IB Math AA SL — Topic 3

Geometry Toolkit

Coordinate Geometry · Radian Measure · Arcs & Sectors

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

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Name:_____ Date:_____

Score:_____/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

Key Formulae

- **Distance:** $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- **Midpoint:** $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- **Gradient:** $m = \frac{y_2 - y_1}{x_2 - x_1}$ **Perpendicular gradient:** $m_\perp = -\frac{1}{m}$
- **Arc length (radians):** $l = r\theta$ **Arc length (degrees):** $l = \frac{\theta}{360} \times 2\pi r$
- **Sector area (radians):** $A = \frac{1}{2}r^2\theta$ **Sector area (degrees):** $A = \frac{\theta}{360} \times \pi r^2$
- **Sector perimeter (radians):** $P = r\theta + 2r = r(\theta + 2)$
- **Segment area (radians):** $A_{\text{seg}} = \frac{1}{2}r^2(\theta - \sin\theta)$

GDC Tips

- **Degree/radian mode:** Before any trig calculation, confirm the mode matches the angle units given; go to Settings (or MODE) and select RAD or DEG explicitly.
- **Solving sector equations numerically:** Use Equation Solver or solve(in the CAS/numeric menu to find r or θ when perimeter and area conditions give a non-linear system.
- **Storing intermediate values:** Store exact decimals (e.g. an arc length) with STO$\rightarrow$ before rounding, then recall them in the next step to avoid accumulated rounding error.
- **Converting between degrees and radians:** Use $\times\pi \div 180$ or the built-in $^\circ$ and ^{r} conversion keys; verify by checking $180^\circ = \pi$.
- **Graphing to find intersection of sector constraint curves:** Enter both expressions (e.g. perimeter and area) as y_1, y_2 and use Intersection under G-Solve to read off r or θ.

Common Mistakes to Avoid

1. **Forgetting the two radii in the sector perimeter.** The perimeter of a sector is the arc length plus both straight sides: $P = r\theta + 2r$. Writing $P = r\theta$ alone loses the two radii and is the single most common error in sector problems.
2. **Mixing degree and radian formulas.** Using $A = \frac{1}{2}r^2\theta$ with θ still in degrees (or vice versa) gives a completely wrong answer. Always convert θ to radians before applying the radian-form formulas, or use the degree-form versions explicitly.
3. **Confusing segment area with sector area.** A segment is bounded by a chord, not two radii. Its area is $\frac{1}{2}r^2(\theta - \sin\theta)$; omitting the $-\sin\theta$ term gives the sector area, not the segment area.
4. **Rounding the angle or radius too early in multi-step problems.** Sector area and arc length are sensitive to r and θ. Premature rounding (e.g. truncating r to 3 s.f. before computing the area) propagates error; keep full precision until the final answer.
5. **Applying the distance or midpoint formula with coordinates in the wrong order.** Although d is symmetric, the midpoint and gradient formulas are not immune to sign errors when $x_1 > x_2$ or coordinates are negative. Write the subtraction explicitly as $(x_2 - x_1)$, not $|x_1 - x_2|$, especially for gradient.
6. **Using the wrong perpendicular-gradient relationship.** The perpendicular gradient is $-1/m$, not $1/m$. A positive gradient gives a negative perpendicular gradient; forgetting the negative sign produces a line that is a reflection of the correct normal rather than truly perpendicular.

Section A — Easy EASY (10 questions × 3 marks = 30 marks)

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1. EASY No Calculator [3 marks]

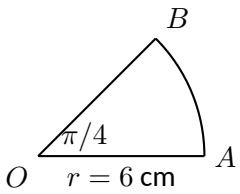
The points $A(1, 3)$ and $B(5, 9)$ are in the coordinate plane. Find the distance AB.

2. EASY Calculator [3 marks]

The points $P(2, 7)$ and $Q(8, 3)$ are in the coordinate plane. Find the coordinates of the midpoint of PQ.

3. EASY No Calculator [3 marks]

A sector OAB has radius $r = 6$ cm and sector angle $\theta = \frac{\pi}{4}$ radians, where θ is the angle at the centre O. Find the arc length AB.



4. EASY Calculator **[3 marks]**

A sector has radius 9 cm and arc length 12 cm. Find the sector angle in radians.

5. EASY No Calculator **[3 marks]**

Convert $225°$ to radians, giving your answer as an exact multiple of π.

6. EASY Calculator **[3 marks]**

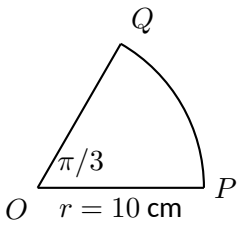
A sector OPQ has radius 5 cm and sector angle 0.8 radians. Find the area of the sector.

