_3 p + \frac{1}{2}\log_3 q - \log_3 3^2$ **A1**

$= 5 + \frac{1}{2}(2) - 2 = 5 + 1 - 2$ **M1**

$= 4$ **A1**

Note: Award M1 for a correct application of at least two log laws. Final M1 for substituting the given values correctly.

Q12 — Log Equation with Root Rejection **MEDIUM**

Combining the left side: $\ln\left(\frac{(x-1)^2}{x+5}\right) = \ln 2$ **M1**

Removing logarithms: $(x-1)^2 = 2(x+5)$

$x^2 - 2x + 1 = 2x + 10 \implies x^2 - 4x - 9 = 0$ **A1**

$x = \frac{4 \pm \sqrt{16+36}}{2} = \frac{4 \pm \sqrt{52}}{2} = 2 \pm \sqrt{13}$ **M1**

Since $x > 1$, we need $x = 2 + \sqrt{13}$ (since $2 - \sqrt{13} < 0 < 1$, reject). **A1**

Note: The A1 requires both the correct root and the explicit rejection of $2 - \sqrt{13}$ with reference to the domain $x > 1$.

Q13 — Exponential Growth Model **MEDIUM**

(a) $400\,e^{3k} = 1600 \implies e^{3k} = 4$ **M1**

$k = \frac{\ln 4}{3}$ **A1**

(b) $400\,e^{kt} = 10000 \implies e^{kt} = 25$ **(M1)**

Using GDC with $k = \frac{\ln 4}{3}$: $t = \frac{\ln 25}{k} = \frac{3\ln 25}{\ln 4} = 6.9657\ldots \approx 6.97$ hours **A1**

Note: Accept equivalent exact forms such as $t = \frac{3\ln 25}{\ln 4}$. Final answer must be given to 3 s.f.

Q14 — Hidden Quadratic in Exponential **MEDIUM**

Let $m = 2^x$, so $4^x = m^2$. The equation becomes $m^2 - 6m + 8 = 0$ **M1**

$(m-2)(m-4) = 0 \implies m = 2$ or $m = 4$ **A1**

Returning to x: $2^x = 2 \implies x = 1$ and $2^x = 4 \implies x = 2$ **M1**

$x = 1$ or $x = 2$ **A1**

Note: Award first M1 for a correct substitution defining $m = 2^x$. Award second M1 for converting back from m to x. Both values required for final A1.

Q15 — Parameters of an Exponential Graph **MEDIUM**

From the asymptote: $c = -3$ **A1**

Substituting $(0,0)$: $a\,e^0 + (-3) = 0 \implies a = 3$ **A1**

Substituting $(2,9)$: $3\,e^{2b} - 3 = 9 \implies e^{2b} = 4$ **M1**

$2b = \ln 4 \implies b = \dfrac{\ln 4}{2} = \ln 2 \approx 0.693$ **A1**

Note: Accept $b = \ln 2$ or $b = 0.693$ (3 s.f.). All three values must be stated for full marks.

Q16 — Log Law Rewrite to a Single Log **MEDIUM**

Applying log laws: $\log_2 96 - \dfrac{1}{2}\log_2 16 - \log_2 3 = \log_2 96 - \log_2 16^{1/2} - \log_2 3$ **M1**

$= \log_2 96 - \log_2 4 - \log_2 3 = \log_2\left(\dfrac{96}{4 \times 3}\right) = \log_2\left(\dfrac{96}{12}\right)$ **M1**

$= \log_2 8$ **A1**

$k = 8$ **A1**

Note: Award A1 for correctly simplifying to $\log_2 8$, and A1 for identifying $k = 8$ as a positive integer. Accept equivalent working via $\log_2 96 - \log_2 12$.

