IB Math AA SL — Topic 1

Sequences & Series

Arithmetic & Geometric Sequences · Sigma Notation · Applications & Finance

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

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Name:______________________________ Date:_______________

Score:__________/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

Key Formulae

- Arithmetic sequence: $u_n = u_1 + (n-1)d$
- Arithmetic series: $S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$
- Geometric sequence: $u_n = u_1 r^{n-1}$
- Geometric series: $S_n = \frac{u_1(r^n - 1)}{r-1} = \frac{u_1(1-r^n)}{1-r}, \quad r \neq 1$
- Infinite geometric series: $S_\infty = \frac{u_1}{1-r}, \quad |r| < 1$
- Compound interest: $FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$, where k = compounding periods per year
- Sigma notation: $\sum_{i=m}^{n} u_i = u_m + u_{m+1} + \cdots + u_n$

GDC Tips

- **Sequence tables.** Use List & Spreadsheet or seq() to generate terms and spot patterns; confirm the u_n formula by comparing columns.
- **Least-n threshold problems.** Store S_n or u_n as a function, generate a table of values, and read off the first row satisfying the inequality (bracket with $n-1$ and n to show both sides).
- **Finance Solver (TVM).** Enter $N, I\%, PV, PMT, FV, P/Y, C/Y$; leave the unknown blank and solve. Pay attention to the sign convention — cash out is negative.
- **Sigma sums.** Evaluate $\sum$ directly with sum(seq(expression, variable, start, end)) to check by-hand results or tackle large limits.
- **Equation solving for n.** Use solve() or nSolve() when $r^n = k$ arises; remember n must be a positive integer, so round up or down according to the context.
- **Regression for sequences.** If context data are given, run LinReg ($y = ax + b$) or ExpReg ($y = ab^x$) to identify AP or GP structure, then read off d or r.

Common Mistakes to Avoid

1. **Forgetting to verify $|r| < 1$ for S_∞.** The infinite sum formula only converges when $|r| < 1$. A quadratic in r often yields two roots; the root with $|r| \geq 1$ must be explicitly rejected with a reason, earning the R1 mark.
2. **Off-by-one errors with u_n vs. S_n.** Recovering u_n from a given S_n formula requires $u_n = S_n - S_{n-1}$ for $n \geq 2$, and $u_1 = S_1$ separately. Using S_n directly as a term loses the method mark.
3. **Rounding direction in least-n problems.** When finding the least n such that $S_n > T$ or $u_n < \varepsilon$, candidates must show evaluations at both $n-1$ (failing) and n (satisfying the condition); rounding a logged solution without these bracketing values loses accuracy marks.
4. **Money answers not given to two decimal places.** Finance questions require the final answer rounded to two decimal places (nearest cent). Leaving a full-precision GDC display, or rounding to 3 significant figures, loses the final A1.
5. **Mixing up S_n and u_n in word problems.** "Total amount after n years" is S_n; "amount in year n" is u_n. Salary, depreciation, and instalment problems frequently require one and candidates substitute the other.
6. **Using the wrong regression direction or variable for applications.** When a problem gives time as the independent variable and asks for a predicted value, candidates must regress on the correct axis. Swapping x and y gives a different line and incorrect predictions; extrapolation beyond the data range must also be flagged as unreliable.

Section A — Easy EASY (10 questions $\times$ 3 marks = 30 marks)

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1. EASY No Calculator **[3 marks]**

An arithmetic sequence has first term $u_1 = 7$ and common difference $d = 4$. Find the tenth term of the sequence.

2. EASY No Calculator **[3 marks]**

A geometric sequence has first term $u_1 = 5$ and common ratio $r = 2$. Find the sum of the first six terms of the sequence.

3. EASY Calculator **[3 marks]**

Find the value of $\displaystyle\sum_{k=1}^{5}(3k+1)$.

4. EASY Calculator **[3 marks]**

The following table shows the first four terms of a sequence.

n	1	2	3	4
u_n	48	24	12	6

Write down the common ratio of the sequence. Hence find the sum to infinity of the sequence.

5. EASY Calculator **[3 marks]**

An arithmetic sequence has $u_1 = 3$ and $u_5 = 19$. Find the common difference d. Hence find the sum of the first 12 terms of the sequence.

6. EASY No Calculator **[3 marks]**

A geometric sequence has first term $u_1 = 200$ and common ratio $r = 0.6$, where $r \in \mathbb{R}$. Find the sum to infinity of the sequence.

