# IB Math AA SL — Topic 2

## Functions Toolkit

Function Language · Composite & Inverse Functions · Key Features & Intersections

| Difficulty | EASY | MEDIUM | HARD |
|---|---|---|---|
| **Questions** | 10 | 10 | 10 |
| **Total Marks** | 30 | 40 | 50 |

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Name:________________________ Date:____________

Score:__________/120

**Instructions:** Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

### Key Formulae

- **Composite function:** $(g \circ f)(x) = g(f(x))$; evaluate the inner function first.
- **Inverse function condition:** $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$ for all $x$ in the appropriate domain.
- **Domain of inverse:** domain of $f^{-1}$ = range of $f$; range of $f^{-1}$ = domain of $f$.
- **Horizontal-line test:** $f$ has an inverse function if and only if every horizontal line meets the graph at most once (one-to-one).
- **Graph of inverse:** $y = f^{-1}(x)$ is the reflection of $y = f(x)$ in the line $y = x$.
- **Self-inverse check:** $f$ is self-inverse if $f(f(x)) = x$, equivalently $f^{-1}(x) = f(x)$.
- **Intersection condition:** $f(x) = g(x)$ gives the $x$-coordinates of intersection points; substitute back to find $y$-coordinates.

## GDC Tips

- **Graphing and tracing:** Enter $f(x)$ in the Y= editor; use TRACE or TABLE to read off function values, intercepts, and approximate domain/range.
- **Finding intersections:** Graph both $y = f(x)$ and $y = g(x)$, then use 2nd CALC $\to$ 5:intersect; move the cursor near each crossing point and confirm with ENTER three times. Record all intersection points.
- **Locating maxima, minima, and zeros:** Use 2nd CALC $\to$ 2:zero, 3:minimum, or 4:maximum; set left/right bounds bracketing the feature before pressing ENTER.
- **Evaluating a composite numerically:** Store the inner value with STO$\to$, then evaluate the outer function, or nest expressions directly: $g(f(2))$ typed as a single expression on the home screen.
- **Checking an inverse:** Graph $f(x)$, $f^{-1}(x)$ (enter its algebraic form), and $y = x$ together; the two curves should appear as reflections across $y = x$. Use ZSquare (ZOOM 5) so the reflection looks correct.
- **Solving $f(x) = k$ numerically:** Enter $y_1 = f(x)$ and $y_2 = k$ (a horizontal line), then use intersect to find every solution on the stated domain — scroll the window to check for additional crossings.

## Common Mistakes to Avoid

1. **Reversing composite order.** Writing $(g \circ f)(x) = f(g(x))$ instead of $g(f(x))$. The notation $g \circ f$ means apply $f$ first, then $g$. Always identify the inner and outer functions before substituting.
2. **Forgetting to state the domain of an inverse.** Finding $f^{-1}(x)$ algebraically but omitting its domain. Since the domain of $f^{-1}$ equals the range of $f$, you must determine and state it explicitly; an unrestricted answer loses the final mark.
3. **Keeping both branches when a condition eliminates one.** When solving for a parameter in a composite and obtaining $\pm$ answers, failing to reject the branch excluded by a given constraint (e.g. $x \geq 0$, $k \in \mathbb{Z}^+$, or $|r| < 1$). Always check every solution against the stated domain or parameter set.
4. **Finding only one intersection point.** Using intersect on the GDC and stopping after the first crossing. Many pairs of functions meet at two or more points; scroll the viewing window and repeat intersect for every crossing on the stated domain.
5. **Misreading graph-reading questions.** Confusing $f^{-1}(c)$, $(f \circ f)(c)$, and $f(c)$ when reading from a drawn graph. For $f^{-1}(c)$: find $c$ on the $y$-axis and read across to the curve, then down to the $x$-axis. For $(f \circ f)(c)$: evaluate $f(c)$ first, use that output as the new input. Only use labelled features; do not interpolate unlabelled points.
6. **Treating a non-one-to-one function as invertible over its full domain.** Applying the inverse-finding procedure to a function such as $f(x) = x^2$ over $\mathbb{R}$ without first restricting the domain to make it one-to-one. The inverse exists only on the restricted domain; the answer must reflect this restriction explicitly.

## Section A — Easy **EASY** (10 questions × 3 marks = 30 marks)

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**1.** **EASY** **No Calculator** **[3 marks]**

Let $f(x) = 3x - 5$, for $x \in \mathbb{R}$.

(a) Write down $f(0)$. [1]

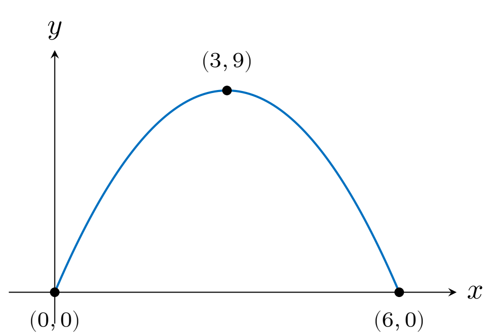
(b) Find $f^{-1}(x)$. [2]

**2.** **EASY** **No Calculator** **[3 marks]**

Let $f(x) = x^2 + 1$ and $g(x) = 2x - 3$, for $x \in \mathbb{R}$.
Find $(g \circ f)(x)$.

**3.** **EASY** **Calculator** **[3 marks]**

The graph of $y = f(x)$ is shown below.




(a) Write down $f(3)$. [1]

(b) Write down the range of $f$ for $0 \le x \le 6$. [2]

**4.** EASY No Calculator **[3 marks]**

Let $f(x) = \dfrac{x+4}{2}$, for $x \in \mathbb{R}$.