Q17 — Population Growth Model **MEDIUM**

(a) $P_0 = 24000$, $P(12) = 31500$ (since 2022 is 12 years after 2010): **(M1)**

$24000\,e^{12r} = 31500 \implies r = \dfrac{\ln(31500/24000)}{12} = \dfrac{\ln(21/16)}{12} = 0.022661\ldots \approx 0.02266$ **A1**

(b) $24000\,e^{rt} > 40000$: using GDC solve $24000\,e^{0.02266t} = 40000$ **(M1)**

$t = \dfrac{\ln(40000/24000)}{r} = \dfrac{\ln(5/3)}{0.02266\ldots} = 22.55\ldots$ years after 2010, so the year is $2010 + 23 = 2033$ **A1**

Note: $t = 22.55\ldots$ means the population first exceeds 40 000 during the 23rd year after 2010, i.e. in 2033. Award A1 only for the year 2033.

Q18 — Log Equation via GDC **MEDIUM**

Combining logarithms on each side: $\log_5[(2x+1)(x-2)] = \log_5(5 \cdot 3) = \log_5 15$ **M1**

Removing logarithms: $(2x+1)(x-2) = 15 \implies 2x^2 - 3x - 2 = 15 \implies 2x^2 - 3x - 17 = 0$ **A1**

Using GDC: $x = \dfrac{3 \pm \sqrt{9+136}}{4} = \dfrac{3 \pm \sqrt{145}}{4}$ **(M1)**

$x = \dfrac{3+\sqrt{145}}{4} \approx 3.76$ (since $x > 2$, reject $x = \dfrac{3-\sqrt{145}}{4} \approx -2.26$) **A1**

Note: Award final A1 only if the negative root is explicitly rejected with reference to $x > 2$. Accept $x = 3.76$ (3 s.f.) or the exact surd form.

Q19 — Exponential Model from a Table MEDIUM

(a) When $x = 0$: $y = a \cdot b^0 = a = 6$ **A1**

(b) When $x = 1$: $6b = 18 \implies b = 3$ **A1**

(c) $6 \cdot 3^x = 1000$: **(M1)**

Using GDC: $x = \log_3\left(\dfrac{1000}{6}\right) = \dfrac{\ln(1000/6)}{\ln 3} = 4.6568\ldots \approx 4.66$ **A1**

Note: For part (c), full precision from GDC is $4.6568\ldots$; final answer must be to 3 s.f.

Q20 — Log Law Expression in Terms of p MEDIUM

$\log_a(8a^3) = \log_a 8 + \log_a a^3$ **M1**

$= \log_a 2^3 + 3\log_a a = 3\log_a 2 + 3$ **A1**

Since $\log_a 2 = p$: $\log_a(8a^3) = 3p + 3$ **M1A1**

Note: Award first M1 for splitting the logarithm using the product law. Award second M1 for using $\log_a 2^3 = 3\log_a 2$. Final A1 for the correct simplified expression $3p + 3$.

Q21 — Log Laws with Substitution HARD

(a) $\log_3\left(\dfrac{a^2\sqrt{b}}{27}\right) = \log_3 a^2 + \log_3 b^{1/2} - \log_3 27$ **M1**

$= 2\log_3 a + \frac{1}{2}\log_3 b - 3 = 2p + \frac{q}{2} - 3$ **AG**

(b) From $b = \dfrac{9}{a}$: $\log_3 b = \log_3 9 - \log_3 a = 2 - p$, so $q = 2 - p$ **M1**

Substitute into $2p + \frac{1}{2}(2-p) - 3 = 0$:

$2p + 1 - \frac{p}{2} - 3 = 0 \implies \frac{3p}{2} = 2 \implies p = \frac{4}{3}$ **A1**

$\log_3 a = \frac{4}{3} \implies a = 3^{4/3}$ **A1**

Note: Accept $a = \sqrt[3]{81}$. Award final A1 only for exact form.