7. EASY Calculator **[3 marks]**

Lena invests $4000 in a savings account that pays compound interest at a rate of 3.5% per annum. Find the value of Lena's investment at the end of 5 years. Give your answer correct to two decimal places.

8. EASY Calculator **[3 marks]**

The nth term of a sequence is given by $u_n = 2n - 5$, where $n \in \mathbb{Z}^+$.

(a) Write down the first three terms of the sequence. [1]

(b) Find the value of $\sum_{n=1}^{4} u_n$. [2]

9. EASY Calculator **[3 marks]**

A car is purchased for \$18 000. Its value depreciates at a rate of 15% per year. Find the value of the car after 4 years. Give your answer correct to two decimal places.

10. EASY No Calculator **[3 marks]**

An arithmetic sequence is shown in the table below.

n	1	2	3	4
u_n	6	11	16	21

Find the sum of the first 20 terms of the sequence.

Section B — Medium MEDIUM (10 questions × 4 marks = 40 marks)

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11. MEDIUM No Calculator **[4 marks]**

An arithmetic sequence has first term $u_1 = 7$ and common difference $d = 4$, where $n \in \mathbb{Z}^+$.

(a) Write down the fifth term of the sequence. [1]

(b) Find the value of n such that $u_n = 107$. [1]

(c) Find the sum of the first 25 terms of the sequence. [2]

12. MEDIUM No Calculator **[4 marks]**

A geometric sequence has terms $u_1, u_2, u_3, \ldots$ where $u_1 = 80$ and common ratio $r = \frac{3}{4}$, with $r \in \mathbb{R}$.

(a) Find u_4. [1]

(b) Find the sum to infinity of the sequence. [2]

(c) Find the sum of the first 6 terms, giving your answer as an exact fraction. [1]

13. MEDIUM No Calculator **[4 marks]**

The sum of the first n terms of a sequence is given by $S_n = 3n^2 + 5n$, where $n \in \mathbb{Z}^+$.

(a) Find u_1. [1]

(b) Show that the sequence is arithmetic. [2]

(c) Find the common difference. [1]

14. MEDIUM Calculator **[4 marks]**

The following table shows part of a payment schedule for a training fund. The amounts paid each month form a geometric sequence, where $a > 0$.

Month	1	2	3	4
Payment ($)	a	60	75	b

(a) Find the value of a and the value of b. [2]

(b) Find the total amount paid over the first 8 months. Give your answer correct to the nearest dollar. [2]

15. MEDIUM No Calculator **[4 marks]**

Find the value of $\displaystyle\sum_{k=3}^{10}(2k-1)$.

16. MEDIUM Calculator **[4 marks]**

Three consecutive terms of a geometric sequence are $k-2$, $k+4$, and $3k+2$, where $k \in \mathbb{R}$ and $k > 0$.

(a) Find the value of k. [3]

(b) Find the common ratio. [1]

17. MEDIUM Calculator **[4 marks]**

Priya invests \$5000 in an account that pays compound interest at a rate of 3.2% per annum, compounded monthly, where t represents time in years, $t \geq 0$.

(a) Find the value of the investment after 4 years. Give your answer correct to two decimal places. [2]

(b) Find the number of complete years it takes for the investment to first exceed \$6500. [2]

18. MEDIUM Calculator **[4 marks]**

The nth term of a geometric sequence is given by $u_n = 3 \times 2^{n-1}$, where $n \in \mathbb{Z}^+$.

(a) Write down the first term and the common ratio. [1]

(b) Find the least value of n such that $S_n > 3000$, where S_n denotes the sum of the first n terms. [3]

The following table shows selected values of S_n.

n	9	10	11	12
S_n	1533	3069	6141	12285

19. **MEDIUM** **Calculator** **[4 marks]**

A car is purchased for \$24 000. Its value depreciates by 18% each year.

(a) Find the value of the car after 3 years. Give your answer correct to two decimal places. [2]

(b) Find the number of complete years after which the value of the car first falls below \$8000. [2]

20. **MEDIUM** **Calculator** **[4 marks]**

Seats in a concert hall are arranged in rows. The first row has 20 seats, and each subsequent row has 3 more seats than the row before it, where $n \in \mathbb{Z}^+$ represents the row number.

(a) Find the number of seats in the 15th row. [2]

(b) The hall has 18 rows in total. Find the total number of seats in the hall. [2]

Section C — Hard **HARD** (10 questions $\times$ 5 marks = 50 marks)

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21. **HARD** **No Calculator** **[5 marks]**

The first three terms of an arithmetic sequence are $k+3$, $3k-1$, and $6k-7$, where $k \in \mathbb{R}$.