7. EASY Calculator **[3 marks]**

The points $C(-3, 2)$ and $D(5, 2)$ are in the coordinate plane. Find the gradient of the line CD.

8. **EASY** **No Calculator** **[3 marks]**

A sector has radius 10 cm and sector angle $\frac{\pi}{3}$ radians. Find the perimeter of the sector.



9. **EASY** **Calculator** **[3 marks]**

Convert $\frac{3\pi}{5}$ radians to degrees.

10. **EASY** **Calculator** **[3 marks]**

A sector has area 24 cm^2 and radius 4 cm. Find the sector angle θ in radians, where $0 < \theta < 2\pi$.

Section B — Medium MEDIUM (10 questions × 4 marks = 40 marks)

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11. MEDIUM No Calculator [4 marks]

The points $A(1, 4)$ and $B(7, -2)$ are given, where $x, y \in \mathbb{R}$.

(a) Find the gradient of the line AB. [1]

(b) Find the equation of the perpendicular bisector of AB, giving your answer in the form $y = mx + c$. [3]

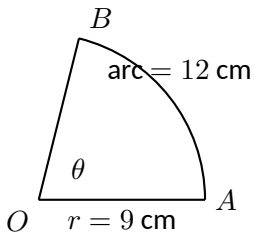
12. MEDIUM No Calculator [4 marks]

A sector OAB has radius $r = 9$ cm and arc length 12 cm.

(a) Find the angle θ at the centre O, in radians. [1]

(b) Find the area of the sector. [1]

(c) Find the perimeter of the sector. [2]



13. MEDIUM Calculator **[4 marks]**

Point P lies on the circle with centre $C(3, 1)$ and radius 5. The point Q has coordinates $(9, 9)$. Find the length PQ, given that P is the point on the circle closest to Q.

14. MEDIUM No Calculator **[4 marks]**

An angle of $\frac{5\pi}{4}$ radians is converted to degrees, and a second angle of $252°$ is converted to radians.

(a) Express $\frac{5\pi}{4}$ radians in degrees. [1]

(b) Express $252°$ in radians, giving your answer as an exact multiple of π. [2]

(c) Hence find the sum of the two angles in radians, giving your answer as an exact multiple of π. [1]

15. MEDIUM Calculator **[4 marks]**

A sector has perimeter 30 cm and area 54 cm^2, where $r > 0$ and $0 < \theta < 2\pi$.

(a) Write down two equations in terms of r and θ. [2]

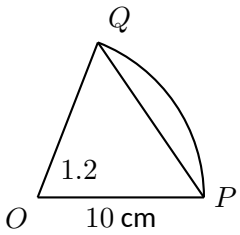
(b) Find the values of r and θ. [2]

16. **MEDIUM** **Calculator** **[4 marks]**

The diagram shows a sector OPQ with $OP = 10$ cm and $\angle POQ = 1.2$ radians. The chord PQ divides the sector into a triangle OPQ and a segment.

(a) Find the area of triangle OPQ. [2]

(b) Find the area of the segment. [2]



17. **MEDIUM** **Calculator** **[4 marks]**

The line ℓ passes through the points $M(-2, 5)$ and $N(4, -1)$, where $x, y \in \mathbb{R}$.

(a) Find the equation of line ℓ, giving your answer in the form $ax + by + d = 0$, where $a, b, d \in \mathbb{Z}$. [2]

(b) The point $K(k, 3)$ lies on ℓ. Find the value of k. [2]

18. MEDIUM No Calculator **[4 marks]**

A circular lawn has a sprinkler at its centre O. The sprinkler waters a sector of radius 8 m with central angle $140°$.

(a) Convert $140°$ to radians, giving your answer as an exact multiple of π. [1]

(b) Find the area watered by the sprinkler, giving your answer in the form $k\pi$ m^2 where $k \in \mathbb{Q}$. [2]

(c) Find the arc length of the watered sector, giving your answer in the form $k\pi$ m where $k \in \mathbb{Q}$. [1]

19. MEDIUM Calculator **[4 marks]**

Triangle ABC has vertices $A(0, 6)$, $B(-3, 0)$ and $C(5, 0)$, where $x, y \in \mathbb{R}$.

