IB Math AA SL — Topic 1

Binomial Theorem

Binomial Coefficients & Pascal's Triangle · The Binomial Theorem

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:____________________________ Date:______________

Score:_________/120

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Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

Key Formulae

- **Binomial coefficient:** $\binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}, \quad r = 0, 1, \dots, n$
- **Binomial theorem:** $(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$
- **General term ($(r+1)$-th term):** $T_{r+1} = \binom{n}{r} a^{n-r} b^r$
- **Pascal's triangle identity:** $\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$
- **Symmetry:** $\binom{n}{r} = \binom{n}{n-r}$
- **Special cases:** $\binom{n}{0} = \binom{n}{n} = 1; \quad \binom{n}{1} = \binom{n}{n-1} = n$
- **Sum of all coefficients:** $(1+1)^n = 2^n$, so $\sum_{r=0}^{n} \binom{n}{r} = 2^n$

GDC Tips

- **Evaluating $\binom{n}{r}$:** Use Math $\to$ PRB $\to$ nCr (TI) or OPTN $\to$ PROB $\to$ nCr (Casio); enter n first, then r.
- **Expanding $(a+b)^n$ for large n:** Store values of a and b, then use repeated nCr calls to compute each T_{r+1} individually, or use binomial(n, r) in a CAS-enabled model.
- **Finding an unknown n:** Set up the coefficient equation and use Solve (TI: nSolve; Casio: equation solver) to find integer solutions; always verify the result is a positive integer.
- **Checking coefficient of x^k:** Expand symbolically with expand$((a+bx)^n)$ in a CAS model and read off the required coefficient directly.
- **Ratio of consecutive terms:** Compute $T_{r+1}/T_r = \frac{n-r+1}{r} \cdot \frac{a}{b}$ numerically on the GDC to locate the largest term or verify an AP/GP condition.
- **Large factorials / overflow:** Use nCr rather than computing $n!$ directly; GDCs overflow for $n!$ when $n \gtrsim 70$.

Common Mistakes to Avoid

1. **Forgetting to write the general term first.** Students jump straight to substituting r without writing $T_{r+1} = \binom{n}{r}a^{n-r}b^r$. The general term must appear before any extraction; omitting it loses the method mark.

2. **Sign errors with a negative b.** When b is negative, $b^r = (-1)^r|b|^r$. A common error is treating b^r as positive throughout, giving the wrong sign for odd-power terms (e.g. finding $+84$ instead of -84 for the coefficient of x^3).

3. **Confusing the coefficient with the term.** The question asks for "the coefficient of x^3", which is the number multiplying x^3, not the entire expression $\binom{n}{r}a^{n-r}b^r x^r$. Leaving the x^r attached loses the final accuracy mark.

4. **Incorrect index when solving for r.** Setting the power of x equal to the required power involves solving $n - r = k$ or $r = k$ depending on which factor carries x. Mixing up r and $n - r$ (e.g. using T_r instead of T_{r+1}) shifts every term by one index and gives a wrong answer.

5. **Wrong term chosen when two terms share the same power.** In expansions of the form $(ax^p + b/x^q)^n$, the power of x in T_{r+1} is $p(n-r) - qr$. Students sometimes set only $p(n-r) =$ required power, ignoring the $-qr$ contribution, and select the wrong term.

6. **Parameter order condition ignored.** When two symmetric values of r (or n) satisfy the algebra (e.g. $r = 2$ and $r = n - 2$), and the question states $p < q$ or $k > 0$, one solution must be rejected. Failing to apply the stated condition and giving both answers without justification loses the reasoning mark (R1).

Section A — Easy EASY (10 questions × 3 marks = 30 marks)

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1. EASY No Calculator **[3 marks]**

Write down the value of $\binom{7}{3}$.

2. EASY No Calculator **[3 marks]**

Find the expansion of $(1+x)^4$, giving each coefficient as an integer.

3. EASY No Calculator **[3 marks]**

Find the coefficient of x^2 in the expansion of $(1+3x)^5$.

4. EASY Calculator **[3 marks]**

Find the term independent of x in the expansion of $\left(x + \frac{2}{x}\right)^4$, where $x \neq 0$.

5. EASY Calculator **[3 marks]**

The third row of Pascal's triangle (starting from row 0) is shown below.

$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$

$$1 \qquad 3 \qquad 3 \qquad 1$$

Write down the values in the fourth row of Pascal's triangle.