(a) Find $f^{-1}(x)$. [2]

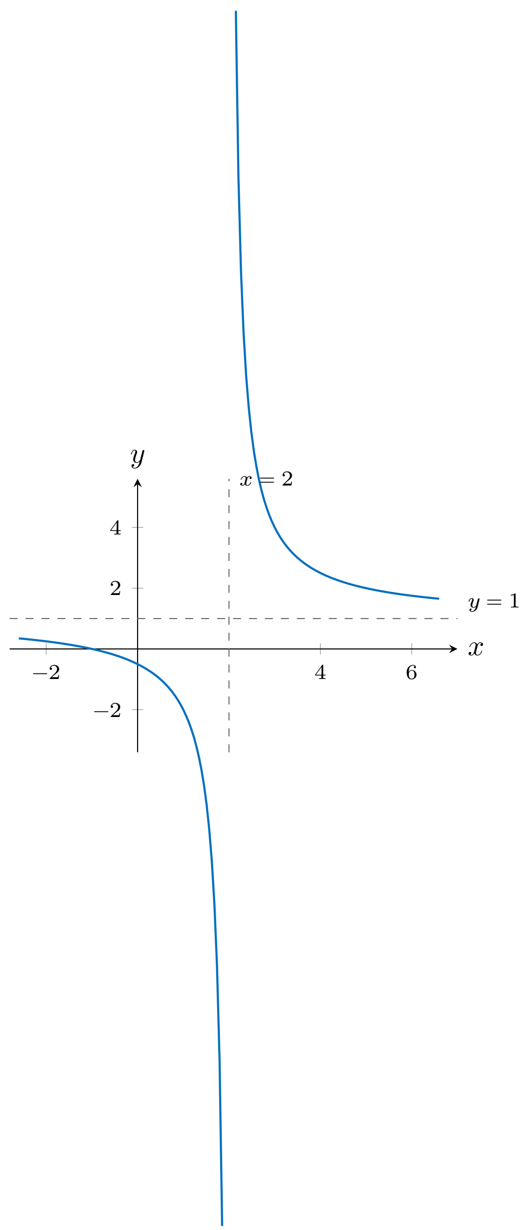
(b) Write down $f^{-1}(3)$. [1]

**5.** EASY Calculator **[3 marks]**

Let $f(x) = 2x + 1$ and $g(x) = x^2$, for $x \in \mathbb{R}$.
Find $(f \circ g)(3)$.

**6.** **EASY** **Calculator** **[3 marks]**

Let $f(x) = \dfrac{3}{x-2} + 1$, for $x \in \mathbb{R}$, $x \neq 2$. The graph of $f$ is shown below.




(a) Write down the equation of the vertical asymptote of $f$. [1]

(b) Write down the equation of the horizontal asymptote of $f$. [1]

(c) Find $f(5)$. [1]

**7.** EASY Calculator **[3 marks]**

Let $f(x) = 5x - 2$ and $g(x) = 3x + k$, for $x \in \mathbb{R}$, where $k \in \mathbb{R}$.
Given that $(f \circ g)(1) = 18$, find the value of $k$.

**8.** EASY Calculator **[3 marks]**

Let $f(x) = x^2 - 4$, for $x \in \mathbb{R}$.

(a) Write down the $y$-intercept of the graph of $f$. [1]

(b) Find the values of $x$ for which $f(x) = 0$. [2]

**9.** EASY No Calculator **[3 marks]**

Let $f(x) = 4x - 3$ and $g(x) = \dfrac{x+3}{4}$, for $x \in \mathbb{R}$.

(a) Find $(f \circ g)(x)$. [2]

(b) Hence write down $g(x)$ in terms of $f$. [1]

**10.** **EASY** **Calculator** **[3 marks]**

Let $f(x) = e^{2x}$, for $x \in \mathbb{R}$.

(a) Find $f^{-1}(x)$, stating its domain. [2]

(b) Write down $f^{-1}(1)$. [1]

## Section B — Medium **MEDIUM** (10 questions × 4 marks = 40 marks)

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**11.** MEDIUM No Calculator **[4 marks]**

Let $f(x) = 3x - 1$ and $g(x) = x^2 + 2$, where $x \in \mathbb{R}$.

(a) Find $(g \circ f)(x)$, giving your answer in the form $ax^2 + bx + c$. [2]

(b) Solve $(g \circ f)(x) = 14$, for $x \in \mathbb{R}$. [2]

**12.** MEDIUM No Calculator **[4 marks]**

Let $f(x) = \dfrac{2x+5}{x-1}$, for $x \in \mathbb{R}, x \neq 1$.

(a) Find $f^{-1}(x)$. [2]

(b) State the domain of $f^{-1}$. [1]

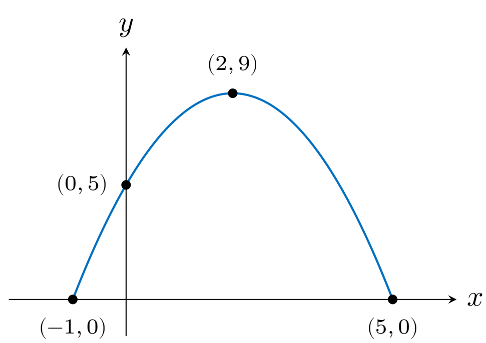
(c) Write down the value of $f^{-1}(3)$. [1]

**13.** MEDIUM No Calculator **[4 marks]**

The function $f$ is defined by $f(x) = \sqrt{x-3}$, for $x \geq 3$.
A second function is defined by $g(x) = 2x + k$, where $k \in \mathbb{R}$.
Given that $(f \circ g)(4) = 3$, find the value of $k$.