(a) Find the coordinates of the midpoint M of BC. [1]

(b) Find the length of the median AM. [2]

(c) Find the gradient of AM. [1]

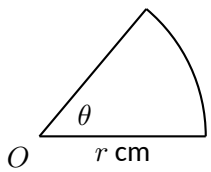
20. MEDIUM Calculator **[4 marks]**

A sector of radius r cm and central angle θ radians has arc length equal to $\frac{3}{2}r$, where $r > 0$.

(a) Write down the value of θ. [1]

(b) Find the area of the sector in terms of r. [1]

(c) Given that the area of the sector is 48 cm^2, find the perimeter of the sector. [2]



Section C — Hard HARD (10 questions × 5 marks = 50 marks)

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21. HARD No Calculator [5 marks]

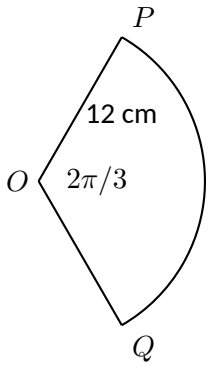
A sector OAB has radius r cm and angle θ radians, where $r > 0$ and $0 < \theta < 2\pi$. The perimeter of the sector equals the area of the sector numerically. The arc length of the sector equals $\frac{3}{2}r$ cm. Find the exact values of r and θ.

22. HARD Calculator [5 marks]

The points $A(-2, 3)$, $B(4, k)$ and $C(10, -5)$ are such that AB is perpendicular to BC. Find the two possible values of the distance BC.

23. **HARD** **No Calculator** **[5 marks]**

A circle has centre O and radius 12 cm. A chord PQ subtends an angle of $\frac{2\pi}{3}$ radians at the centre. Find the exact area of the minor segment cut off by chord PQ.



24. **HARD** **No Calculator** **[5 marks]**

A sector has radius r cm and angle θ radians. Its perimeter is 30 cm and its area is 50 cm^2. Show that $r^2 - 15r + 50 = 0$. Hence find the possible values of r and the corresponding values of θ, rejecting any value that is not valid for a sector.

25. **HARD** **Calculator** **[5 marks]**

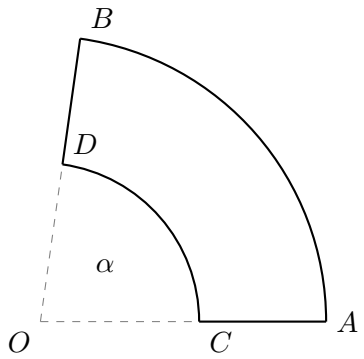
The points $P(2, -1)$, $Q(a, 3)$ and $R(6, b)$ lie on a straight line. The distance $PR = 4\sqrt{5}$. Given that $a < 6$, find the values of a and b.

26. **HARD** **Calculator** **[5 marks]**

A garden feature consists of a large sector OAB of radius $R = 9$ m and angle α radians, from which a smaller sector OCD of radius $r = 5$ m and the same angle α is removed, as shown. The perimeter of the remaining shape is 28 m.

(a) Find the exact value of α. [2]

(b) Hence find the exact area of the remaining shape. [3]



27. **HARD** **Calculator** **[5 marks]**

The triangle ABC has vertices $A(-1, 4)$, $B(3, -2)$ and $C(7, 2)$. Find the area of triangle ABC.

28. **HARD** **No Calculator** **[5 marks]**

A point T moves along the arc of a circle of radius 9 cm, centre O. The arc subtends an angle θ at O, where θ is measured in degrees. At a particular moment, the arc length equals 6π cm. Find θ exactly. Hence find the area of the sector, giving your answer in terms of π.

29. **HARD** **Calculator** **[5 marks]**

The line ℓ_1 passes through $A(1, 5)$ and $B(4, -1)$. The line ℓ_2 passes through $C(0, k)$ and is perpendicular to ℓ_1. The lines ℓ_1 and ℓ_2 intersect at point D. Given that $CD = 2 \cdot AD$, find the possible values of k.

30. **HARD** **No Calculator** **[5 marks]**

A sector OPQ has radius 5 cm and angle θ radians, where $0 < \theta < 2\pi$. A triangle OPQ is formed by joining P and Q with a straight line. The area of the triangle OPQ is $\frac{25\sqrt{3}}{4}$ cm^2. Find all possible values of the perimeter of the sector, giving exact answers.

Mark Schemes

Q1 — Distance Between Two Points EASY

Using the distance formula with $A(1,3)$ and $B(5,9)$: $AB = \sqrt{(5-1)^2+(9-3)^2}$ **M1**

$AB = \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$ **A1**

$AB = 2\sqrt{13}$ cm **A1**

Note: Accept $\sqrt{52}$. Award final A1 only for exact simplified surd.

