IB Math AA SL — Topic 1

Binomial Theorem

Binomial Coefficients & Pascal's Triangle · The Binomial Theorem

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:______________________________ Date:_______________

Score:__________/120

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Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

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Key Formulae

- **Binomial coefficient:** $\binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}, \quad r = 0, 1, \dots, n$
- **Binomial theorem:** $(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$
- **General term** ($(r+1)$-th term): $T_{r+1} = \binom{n}{r} a^{n-r} b^r$
- **Pascal's triangle identity:** $\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$
- **Symmetry:** $\binom{n}{r} = \binom{n}{n-r}$
- **Special cases:** $\binom{n}{0} = \binom{n}{n} = 1; \quad \binom{n}{1} = \binom{n}{n-1} = n$
- **Sum of all coefficients:** $(1+1)^n = 2^n$, so $\sum_{r=0}^{n} \binom{n}{r} = 2^n$

GDC Tips

- **Evaluating $\binom{n}{r}$:** Use `Math` $\to$ `PRB` $\to$ `nCr` (TI) or `OPTN` $\to$ `PROB` $\to$ `nCr` (Casio); enter n first, then r.
- **Expanding $(a+b)^n$ for large n:** Store values of a and b, then use repeated `nCr` calls to compute each T_{r+1} individually, or use `binomial(n,r)` in a CAS-enabled model.
- **Finding an unknown n:** Set up the coefficient equation and use `Solve` (TI: `nSolve`; Casio: equation solver) to find integer solutions; always verify the result is a positive integer.
- **Checking coefficient of x^k:** Expand symbolically with `expand((a+b x)^n)` in a CAS model and read off the required coefficient directly.
- **Ratio of consecutive terms:** Compute $T_{r+1}/T_r = \frac{n-r+1}{r} \cdot \frac{b}{a}$ numerically on the GDC to locate the largest term or verify an AP/GP condition.
- **Large factorials / overflow:** Use `nCr` rather than computing $n!$ directly; GDCs overflow for $n!$ when $n \gtrsim 70$.

Common Mistakes to Avoid

1. **Forgetting to write the general term first.** Students jump straight to substituting r without writing $T_{r+1} = \binom{n}{r}a^{n-r}b^r$. The general term must appear before any extraction; omitting it loses the method mark.

2. **Sign errors with a negative b.** When b is negative, $b^r = (-1)^r|b|^r$. A common error is treating b^r as positive throughout, giving the wrong sign for odd-power terms (e.g. finding $+84$ instead of -84 for the coefficient of x^3).

3. **Confusing the coefficient with the term.** The question asks for "the coefficient of x^3", which is the number multiplying x^3, not the entire expression $\binom{n}{r}a^{n-r}b^r x^3$. Leaving the x^r attached loses the final accuracy mark.

4. **Incorrect index when solving for r.** Setting the power of x equal to the required power involves solving $n - r = k$ or $r = k$ depending on which factor carries x. Mixing up r and $n - r$ (e.g. using T_r instead of T_{r+1}) shifts every term by one index and gives a wrong answer.

5. **Wrong term chosen when two terms share the same power.** In expansions of the form $(ax^p + b/x^q)^n$, the power of x in T_{r+1} is $p(n-r) - qr$. Students sometimes set only $p(n-r) =$ required power, ignoring the $-qr$ contribution, and select the wrong term.

6. **Parameter order condition ignored.** When two symmetric values of r (or n) satisfy the algebra (e.g. $r = 2$ and $r = n - 2$), and the question states $p < q$ or $k > 0$, one solution must be rejected. Failing to apply the stated condition and giving both answers without justification loses the reasoning mark (R1).

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Section A — Easy

EASY

1. Write down the value of $\begin{pmatrix} 7 \\ 3 \end{pmatrix}$. *Calculator not allowed* **[3 marks]**

2. Find the expansion of $(1+x)^4$, giving each coefficient as an integer. *Calculator not allowed* **[3 marks]**

3. Find the coefficient of x^2 in the expansion of $(1+3x)^5$. *Calculator not allowed* **[3 marks]**

4. Find the term independent of x in the expansion of $\left(x+\dfrac{2}{x}\right)^4$, where $x \neq 0$. *Calculator allowed* **[3 marks]**

5. The third row of Pascal's triangle (starting from row 0) is shown below.

$\binom{3}{0}$	$\binom{3}{1}$	$\binom{3}{2}$	$\binom{3}{3}$
1	3	3	1

Write down the values in the fourth row of Pascal's triangle. *Calculator allowed* **[3 marks]**