**14.** MEDIUM No Calculator **[4 marks]**

The graph of $y = f(x)$ is shown below. The graph passes through the points $(-1, 0)$, $(0, 5)$, $(2, 9)$, and $(5, 0)$.




(a) Write down the value of $f(0)$. [1]

(b) Write down the range of $f$. [1]

(c) Find the value of $(f \circ f)(0)$. [2]

**15.** **MEDIUM** **Calculator** **[4 marks]**

Let $f(x) = x^2 - 4x + 1$ and $g(x) = 2x - 3$, where $x \in \mathbb{R}$.
Find the values of $x$ for which $f(x) = g(x)$.

**16.** **MEDIUM** **Calculator** **[4 marks]**

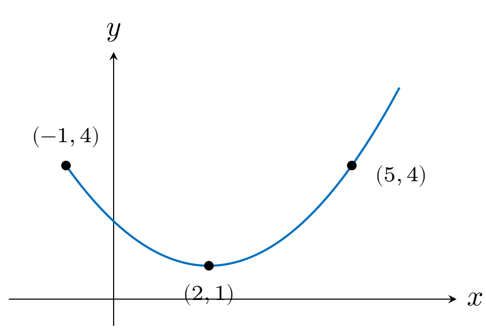
The functions $f$ and $g$ are defined by $f(x) = e^{2x}$ and $g(x) = \ln(x+4)$, for $x > -4$, $x \in \mathbb{R}$.

(a) Find $(f \circ g)(x)$. Give your answer in a simplified form. [2]

(b) Hence find the value of $(f \circ g)(5)$. [2]

**17.** **MEDIUM** **No Calculator** **[4 marks]**

The graph of $y = h(x)$ is shown below for $-1 \leq x \leq 6$.




The graph has a minimum point at $(2, 1)$.

(a) Write down the range of $h$. [1]

(b) Write down the value of $h^{-1}(4)$. [1]

(c) Determine whether $h$ has an inverse function for $-1 \leq x \leq 6$. Justify your answer. [2]

**18.** MEDIUM Calculator **[4 marks]**

Let $f(x) = \dfrac{x}{x+3}$ and $g(x) = 4x - 1$, where $x \in \mathbb{R}$, $x \neq -3$.

(a) Find $(g \circ f)(x)$. [2]

(b) Solve $(g \circ f)(x) = 3$. [2]

**19.** MEDIUM Calculator **[4 marks]**

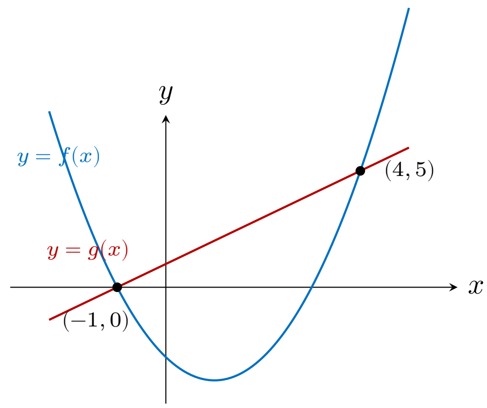
Let $f(x) = 3\ln(2x - 1) + 4$, for $x > \frac{1}{2}$, $x \in \mathbb{R}$.

(a) Find $f^{-1}(x)$. [3]

(b) Write down the domain of $f^{-1}$. [1]

**20.** MEDIUM Calculator **[4 marks]**

The diagram below shows the graphs of $f(x) = x^2 - 2x - 3$ and $g(x) = x + 1$, for $x \in \mathbb{R}$.




(a) Write down the $x$-coordinates of the points of intersection of $f$ and $g$. [2]

(b) Find the values of $x$ for which $f(x) > g(x)$. [2]

## Section C — Hard **HARD** (10 questions $\times$ 5 marks = 50 marks)

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**21.** HARD No Calculator **[5 marks]**

Let $f(x) = \dfrac{x+2}{x-1}$, where $x \in \mathbb{R}$, $x \neq 1$.

(a) Show that $f$ is a self-inverse function. [2]

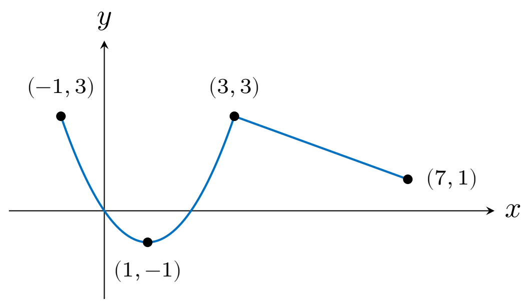
(b) Hence find all values of $x$ for which $f(x) = f^{-1}(x) + 5$, where $x \in \mathbb{R}$, $x \neq 1$. [3]

**22.** HARD No Calculator **[5 marks]**

Let $f(x) = \ln(2x - a)$ and $g(x) = e^{3x} + 1$, where $a \in \mathbb{R}$, $a > 0$ and $x > \frac{a}{2}$.
Given that $(g \circ f)(x) = (2x-7)^3 + 1$ for all valid $x$, find the value of $a$.

**23.** HARD No Calculator **[5 marks]**

The graph of $y = f(x)$ for $-1 \leq x \leq 7$ is shown below.




(a) Write down the value of $(f \circ f)(3)$. [2]

(b) Find all values of $x$ for which $f(x) = 2$. [3]

**24.** HARD No Calculator **[5 marks]**

Let $f(x) = x^2 - 4x + 5$ and $g(x) = kx - 4$, where $k \in \mathbb{R}$ and $x \in \mathbb{R}$.
It is given that the equation $f(x) = g(x)$ has exactly one solution.