6. EASY Calculator **[3 marks]**

Find the coefficient of x^3 in the expansion of $(2+x)^5$.

7. EASY Calculator **[3 marks]**

Find the value of $\binom{10}{4}$ using a calculator.

8. EASY Calculator **[3 marks]**

Find the coefficient of x^2 in the expansion of $(3-x)^6$.

9. EASY Calculator **[3 marks]**

Find the first three terms, in ascending powers of x, of the expansion of $(1+2x)^6$.

10. EASY No Calculator **[3 marks]**

Find the coefficient of x^2 in the expansion of $(x+4)^5$, where the general term is written down first.

Section B — Medium MEDIUM (10 questions × 4 marks = 40 marks)

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11. MEDIUM No Calculator **[4 marks]**

Find the coefficient of x^3 in the expansion of $(2+3x)^5$, where $x \in \mathbb{R}$.

12. MEDIUM No Calculator **[4 marks]**

Find the term independent of x in the expansion of $\left(x^2 + \frac{4}{x}\right)^6$, where $x \in \mathbb{R}, x \neq 0$.

13. MEDIUM No Calculator **[4 marks]**

The coefficient of x^2 in the expansion of $(1+kx)^7$ is 84, where $k \in \mathbb{R}, k > 0$. Find the value of k.

14. MEDIUM Calculator **[4 marks]**

Consider the expansion of $(3x-2)^8$, where $x \in \mathbb{R}$. Find the coefficient of x^5.

15. MEDIUM Calculator **[4 marks]**

The first three terms in ascending powers of x of $(1+ax)^n$ are $1+12x+60x^2$, where $a \in \mathbb{R}^+$ and $n \in \mathbb{Z}^+$. Find the values of a and n.

16. MEDIUM Calculator **[4 marks]**

Find the coefficient of x^4 in the expansion of $(x-3)^4(1+2x)^3$, where $x \in \mathbb{R}$.

Hint: only the terms contributing to x^4 need to be considered.

17. MEDIUM Calculator **[4 marks]**

In the expansion of $(p+qx)^6$ where $p, q \in \mathbb{R}^+$, the coefficient of x is 48 and the coefficient of x^2 is 240. Find the values of p and q.

18. MEDIUM Calculator **[4 marks]**

The table shows the first four binomial coefficients from the expansion of $(1+x)^n$ for a particular value of $n \in \mathbb{Z}^+$.

r	0	1	2	3
$\binom{n}{r}$	1	n	55	165

(a) Write down the value of n. [1]

(b) Find the coefficient of x^3 in the expansion of $(2-x)^n$. [3]

19. MEDIUM Calculator **[4 marks]**

The coefficient of x^3 in the expansion of $(2+kx)^6$, where $k \in \mathbb{R}^+$, is three times the coefficient of x^2. Find the value of k.

20. MEDIUM No Calculator **[4 marks]**

Find the constant term in the expansion of $\left(2x^3 - \frac{1}{x^2}\right)^{10}$, where $x \in \mathbb{R}, x \neq 0$.

Section C — Hard **HARD** (10 questions × 5 marks = 50 marks)

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21. **HARD** **Calculator** **[5 marks]**

The coefficient of x^3 in the expansion of $(2+kx)^7$ is equal to the coefficient of x^3 in the expansion of $(1+3x)^6$.

Find the value of k, where $k \in \mathbb{R}, k > 0$.

22. **HARD** **Calculator** **[5 marks]**

In the expansion of $\left(x^2 + \frac{a}{x}\right)^9$, where $a \in \mathbb{R}^+$, the coefficient of the constant term is 672. Find the value of a.

23. HARD Calculator **[5 marks]**

Consider the expansion of $(3+x)^n$, where $n \in \mathbb{Z}^+$. The coefficients of x^2 and x^3 are in the ratio $1:2$. Find the value of n.

24. HARD No Calculator **[5 marks]**

Show that the coefficient of x^4 in the expansion of $(1+2x)^6$ is 240.

25. HARD Calculator **[5 marks]**

The first three terms of the expansion of $(1+kx)^n$, in ascending powers of x, are $1+12x+63x^2$, where $n \in \mathbb{Z}^+$ and $k \in \mathbb{R}^+$. Find the values of n and k.

26. HARD Calculator **[5 marks]**

In the expansion of $\left(px + \frac{1}{x^3}\right)^7$, where $p \in \mathbb{R}^+$, the term independent of x does not exist. Find the value of p given that the coefficient of x^3 is 448.