6. Find the coefficient of x^3 in the expansion of $(2 + x)^5$. *Calculator allowed* **[3 marks]**

7. Find the value of $\binom{10}{4}$ using a calculator. *Calculator allowed* **[3 marks]**

8. Find the coefficient of x^2 in the expansion of $(3 - x)^6$. *Calculator allowed* **[3 marks]**

9. Find the first three terms, in ascending powers of x, of the expansion of $(1 + 2x)^6$. *Calculator allowed* **[3 marks]**

10. Find the coefficient of x^2 in the expansion of $(x+4)^5$, where the general term is written down first. *Calculator not allowed* **[3 marks]**

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Section B — Medium

MEDIUM

11. Find the coefficient of x^3 in the expansion of $(2+3x)^5$, where $x \in \mathbb{R}$. *Calculator not allowed* **[4 marks]**

12. Find the term independent of x in the expansion of $\left(x^2 + \dfrac{4}{x}\right)^6$, where $x \in \mathbb{R}$, $x \neq 0$. *Calculator not allowed* **[4 marks]**

13. The coefficient of x^2 in the expansion of $(1+kx)^7$ is 84, where $k \in \mathbb{R}$, $k > 0$. Find the value of k. *Calculator not allowed* **[4 marks]**

14. Consider the expansion of $(3x - 2)^8$, where $x \in \mathbb{R}$. Find the coefficient of x^5. *Calculator allowed* **[4 marks]**

15. The first three terms in ascending powers of x of $(1 + ax)^n$ are $1 + 12x + 60x^2$, where $a \in \mathbb{R}^+$ and $n \in \mathbb{Z}^+$. Find the values of a and n. *Calculator allowed* **[4 marks]**

16. Find the coefficient of x^4 in the expansion of $(x - 3)^4(1 + 2x)^3$, where $x \in \mathbb{R}$.
Hint: only the terms contributing to x^4 need to be considered. *Calculator allowed* **[4 marks]**

17. In the expansion of $(p + qx)^6$ where $p, q \in \mathbb{R}^+$, the coefficient of x is 48 and the coefficient of x^2 is 240. Find the values of p and q. *Calculator allowed* **[4 marks]**

18. The table shows the first four binomial coefficients from the expansion of $(1+x)^n$ for a particular value of $n \in \mathbb{Z}^+$.

r	0	1	2	3
$\binom{n}{r}$	1	n	55	165

(a) Write down the value of n. *Calculator allowed* **[1 marks]**

(b) Find the coefficient of x^3 in the expansion of $(2-x)^n$. *Calculator allowed* **[3 marks]**

19. The coefficients of x^2 and x^3 in the expansion of $(2+kx)^6$, where $k \in \mathbb{R}^+$, are consecutive terms of an arithmetic sequence with the coefficient of x^2 being the smaller term. Find the value of k. *Calculator allowed* **[4 marks]**

20. Find the constant term in the expansion of $\left(2x^3 - \frac{1}{x^2}\right)^{10}$, where $x \in \mathbb{R}$, $x \neq 0$. *Calculator not allowed* **[4 marks]**

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Section C — Hard

HARD

21. The coefficient of x^3 in the expansion of $(2 + kx)^7$ is equal to the coefficient of x^3 in the expansion of $(1 + 3x)^6$. Find the value of k, where $k \in \mathbb{R}$, $k > 0$. *Calculator allowed* **[5 marks]**

22. In the expansion of $\left(x^2 + \dfrac{a}{x}\right)^9$, where $a \in \mathbb{R}^+$, the coefficient of the constant term is 672. Find the value of a. *Calculator allowed* **[5 marks]**

23. Consider the expansion of $(3+x)^n$, where $n \in \mathbb{Z}^+$. The coefficients of x^2 and x^3 are in the ratio $1:2$. Find the value of n. *Calculator allowed* **[5 marks]**

24. Show that the coefficient of x^4 in the expansion of $(1+2x)^6$ is 240. *Calculator not allowed* **[5 marks]**

25. The first three terms of the expansion of $(1+kx)^n$, in ascending powers of x, are $1+12x+63x^2$, where $n \in \mathbb{Z}^+$ and $k \in \mathbb{R}^+$. Find the values of n and k. *Calculator allowed* **[5 marks]**

26. In the expansion of $\left(px + \frac{1}{x^3}\right)^8$, where $p \in \mathbb{R}^+$, the term independent of x does not exist, but the coefficient of x^4 is 1120. Find the value of p. *Calculator allowed* **[5 marks]**

27. Consider the expansion of $\left(2x^3 - \frac{1}{x^2}\right)^{10}$, where $x \in \mathbb{R}, x \neq 0$. Find the coefficient of x^5 in this expansion.
Calculator not allowed **[5 marks]**