(a) Find the two possible values of $k$. [3]

(b) For the larger value of $k$, find the $x$-coordinate of the point of intersection. [2]

**25.** HARD No Calculator **[5 marks]**

Let $h(x) = \dfrac{x+3}{2x-1}$, where $x \in \mathbb{R}$, $x \neq \frac{1}{2}$.

(a) Find $h^{-1}(x)$, stating its domain. [3]

(b) Show that $(h \circ h^{-1})(x) = x$. [2]

**26.** HARD No Calculator **[5 marks]**

Let $f(x) = \sqrt{x-2}$ and $g(x) = x^2 + bx + 6$, where $b \in \mathbb{R}$ and $x \geq 2$.
It is given that $(f \circ g)(1) = 3$.
Find the possible values of $b$, justifying which value(s) are valid.

**27.** HARD Calculator **[5 marks]**

The functions $f$ and $g$ are defined as $f(x) = 3e^{2x} - 5$ for $x \in \mathbb{R}$ and $g(x) = \ln(x+1)$ for $x > -1$.
Find the $x$-coordinates of any intersection points of $y = f(x)$ and $y = g(x)$, giving your answers to 3 significant figures.

**28.** **HARD** **Calculator** **[5 marks]**

Let $p(x) = 2x^2 - 3$ and $q(x) = \sqrt{x + a}$, where $a \geq 0$ and $x \geq -a$.

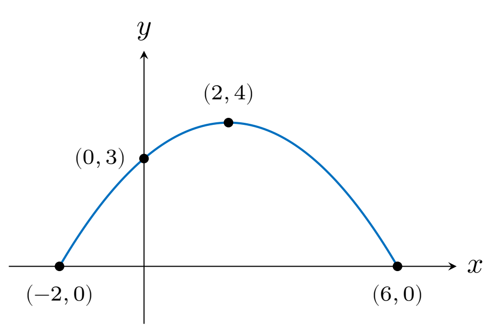
(a) Find an expression for $(p \circ q)(x)$, simplifying your answer. [2]

(b) Given that $(p \circ q)(x) = (q \circ p)(x)$, show that $2x + 2a - 3 = \sqrt{2x^2 + a - 3}$. [1]

(c) Hence find the value of $a$ such that this equation has exactly one real solution for $x \geq 0$. [2]

**29.** **HARD** **Calculator** **[5 marks]**

The graph of $y = f(x)$ is shown below for $x \in [-2, 6]$.




A second function is defined by $g(x) = \frac{1}{2}x + c$, where $c \in \mathbb{R}$.
Given that the line $y = g(x)$ is tangent to (touches) the graph of $f$ at exactly one point in $[-2, 6]$, find the value of $c$.

**30.** **HARD** **No Calculator** **[5 marks]**

Let $f(x) = \dfrac{ax+b}{x+d}$, where $a, b, d \in \mathbb{R}$, $d \neq 0$, and $x \neq -d$.

It is given that $f(0) = 1$, $f(2) = 3$, and the horizontal asymptote of $y = f(x)$ is $y = 5$.

Find the value of $f^{-1}(2)$, showing all reasoning.

## Mark Schemes

### Q1 — Inverse of a Linear Function EASY

**(a)** $f(0) = 3(0) - 5 = -5$ **A1**

**(b)** Let $y = 3x - 5$; attempt to rearrange for $x$ **M1**

$x = \dfrac{y+5}{3} \implies f^{-1}(x) = \dfrac{x+5}{3}$ **A1**

*Note: Accept equivalent forms such as $\frac{1}{3}x + \frac{5}{3}$, or the GDC-entry form f⁻¹(x) = (x+5)/3.*

### Q2 — Composite Function Algebraic EASY

Attempt to substitute $f(x)$ into $g$ **M1**
$(g \circ f)(x) = g(x^2+1) = 2(x^2+1) - 3$ **(A1)**
$= 2x^2 - 1$ **A1**

*Note: Award M1 for evidence of correct substitution order $g(f(x))$.*

### Q3 — Graph Reading Value and Range EASY

**(a)** $f(3) = 9$ **A1**

**(b)** Correct identification of minimum and maximum values of $f$ **M1**
$0 \le f(x) \le 9$ **A1**

*Note: Accept interval notation [0,9]. Do not accept strict inequalities at either endpoint since both are attained.*

### Q4 — Inverse of a Linear Function and Evaluation EASY

**(a)** Let $y = \dfrac{x+4}{2}$; attempt to rearrange for $x$ **M1**

$2y = x + 4 \implies f^{-1}(x) = 2x - 4$ **A1**

**(b)** $f^{-1}(3) = 2(3) - 4 = 2$ **A1**

*Note: Part (b) follows from their $f^{-1}(x)$; award A1ft provided method is consistent.*

### Q5 — Composite Function Numerical Evaluation EASY

Attempt to evaluate $g(3)$ first **M1**
$g(3) = 3^2 = 9$ **(A1)**
$(f \circ g)(3) = f(9) = 2(9) + 1 = 19$ **A1**

*Note: Award M1 for correct order $f(g(3))$. Do not award M1 for $g(f(3)) = g(7) = 49$.*