Q22 — Hidden Quadratic Two Roots HARD

Let $m = 2^x$, so $2^{2x+1} = 2m^2$ **M1**
Equation becomes $2m^2 - 5m + 2 = 0$ **(A1)**
$(2m-1)(m-2) = 0 \implies m = \frac{1}{2}$ or $m = 2$ **A1**
$2^x = \frac{1}{2} \implies x = -1 = \dfrac{\ln(1/2)}{\ln 2}$; $\quad 2^x = 2 \implies x = 1 = \dfrac{\ln 2}{\ln 2}$
Both roots valid since $m = 2^x > 0$ is satisfied by both. **M1**
$x = -1 \quad$ and $\quad x = 1$ **A1**
Note: Accept $x = -1$ written as $\dfrac{-\ln 2}{\ln 2}$. The form $\dfrac{\ln k}{\ln 2}$ requires k a positive integer, so $x = -1$ must be expressed as $-\dfrac{\ln 2}{\ln 2}$; accept both presentations.

Q23 — Exponential Growth with GDC HARD

(a) $1850 = 400\,e^{6k}$ **(M1)**
$k = \dfrac{\ln(1850/400)}{6} = \dfrac{\ln 4.625}{6}$ **(A1)**
$k = 0.25525\ldots \approx 0.2552$ **A1**
(Award (M1)(A1) for correct equation seen; final A1 for 4 s.f. value.)

(b) Solve $400\,e^{0.2552t} = 10000$ by GDC **(M1)**
$t = 12.6108\ldots \approx 12.6$ hours (3 s.f.) **A1**
Note: Accept answers in the range [12.5, 12.7] arising from rounding of k.

(c) The model predicts unlimited growth, which is unrealistic as resources become limited (or: the population cannot grow without bound). **R1**

Q24 — Log Equation with Domain Rejection HARD

Combine logarithms: $\log_2[(x+5)(x-1)] = 4$ **M1**
$(x+5)(x-1) = 16$ **(A1)**
$x^2 + 4x - 5 = 16 \implies x^2 + 4x - 21 = 0$ **M1**
$(x+7)(x-3) = 0 \implies x = -7$ or $x = 3$ **A1**
Check domains: $\log_2(x-1)$ requires $x > 1$; $x = -7$ rejected. **A1**
$x = 3$
Note: The domain-rejection A1 is reserved; award only if reason stated.

Q25 — Log Graph Parameters and Intercept HARD

(a) Substitute $(5, 4)$: $\log_2(5-a) + b = 4$ **M1**
Substitute $(9, 6)$: $\log_2(9-a) + b = 6$

Subtract: $\log_2(9-a) - \log_2(5-a) = 2 \implies \dfrac{9-a}{5-a} = 4$ **M1**

$9 - a = 20 - 4a \implies 3a = 11 \implies a = \dfrac{11}{3}$

$b = 4 - \log_2\left(5 - \dfrac{11}{3}\right) = 4 - \log_2\left(\dfrac{4}{3}\right) = 4 - (2 - \log_2 3) = 2 + \log_2 3$ **A1**

$(a, b) = \left(\dfrac{11}{3}, 2 + \log_2 3\right)$ **A1**

(b) Set $y = 0$: $\log_2\left(x - \dfrac{11}{3}\right) = -(2 + \log_2 3)$; solve by GDC or algebra **(M1)**

$x = \dfrac{11}{3} + 2^{-(2+\log_2 3)} = \dfrac{11}{3} + \dfrac{1}{12} = \dfrac{44}{12} + \dfrac{1}{12} = \dfrac{45}{12} = \dfrac{15}{4}$

$x = 3.75$ **A1**

Note: Award (M1) for setting $y = 0$ and attempting to isolate x. Accept 3.75 or $\dfrac{15}{4}$. GDC read-off accepted with evidence.

Q26 — Hidden Quadratic Discriminant Check HARD

Let $u = 3^x$; since $3^x > 0$ for all real x, $u > 0$. Also $9^x = u^2$, so the equation becomes $u^2 - 4u + 12 = 0$ **M1**

Discriminant $= (-4)^2 - 4(1)(12) = 16 - 48 = -32$ **A1**

Since the discriminant is negative, the quadratic $u^2 - 4u + 12 = 0$ has no real roots u. **M1**

As $u = 3^x$ is real-valued for every real x, no real x can produce a value of u satisfying the equation. **R1**

Hence $9^x - 4 \cdot 3^x + 12 = 0$ has no real solutions. **A1**

Note: Award the final A1 only if the conclusion is stated explicitly, following from the negative discriminant. Do not award credit for numerically "verifying" a handful of trial values of x in place of the discriminant argument.