28. In the expansion of $(1+x)^n(1+x)^m$, where $m, n \in \mathbb{Z}^+$, the coefficient of x^2 equals 55. Given also that $n+m=10$, find the values of n and m, where $n<m$. *Calculator not allowed* **[5 marks]**

29. In the binomial expansion of $(k+x)^5$, where $k \in \mathbb{Z}^+$, the coefficients of x^2, x^3, and x^4 form three consecutive terms of a geometric sequence. Find all possible values of k. *Calculator allowed* **[5 marks]**

Term	x^2	x^3	x^4
Coefficient	$10k^3$	$10k^2$	$5k$

30. Find the term independent of x in the expansion of $\left(x - \dfrac{2}{x^2}\right)^6$, where $x \in \mathbb{R}$, $x \neq 0$. *Calculator not allowed* **[5 marks]**

Worked Solutions

Q1 — Binomial coefficient evaluation [EASY]

Attempt to evaluate $\binom{7}{3}$ using the formula or Pascal's triangle **M1**

$\binom{7}{3} = \frac{7!}{3!\,4!} = \frac{7\times 6\times 5}{3\times 2\times 1} = 35$ **A2**

Note: Award M1A1 for a correct but unsimplified expression such as $\frac{7\times 6\times 5}{6}$ *seen.*

Q2 — Full expansion of $(1+x)^4$ [EASY]

Attempt to expand $(1+x)^4$ using binomial coefficients from Pascal's triangle or $\binom{4}{r}$ **M1**

$(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$ **A2**

Note: Award A1 for exactly three correct terms if the expansion is otherwise incomplete.

Q3 — Coefficient of x^2 in $(1+3x)^5$ [EASY]

General term: $\binom{5}{r}(3x)^r$; for x^2, set $r = 2$ **M1**

$\binom{5}{2}(3)^2 = 10\times 9 = 90$ **A2**

Note: Award M1A1 for $\binom{5}{2}\times 9$ *seen without final evaluation.*

Q4 — Constant term in $(x+\frac{2}{x})^4$ [EASY]

General term: $T_{r+1} = \binom{4}{r}x^{4-r}\left(\frac{2}{x}\right)^r = \binom{4}{r}2^r\,x^{4-2r}$ **M1**

For constant term: $4 - 2r = 0 \Rightarrow r = 2$ **(A1)**

$T_3 = \binom{4}{2}\times 2^2 = 6\times 4 = 24$ **A1**

Note: Award final A1 only if a clear method for finding the constant term is shown.

Q5 — Fourth row of Pascal's triangle [EASY]

Use the property that each entry is the sum of the two entries above it **M1**

$1, \quad 4, \quad 6, \quad 4, \quad 1$ **A2**

Note: Award A1 if four of the five values are correct. Accept entries written as $\binom{4}{0}, \binom{4}{1}, \binom{4}{2}, \binom{4}{3}, \binom{4}{4}$ *evaluated correctly.*

Q6 — Coefficient of x^3 in $(2+x)^5$ [EASY]

General term: $T_{r+1} = \binom{5}{r} 2^{5-r} x^r$; for x^3, set $r = 3$ **M1**

$\binom{5}{3} \times 2^2 = 10 \times 4 = 40$ **A2**

Note: Award M1A1 for $\binom{5}{3} \times 4$ *seen without evaluation.*

Q7 — GDC evaluation of $\binom{10}{4}$ [EASY]

Attempt to use GDC nCr function with $n = 10$, $r = 4$ **(M1)**

$\binom{10}{4} = 210$ **A2**

Note: Award (M1)A2 for the correct answer with GDC-evidenced method. Award (M1)A1 for $\frac{10!}{4!\,6!}$ *seen unsimplified.*

Q8 — Coefficient of x^2 in $(3-x)^6$ [EASY]

General term: $T_{r+1} = \binom{6}{r} 3^{6-r} (-x)^r = \binom{6}{r} 3^{6-r} (-1)^r x^r$ **M1**

For x^2, set $r = 2$: $\binom{6}{2} \times 3^4 \times (-1)^2 = 15 \times 81 \times 1 = 1215$ **A2**

Note: Award M1A1 for 15×81 *seen without final simplification. The factor* $(-1)^2 = 1$ *must be correctly* *handled.*

Q9 — First three terms of $(1+2x)^6$ [EASY]

Attempt to find at least two terms using the binomial theorem **M1**

$T_1 = 1; \quad T_2 = \binom{6}{1}(2x) = 12x; \quad T_3 = \binom{6}{2}(2x)^2 = 15 \times 4x^2 = 60x^2$ **A1**

First three terms: $1 + 12x + 60x^2$ **A1**

Note: Award M1A1A0 if exactly two of the three terms are correct. Condone listing as separate terms rather than a sum.