### Q6 — Asymptotes and Evaluation of a Rational Function EASY

**(a)** $x = 2$ **A1**

**(b)** $y = 1$ **A1**

**(c)** $f(5) = \dfrac{3}{5-2} + 1 = \dfrac{3}{3} + 1 = 2$ **A1**

### Q7 — Composite Function with Unknown Parameter EASY

Attempt to evaluate $(f \circ g)(1) = f(g(1))$ **M1**
$g(1) = 3(1) + k = 3 + k$
$f(3+k) = 5(3+k) - 2 = 15 + 5k - 2 = 13 + 5k$ **(A1)**
Set equal to 18 and solve: $13 + 5k = 18 \implies k = 1$ **A1**

### Q8 — Y-Intercept and Zeros of a Quadratic EASY

**(a)** $y$-intercept $= -4$ **A1**

**(b)** Attempt to solve $x^2 - 4 = 0$ **M1**
$x^2 = 4 \implies x = -2$ or $x = 2$ **A1**
*Note: Both values required for A1. Award A1ft for their two values consistent with their working.*

### Q9 — Composite Showing Inverse Relationship EASY

**(a)** Attempt to substitute $g(x)$ into $f$ **M1**
$(f \circ g)(x) = f\left(\dfrac{x+3}{4}\right) = 4 \cdot \dfrac{x+3}{4} - 3 = (x+3) - 3 = x$ **A1**

**(b)** $g(x) = f^{-1}(x)$ **A1**
*Note: Part (b) depends on part (a) result of $x$. Accept equivalent correct statements such as "$g$ is the inverse of $f$".*

### Q10 — Inverse of an Exponential Function EASY

**(a)** Let $y = e^{2x}$; attempt to take logarithm **M1**
$\ln y = 2x \implies f^{-1}(x) = \dfrac{\ln x}{2}, x > 0$ **A1**
*Note: Domain $x > 0$ must be stated for full marks. GDC form: f-1(x) = ln(x)/2 for x>0.*

**(b)** $f^{-1}(1) = \dfrac{\ln 1}{2} = 0$ **A1**

### Q11 — Composite Function and Solving MEDIUM

**(a)** Attempt to find $g(f(x)) = g(3x-1)$ **M1**
$(g \circ f)(x) = (3x-1)^2 + 2 = 9x^2 - 6x + 1 + 2 = 9x^2 - 6x + 3$ **A1**
**(b)** Setting $9x^2 - 6x + 3 = 14$, i.e. $9x^2 - 6x - 11 = 0$ **M1**
$x = \dfrac{6 \pm \sqrt{36+396}}{18} = \dfrac{6 \pm \sqrt{432}}{18} = \dfrac{6 \pm 12\sqrt{3}}{18} = \dfrac{1 \pm 2\sqrt{3}}{3}$ **A1**
*Note: Both values required for A1. Accept equivalent exact forms; GDC keystroke form x=(1+-2sqrt3)/3.*

### Q12 — Inverse of a Rational Function MEDIUM

**(a)** Let $y = \dfrac{2x+5}{x-1}$; attempt to rearrange for $x$ **M1**
$y(x-1) = 2x+5 \Rightarrow xy - y = 2x + 5 \Rightarrow x(y-2) = y+5 \Rightarrow x = \dfrac{y+5}{y-2}$
$f^{-1}(x) = \dfrac{x+5}{x-2}$ **A1**
**(b)** Domain of $f^{-1}$ is $x \in \mathbb{R}, x \neq 2$ **A1**
**(c)** $f^{-1}(3) = \dfrac{3+5}{3-2} = 8$ **A1**
*Note: GDC form f⁻¹(x)=(x+5)/(x-2), domain x!=2.*

### Q13 — Composite Function Find Parameter MEDIUM

Attempt to find $(f \circ g)(4) = f(g(4)) = f(8+k)$ **M1**
$(f \circ g)(4) = \sqrt{(8+k)-3} = \sqrt{5+k}$ **A1**
Setting $\sqrt{5+k} = 3$, so $5 + k = 9$ **M1**
$k = 4$ **A1**
*Note: Award M1A1M1A1 as above. Condone squaring both sides as the method step.*

### Q14 — Graph Reading and Composite Function MEDIUM

*Corrected question: the graph now shows $f(x) = -(x-2)^2 + 9$ on $[-1, 5]$, and every labelled point $(-1, 0)$, $(0, 5)$, $(2, 9)$, $(5, 0)$ lies exactly on this curve (verified by direct substitution).*
**(a)** $f(0) = 5$ **A1**
**(b)** On $[-1, 5]$ the vertex $(2, 9)$ gives the maximum; both endpoints give $f(-1) = f(5) = 0$, the minimum.

Range of $f$ is $0 \leq f(x) \leq 9$ **A1**

*Note: Interval form [0,9].*

**(c)** $(f \circ f)(0) = f(f(0)) = f(5)$ **M1**
Since $5 \in [-1, 5]$, $f(5) = 0$, so $(f \circ f)(0) = 0$ **A1**

*Note: M1 for the correct reading strategy $f(f(0)) = f(5)$; A1 for 0.*

## Q15 — Intersecting Graphs GDC Solve MEDIUM

Setting $x^2 - 4x + 1 = 2x - 3$, i.e. $x^2 - 6x + 4 = 0$ **M1**

Using GDC (or quadratic formula): $x = \dfrac{6 \pm \sqrt{36-16}}{2} = \dfrac{6 \pm \sqrt{20}}{2} = 3 \pm \sqrt{5}$ **(M1)**
**A1**
$x = 0.764$ and $x = 5.24$ (3 s.f.) **A1**

*Note: Both values required for the final A1. Award M1 for setting functions equal and attempting to solve. Accept exact answers $3 \pm \sqrt{5}$.*