Q2 — Midpoint of a Line Segment EASY

Using the midpoint formula with $P(2,7)$ and $Q(8,3)$: $M = \left(\frac{2+8}{2}, \frac{7+3}{2}\right)$ **M1**

$M = (5,5)$ **A1A1**

Note: Award A1 for each correct coordinate. Accept $(5,5)$ seen from GDC.

Q3 — Arc Length in Radians EASY

Using the arc length formula $l = r\theta$: **M1**

$l = 6 \times \frac{\pi}{4} = \frac{6\pi}{4} = 3\pi/2$ cm **A2**

Note: Award A2 for $3\pi/2$. Award A1 for $6\pi/4$ unsimplified. Accept 1.5π.

Q4 — Sector Angle from Arc Length EASY

Using $l = r\theta$: $12 = 9\theta$ **M1**

$\theta = \frac{12}{9} = 4/3 \approx 1.33$ radians **A1A1**

Note: Award A1 for correct equation, A1 for $\theta = 4/3$ (1.33 to 3 s.f.). Accept exact fraction or 1.33.

Q5 — Degrees to Radians Conversion EASY

Using the conversion factor $\frac{\pi}{180}$: $225^\circ \times \frac{\pi}{180} = \frac{225\pi}{180}$ **M1A1**

$= 5\pi/4$ **A1**

Note: Award M1 for multiplying by $\pi/180$. Final A1 for fully simplified $5\pi/4$.

Q6 — Sector Area in Radians EASY

Using the sector area formula $A = \frac{1}{2}r^2\theta$: $A = \frac{1}{2} \times 5^2 \times 0.8$ **M1(A1)**

$A = \frac{1}{2} \times 25 \times 0.8 = 10$ cm^2 **A1**

Note: Award M1 for correct substitution into $\frac{1}{2}r^2\theta$. Final A1 for 10 cm^2.

Q7 — Gradient of a Horizontal Line EASY

Using the gradient formula with $C(-3, 2)$ and $D(5, 2)$: $m = \frac{2-2}{5-(-3)}$ **M1**

$m = \frac{0}{8} = 0$ **A1A1**

Note: Award M1 for correct substitution into gradient formula. Award A1A1 for $m = 0$ clearly stated. Condone unsimplified $0/8$ for first A1.

Q8 — Perimeter of a Sector EASY

Arc length: $l = r\theta = 10 \times \frac{\pi}{3} = \frac{10\pi}{3}$ **M1**

Perimeter $= l + 2r$: $P = \frac{10\pi}{3} + 2(10) = 10\pi/3 + 20$ cm **M1A1**

Note: Award first M1 for correct arc length. Award second M1 for adding $2r$ to arc length. A1 for $10\pi/3 + 20$. Condone equivalent exact forms.

Q9 — Radians to Degrees Conversion EASY

Using the conversion factor $\frac{180^\circ}{\pi}$: $\frac{3\pi}{5} \times \frac{180^\circ}{\pi} = \frac{3 \times 180^\circ}{5}$ **M1(A1)**

$= 108^\circ$ **A1**

Note: Award M1 for multiplying by $180/\pi$. Final A1 for 108°, i.e. 108.

Q10 — Sector Angle from Area EASY

Using the sector area formula $A = \frac{1}{2}r^2\theta$: $24 = \frac{1}{2} \times 4^2 \times \theta = 8\theta$ **M1(A1)**

$\theta = \frac{24}{8} = 3$ radians **A1**

Note: Award M1 for correct substitution into $\frac{1}{2}r^2\theta$. Final A1 for $\theta = 3$. Confirm $0 < 3 < 2\pi$, so the answer

is valid.

Q11 — Perpendicular Bisector MEDIUM

(a) gradient of $AB = \dfrac{-2-4}{7-1} = \dfrac{-6}{6}$ **M1**

$= -1$ **A1**

(b) gradient of perpendicular bisector $= 1$ **A1ft**

midpoint of $AB = \left(\dfrac{1+7}{2}, \dfrac{4+(-2)}{2}\right) = (4, 1)$

equation: $y - 1 = 1(x - 4)$ **M1**

$y = x - 3$ **A1**

Note: Award M1 for using the midpoint $(4, 1)$ with any perpendicular gradient. Accept equivalent forms before final simplification, but require $y = x - 3$ for the final A1.