Q10 — Coefficient of x^2 in $(x+4)^5$ [EASY]

General term: $T_{r+1} = \binom{5}{r} x^{5-r} \cdot 4^r$ **M1**

For x^2: $5 - r = 2 \Rightarrow r = 3$ **(A1)**

$\binom{5}{3} \times 4^3 = 10 \times 64 = 640$ **A1**

Note: The general term must be written down before extraction for M1 to be awarded. Award final A1 for 640 only.

Q11 — Coefficient of x^3 in $(2+3x)^5$ [MEDIUM]

The general term of $(2+3x)^5$ is

$$T_{r+1} = \binom{5}{r}(2)^{5-r}(3x)^r = \binom{5}{r}2^{5-r} \cdot 3^r \cdot x^r.$$

M1

For the term in x^3, set $r = 3$:

$$T_4 = \binom{5}{3}(2)^2(3)^3x^3 = 10 \times 4 \times 27 \cdot x^3.$$

A1

Coefficient of $x^3 = 10 \times 4 \times 27 = 1080.$ **A2**

Note: Award A1A0 if the answer $1080x^3$ is given rather than the coefficient 1080.

Q12 — Constant term in $(x^2 + 4/x)^6$ [MEDIUM]

The general term is

$$T_{r+1} = \binom{6}{r}(x^2)^{6-r}\left(\frac{4}{x}\right)^r = \binom{6}{r}4^r \cdot x^{12-2r-r} = \binom{6}{r}4^r \cdot x^{12-3r}.$$

M1

For the term independent of x, set $12 - 3r = 0$, giving $r = 4$. **A1**

$$T_5 = \binom{6}{4}4^4 = 15 \times 256 = 3840.$$

A2

Note: Award M1 for a correct general term with the power of x simplified. Award A1 for correctly finding $r = 4$. Award the final A2 (or A1A1) for 3840.

Q13 — Finding k from coefficient of x^2 in $(1 + kx)^7$ [MEDIUM]

The general term of $(1 + kx)^7$ is $\binom{7}{r}(kx)^r$. **M1**

The coefficient of x^2 corresponds to $r = 2$:

$$\binom{7}{2}k^2 = 21k^2.$$

A1

Setting $21k^2 = 84$ gives $k^2 = 4$, so $k = \pm 2$. **A1**

Since $k > 0$, $k = 2$. **A1**

Note: Award the final A1 only if the positive root is selected with reference to $k > 0$. If $k = \pm 2$ is stated without selecting the positive root, award A1A1A0.

Q14 — Coefficient of x^5 in $(3x - 2)^8$ [MEDIUM]

The general term of $(3x - 2)^8$ is

$$T_{r+1} = \binom{8}{r}(3x)^{8-r}(-2)^r = \binom{8}{r}3^{8-r}(-2)^r x^{8-r}.$$

M1

For the term in x^5, set $8 - r = 5$, giving $r = 3$. **(A1)**

$$T_4 = \binom{8}{3}(3)^5(-2)^3 = 56 \times 243 \times (-8).$$

(A1)

Coefficient of $x^5 = 56 \times 243 \times (-8) = -108\,864.$ **A1**

Note: Award (A1) for $r = 3$. Award (A1) for a correct unsimplified expression. The trap is losing the negative sign from $(-2)^3$; award A0 for $+108864$.

Q15 — Finding a and n from first three terms [MEDIUM]

The first three terms of $(1 + ax)^n$ are

$$1 + nax + \frac{n(n-1)}{2}a^2x^2.$$

M1

Equating coefficients:

$$na = 12 \quad \text{and} \quad \frac{n(n-1)}{2}a^2 = 60.$$

A1

Dividing the second equation by the square of the first:

$$\frac{\frac{n(n-1)}{2}a^2}{n^2a^2} = \frac{60}{144} \implies \frac{n-1}{2n} = \frac{5}{12} \implies 12(n-1) = 10n \implies n = 6.$$

Then $a = \dfrac{12}{6} = 2.$ **A1A1**

Note: Award M1 for writing both binomial coefficients correctly. Award A1 for setting up both equations. Award A1 for $n = 6$ and A1 for $a = 2$. Both values must be stated.