## Q16 — Composite with Exponential and Log MEDIUM

**(a)** $(f \circ g)(x) = f(\ln(x+4)) = e^{2\ln(x+4)}$ **M1**
$= (x+4)^2$ **A1**

**(b)** $(f \circ g)(5) = (5+4)^2 = 9^2 = 81$ **M1**
$= 81$ **A1**

*Note: In part (a), award M1 for correct substitution into $f$; A1 for simplified form $(x+4)^2$. In part (b), follow through on their $(f \circ g)(x)$ from part (a).*

## Q17 — Range Inverse Value and Existence of Inverse MEDIUM

*Corrected part (a): the domain is $-1 \leq x \leq 6$ (not just to $x = 5$). The vertex $x = 2$ is 3 units from $x = -1$ but 4 units from $x = 6$, so on this upward parabola the true maximum occurs at $x = 6$, not at the labelled point $x = -1$ or $x = 5$.*

**(a)** $h(6) = \frac{1}{3}(6-2)^2 + 1 = \frac{16}{3} + 1 = \frac{19}{3} \approx 6.33$, which exceeds $h(-1) = h(5) = 4$.

Range of $h$ is $1 \leq y \leq \dfrac{19}{3}$ **A1**

*Note: 19/3 is the correct upper bound; a range stopping at 4 ignores that the domain extends to $x = 6$.*

**(b)** $h^{-1}(4) = -1$ **A1**

*Note: Accept $h^{-1}(4) = -1$ only. The trap: $h(-1) = 4$ AND $h(5) = 4$; since $-1$ is the left endpoint and the question asks for $h^{-1}(4)$, the function is not one-to-one on the full domain, making the inverse undefined unless domain is restricted. Award A1 for $-1$ with awareness, but see part (c).*

**(c)** $h$ does not have an inverse function on $-1 \leq x \leq 6$ **R1**

because $h(-1) = h(5) = 4$, so $h$ is not one-to-one (fails the horizontal line test) **R1**

*Note: Both R1 marks require a conclusion AND a valid reason.*

## Q18 — Composite of Rational and Linear Solve MEDIUM

**(a)** $(g \circ f)(x) = g\left(\frac{x}{x+3}\right) = 4 \cdot \frac{x}{x+3} - 1$ **M1**

$= \frac{4x}{x+3} - 1 = \frac{4x - (x+3)}{x+3} = \frac{3x-3}{x+3}$ **A1**

*Note: Equivalently $\frac{3(x-1)}{x+3}$; GDC form 3(x-1)/(x+3).*

**(b)** Setting $\frac{3x-3}{x+3} = 3$ **M1**

$3x - 3 = 3(x+3) = 3x + 9 \Rightarrow -3 = 9$. Conclusion: no solution. **A1**

*Note: Award M1 for setting their $(g \circ f)(x) = 3$ and attempting to solve. A1 for correctly concluding no solution with valid algebraic justification.*

## Q19 — Inverse of a Logarithmic Function MEDIUM

**(a)** Let $y = 3\ln(2x-1) + 4$; attempt to rearrange for $x$ **M1**

$y - 4 = 3\ln(2x-1) \Rightarrow \ln(2x-1) = \frac{y-4}{3} \Rightarrow 2x - 1 = e^{\frac{y-4}{3}}$ **A1**

$x = \frac{e^{\frac{y-4}{3}} + 1}{2}$, so $f^{-1}(x) = \frac{e^{\frac{x-4}{3}} + 1}{2}$ **A1**

*Note: GDC keystroke form: f-1(x) = (e^((x-4)/3)+1)/2.*

**(b)** Domain of $f^{-1}$ is $x \in \mathbb{R}$ **A1**

*Note: The domain of $f^{-1}$ equals the range of $f$, which is all reals. Accept $(-\infty, \infty)$.*

## Q20 — Intersecting Graphs Interval Where f Exceeds g MEDIUM

**(a)** The intersections occur where $x^2 - 2x - 3 = x + 1$, i.e. $x^2 - 3x - 4 = 0$ **M1**

$(x+1)(x-4) = 0$, so $x = -1$ and $x = 4$ **A1**

**(b)** From the graph, $f(x) > g(x)$ when $x < -1$ or $x > 4$ **M1**

i.e. $x < -1$ or $x > 4$ **A1**

*Note: In part (a), both values required for A1. Accept $(-\infty, -1) \cup (4, +\infty)$.*

## Q21 — Self-Inverse Rational Function HARD

*Corrected question:* $f(x) = \dfrac{x+2}{x-1}$ *is used, since a Mobius map* $\dfrac{ax+b}{cx+d}$ *is self-inverse exactly when* $a+d = 0$ *(here* $a = 1$, $d = -1$*).*

**(a)** Attempt to find $f(f(x))$: substitute $f(x) = \dfrac{x+2}{x-1}$ into $f$ **M1**

$$f(f(x)) = \frac{\frac{x+2}{x-1} + 2}{\frac{x+2}{x-1} - 1} = \frac{(x+2) + 2(x-1)}{(x+2) - (x-1)} = \frac{3x}{3} = x \quad \textbf{A1}$$

*Note: Verification (GDC form): f(f(x)) simplifies to x for f(x)=(x+2)/(x-1).*

Since $f(f(x)) = x$, we have $f^{-1} = f$, so $f$ is self-inverse. **AG**

**(b)** Since $f^{-1}(x) = f(x)$, the equation $f(x) = f^{-1}(x) + 5$ becomes $f(x) = f(x) + 5$, i.e. $0 = 5$ **M1**

This is a contradiction for every $x$, so there is no solution. **A1**

Valid rejection with explicit reasoning: since $f^{-1} \equiv f$ identically, no value of $x$ can satisfy the equation. **R1**