(a) Find the value of k. [2]

(b) Find the sum of the first 20 terms of the sequence. [3]

22. **HARD** **No Calculator** **[5 marks]**

Three consecutive terms of a geometric sequence are $t-6$, $t-4$, and $2t-5$, where $t \in \mathbb{R}$, $t > 0$.

Show that $t^2 - 9t + 14 = 0$. [2]

Hence find the common ratio of the sequence, rejecting any value of t that leads to $|r| \geq 1$. [3]

23. **HARD** **Calculator** **[5 marks]**

Lena invests \$8000 in an account paying compound interest at a rate of $r\%$ per year. After 6 years the investment is worth \$11 245.08.

(a) Find the value of r. [2]

(b) Lena instead considers a second account that pays simple interest at a fixed rate of 5.84% per year. Find the number of complete years it takes for the simple-interest account to overtake the compound-interest account. [3]

24. **HARD** **No Calculator** **[5 marks]**

An arithmetic sequence has first term a and common difference d, where $d \neq 0$. The 2nd, 3rd, and 6th terms of the sequence form a geometric sequence.

Show that $\dfrac{a}{d} = -\dfrac{1}{2}$.

25. HARD No Calculator **[5 marks]**

The nth partial sum of a series is given by $S_n = 5n^2 - 3n$ for $n \in \mathbb{Z}^+$.

(a) Find the first term and the common difference of the series. [3]

(b) Find the value of $\sum_{k=11}^{30} u_k$. [2]

26. HARD Calculator **[5 marks]**

In a geometric sequence, the sum of the first four terms is 45 and the sum of the first eight terms is 765.

Find the first term and common ratio of the sequence, justifying any rejection of candidate values.

27. HARD No Calculator **[5 marks]**

The table below shows a payment schedule for a training programme. Each week, a participant completes a number of sessions. The number of sessions in week 1 is p, and the numbers form an arithmetic sequence with common difference d.

Week	1	2	3	4	5	6
Sessions	p	$p+d$	$p+2d$	$p+3d$	$p+4d$	$p+5d$

The total number of sessions over 6 weeks is 90, and the number of sessions in week 6 is double the number in week 1.

(a) Find the values of p and d. [3]

(b) Find the total number of sessions completed in weeks 4 through 8, given the arithmetic pattern continues. [2]

28. HARD Calculator **[5 marks]**

Find the least value of $n \in \mathbb{Z}^+$ such that $\sum_{k=1}^{n} 3 \cdot \left(\frac{4}{5}\right)^{k-1}$ differs from its sum to infinity by less than 0.01.

29. HARD Calculator **[5 marks]**

A company offers two salary schemes to a new employee, each starting with an annual salary of $32 000 in year 1.

Scheme A: salary increases by $1800 each year.
Scheme B: salary increases by 4.5% each year.

(a) Find the total earnings under each scheme over 12 years. Give your answers correct to the nearest dollar. [3]

(b) Determine the first year in which Scheme B pays a higher annual salary than Scheme A, and hence state which scheme gives greater total earnings over 20 years. [2]

30. HARD No Calculator **[5 marks]**

A geometric series has first term 12 and common ratio r, where $r \in \mathbb{R}$, $r \neq 0$. The sum of the first n terms is denoted S_n and the sum to infinity is denoted S_∞.

It is given that $S_6 = \dfrac{9}{8} S_3$.

(a) Find the value of r. [3]

(b) For the value of r for which S_∞ exists, find $S_\infty - S_6$, expressing your answer as a fraction. [2]

Mark Schemes

Q1 — Arithmetic Sequence Tenth Term EASY

Using $u_n = u_1 + (n-1)d$ with $u_1 = 7$, $d = 4$, $n = 10$: **M1**

$u_{10} = 7 + (10-1)(4) = 7 + 36$

$u_{10} = 43$ **A1**

Note: Award M1 for correct substitution into the arithmetic term formula. Award A1 for the correct answer.

Q2 — Geometric Series Sum of Six Terms EASY

Using $S_n = u_1 \frac{r^n - 1}{r - 1}$ with $u_1 = 5$, $r = 2$, $n = 6$: **M1**

$S_6 = 5 \times \frac{2^6 - 1}{2 - 1} = 5 \times \frac{63}{1}$

$S_6 = 315$ **A1**

Note: Award M1 for correct substitution into the geometric series formula. Award A1 for the correct answer.