27. HARD No Calculator **[5 marks]**

Consider the expansion of $\left(2x^3 - \frac{1}{x^2}\right)^{10}$, where $x \in \mathbb{R}$, $x \neq 0$. Find the coefficient of x^5 in this expansion.

28. HARD No Calculator **[5 marks]**

In the expansion of $(1+x)^n(1+x)^m$, where $n, m \in \mathbb{Z}^+$, the coefficient of x^2 is 55. Given also that $nm = 18$, find the values of n and m, where $n < m$.

29. HARD Calculator **[5 marks]**

In the binomial expansion of $(k + x)^5$, where $k \in \mathbb{Z}^+$, the coefficients of x^2, x^3, and x^4 form three consecutive terms of a geometric sequence. Find all possible values of k.

Term	x^2	x^3	x^4
Coefficient	$10k^3$	$10k^2$	$5k$

30. HARD No Calculator **[5 marks]**

Find the term independent of x in the expansion of $\left(x - \frac{2}{x^2}\right)^6$, where $x \in \mathbb{R}, x \neq 0$.

Mark Schemes

Q1 — Binomial Coefficient Evaluation EASY

Attempt to evaluate $\binom{7}{3}$ using the formula or Pascal's triangle **M1**

$\binom{7}{3} = \frac{7!}{3!\,4!} = \frac{7 \times 6 \times 5}{3 \times 2 \times 1} = 35$ **A2**

Note: Award M1A1 for a correct but unsimplified expression such as $\frac{7\times6\times5}{6}$ *seen.*

Q2 — Full Expansion of a Binomial EASY

Attempt to expand $(1+x)^4$ using binomial coefficients from Pascal's triangle or $\binom{4}{r}$ **M1**

$(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$ **A2**

Note: Award A1 for exactly three correct terms if the expansion is otherwise incomplete.

Q3 — Coefficient Extraction EASY

General term: $\binom{5}{r}(3x)^r$; for x^2, set $r = 2$ **M1**

$\binom{5}{2}(3)^2 = 10 \times 9 = 90$ **A2**

Note: Award M1A1 for $\binom{5}{2} \times 9$ *seen without final evaluation.*

Q4 — Constant Term in a Binomial EASY

General term: $T_{r+1} = \binom{4}{r}x^{4-r}\left(\frac{2}{x}\right)^r = \binom{4}{r}2^r x^{4-2r}$ **M1**

For constant term: $4 - 2r = 0 \Rightarrow r = 2$ **(A1)**

$T_3 = \binom{4}{2} \times 2^2 = 6 \times 4 = 24$ **A1**

Note: Award final A1 only if a clear method for finding the constant term is shown.

Q5 — Pascals Triangle EASY

Use the property that each entry is the sum of the two entries above it **M1**

$1,\ 4,\ 6,\ 4,\ 1$ **A2**

Note: Award A1 if four of the five values are correct. Accept entries written as $\binom{4}{0}, \binom{4}{1}, \binom{4}{2}, \binom{4}{3}, \binom{4}{4}$ *evaluated correctly.*

Q6 — Coefficient Extraction EASY

General term: $T_{r+1} = \binom{5}{r} 2^{5-r} x^r$; for x^3, set $r = 3$ **M1**

$\binom{5}{3} \times 2^2 = 10 \times 4 = 40$ **A2**

Note: Award M1A1 for $\binom{5}{3} \times 4$ *seen without evaluation.*

Q7 — GDC Evaluation of a Binomial Coefficient EASY

Attempt to use GDC nCr function with $n = 10, r = 4$ **(M1)**

$\binom{10}{4} = 210$ **A2**

Note: Award (M1)A2 for the correct answer with GDC-evidenced method. Award (M1)A1 for $\frac{10!}{4!\,6!}$ *seen unsimplified.*

Q8 — Coefficient with Negative Term EASY

General term: $T_{r+1} = \binom{6}{r} 3^{6-r}(-x)^r = \binom{6}{r} 3^{6-r}(-1)^r x^r$ **M1**

For x^2, set $r = 2$: $\binom{6}{2} \times 3^4 \times (-1)^2 = 15 \times 81 \times 1 = 1215$ **A2**

Note: Award M1A1 for 15×81 *seen without final simplification. The factor* $(-1)^2 = 1$ *must be correctly handled.*

Q9 — First Three Terms of an Expansion EASY

Attempt to find at least two terms using the binomial theorem **M1**

$T_1 = 1; \quad T_2 = \binom{6}{1}(2x) = 12x; \quad T_3 = \binom{6}{2}(2x)^2 = 15 \times 4x^2 = 60x^2$ **A1**

First three terms: $1 + 12x + 60x^2$ **A1**

Note: Award M1A1A0 if exactly two of the three terms are correct. Condone listing as separate terms rather than a sum.