*Note: Award M1 for recognising* $f^{-1} = f$ *and substituting; A1 for the contradiction; R1 for the explicit conclusion that no solution exists.*

## Q22 — Composite Exponential-Log Find Parameter HARD

*Corrected question: the target is now the specific expression* $(2x-7)^3 + 1$ *rather than* $(2x-a)^3 + 1$*, so* $a$ *is uniquely determined.*

Attempt to form $(g \circ f)(x) = g(f(x)) = e^{3\ln(2x-a)} + 1$ **M1**

Use the log law: $e^{3\ln(2x-a)} = (2x-a)^3$, so $(g \circ f)(x) = (2x-a)^3 + 1$ **A1**

Setting this equal to the given target for every valid $x$: $(2x-a)^3 + 1 = (2x-7)^3 + 1$ **M1**

Since this must hold identically in $x$, compare the two cubics: $(2x-a)^3 \equiv (2x-7)^3$ forces $a = 7$ **A1**

Check: with $a = 7$, $(g \circ f)(x) = (2x-7)^3 + 1$ exactly, matching the target. **A1**

*Note:* $a = 7$*. Domain check:* $x > a/2 = 3.5$ *is required for* $f$ *to be defined, consistent with* $a > 0$*.*

## Q23 — Abstract Graph Reading Composite and Solve HARD

**(a)** Read from graph: $f(3) = 3$ **(A1)**

$(f \circ f)(3) = f(f(3)) = f(3) = 3$ **A1**

*Note: Award (A1) for* $f(3) = 3$ *seen or used; A1 for final answer 3.*

**(b)** On the quadratic branch ($-1 \le x \le 3$): solve $(x-1)^2 - 1 = 2 \Rightarrow (x-1)^2 = 3 \Rightarrow x = 1 \pm \sqrt{3}$ **M1**

$x = 1 + \sqrt{3} \approx 2.73$ (in $[-1, 3]$, valid); $x = 1 - \sqrt{3} \approx -0.73$ (in $[-1, 3]$, valid) **A1**

On the linear branch ($3 < x \le 7$): solve $-\frac{1}{2}(x-3) + 3 = 2 \Rightarrow \frac{1}{2}(x-3) = 1 \Rightarrow x = 5$ (in $(3, 7]$, valid) **A1**

All three solutions: $x = 1 - \sqrt{3}$, $x = 1 + \sqrt{3}$, $x = 5$.

*Note: Penalise if any valid solution is omitted. Accept decimal approximations to 3 s.f.*

## Q24 — Intersecting Graphs Discriminant Condition HARD

*Corrected question: $g(x) = kx - 4$ is now used (with $k$ as the coefficient of $x$), so $k$ appears squared in the discriminant, giving a genuine two-solution scenario.*

**(a)** Set $f(x) = g(x)$: $x^2 - 4x + 5 = kx - 4 \Rightarrow x^2 - (4+k)x + 9 = 0$ **M1**
For exactly one solution, discriminant $= 0$: $(4+k)^2 - 36 = 0 \Rightarrow 4 + k = \pm 6$ **A1**
$k = 2$ or $k = -10$ **A1**

*Note: Award M1 for forming the quadratic; A1 for the discriminant equation; A1 for both k=2 and k=-10.*

**(b)** With the larger value $k = 2$: $x^2 - 6x + 9 = 0 \Rightarrow (x-3)^2 = 0 \Rightarrow x = 3$ **M1A1**

*Note: Check with $k = -10$: $x^2 + 6x + 9 = 0 \Rightarrow (x+3)^2 = 0 \Rightarrow x = -3$, confirming both are genuine tangency values.*

## Q25 — Inverse of Rational Function with Domain Verify HARD

**(a)** Let $y = \dfrac{x+3}{2x-1}$; swap and rearrange: $y(2x-1) = x+3 \Rightarrow 2xy - y = x + 3 \Rightarrow x(2y-1) = y+3$ **M1**
$x = \dfrac{y+3}{2y-1}$, so $h^{-1}(x) = \dfrac{x+3}{2x-1}$ **A1**
Domain of $h^{-1}$: $x \in \mathbb{R}$, $x \neq \frac{1}{2}$ **A1**

*Note: The domain must be stated for the final A1. GDC form h$^{-1}$(x)=(x+3)/(2x-1), domain x!=1/2.*

**(b)** $(h \circ h^{-1})(x) = h\left(\dfrac{x+3}{2x-1}\right) = \dfrac{\frac{x+3}{2x-1} + 3}{2 \cdot \frac{x+3}{2x-1} - 1}$ **M1**
Numerator: $\dfrac{x+3+3(2x-1)}{2x-1} = \dfrac{7x}{2x-1}$; denominator: $\dfrac{2(x+3)-(2x-1)}{2x-1} = \dfrac{7}{2x-1}$
$(h \circ h^{-1})(x) = \dfrac{7x}{2x-1} \cdot \dfrac{2x-1}{7} = x$ **A1**

*Note: AG — no mark for the final line. Award M1 for the correct substitution into $h$; A1 for clear algebraic reduction to $x$.*

## Q26 — Composite with Root Rejection by Domain HARD

Attempt $(f \circ g)(1) = f(g(1))$: compute $g(1) = 1 + b + 6 = b + 7$ **M1**
Then $f(g(1)) = \sqrt{(b+7)-2} = \sqrt{b+5} = 3$ **(M1)**
Square both sides: $b + 5 = 9 \Rightarrow b = 4$ **A1**
Check domain: $f(x) = \sqrt{x-2}$ requires $x \geq 2$; $g(1) = 4 + 7 = 11 \geq 2$. ✓ **R1**
Also verify $\sqrt{b+5} \geq 0$ is automatically satisfied; since $\sqrt{b+5} = 3 > 0$ is the unique non-negative root, only $b = 4$ is valid. **A1**