Q3 — Sigma Notation Evaluation EASY

Expanding: $\sum_{k=1}^{5}(3k+1) = 4 + 7 + 10 + 13 + 16$ **M1**

$= 50$ **A2**

Note: Award M1 for a correct method (expanding terms or using the arithmetic series formula). Award A2 for the correct answer 50.

Q4 — Geometric Sequence Sum to Infinity EASY

$r = \frac{24}{48} = \frac{1}{2}$ **A1**

Since $|r| = \frac{1}{2} < 1$, using $S_\infty = \frac{u_1}{1-r}$: **M1**

$S_\infty = \frac{48}{1 - \frac{1}{2}} = \frac{48}{\frac{1}{2}} = 96$ **A1**

Note: $r = 24/48 = 1/2$. The condition $|r| < 1$ must be acknowledged for the sum to infinity to exist.

Q5 — Arithmetic Sequence Common Difference and Sum EASY

$u_5 = u_1 + 4d \Rightarrow 19 = 3 + 4d \Rightarrow d = 4$ **A1**

Using $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ with $n = 12$, $u_1 = 3$, $d = 4$: **M1**

$S_{12} = \frac{12}{2}(2(3) + 11(4)) = 6(6 + 44) = 300$ **A1**

Q6 — Geometric Series Sum to Infinity EASY

Since $|r| = 0.6 < 1$, the sum to infinity exists.

Using $S_\infty = \frac{u_1}{1-r}$: **M1**

$S_\infty = \frac{200}{1-0.6} = \frac{200}{0.4}$

$S_\infty = 500$ **A2**

Q7 — Compound Interest Future Value EASY

Using $FV = PV \times \left(1 + \frac{r}{100}\right)^n$ with $PV = 4000$, $r = 3.5$, $n = 5$: **M1**

$FV = 4000 \times (1.035)^5 = 4000 \times 1.187866\ldots$ **(A1)**

$FV = \$4750.75$ **A1**

Note: Award (A1) for the unrounded value $4750.7452\ldots$. Award A1 for the final answer given correct to two decimal places. Final A1 lost if not rounded to two decimal places.

Q8 — nth Term Formula and Sigma Sum EASY

(a) $u_1 = 2(1) - 5 = -3$; $u_2 = -1$; $u_3 = 1$ **A1**

(b) $u_4 = 2(4) - 5 = 3$

$\sum_{n=1}^{4} u_n = -3 + (-1) + 1 + 3$ **M1**

$= 0$ **A1**

Note: Accept any valid method including use of the arithmetic series formula.

Q9 — Depreciation Future Value EASY

Using $V = V_0 \times (1 - 0.15)^n$ with $V_0 = 18000$, $n = 4$: **M1**

$V = 18000 \times (0.85)^4 = 18000 \times 0.52200625$ **(A1)**

$V = \$9396.11$ **A1**

Note: Award (A1) for the unrounded value 9396.1125. Award A1 for the final answer given correct to two *decimal places.*

Q10 — Arithmetic Series Sum of 20 Terms EASY

From the table: $u_1 = 6$, $d = 5$.
Using $S_n = \dfrac{n}{2}(2u_1 + (n-1)d)$ with $n = 20$: **M1**
$S_{20} = \dfrac{20}{2}(2(6) + 19(5)) = 10(12 + 95) = 10 \times 107$ **A1**
$S_{20} = 1070$ **A1**
Note: Accept the equivalent formula $S_n = \frac{n}{2}(u_1 + u_n)$ provided u_{20} is correctly found as 101.

Q11 — Arithmetic Sequence Term and Sum MEDIUM

(a) $u_5 = 7 + 4(4) = 23$ **A1**
(b) $7 + (n-1)(4) = 107 \Rightarrow 4(n-1) = 100 \Rightarrow n = 26$ **A1**
(c) Attempt to use $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ with $n = 25$ **M1**
$S_{25} = \frac{25}{2}(2(7) + 24(4)) = \frac{25}{2}(110) = 1375$ **A1**
Note: Accept $S_{25} = \frac{25}{2}(u_1 + u_{25}) = \frac{25}{2}(7 + 103) = 1375$.

Q12 — Geometric Sequence Sum to Infinity MEDIUM

(a) $u_4 = 80 \times \left(\frac{3}{4}\right)^3 = 80 \times \frac{27}{64} = 135/4$ **A1**
(b) Since $|r| = \frac{3}{4} < 1$, the sum to infinity exists. **R1**
$S_\infty = \dfrac{80}{1 - \frac{3}{4}} = \dfrac{80}{\frac{1}{4}} = 320$ **A1**
(c) $S_6 = \dfrac{80\left(1 - \left(\frac{3}{4}\right)^6\right)}{1 - \frac{3}{4}} = \dfrac{80\left(1 - \frac{729}{4096}\right)}{\frac{1}{4}} = 16835/64$ **A1**
Note: Accept $\frac{16835}{64}$ or equivalent exact fraction. Condone unsimplified form.