Q12 — Arc Length and Sector Perimeter MEDIUM

(a) $\theta = \dfrac{l}{r} = \dfrac{12}{9}$ **M1**

$\theta = 4/3$ radians **A1**

(b) Area $= \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(9)^2\left(\dfrac{4}{3}\right) = \dfrac{1}{2}(81)\left(\dfrac{4}{3}\right) = 54$ cm^2 **A1**

(c) Perimeter $= r\theta + 2r = 12 + 18 = 30$ cm **M1A1**

Note: Award M1 for the formula $P = l + 2r$ (i.e. arc length plus both radii). A common trap is to omit the two radii; do not award the final A1 if only the arc length is given.

Q13 — Closest Point on a Circle to an External Point MEDIUM

Distance $CQ = \sqrt{(9-3)^2 + (9-1)^2}$ **M1**

$= \sqrt{36 + 64} = \sqrt{100} = 10$ **A1**

P lies on segment CQ, so $CP = 5$ (radius).

$PQ = CQ - CP = 10 - 5$ **M1**

$PQ = 5$ **A1**

Note: Award M1 for a correct distance formula attempt for CQ. The second M1 requires recognition that the closest point lies on line CQ between C and Q, so $PQ = CQ - r$.

Q14 — Degree-Radian Conversion MEDIUM

(a) $\frac{5\pi}{4} \times \frac{180}{\pi} = 225°$, i.e. 225 **A1**

(b) $252° \times \frac{\pi}{180}$ **M1**

$= \frac{252\pi}{180} = 7\pi/5$ radians **A1**

(c) Sum $= \frac{5\pi}{4} + \frac{7\pi}{5} = \frac{25\pi}{20} + \frac{28\pi}{20} = 53\pi/20$ radians **A1**

Note: Award A1 in (a) for $225°$ only. In (b) award M1 for multiplying by $\pi/180$; require fully simplified fraction $7\pi/5$ for A1. In (c) follow through on their answer from (b) provided it is a multiple of π.

Q15 — Sector Algebra for Perimeter and Area MEDIUM

(a) Perimeter: $r\theta + 2r = 30$ **A1**

Area: $\frac{1}{2}r^2\theta = 54$ **A1**

(b) From area equation: $r^2\theta = 108$, so $\theta = \frac{108}{r^2}$.

Substitute into perimeter: $r \cdot \frac{108}{r^2} + 2r = 30 \Rightarrow \frac{108}{r} + 2r = 30$ **M1**

$2r^2 - 30r + 108 = 0 \Rightarrow r^2 - 15r + 54 = 0 \Rightarrow (r-6)(r-9) = 0$

$r = 6$ gives $\theta = 3$; $r = 9$ gives $\theta = 4/3$

Both solutions valid; $r = 6, \theta = 3$ or $r = 9, \theta = 4/3$ **A1**

Note: Award M1 for eliminating one variable to obtain a quadratic in r (or θ). Award A1 for both solution pairs stated. Award (M1)(A1) if GDC solve is evidenced with both pairs.

Q16 — Segment Area MEDIUM

(a) Area of triangle $= \frac{1}{2}r^2 \sin\theta = \frac{1}{2}(10)^2 \sin(1.2)$ **M1**

$= 50\sin(1.2) = 46.6\ldots \approx 46.6$ cm^2 **A1**

(b) Area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2}(100)(1.2) = 60$ cm^2 **M1**

Area of segment $= 60 - 46.6\ldots = 13.4$ cm^2 **A1**

Note: Award M1 in (b) for sector area formula with their r and θ. Accept 13.4 cm^2 (3 s.f.); full precision 13.398…cm^2. Follow through on their triangle area from (a).

Q17 — Line Equation and Point on Line MEDIUM

(a) gradient $= \dfrac{-1-5}{4-(-2)} = \dfrac{-6}{6} = -1$ **M1**

$y - 5 = -1(x+2) \Rightarrow y = -x + 3 \Rightarrow x + y - 3 = 0$ **A1**

(b) Substitute $K(k, 3)$: $k + 3 - 3 = 0$ **M1**

$k = 0$ **A1**

Note: Award M1 in (a) for a correct gradient calculation. In (b), award M1 for substituting $(k, 3)$ into their line equation from (a). FT on their equation provided it is linear. Accept $x + y = 3$ or equivalent integer forms for the A1 in (a).

Q18 — Sector with Degrees Converted to Radians MEDIUM

(a) $140^\circ = 140 \times \dfrac{\pi}{180} = 7\pi/9$ radians **A1**

(b) Area $= \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(8)^2 \cdot \dfrac{7\pi}{9}$ **M1**

$= \dfrac{1}{2} \cdot 64 \cdot \dfrac{7\pi}{9} = 224\pi/9$ m^2 **A1**

(c) Arc length $= r\theta = 8 \cdot \dfrac{7\pi}{9} = 56\pi/9$ m **A1**

Note: Award A1 in (a) for $7\pi/9$ only (unsimplified equivalents such as $14\pi/18$ do not earn the mark). In (b) and (c) follow through on their radian value from (a).