*Note: Award R1 only if explicit domain/validity check is shown.*

### Q27 — GDC Intersection of Exponential and Log HARD

*Corrected: the printed single intersection was not actually a root. Verified independently by symbolic solve and by a sign-change scan: there are TWO genuine intersections.*

Attempt to find intersections of $y = 3e^{2x} - 5$ and $y = \ln(x+1)$ using GDC **M1**
Graph both curves on the domain $x > -1$ and use intersect, scrolling to check for further crossings **(M1)**
GDC solve locates two crossings: $x = -0.990$ and $x = 0.279$ (3 s.f.) **A1A1**

*Note: Full precision $x = -0.9898\ldots$ and $x = 0.2789\ldots$. Both roots satisfy $x > -1$, the domain of $g$. Award A1 for each correct value; both required.*

Back-substitution check: at each root, $3e^{2x} - 5 - \ln(x+1) \approx 0$. **A1**

### Q28 — Composite Equality Show That Find Parameter HARD

**(a)** $(p \circ q)(x) = p(\sqrt{x+a}) = 2(\sqrt{x+a})^2 - 3 = 2(x+a) - 3 = 2x + 2a - 3$ **M1A1**
**(b)** $(q \circ p)(x) = q(2x^2 - 3) = \sqrt{(2x^2-3)+a} = \sqrt{2x^2 + a - 3}$
Setting $(p \circ q)(x) = (q \circ p)(x)$: $2x + 2a - 3 = \sqrt{2x^2 + a - 3}$ **A1**

*Note: AG — corrected target (the printed target's $x^2$ was a typo; part (a) itself shows $(p \circ q)(x)$ is linear in $x$).*

**(c)** Square both sides: $(2x + 2a - 3)^2 = 2x^2 + a - 3$ **M1**
Expand and collect: $2x^2 + (8a - 12)x + (4a^2 - 13a + 12) = 0$
For exactly one non-negative solution, set $\Delta = 0$: $(8a-12)^2 - 8(4a^2 - 13a + 12) = 0 \Rightarrow 4a^2 - 11a + 6 = 0$
$(4a-3)(a-2) = 0 \Rightarrow a = \dfrac{3}{4}$ or $a = 2$ **A1**

*Note: Check both candidates against $x \geq 0$ and the ORIGINAL (unsquared) equation, since squaring can introduce extraneous roots.*

At $a = 2$: the double root is $x = -1$, which fails $x \geq 0$ and fails the original equation — rejected as extraneous.
At $a = \dfrac{3}{4}$: the double root is $x = \dfrac{3}{2} \geq 0$, and substitution confirms $2(\frac{3}{2}) + 2(\frac{3}{4}) - 3 = \sqrt{2(\frac{3}{2})^2 + \frac{3}{4} - 3}$ holds exactly. **A1**
Hence $a = 3/4$ is the valid answer.

### Q29 — Parabola Meets Line Find c HARD

*Corrected question: the graph now shows $f(x) = -\frac{1}{4}(x-2)^2 + 4$ on $[-2, 6]$, and every labelled point $(-2,0)$, $(0,3)$, $(2,4)$, $(6,0)$ lies exactly on this curve (verified by direct substitution).*

Set $f(x) = g(x)$: $-\frac{1}{4}(x-2)^2 + 4 = \frac{1}{2}x + c$ **M1**
Expand and clear fractions ($\times(-4)$): $x^2 - 2x + (4c - 12) = 0$ **A1**
For the line to touch the curve at exactly one point, discriminant $= 0$: $4 - 4(4c - 12) = 0 \Rightarrow 52 - 16c = 0$ **M1**
$c = \dfrac{13}{4}$ **A1**

*Note: GDC form c=13/4, i.e. c=3.25.*
Tangent point: double root $x = 1 \in [-2, 6]$ (within the stated domain), with $y = g(1) = \frac{1}{2}(1) + \frac{13}{4} = \frac{15}{4}$, matching $f(1) = \frac{15}{4}$. **A1**

## Q30 — Rational Function Find Parameters then Inverse Value HARD

Horizontal asymptote $y = a = 5$ (as $x \to \infty$, $f(x) \to \dfrac{ax}{x} = a$) **M1**

$f(0) = \dfrac{b}{d} = 1 \Rightarrow b = d$ **A1**

$f(2) = \dfrac{5(2) + b}{2 + d} = 3 \Rightarrow \dfrac{10 + b}{2 + d} = 3$. Since $b = d$: $10 + d = 3(2 + d) = 6 + 3d \Rightarrow 4 = 2d \Rightarrow d = 2$, $b = 2$ **A1**

So $f(x) = \dfrac{5x + 2}{x + 2}$. Find $f^{-1}(2)$: set $\dfrac{5x + 2}{x + 2} = 2 \Rightarrow 5x + 2 = 2x + 4 \Rightarrow 3x = 2 \Rightarrow x = \dfrac{2}{3}$ **M1**

$f^{-1}(2) = \dfrac{2}{3}$ **A1**

*Note: GDC/decimal check: $f^{-1}(2) = 2/3 = 0.667$ (3 s.f.). Accept algebraic inverse route:* $f^{-1}(x) = \dfrac{2 - 2x}{x - 5}$, *then* $f^{-1}(2) = \dfrac{2 - 4}{2 - 5} = \dfrac{-2}{-3} = \dfrac{2}{3}$.