Q13 — Sum Formula to Recover Arithmetic Terms MEDIUM

(a) $u_1 = S_1 = 3(1)^2 + 5(1) = 8$ **A1**
(b) Attempt to find $u_n = S_n - S_{n-1}$ for $n \geq 2$ **M1**
$u_n = (3n^2 + 5n) - (3(n-1)^2 + 5(n-1)) = 6n + 2$
Since $u_n = 6n + 2$ is linear in n, the sequence is arithmetic. AG
(c) $d = u_n - u_{n-1} = 6(n+1) + 2 - (6n + 2) = 6$ **A1**

Note: Check: $u_2 = S_2 - S_1 = (12+10) - 8 = 14$; $d = 14 - 8 = 6$.

Q14 — Geometric Payment Schedule **MEDIUM**

(a) Common ratio: $r = \frac{75}{60} = \frac{5}{4}$ **(A1)**

$a = \frac{60}{r} = \frac{60}{\frac{5}{4}} = 48, \quad b = 75 \times \frac{5}{4} = 93.75$ **A1**

(b) Attempt $S_8 = \frac{48\left(\left(\frac{5}{4}\right)^8 - 1\right)}{\frac{5}{4} - 1}$ **M1**

$S_8 = \frac{48(5.96046\ldots - 1)}{0.25} = 952.41\ldots$

Total $= \$952$ **A1**

Note: Unrounded value $952.41\ldots$ *must be seen before rounding.*

Q15 — Sigma Notation Evaluation **MEDIUM**

Recognise the sum as arithmetic with $u_k = 2k - 1$; first term $k = 3$: $u_3 = 5$, last term $k = 10$: $u_{10} = 19$, number of terms $= 8$ **M1**

$\sum_{k=3}^{10}(2k-1) = \frac{8}{2}(5+19) = 4 \times 24 = 96$ **A1 A1 A1**

Note: M1 requires a valid method attempted. Marking note: this question awards M1 for setting up the sum correctly, A1 for identifying first and last terms, A1 for the sum formula or full expansion, A1 for 96.

Q16 — Parameterised Geometric Sequence **MEDIUM**

(a) For a geometric sequence, the ratio between consecutive terms is constant: **M1**

$(k+4)^2 = (k-2)(3k+2)$

$k^2 + 8k + 16 = 3k^2 - 4k - 4$

$0 = 2k^2 - 12k - 20 \Rightarrow k^2 - 6k - 10 = 0$

Attempt to solve: $k = \frac{6 \pm \sqrt{36+40}}{2} = \frac{6 \pm \sqrt{76}}{2}$

(via GDC): $k = 7.359\ldots$ or $k = -1.359\ldots$ **A1**

Since $k > 0$, reject $k = -1.359\ldots$; therefore $k = 7.36$ (3 s.f.) **A1**

(b) $r = \frac{k+4}{k-2} = \frac{11.359\ldots}{5.359\ldots} = 2.12$ (3 s.f.) **A1**

Note: Accept exact forms $k = 3 + \sqrt{19}$ *for part (a) and* $r = \frac{7+\sqrt{19}}{1+\sqrt{19}}$ *for part (b).*

Q17 — Compound Interest Value and Time **MEDIUM**

(a) $A = 5000\left(1+\dfrac{0.032}{12}\right)^{12\times 4}$ **M1**

$A = 5000(1.0026\overline{6})^{48} = 5000 \times 1.136359\ldots$

$A = \$5681.80$ **A1**

Note: Answer must be given correct to two decimal places. The monthly factor $1 + 0.032/12$ must not be rounded before raising to the power 48, or the final cents will be wrong.

(b) Solve $5000\left(1+\dfrac{0.032}{12}\right)^{12t} > 6500$ for t integer **(M1)**

Via GDC: $t = 8$ gives \$6456.56...< 6500; $t = 9$ gives \$6666.23...> 6500

Least number of complete years $= 9$ **A1**

Note: Both bracketing values $t = 8$ and $t = 9$ must be shown or implied for full marks.