Q19 — Median of a Triangle MEDIUM

(a) $M = \left(\dfrac{-3+5}{2}, \dfrac{0+0}{2}\right) = (1, 0)$ **A1**

(b) $AM = \sqrt{(1-0)^2 + (0-6)^2}$ **M1**

$= \sqrt{1+36} = \sqrt{37} \approx 6.08$ **A1**

(c) gradient of $AM = \dfrac{0-6}{1-0} = -6$ **A1**

Note: Award M1 in (b) for a correct substitution into the distance formula with their M. Accept $\sqrt{37}$ or 6.08 (3 s.f.). In (c) follow through on their M from (a).

Q20 — Sector with Arc Length Proportional to Radius MEDIUM

(a) $l = r\theta \Rightarrow \dfrac{3}{2}r = r\theta \Rightarrow \theta = 3/2$ **A1**

(b) Area $= \dfrac{1}{2}r^2\theta = \dfrac{1}{2}r^2 \cdot \dfrac{3}{2} = 3r^2/4$ **A1**

(c) $\frac{3r^2}{4} = 48 \Rightarrow r^2 = 64 \Rightarrow r = 8$ cm **M1**

Perimeter $= r\theta + 2r = 8 \cdot \frac{3}{2} + 2(8) = 12 + 16 = 28$ cm **A1**

Note: Award M1 for solving $3r^2/4 = 48$ to obtain $r = 8$. Reject $r = -8$ (since $r > 0$). The final A1 requires the perimeter to include both radii: $l + 2r$; award A1ft on their $r > 0$.

Q21 — Simultaneous Sector Algebra and Exact Answers HARD

Arc length equals $\frac{3}{2}r$: $r\theta = \frac{3}{2}r$, giving $\theta = 3/2$ (since $r > 0$). **A1**

Perimeter equals area numerically: Perimeter $= r\theta + 2r$, Area $= \frac{1}{2}r^2\theta$. **M1**

Setting $r\theta + 2r = \frac{1}{2}r^2\theta$ and substituting $\theta = \frac{3}{2}$: **M1**

$$\frac{3r}{2} + 2r = \frac{1}{2}r^2 \cdot \frac{3}{2}$$

$$\frac{7r}{2} = \frac{3r^2}{4}$$

Dividing both sides by r (since $r > 0$): $\frac{7}{2} = \frac{3r}{4}$ **M1**

$r = 14/3$ **A1**

$\theta = 3/2$ (already found above).

Note: Award the first A1 for $\theta = 3/2$ seen anywhere. Do not accept $r = 0$.

Q22 — Perpendicular Gradient Condition and Two Distances HARD

Gradient of AB: $m_{AB} = \frac{k-3}{4-(-2)} = \frac{k-3}{6}$ **M1**

Gradient of BC: $m_{BC} = \frac{-5-k}{10-4} = \frac{-5-k}{6}$

Perpendicularity condition $m_{AB} \times m_{BC} = -1$: **M1**

$$\frac{(k-3)(-5-k)}{36} = -1 \Rightarrow (k-3)(-5-k) = -36$$

Expanding: $-k^2 - 2k + 15 = -36 \Rightarrow k^2 + 2k - 51 = 0$ **(A1)**

$$k = \frac{-2 \pm \sqrt{4+204}}{2} = \frac{-2 \pm \sqrt{208}}{2} = -1 \pm 2\sqrt{13}$$ **(M1)**

$k = -8.21$ or $k = 6.21$ (3 s.f.)

Distance $BC = \sqrt{(10-4)^2 + (-5-k)^2}$:

For $k = -8.21$: $BC = \sqrt{36 + (3.21)^2} \approx 6.81$ cm

For $k = 6.21$: $BC = \sqrt{36 + (11.21)^2} \approx 12.7$ cm **A1**

Note: Unlike a distance between two fixed points, BC genuinely depends on k here because B is the vertex of the right angle; the two roots of the quadratic give two different lengths. Award final A1 for both

3 s.f. values 6.81 *and* 12.7 *cm (exact equivalents* $2\sqrt{26-4\sqrt{13}}$ *and* $2\sqrt{26+4\sqrt{13}}$ *also accepted). Full precision:* $BC = 6.8052\ldots$ *and* $12.7157\ldots$

Q23 — Minor Segment and Exact Answer HARD

The minor segment corresponds to the angle $\dfrac{2\pi}{3}$ at the centre.