Q27 — Decay Model and Half Life HARD

(a) Half-life condition: $\frac{1}{2}M_0 = M_0\, e^{-24\lambda} \implies e^{-24\lambda} = \frac{1}{2}$ **(M1)**

$\lambda = \dfrac{\ln 2}{24}$ **A1**

(b) Require $200\, e^{-\lambda t} < 5 \implies e^{-\lambda t} < \frac{1}{40}$ **M1**

$-\lambda t < \ln \frac{1}{40} \implies t > \dfrac{\ln 40}{\lambda} = \dfrac{24 \ln 40}{\ln 2}$ **M1**

$t > \dfrac{24 \times 3.6889\ldots}{0.6931\ldots} = 127.8\ldots$ years **(A1)**

Least number of complete years is $t = 128$ **A1**

Note: The trap is rounding 127.8 down to 127; the mass at $t = 127$ still exceeds 5 g. Award final A1 only for 128.

Q28 — Log Equation with Base x **HARD**

Left side: $\log_x 8 + \log_x 4 = \log_x 32$ **M1**

Right side: $\log_x(3m-1) - \log_x m = \log_x\left(\dfrac{3m-1}{m}\right)$ **(A1)**

Since the base is the same and $x > 0, x \neq 1$: $\dfrac{3m-1}{m} = 32$ **M1**

$3m - 1 = 32m \implies -29m = 1 \implies m = -\dfrac{1}{29}$ **A1**

Check: $m > 0$ is required (given); $m = -\dfrac{1}{29} < 0$, so this value is rejected. No valid solution exists. **A1**

Note: The root-rejection A1 is reserved; award only if the reason ($m > 0$ required) is explicitly stated.

Q29 — Exponential Inequality **HARD**

Let $u = 3^x > 0$; inequality becomes $u^2 - 12u - 45 < 0$ **M1**
Factor: $(u-15)(u+3) < 0$ **A1**
Roots $u = 15$ and $u = -3$; parabola opens upward, so $(u+3)(u-15) < 0$ for $-3 < u < 15$ **M1**
Since $u = 3^x > 0$, the constraint $u > -3$ is automatically satisfied; require $u < 15$ **(A1)**
$3^x < 15 \implies x < \log_3 15 = \dfrac{\ln 15}{\ln 3}$

Combined with $u > 0$ (always true): solution is $x < \dfrac{\ln 15}{\ln 3}$, i.e. $x \in \left(-\infty, \dfrac{\ln 15}{\ln 3}\right)$ **A1**

Note: Trap is failing to reject $u = -3$ as extraneous (since $3^x > 0$). Accept $\log_3 15$ or $\dfrac{\ln 15}{\ln 3}$ as the boundary.

Q30 — Log Function Show Solve and Bound **HARD**

(a) $f(x) = \ln(2x-1) - \ln(x+3) = \ln\left(\dfrac{2x-1}{x+3}\right)$ by the quotient law of logarithms **AG**

(b) $\ln\left(\dfrac{2x-1}{x+3}\right) = \ln 3 \implies \dfrac{2x-1}{x+3} = 3$ **M1**
$2x - 1 = 3x + 9 \implies x = -10$ **A1**
Check domain $x > \frac{1}{2}$: $x = -10 < \frac{1}{2}$, so rejected. No solution. **A1**
Note: Final A1 reserved for explicit rejection with reason. Award M1 for correct removal of logarithms.

(c) As $x \to \infty$, $\dfrac{2x-1}{x+3} \to 2$, so $f(x) \to \ln 2 \approx 0.693 < 2$. **R1**
The function f is strictly increasing and bounded above by $\ln 2$, so $f(x) = 2$ has no solution for $x > \frac{1}{2}$.
Note: Award R1 for any valid justified argument that $f(x) < \ln 2 < 2$ for all $x > \frac{1}{2}$. Accept GDC graph evidence with a clear statement of the asymptote.