Q18 — Geometric Sequence Least n Threshold **MEDIUM**

(a) First term $u_1 = 3$, common ratio $r = 2$ **A1**

(b) $S_n = \dfrac{3(2^n - 1)}{2-1} = 3(2^n - 1)$; attempt to find least n with $S_n > 3000$ **M1**

From the table: $S_9 = 1533 \leq 3000$ and $S_{10} = 3069 > 3000$ **A1**

Least value of $n = 10$ **A1**

Note: Both bracketing values $S_9 = 1533$ and $S_{10} = 3069$ must be identified for the second A1.

Q19 — Depreciation Value and Threshold Year **MEDIUM**

(a) $V = 24000 \times (0.82)^3$ **M1**

$V = 24000 \times 0.551368 = \$13232.83$ (2 d.p.) **A1**

(b) Solve $24000 \times (0.82)^n < 8000$ for least integer n **(M1)**

Via GDC: $n = 5$: $24000(0.82)^5 = 8898.0\ldots > 8000$

$n = 6$: $24000(0.82)^6 = 7296.4\ldots < 8000$

Least number of complete years $= 6$ **A1**

Note: Both bracketing evaluations must be seen or implied.

Q20 — Arithmetic Seating Arrangement **MEDIUM**

(a) $u_n = 20 + (n-1)(3)$; $u_{15} = 20 + 14 \times 3$ **M1**

$u_{15} = 20 + 42 = 62$ **A1**

(b) $S_{18} = \dfrac{18}{2}(2(20) + 17(3))$ **M1**

$S_{18} = 9(40 + 51) = 9 \times 91 = 819$ **A1**

Note: Accept $S_{18} = \frac{18}{2}(u_1 + u_{18}) = 9(20 + 71) = 819$.

Q21 — Arithmetic Sequence from Three Terms HARD

(a) Setting up the equal-difference condition: $(3k-1)-(k+3)=(6k-7)-(3k-1)$ **M1**
$2k-4=3k-6 \Rightarrow k=2$ **A1**

(b) First term: $u_1 = k+3 = 5$; second term $= 3(2)-1 = 5$; third term $= 6(2)-7 = 5$.
Attempt to find d: $d = (3k-1)-(k+3) = 2(2)-4 = 0$.
Note: Students must recognise $d = 0$ is valid here and proceed.
$S_{20} = \dfrac{20}{2}(2(5)+19(0))$ **M1**
$S_{20} = 10 \times 10 = 100$ **A1**

Q22 — Geometric Sequence Parameter Root Rejection HARD

Show that:
Using the equal-ratio condition: $\dfrac{t-4}{t-6} = \dfrac{2t-5}{t-4}$ **M1**
$(t-4)^2 = (t-6)(2t-5)$
$t^2 - 8t + 16 = 2t^2 - 17t + 30$
$0 = t^2 - 9t + 14$ AG

Hence:
Factorising: $(t-2)(t-7) = 0$ **M1**
$t = 2$ or $t = 7$. Both satisfy $t > 0$, so the common ratio must be used to decide which value is valid.
For $t = 2$: terms are $-4, -2, -1$; $r = \dfrac{-2}{-4} = \dfrac{1}{2}$.
For $t = 7$: terms are $1, 3, 9$; $r = \dfrac{3}{1} = 3$.
Since $|r| = 3 \geq 1$ for $t = 7$, this value of t is rejected. **R1**
For $t = 2$, $|r| = \frac{1}{2} < 1$, so this value is accepted.
$r = \dfrac{1}{2}$ **A1**
Note: The show-that mark is M1 (the AG line itself carries no separate mark). The hence part awards M1 for factorising/solving, R1 for the explicit $|r| \geq 1$ rejection with reason, and A1 for the final common ratio.

Q23 — Compound Interest Rate and Scheme Comparison HARD

(a) Setting up the compound interest equation: $8000\left(1+\dfrac{r}{100}\right)^6 = 11245.08$ **(M1)**
$\left(1+\dfrac{r}{100}\right)^6 = \dfrac{11245.08}{8000} = 1.405635$
$r = 100\left(1.405635^{1/6}-1\right) = 5.8389\ldots \approx 5.84$ (3 s.f.) **A1**

(b) Simple interest value after n years: $V_S = 8000(1 + 0.0584n)$
Compound interest value: $V_C = 8000(1.0584)^n$
Require $8000(1 + 0.0584n) > 8000(1.0584)^n$ **M1**
Using GDC table: at $n = 1$, $V_S = 8467.20$, $V_C = 8467.20$ (equal); for $n \geq 1$, compound grows faster than simple since $r > 0$, so $V_C \geq V_S$ for all $n \geq 1$. **(M1)**
Note: Award M1 for a correct inequality or equation set up. Award (M1) for GDC evidence. Award A1 for the correct conclusion.
The simple interest account never overtakes the compound interest account for any complete year $n \geq 1$.
A1