Q16 — Coefficient of x^4 in $(x-3)^4(1+2x)^3$ [MEDIUM]

METHOD 1

Relevant terms from $(x-3)^4$: $\binom{4}{4}x^4 = x^4$ and $\binom{4}{3}x^3(-3) = -12x^3$ and $\binom{4}{2}x^2(9) = 54x^2$. **M1**

Relevant terms from $(1+2x)^3$: constant term $= 1$; coefficient of x is $\binom{3}{1}(2) = 6$; coefficient of x^2 is $\binom{3}{2}(4) = 12$. **A1**

Contributions to x^4:

$$(x^4)(1) + (-12x^3)(6x) + (54x^2)(12x^2) = 1 - 72 + 648.$$

A1

Coefficient of $x^4 = 1 - 72 + 648 = 577.$ **A1**

Note: Award M1 for identifying all three pairs of terms that contribute to x^4. FT arithmetic in the final A1 provided the method is correct.

Q17 — Finding p and q from two coefficients in $(p+qx)^6$ [MEDIUM]

Coefficient of x: $\binom{6}{1}p^5q = 6p^5q = 48$, so $p^5q = 8$. **M1**

Coefficient of x^2: $\binom{6}{2}p^4q^2 = 15p^4q^2 = 240$, so $p^4q^2 = 16$. **A1**

Dividing $p^4q^2 = 16$ by $p^5q = 8$:

$$\frac{q}{p} = 2 \implies q = 2p.$$

Substituting into $p^5q = 8$: $p^5(2p) = 8 \implies 2p^6 = 8 \implies p^6 = 4 \implies p = 4^{1/6} = \sqrt{2}$. **A1**

Then $q = 2\sqrt{2}$. **A1**

Note: Award M1 for setting up both coefficient equations correctly. Award A1 for both simplified equations. Award A1A1 for the correct values $p = \sqrt{2}$, $q = 2\sqrt{2}$. Accept $p = 2^{1/2}$, $q = 2^{3/2}$.

Q18 — Pascal row identification and coefficient in $(2-x)^n$ [MEDIUM]

(a)

From $\binom{n}{2} = 55$: $\dfrac{n(n-1)}{2} = 55 \implies n(n-1) = 110 \implies n = 11.$ **A1**

Note: Verification with $\binom{11}{3} = 165$ *is implied; no further working required.*

(b)

The general term of $(2-x)^{11}$ is $\binom{11}{r}(2)^{11-r}(-x)^r = \binom{11}{r}2^{11-r}(-1)^r x^r$. **M1**

For $r = 3$:

$$\binom{11}{3}(2)^8(-1)^3 = 165 \times 256 \times (-1).$$

A1

Coefficient of $x^3 = 165 \times 256 \times (-1) = -42\,240$. **A1**

Note: In part (b) FT their value of n from part (a). The trap is omitting the factor $(-1)^3$; award A1A0 for $+42240$.

Q19 — AP condition on coefficients in $(2+kx)^6$ [MEDIUM]

The coefficient of x^2 in $(2+kx)^6$ is $\binom{6}{2}(2)^4k^2 = 15 \times 16 \times k^2 = 240k^2$. **M1**

The coefficient of x^3 is $\binom{6}{3}(2)^3k^3 = 20 \times 8 \times k^3 = 160k^3$. **A1**

For an arithmetic sequence with $240k^2$ as the smaller term:

$$160k^3 - 240k^2 = 240k^2 - \text{(previous term)}.$$

Since the two given consecutive terms form part of an AP, their common difference gives:

$$160k^3 - 240k^2 = d \quad \text{and the condition is that they are consecutive, so}$$

$$2(240k^2) = \text{(first term)} + \text{(third term)},$$

but since only the two coefficients are stated to be consecutive terms, the AP condition is that they have a constant difference with $240k^2 < 160k^3$:

$$\frac{160k^3}{240k^2} = \text{constant ratio is not required; the AP condition: } 160k^3 - 240k^2 = c.$$

The condition that these two values are consecutive terms of an AP (i.e. the difference is constant, requiring a third term) is interpreted as: the two coefficients satisfy $160k^3 = 240k^2 + d$ for some $d > 0$. Since the problem states they are consecutive AP terms, we use the defining condition directly:

$$160k^3 - 240k^2 > 0 \implies k > \frac{3}{2}.$$

For the two terms to be in AP (with equal common difference), a third term is needed. With only two terms stated, the AP condition means their ratio is not fixed; instead we note: for the coefficients of x^2 and x^3 to be consecutive AP terms with x^2 smaller, use $160k^3 - 240k^2 = 240k^2$ (the common difference equals the smaller term, i.e. the first term is 0):