Sector area: $A_{\text{sector}} = \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(12)^2\left(\dfrac{2\pi}{3}\right) = \dfrac{1}{2}(144)\left(\dfrac{2\pi}{3}\right) = 48\pi$ **M1A1**

Triangle area: $A_{\triangle} = \dfrac{1}{2}r^2\sin\theta = \dfrac{1}{2}(144)\sin\left(\dfrac{2\pi}{3}\right) = 72\cdot\dfrac{\sqrt{3}}{2} = 36\sqrt{3}$ **M1A1**

Segment area: $A_{\text{segment}} = 48\pi - 36\sqrt{3}\ \text{cm}^2$ **A1**

Note: The sector and triangle areas must be shown separately before subtraction. Accept $(48\pi - 36\sqrt{3})$ *cm*^{2} *exactly. Do not accept decimal approximations.*

Q24 — Sector Simultaneous Equations and Root Rejection HARD

Setting up simultaneous equations: Perimeter: $r\theta + 2r = 30$, so $\theta = \dfrac{30-2r}{r}$ **M1**

Area: $\dfrac{1}{2}r^2\theta = 50$, so $r^2\theta = 100$

Substituting $\theta = \dfrac{30-2r}{r}$ into $r^2\theta = 100$: **M1**

$$r^2\cdot\frac{30-2r}{r} = 100 \Rightarrow r(30-2r) = 100 \Rightarrow 30r - 2r^2 = 100$$

$$\Rightarrow 2r^2 - 30r + 100 = 0 \Rightarrow r^2 - 15r + 50 = 0$$ **AG**

Solving and rejecting: $(r-5)(r-10) = 0 \Rightarrow r = 5$ or $r = 10$ **A1**

For $r = 5$: $\theta = \dfrac{30-10}{5} = 4$; for $r = 10$: $\theta = \dfrac{30-20}{10} = 1$. **A1**

Both satisfy $0 < \theta < 2\pi$, so both are valid: $r = 5, \theta = 4$ and $r = 10, \theta = 1$. **R1**

Note: The AG line $r^2 - 15r + 50 = 0$ *must follow from correct working; do not award AG if perimeter formula omits* $2r$*. Award R1 only if a validity check is explicitly stated.*

Q25 — Collinear Points and Distance Condition HARD

Gradient condition (collinearity): gradient $PR = \dfrac{b-(-1)}{6-2} = \dfrac{b+1}{4}$; gradient $PQ = \dfrac{3-(-1)}{a-2} = \dfrac{4}{a-2}$ **M1**

Setting equal: $\dfrac{b+1}{4} = \dfrac{4}{a-2}$

Distance condition: $PR = \sqrt{(6-2)^2 + (b+1)^2} = \sqrt{16 + (b+1)^2} = 4\sqrt{5}$ **M1**

$16 + (b+1)^2 = 80 \Rightarrow (b+1)^2 = 64 \Rightarrow b+1 = \pm 8$ **(A1)**

$b = 7$ or $b = -9$

Using gradient: when $b = 7$: $\dfrac{8}{4} = \dfrac{4}{a-2} \Rightarrow a - 2 = 2 \Rightarrow a = 4$. **M1**

When $b = -9$: $\dfrac{-8}{4} = \dfrac{4}{a-2} \Rightarrow a - 2 = -2 \Rightarrow a = 0$.

Since $a < 6$, both are initially valid; both satisfy the constraint.

Both solutions: $(a, b) = (4, 7)$ or $(a, b) = (0, -9)$. **A1**

Note: Award final A1 for both pairs correct. Accept either order. Full precision for GDC check: distances confirmed $4\sqrt{5} \approx 8.944$.

Q26 — Annular Sector Solve for Alpha Then Area HARD

(a) Perimeter of remaining shape consists of two straight edges of length $R - r$, one outer arc $R\alpha$, and one inner arc $r\alpha$: **M1**

$2(R-r) + R\alpha + r\alpha = 2(R-r) + \alpha(R+r) = 28$

Substituting $R = 9$, $r = 5$: $2(4) + \alpha(14) = 28 \Rightarrow 8 + 14\alpha = 28$ **A1**

$14\alpha = 20 \Rightarrow \alpha = 20/14 = 10/7$ **A1**

(b) Area of remaining shape $= \dfrac{1}{2}R^2\alpha - \dfrac{1}{2}r^2\alpha = \dfrac{1}{2}\alpha(R^2 - r^2)$ **M1**

$= \dfrac{1}{2}\left(\dfrac{10}{7}\right)(81 - 25) = \dfrac{1}{2}\left(\dfrac{10}{7}\right)(56)$ **A1**

$= 40$ m^2 **A1**

Note: With R and r given explicitly, the perimeter equation alone determines α uniquely, and the area then follows without ambiguity. Full precision: $\alpha = 10/7 = 1.42857\ldots$, area $= 40$ m^2 exactly.