Q24 — AP Terms Forming GP Show That HARD

The 2nd, 3rd, and 6th terms of the AP are: **M1**
$u_2 = a + d$, $u_3 = a + 2d$, $u_6 = a + 5d$
For a geometric sequence: $(a + 2d)^2 = (a + d)(a + 5d)$ **M1**
Expanding left side: $a^2 + 4ad + 4d^2$ **(A1)**
Expanding right side: $a^2 + 6ad + 5d^2$ **(A1)**
Setting equal: $a^2 + 4ad + 4d^2 = a^2 + 6ad + 5d^2$
$-2ad - d^2 = 0 \Rightarrow d(2a + d) = 0$
Since $d \neq 0$: $2a + d = 0 \Rightarrow \dfrac{a}{d} = -\dfrac{1}{2}$ **A1**
Note: Award M1 for writing the three AP terms; M1 for setting up the GP condition; (A1)(A1) for expanding both sides correctly; A1 for completing the simplification with the $d \neq 0$ justification.

Q25 — Partial Sum Formula Recover Terms and Sigma HARD

(a) First term: $u_1 = S_1 = 5(1)^2 - 3(1) = 2$ **A1**
For $n \geq 2$: $u_n = S_n - S_{n-1} = (5n^2 - 3n) - (5(n-1)^2 - 3(n-1))$ **M1**
$= 10n - 8$
Check: $u_1 = 10(1) - 8 = 2$; $u_2 = S_2 - S_1 = (20 - 6) - 2 = 12$; $10(2) - 8 = 12$ **A1**
Common difference: $d = u_2 - u_1 = 12 - 2 = 10$ **A1**

(b) $\displaystyle\sum_{k=11}^{30} u_k = S_{30} - S_{10}$ **M1**
$S_{30} = 5(900) - 3(30) = 4410; \quad S_{10} = 5(100) - 3(10) = 470$
$\displaystyle\sum_{k=11}^{30} u_k = 4410 - 470 = 3940$ **A1**

Q26 — Geometric Series Two Sum Conditions HARD

Let first term be a and common ratio r.

$S_4 = \dfrac{a(r^4-1)}{r-1} = 45$ and $S_8 = \dfrac{a(r^8-1)}{r-1} = 765$ **M1**

Dividing: $\dfrac{S_8}{S_4} = \dfrac{r^8-1}{r^4-1} = r^4+1 = \dfrac{765}{45} = 17$ **M1**

$r^4 = 16 \Rightarrow r = \pm 2$ **A1**

For $r = 2$: $a(1+2+4+8) = 45 \Rightarrow 15a = 45 \Rightarrow a = 3$

For $r = -2$: $a(1-2+4-8) = 45 \Rightarrow -5a = 45 \Rightarrow a = -9$ **A1**

Both solutions are valid geometric sequences; $r = \pm 2$ are real, nonzero, and $\neq 1$, and no contextual restriction eliminates either. **R1**

Note: Award M1 for forming both sum equations; M1 for dividing to eliminate a; A1 for $r = \pm 2$; A1 for both values of a; R1 for explicit justification that both are retained.

Q27 — AP Application Sessions Schedule HARD

(a) From the total: $S_6 = \dfrac{6}{2}(2p+5d) = 90 \Rightarrow 2p+5d = 30$ **M1**

From the doubling condition: $p + 5d = 2p \Rightarrow p = 5d$ **A1**

Substituting: $2(5d) + 5d = 30 \Rightarrow 15d = 30 \Rightarrow d = 2, p = 10$ **A1**

(b) Weeks 4 through 8 correspond to terms u_4, u_5, u_6, u_7, u_8. **M1**

$\displaystyle\sum_{k=4}^{8} u_k = S_8 - S_3 = \frac{8}{2}(2(10)+7(2)) - \frac{3}{2}(2(10)+2(2))$

$= 4(34) - 1.5(24) = 136 - 36 = 100$ **A1**

Note: Accept direct summation of $u_4 = 16, u_5 = 18, u_6 = 20, u_7 = 22, u_8 = 24$.

Q28 — Least n for Geometric Series Threshold HARD

The series is geometric with $a = 3$ and $r = \dfrac{4}{5}$.