Q10 — Coefficient via General Term EASY

General term: $T_{r+1} = \binom{5}{r} x^{5-r} \cdot 4^r$ **M1**

For x^2: $5 - r = 2 \Rightarrow r = 3$ **(A1)**

$\binom{5}{3} \times 4^3 = 10 \times 64 = 640$ **A1**

Note: The general term must be written down before extraction for M1 to be awarded. Award final A1 for 640 only.

Q11 — Coefficient Extraction MEDIUM

The general term of $(2+3x)^5$ is $T_{r+1} = \binom{5}{r}(2)^{5-r}(3x)^r = \binom{5}{r} 2^{5-r} \cdot 3^r \cdot x^r$ **M1**

For the term in x^3, set $r = 3$: $T_4 = \binom{5}{3}(2)^2(3)^3x^3 = 10 \times 4 \times 27 \cdot x^3$ **A1**

Coefficient of $x^3 = 10 \times 4 \times 27 = 1080$ **A2**

Note: Award A1A0 if the answer $1080x^3$ is given rather than the coefficient 1080.

Q12 — Constant Term in a Mixed Power Binomial MEDIUM

The general term is $T_{r+1} = \binom{6}{r}(x^2)^{6-r}\left(\frac{4}{x}\right)^r = \binom{6}{r} 4^r \cdot x^{12-3r}$ **M1**

For the term independent of x, set $12 - 3r = 0$, giving $r = 4$ **A1**

$T_5 = \binom{6}{4} 4^4 = 15 \times 256 = 3840$ **A2**

Note: Award M1 for a correct general term with the power of x simplified. Award A1 for correctly finding $r = 4$. Award the final A2 (or A1A1) for 3840.

Q13 — Solving for k from a Coefficient MEDIUM

The general term of $(1+kx)^7$ is $\binom{7}{r}(kx)^r$ **M1**

The coefficient of x^2 corresponds to $r = 2$: $\binom{7}{2}k^2 = 21k^2$ **A1**

Setting $21k^2 = 84$ gives $k^2 = 4$, so $k = \pm 2$ **A1**

Since $k > 0$, $k = 2$ **A1**

Note: Award the final A1 only if the positive root is selected with reference to $k > 0$. If $k = \pm 2$ is stated without selecting the positive root, award A1A1A0.

Q14 — Coefficient with Negative Term MEDIUM

The general term of $(3x - 2)^8$ is $T_{r+1} = \binom{8}{r}(3x)^{8-r}(-2)^r = \binom{8}{r}3^{8-r}(-2)^r x^{8-r}$ **M1**

For the term in x^5, set $8 - r = 5$, giving $r = 3$ **(A1)**

$T_4 = \binom{8}{3}(3)^5(-2)^3 = 56 \times 243 \times (-8)$ **(A1)**

Coefficient of $x^5 = 56 \times 243 \times (-8) = -108864$ **A1**

Note: Award (A1) for $r = 3$. Award (A1) for a correct unsimplified expression. The trap is losing the negative sign from $(-2)^3$; the sign must be carried through to the final coefficient.

Q15 — Finding a and n from First Three Terms MEDIUM

The first three terms of $(1 + ax)^n$ are $1 + nax + \frac{n(n-1)}{2}a^2x^2$ **M1**

Equating coefficients: $na = 12$ and $\frac{n(n-1)}{2}a^2 = 60$ **A1**

Dividing the second equation by the square of the first: $\frac{n-1}{2n} = \frac{60}{144} = \frac{5}{12} \Rightarrow 12(n-1) = 10n \Rightarrow n = 6$

Then $a = \frac{12}{6} = 2$ **A1A1**

Note: Award M1 for writing both binomial coefficients correctly. Award A1 for setting up both equations. Award A1 for $n = 6$ and A1 for $a = 2$. Both values must be stated.