Q27 — Area of Triangle from Coordinates HARD

Using the coordinate area formula: **M1**

$A = \dfrac{1}{2}|x_A(y_B - y_C) + x_B(y_C - y_A) + x_C(y_A - y_B)|$

$= \dfrac{1}{2}|(-1)(-2-2) + 3(2-4) + 7(4-(-2))|$ **M1**

$= \dfrac{1}{2}|(-1)(-4) + 3(-2) + 7(6)|$ **(A1)**

$= \dfrac{1}{2}|4 - 6 + 42|$ **(A1)**

$= \dfrac{1}{2}|40| = 20$ cm^2 **A1**

Note: Accept METHOD 2 using base $\times$ height with the equation of AB and the perpendicular distance from C; both methods give the final answer 20 cm^2.

Q28 — Arc Length in Degrees and Exact Sector Area HARD

Converting arc length formula to degrees: Arc length $= \dfrac{\theta}{360} \times 2\pi r = \dfrac{\theta}{360} \times 2\pi(9) = \dfrac{\theta\pi}{20}$ **M1**

Setting equal to 6π: $\dfrac{\theta\pi}{20} = 6\pi \Rightarrow \theta = 120°$ **A1**

Area of sector: $A = \dfrac{\theta}{360} \times \pi r^2 = \dfrac{120}{360} \times \pi(81) = \dfrac{1}{3} \times 81\pi$ **M1A1**

$A = 27\pi$ cm^2 **A1**

Note: Angles are in degrees throughout this question as declared. Accept 27π cm^2 only; do not accept decimal approximations. Penalise use of radian formula without conversion.

Q29 — Intersection and Distance Ratio HARD

Equation of ℓ_1: gradient $m_1 = \dfrac{-1-5}{4-1} = -2$; equation: $y - 5 = -2(x-1) \Rightarrow y = -2x + 7$ **M1**

Equation of ℓ_2: gradient $m_2 = \dfrac{1}{2}$; passes through $C(0, k)$: $y = \dfrac{1}{2}x + k$ **A1**

Intersection D: $-2x + 7 = \dfrac{1}{2}x + k \Rightarrow x = \dfrac{2(7-k)}{5}$, $y_D = \dfrac{7+4k}{5}$ **M1**

Distance condition $CD = 2 \cdot AD$: $CD^2 = 4\,AD^2$ **(M1)**

Working through the coordinates: $(7-k)^2 = 4(9-2k)^2$

$7 - k = \pm 2(9 - 2k)$

Case 1: $7 - k = 18 - 4k \Rightarrow 3k = 11 \Rightarrow k = 11/3$

Case 2: $7 - k = -18 + 4k \Rightarrow 25 = 5k \Rightarrow k = 5$ **A1**

$k = 11/3$ or $k = 5$.

Note: Award final A1 for both values correct. (Full precision confirmed by GDC.)

Q30 — Sector with Triangle Area and All Solutions HARD

Area of triangle OPQ: $A_\triangle = \dfrac{1}{2}r^2 \sin\theta = \dfrac{1}{2}(25)\sin\theta = \dfrac{25\sqrt{3}}{4}$ **M1**

$\sin\theta = \dfrac{\sqrt{3}}{2}$ **A1**

All solutions in $(0, 2\pi)$: $\theta = \pi/3$ or $\theta = \pi - \dfrac{\pi}{3} = 2\pi/3$ **A1**

Perimeter of sector $= r\theta + 2r = 5\theta + 10$: **M1**

For $\theta = \dfrac{\pi}{3}$: perimeter $= 5 \cdot \dfrac{\pi}{3} + 10 = 5\pi/3 + 10$ cm.

For $\theta = \dfrac{2\pi}{3}$: perimeter $= 5 \cdot \dfrac{2\pi}{3} + 10 = 10\pi/3 + 10$ cm. **A1**

Note: Both values of θ lie in $(0, 2\pi)$ so neither is rejected. Award A1 only if both perimeters are stated in exact form. The perimeter formula must include $2r$; omission of $2r$ loses M1. Do not accept decimal

approximations on this non-calculator question.