Since $|r| = \dfrac{4}{5} < 1$, the sum to infinity exists: $S_\infty = \dfrac{3}{1-\frac{4}{5}} = 15$ **A1**

$S_n = \dfrac{3\left(1-\left(\frac{4}{5}\right)^n\right)}{1-\frac{4}{5}} = 15\left(1-\left(\dfrac{4}{5}\right)^n\right)$

Require $S_\infty - S_n < 0.01$: $15\left(\dfrac{4}{5}\right)^n < 0.01$ **M1**

$\left(\dfrac{4}{5}\right)^n < \dfrac{0.01}{15} = 6.\overline{6} \times 10^{-4}$ **(M1)**

Using GDC table or solving: $n\ln(0.8) < \ln(6.\overline{6}\times 10^{-4})$

$n > \dfrac{\ln(6.\overline{6}\times 10^{-4})}{\ln(0.8)} = 32.78\ldots$

Check: $n = 32$: $15(0.8)^{32} = 15(0.0007923\ldots) = 0.011884 > 0.01$ **(A1)**
$n = 33$: $15(0.8)^{33} = 15(0.0006338\ldots) = 0.009507 < 0.01$ ✓
The least value is $n = 33$. **A1**

Note: Award M1 for forming the inequality $S_\infty - S_n < 0.01$; (M1) for GDC evidence of evaluation near the threshold; (A1) for bracketing evaluations at $n = 32$ (fails) and $n = 33$ (satisfies); A1 for $n = 33$. A1 for $S_\infty = 15$.

Q29 — Salary Scheme Comparison AP vs GP HARD

(a) Scheme A (AP): $a = 32000$, $d = 1800$, $n = 12$

$T_A = \frac{12}{2}(2(32000) + 11(1800)) = 6(83800) = \$502\,800$ **A1**

Scheme B (GP): $a = 32000$, $r = 1.045$, $n = 12$

$T_B = \frac{32000(1.045^{12} - 1)}{0.045}$ **(M1)**

$= \$494\,849$ (nearest dollar) **A1**

Note: Exact value $494\,849.02$, rounds to $494\,849$.

(b) Annual salary under A in year n: $32000 + 1800(n-1)$; under B: $32000(1.045)^{n-1}$
Require $32000(1.045)^{n-1} > 32000 + 1800(n-1)$ **M1**
Using GDC: at year 11, $A = 50000$, $B = 49695.02\ldots$ (A greater);
at year 12, $A = 51800$, $B = 51931.30\ldots$ (B greater).
First year in which Scheme B exceeds Scheme A: year 12.
Since Scheme B overtakes Scheme A already by year 12, and continues to grow faster every year after, Scheme B gives greater total earnings over 20 years. **A1**

Note: $T_{20,A} = \$982\,000$; $T_{20,B} = \$1\,003\,885.53$, confirming Scheme B gives the greater 20-year total. Award M1 for setting up the annual salary comparison; A1 for year 12 with GDC bracketing evidence (year 11 fails, year 12 satisfies) and the conclusion about 20-year totals.

Q30 — Geometric Series Value of r and S-infinity HARD

(a) $S_3 = 12(1 + r + r^2)$ and $S_6 = 12(1 + r + r^2 + r^3 + r^4 + r^5) = S_3(1 + r^3)$ **M1**

Setting $S_6 = \frac{9}{8}S_3$, and noting $S_3 = 12(1 + r + r^2) > 0$ for all real r, so we may divide both sides by S_3: **M1**

$1 + r^3 = \frac{9}{8} \Rightarrow r^3 = \frac{1}{8} \Rightarrow r = \frac{1}{2}$

(the only real cube root; the other two roots of $r^3 = \frac{1}{8}$ are complex)

$r = \frac{1}{2}$ **A1**

(b) Since $|r| = \frac{1}{2} < 1$, S_∞ exists.

$S_\infty = \dfrac{12}{1-\frac{1}{2}} = 24$

$S_6 = S_3 \times \dfrac{9}{8} = 12\left(1+\frac{1}{2}+\frac{1}{4}\right) \times \dfrac{9}{8} = 21 \times \dfrac{9}{8} = \dfrac{189}{8}$ **M1**

$S_\infty - S_6 = 24 - \dfrac{189}{8} = \dfrac{192-189}{8} = \dfrac{3}{8}$ **A1**

Note: Equivalently, $S_\infty - S_6 = \dfrac{ar^6}{1-r} = \dfrac{12 \times \frac{1}{64}}{\frac{1}{2}} = \dfrac{3}{8}$.