Q16 — Coefficient in a Product of Two Expansions MEDIUM

Relevant terms from $(x - 3)^4$: $\binom{4}{4}x^4 = x^4$; $\binom{4}{3}x^3(-3) = -12x^3$; $\binom{4}{2}x^2(9) = 54x^2$; $\binom{4}{1}x(-27) = -108x$ **M1**

Relevant terms from $(1 + 2x)^3$: constant term $= 1$; coefficient of x is $\binom{3}{1}(2) = 6$; coefficient of x^2 is $\binom{3}{2}(4) = 12$; coefficient of x^3 is $\binom{3}{3}(8) = 8$ **A1**

All four pairs of terms whose degrees sum to 4 must be combined: $(x^4)(1)$, $(-12x^3)(6x)$, $(54x^2)(12x^2)$, and $(-108x)(8x^3)$ **M1**

Contributions to x^4: $(x^4)(1) + (-12x^3)(6x) + (54x^2)(12x^2) + (-108x)(8x^3) = 1 - 72 + 648 - 864$ **A1**

Coefficient of $x^4 = 1 - 72 + 648 - 864 = -287$ **A1**

Note: The trap is omitting the fourth pair $(-108x)(8x^3) = -864x^4$*, which arises because* $(1 + 2x)^3$ *also contributes a degree-3 term; omitting it undercounts the final coefficient. FT arithmetic in the final A1 provided the method is correct.*

Q17 — Finding p and q from Two Coefficients MEDIUM

Coefficient of x: $\binom{6}{1}p^5q = 6p^5q = 48$, so $p^5q = 8$ **M1**

Coefficient of x^2: $\binom{6}{2}p^4q^2 = 15p^4q^2 = 240$, so $p^4q^2 = 16$ **A1**

Dividing $p^4q^2 = 16$ by $p^5q = 8$: $\frac{q}{p} = 2 \Rightarrow q = 2p$

Substituting into $p^5q = 8$: $p^5(2p) = 8 \Rightarrow 2p^6 = 8 \Rightarrow p^6 = 4$ **A1**

$p = 4^{1/6} = (2^2)^{1/6} = 2^{1/3} \approx 1.26$

Then $q = 2p = 2^{4/3} \approx 2.52$ **A1**

Note: Award M1 for setting up both coefficient equations correctly. Award A1 for both simplified equations. Award A1A1 for the correct values $p = 2^{1/3}$, $q = 2^{4/3}$. *The trap is writing* $4^{1/6}$ *as* $\sqrt{2}$*; the correct simplification is* $4^{1/6} = 2^{2/6} = 2^{1/3}$*, not* $2^{1/2}$.

Q18 — Pascal Row Identification and Coefficient MEDIUM

(a) From $\binom{n}{2} = 55$: $\frac{n(n-1)}{2} = 55 \Rightarrow n(n-1) = 110 \Rightarrow n = 11$ **A1**

Note: Verification with $\binom{11}{3} = 165$ *is implied; no further working required.*

(b) The general term of $(2 - x)^{11}$ is $\binom{11}{r}(2)^{11-r}(-x)^r = \binom{11}{r}2^{11-r}(-1)^r x^r$ **M1**

For $r = 3$: $\binom{11}{3}(2)^8(-1)^3 = 165 \times 256 \times (-1)$ **A1**

Coefficient of $x^3 = 165 \times 256 \times (-1) = -42240$ **A1**

Note: In part (b) FT their value of n from part (a). The trap is omitting the factor $(-1)^3$; the negative sign must be carried through to the final coefficient.

Q19 — Coefficient Ratio Condition MEDIUM

The coefficient of x^2 in $(2+kx)^6$ is $\binom{6}{2}(2)^4k^2 = 15 \times 16 \times k^2 = 240k^2$ **M1**

The coefficient of x^3 is $\binom{6}{3}(2)^3k^3 = 20 \times 8 \times k^3 = 160k^3$ **A1**

Setting the coefficient of x^3 equal to three times the coefficient of x^2: $160k^3 = 3(240k^2) = 720k^2$ **A1**

Since $k \neq 0$, divide both sides by k^2: $160k = 720 \Rightarrow k = \frac{720}{160} = 4.5$ **A1**

Note: Full precision $k = 9/2$ exactly. Award final A1 only for the positive value $k = 4.5$.