$$160k^3 = 2(240k^2) \implies 160k^3 = 480k^2 \implies k = 3.$$

A1

$k = 3$. **A1**

Note: The AP condition used is $2 \times \text{coeff}(x^2) = \text{coeff}(x) + \text{coeff}(x^3)$, where $\text{coeff}(x) = \binom{6}{1}(2)^5k = 192k$. So $2(240k^2) = 192k + 160k^3$, giving $480k^2 = 192k + 160k^3$, i.e. $160k^3 - 480k^2 + 192k = 0$, i.e. $32k(5k^2 - 15k + 6) = 0$. Since $k > 0$, $k = \frac{15 \pm \sqrt{225-120}}{10} = \frac{15 \pm \sqrt{105}}{10}$. Award M1 for both coefficients correct, A1 for the AP equation, A1A1 for both positive solutions (accept awarding A2 for both values). Accept $k = \frac{15+\sqrt{105}}{10} \approx 2.52$ and $k = \frac{15-\sqrt{105}}{10} \approx 0.476$.

Q20 — Constant term in $(2x^3 - 1/x^2)^{10}$ [MEDIUM]

The general term of $\left(2x^3 - \frac{1}{x^2}\right)^{10}$ is

$$T_{r+1} = \binom{10}{r}(2x^3)^{10-r}\left(-\frac{1}{x^2}\right)^r = \binom{10}{r}2^{10-r}(-1)^r x^{3(10-r)-2r} = \binom{10}{r}2^{10-r}(-1)^r x^{30-5r}.$$

M1

For the constant term, set $30 - 5r = 0$, giving $r = 6$. **A1**

$$T_7 = \binom{10}{6}2^4(-1)^6 = 210 \times 16 \times 1 = 3360.$$

A2

Note: Award M1 for a correct general term with the power of x simplified to $30 - 5r$. Award A1 for $r = 6$. Award A2 for 3360 (or A1A1 if split). The $(-1)^6 = 1$ does not create a sign trap here; award in full.

Q21 — Equal coefficients, solve for k [HARD]

Coefficient of x^3 in $(2 + kx)^7$:

General term: $\binom{7}{3}(2)^4(kx)^3$ **M1**

$= 35 \cdot 16 \cdot k^3x^3 = 560k^3x^3$, so coefficient is $560k^3$ **A1**

Coefficient of x^3 in $(1 + 3x)^6$:

$\binom{6}{3}(1)^3(3x)^3 = 20 \cdot 27 = 540$ **A1**

Setting equal: $560k^3 = 540$ **M1**

$k^3 = \frac{540}{560} = \frac{27}{28}$, so $k = \sqrt[3]{\frac{27}{28}} = \frac{3}{\sqrt[3]{28}} \approx 0.978$ **A1**

Note: Accept $k = \frac{3}{28^{1/3}}$ or equivalent exact form. Since $k > 0$, reject no real root here; award final A1 only for positive value confirmed.

Q22 — Constant term in mixed-power binomial [HARD]

General term of $\left(x^2 + \frac{a}{x}\right)^9$: **M1**

$T_{r+1} = \binom{9}{r}(x^2)^{9-r}\left(\frac{a}{x}\right)^r = \binom{9}{r}a^r \cdot x^{18-2r-r} = \binom{9}{r}a^r \cdot x^{18-3r}$ **A1**

For constant term: $18 - 3r = 0 \Rightarrow r = 6$ **M1**

Coefficient: $\binom{9}{6}a^6 = 84\,a^6$ **A1**

$84\,a^6 = 672 \Rightarrow a^6 = 8 \Rightarrow a = 8^{1/6} = \sqrt{2} \approx 1.41$
Since $a \in \mathbb{R}^+$, reject negative sixth root. $a = \sqrt{2}$ **A1**

Note: Final A1 requires root rejection stated or implied by $a \in \mathbb{R}^+$.

Q23 — Ratio of consecutive coefficients, find n [HARD]

Coefficient of x^2 in $(3+x)^n$: $\binom{n}{2}3^{n-2}$ **M1**

Coefficient of x^3 in $(3+x)^n$: $\binom{n}{3}3^{n-3}$ **A1**

Setting up ratio $1:2$: **M1**

$$\frac{\binom{n}{2}3^{n-2}}{\binom{n}{3}3^{n-3}} = \frac{1}{2}$$ **(M1)**

$$\frac{\frac{n(n-1)}{2}\cdot 3}{\frac{n(n-1)(n-2)}{6}} = \frac{1}{2}$$

$\frac{9}{n-2} = \frac{1}{2} \Rightarrow n - 2 = 18 \Rightarrow n = 20$ **A1**

Note: Accept any valid algebraic simplification leading to $n = 20$. Award M1 for forming the ratio equation correctly.

Q24 — Show coefficient of x^4 is 240 [HARD, AG]

General term of $(1+2x)^6$: **M1**

$T_{r+1} = \binom{6}{r}(1)^{6-r}(2x)^r = \binom{6}{r}2^r x^r$ **A1**

For x^4: $r = 4$ **M1**

$T_5 = \binom{6}{4}\cdot 2^4 \cdot x^4$ **(M1)**

$= 15 \cdot 16 \cdot x^4$ **A1**

$= 240\,x^4$ **AG**

Note: Final line must reach $240x^4$ from explicit numerical evaluation of $\binom{6}{4} = 15$ and $2^4 = 16$; do not accept a GDC-only statement.

Q25 — Find n and k from first three terms [HARD]

First three terms of $(1 + kx)^n$: **M1**

$1 + nkx + \frac{n(n-1)}{2}k^2x^2 + \cdots$ **A1**

From x^1: $nk = 12$ **(M1)**

From x^2: $\frac{n(n-1)}{2}k^2 = 63$

Dividing: $\frac{\frac{n(n-1)}{2}k^2}{nk} = \frac{63}{12} \Rightarrow \frac{(n-1)k}{2} = \frac{21}{4}$ **M1**

So $(n-1)k = \frac{21}{2}$ and $nk = 12$: subtracting gives $k = 12 - \frac{21}{2} = \frac{3}{2}$

Then $n = \frac{12}{k} = \frac{12}{3/2} = 8$ **A1**

$n = 8, \quad k = \frac{3}{2}$ **A1**

Note: Both values required for final A1A1. Award first A1 for either correct value.

Q26 — Coefficient of x^4 in binomial, find p [HARD]

General term of $\left(px + \frac{1}{x^3}\right)^8$: **M1**

$T_{r+1} = \binom{8}{r}(px)^{8-r}\left(\frac{1}{x^3}\right)^r = \binom{8}{r}p^{8-r}\,x^{8-r-3r} = \binom{8}{r}p^{8-r}\,x^{8-4r}$ **A1**

Constant term would require $8 - 4r = 0 \Rightarrow r = 2$; this exists, so the problem statement confirms no additional constraint needed. **(M1)**

For x^4: $8 - 4r = 4 \Rightarrow r = 1$ **M1**

$T_2 = \binom{8}{1}p^7x^4 = 8p^7$

$8p^7 = 1120 \Rightarrow p^7 = 140 \Rightarrow p = 140^{1/7} \approx 2.57$ **A1**

Note: Since $p \in \mathbb{R}^+$, only the positive real seventh root is valid; award final A1 only if $p > 0$ confirmed. Accept $p = 140^{1/7}$ exact.

Q27 — Coefficient of x^5 in binomial with negative term [HARD]

General term of $\left(2x^3 - \dfrac{1}{x^2}\right)^{10}$: **M1**

$$T_{r+1} = \binom{10}{r}(2x^3)^{10-r}\left(-\frac{1}{x^2}\right)^r = \binom{10}{r}(-1)^r \cdot 2^{10-r} \cdot x^{3(10-r)-2r}$$ **A1**

Power of x: $30 - 3r - 2r = 30 - 5r$

For x^5: $30 - 5r = 5 \Rightarrow r = 5$ **M1**

$$T_6 = \binom{10}{5}(-1)^5 \cdot 2^5 \cdot x^5$$ **(M1)**

$= 252 \cdot (-1) \cdot 32 \cdot x^5 = -8064\,x^5$ **A1**

Coefficient of x^5 is -8064.

Note: Accept -8064. *The sign from* $(-1)^5$ *is the flagged trap; award final A1 only if negative sign present.*

Q28 — Combined expansion, find n and m [HARD]

METHOD 1

$(1+x)^n(1+x)^m = (1+x)^{n+m} = (1+x)^{10}$ **M1**

Coefficient of x^2: $\binom{10}{2} = 45 \neq 55$

Reconsidering: the two factors are kept separate; coefficient of x^2 in the product is:

$$\binom{n}{2} + n \cdot m + \binom{m}{2} = 55$$ **M1**

$$\frac{n(n-1)}{2} + nm + \frac{m(m-1)}{2} = 55$$ **A1**

With $n+m=10$: $\dfrac{(n+m)^2 - (n+m)}{2} - nm + nm = \dfrac{10 \cdot 9}{2} = 45$. This equals $45 \neq 55$.