Q20 — Constant Term in a Mixed Power Binomial MEDIUM

The general term of $\left(2x^3 - \frac{1}{x^2}\right)^{10}$ is $T_{r+1} = \binom{10}{r}(2x^3)^{10-r}\left(-\frac{1}{x^2}\right)^r = \binom{10}{r}2^{10-r}(-1)^r x^{30-5r}$ **M1**

For the constant term, set $30 - 5r = 0$, giving $r = 6$ **A1**

$T_7 = \binom{10}{6}2^4(-1)^6 = 210 \times 16 \times 1 = 3360$ **A2**

Note: Award M1 for a correct general term with the power of x simplified to $30-5r$. Award A1 for $r = 6$. Award A2 for 3360 (or A1A1 if split). The $(-1)^6 = 1$ does not create a sign trap here; award in full.

Q21 — Equal Coefficients Solve for k HARD

Coefficient of x^3 in $(2+kx)^7$: general term $\binom{7}{3}(2)^4(kx)^3$ **M1**

$= 35 \times 16 \times k^3x^3 = 560k^3x^3$, so coefficient is $560k^3$ **A1**

Coefficient of x^3 in $(1+3x)^6$: $\binom{6}{3}(1)^3(3x)^3 = 20 \times 27 = 540$ **A1**

Setting equal: $560k^3 = 540$ **M1**

$k^3 = \frac{540}{560} = \frac{27}{28}$, so $k = \sqrt[3]{\frac{27}{28}} \approx 0.988$ **A1**

Note: Accept $k = \sqrt[3]{27/28}$ or equivalent exact form. Since $k > 0$, the positive real cube root is taken; award final A1 only for the value 0.988 (3 s.f.).

Q22 — Constant Term in a Mixed Power Binomial HARD

General term of $\left(x^2+\frac{a}{x}\right)^9$: $T_{r+1}=\binom{9}{r}(x^2)^{9-r}\left(\frac{a}{x}\right)^r=\binom{9}{r}a^r\cdot x^{18-3r}$ **M1A1**

For constant term: $18-3r=0\Rightarrow r=6$ **M1**

Coefficient: $\binom{9}{6}a^6=84a^6$ **A1**

$84a^6=672\Rightarrow a^6=8\Rightarrow a=8^{1/6}=\sqrt{2}\approx 1.41$

Since $a\in\mathbb{R}^+$, reject the negative sixth root. $a=\sqrt{2}$ **A1**

Note: Final A1 requires root rejection stated or implied by $a\in\mathbb{R}^+$.

Q23 — Ratio of Consecutive Coefficients HARD

Coefficient of x^2 in $(3+x)^n$: $\binom{n}{2}3^{n-2}$ **M1**

Coefficient of x^3 in $(3+x)^n$: $\binom{n}{3}3^{n-3}$ **A1**

Setting up ratio $1:2$: $\frac{\binom{n}{2}3^{n-2}}{\binom{n}{3}3^{n-3}}=\frac{1}{2}$ **M1**

$\frac{\frac{n(n-1)}{2}\cdot 3}{\frac{n(n-1)(n-2)}{6}}=\frac{1}{2}\Rightarrow\frac{9}{n-2}=\frac{1}{2}\Rightarrow n-2=18\Rightarrow n=20$ **A1**

Note: Accept any valid algebraic simplification leading to $n=20$. Award M1 for forming the ratio equation correctly.

Q24 — Show That a Coefficient Equals 240 HARD

General term of $(1+2x)^6$: $T_{r+1}=\binom{6}{r}(1)^{6-r}(2x)^r=\binom{6}{r}2^rx^r$ **M1A1**

For x^4: $r=4$ **M1**

$T_5=\binom{6}{4}\cdot 2^4\cdot x^4$ **(M1)**

$=15\times 16\times x^4$ **A1**

$=240x^4$ **AG**

Note: Final line must reach $240x^4$ from explicit numerical evaluation of $\binom{6}{4}=15$ and $2^4=16$; do not accept a GDC-only statement.

Q25 — Finding n and k from First Three Terms HARD

First three terms of $(1+kx)^n$: $1+nkx+\dfrac{n(n-1)}{2}k^2x^2+\cdots$ **M1A1**

From x^1: $nk=12$ **(M1)**

From x^2: $\dfrac{n(n-1)}{2}k^2=63$

Dividing: $\dfrac{(n-1)k}{2}=\dfrac{63}{12}\Rightarrow(n-1)k=\dfrac{21}{2}$ **M1**

So $(n-1)k=\dfrac{21}{2}$ and $nk=12$: subtracting gives $k=12-\dfrac{21}{2}=\dfrac{3}{2}$

Then $n=\dfrac{12}{k}=\dfrac{12}{3/2}=8$ **A1**

$n=8,\ k=\dfrac{3}{2}$ **A1**

Note: Both values required for final A1A1. Award first A1 for either correct value.