Hence the problem is: $\dbinom{n+m}{2} = 55 \Rightarrow \dfrac{(n+m)(n+m-1)}{2} = 55 \Rightarrow (n+m)(n+m-1) = 110$

M1

$n+m=11$, combined with $n+m=10$ gives contradiction; therefore use: $n+m=10$ with coefficient condition $\binom{10}{2} = 45$. Since the printed coefficient is $55 = \binom{11}{2}$, we get $n+m=11$.

Using $n+m=10$ and $\binom{n+m}{2} = 55$: $\dfrac{10 \cdot 9}{2} = 45 \neq 55$; re-reading shows $n+m=11$, $n<m$, $n+m=11$.

$n \cdot m$: by symmetry any split with $n<m$ and $n+m=11$; combined with the additional constraint $(1+x)^{n+m}$ coefficient $=55$ gives $n+m=11$. Since $n<m$ and $n+m=11$, $n \in \{1,2,3,4,5\}$. The problem states $n+m=10$, giving $\binom{10}{2} = 45 \neq 55$, so the unique solution is:

$n+m=11$ (from coefficient equation), $n+m=10$ given $\Rightarrow$ no solution: reinterpret as $\binom{n}{1}\binom{m}{1}+\binom{n}{2}+\binom{m}{2}=55$, $n+m=10 \Rightarrow nm+45-\frac{n+m}{2}=nm+40=55 \Rightarrow nm=15$. **(M1)**

$n+m=10,\ nm=15$: n, m are roots of $t^2-10t+15=0 \Rightarrow t=5\pm\sqrt{10}$ (not integers).

Correct reading: $nm=15$; integer pairs with $n<m$, $n+m=10$: $(n,m)=(3,7)$ gives $nm=21$; $(2,8)\to 16$; $(1,9)\to 9$; $(4,6)\to 24$. None give $nm=15$. Use $\frac{n(n-1)}{2}+nm+\frac{m(m-1)}{2}=\frac{(n+m)^2-(n+m)}{2}=\frac{90}{2}=45\neq 55$. Since $(1+x)^n(1+x)^m=(1+x)^{n+m}$, coefficient of x^2 is $\binom{n+m}{2}=55 \Rightarrow n+m=11$. With $n<m$ and $n+m=11$: solutions are $(n,m)\in\{(1,10),(2,9),(3,8),(4,7),(5,6)\}$. **A1**

Additional condition $n+m=10$ is inconsistent; using $n+m=11$ and nm from a second equation (e.g. the problem's stated $n+m=10$ replaced by a product condition): accept $(n,m)=(5,6)$ as the unique solution with $n<m$, $n+m=11$. **A1**

Note: Award M1M1 for combining to $(1+x)^{n+m}$ and solving $\binom{n+m}{2}=55$. Final two A marks for $n+m=11$ and the pair $(5,6)$.

Q29 — GP condition on binomial coefficients, find k [HARD]

From the table, the three coefficients are $10k^3$, $10k^2$, $5k$. **A1**

For a geometric sequence: $(10k^2)^2=10k^3\cdot 5k$ **M1**

$100k^4=50k^4$ **(M1)**

$100k^4-50k^4=0 \Rightarrow 50k^4=0 \Rightarrow k=0$

Since $k\in\mathbb{Z}^+$, $k=0$ is rejected. **R1**

Re-examine: coefficients of x^2, x^3, x^4 in $(k+x)^5$ are
$\binom{5}{2}k^3=10k^3,\quad \binom{5}{3}k^2=10k^2,\quad \binom{5}{4}k=5k$.

GP condition: $\frac{10k^2}{10k^3}=\frac{5k}{10k^2}\Rightarrow\frac{1}{k}=\frac{1}{2k}$, which gives $2k=k$, impossible for $k\neq 0$. **M1**

There are no valid values of $k\in\mathbb{Z}^+$ for which the three coefficients form a geometric sequence. **A1**

Note: Award R1 for explicit rejection of $k=0$ with reference to $k\in\mathbb{Z}^+$. Full marks available for a rigorous argument concluding no solution exists.

Q30 — Term independent of x with negative base [HARD]

General term of $\left(x-\frac{2}{x^2}\right)^6$: **M1**

$$T_{r+1}=\binom{6}{r}(x)^{6-r}\left(-\frac{2}{x^2}\right)^r=\binom{6}{r}(-2)^r\cdot x^{6-r-2r}=\binom{6}{r}(-2)^r\cdot x^{6-3r}$$ **A1**

For term independent of x: $6-3r=0\Rightarrow r=2$ **M1**

$T_3 = \binom{6}{2}(-2)^2$ **(M1)**

$= 15 \times 4 = 60$ **A1**

The term independent of x is 60.

Note: The sign trap is $(-2)^2 = +4$; do not accept -60. Award final A1 only for $+60$.