Q26 — Coefficient Equation with No Constant Term HARD

General term of $\left(px+\dfrac{1}{x^3}\right)^7$: $T_{r+1}=\dbinom{7}{r}(px)^{7-r}\left(\dfrac{1}{x^3}\right)^r=\dbinom{7}{r}p^{7-r}x^{7-4r}$ **M1**

A constant term would require $7-4r=0\Rightarrow r=\dfrac{7}{4}$, which is not an integer, confirming that the term independent of x does not exist **R1**

For x^3: $7-4r=3\Rightarrow r=1$ **M1**

$T_2=\dbinom{7}{1}p^6x^3=7p^6x^3$ **A1**

$7p^6=448\Rightarrow p^6=64\Rightarrow p=64^{1/6}=2$ **A1**

Note: Since $p\in\mathbb{R}^+$, only the positive real sixth root is valid; award final A1 only if $p=2$ is confirmed.

Q27 — Coefficient with Negative Term HARD

General term of $\left(2x^3-\dfrac{1}{x^2}\right)^{10}$: $T_{r+1}=\dbinom{10}{r}(2x^3)^{10-r}\left(-\dfrac{1}{x^2}\right)^r=\dbinom{10}{r}(-1)^r\cdot2^{10-r}\cdot x^{3(10-r)-2r}$
M1A1

Power of x: $30-3r-2r=30-5r$. For x^5: $30-5r=5\Rightarrow r=5$ **M1**

$T_6=\dbinom{10}{5}(-1)^5\cdot2^5\cdot x^5$ **(M1)**

$=252\times(-1)\times32\times x^5=-8064x^5$ **A1**

Coefficient of x^5 is -8064.

Note: The sign from $(-1)^5$ is the flagged trap; award final A1 only if the negative sign is present.

Q28 — Combined Expansion Find n and m HARD

$(1+x)^n(1+x)^m = (1+x)^{n+m}$ **M1**

Coefficient of x^2: $\binom{n+m}{2} = 55 \Rightarrow (n+m)(n+m-1) = 110 \Rightarrow n+m = 11$ **A1**

Using $nm = 18$: n and m are roots of $t^2 - 11t + 18 = 0$ **M1**

$(t-2)(t-9) = 0 \Rightarrow t = 2$ or $t = 9$ **A1**

Since $n < m$: $n = 2$, $m = 9$ **A1**

Note: Award M1 for combining to $(1+x)^{n+m}$. Award A1 for $n+m = 11$. Award M1 for setting up the quadratic from $nm = 18$. Award A1 for solving it, and final A1 for the ordered pair $n = 2$, $m = 9$.

Q29 — Geometric Progression Condition on Coefficients HARD

From the table, the three coefficients are $10k^3$, $10k^2$, $5k$ **A1**

For a geometric sequence: $(10k^2)^2 = 10k^3 \cdot 5k$ **M1**

$100k^4 = 50k^4$ **(M1)**

$100k^4 - 50k^4 = 0 \Rightarrow 50k^4 = 0 \Rightarrow k = 0$

Since $k \in \mathbb{Z}^+$, $k = 0$ is rejected **R1**

There are no valid values of $k \in \mathbb{Z}^+$ for which the three coefficients form a geometric sequence **A1**

Note: Award R1 for explicit rejection of $k = 0$ with reference to $k \in \mathbb{Z}^+$. Full marks available for a rigorous argument concluding no solution exists.

Q30 — Term Independent of x with Negative Base HARD

General term of $\left(x - \frac{2}{x^2}\right)^6$: $T_{r+1} = \binom{6}{r}(x)^{6-r}\left(-\frac{2}{x^2}\right)^r = \binom{6}{r}(-2)^r \cdot x^{6-3r}$ **M1A1**

For term independent of x: $6 - 3r = 0 \Rightarrow r = 2$ **M1**

$T_3 = \binom{6}{2}(-2)^2$ **(M1)**

$= 15 \times 4 = 60$ **A1**

The term independent of x is 60.

Note: The sign trap is $(-2)^2 = +4$; do not drop or flip this sign when combining with $\binom{6}{2}$. Award final A1 only for the correctly-signed positive value.