IB Math AA SL — Topic 3

Geometry Toolkit

Coordinate Geometry · Radian Measure · Arcs & Sectors

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

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Name:________________________________ Date:________________

Score:__________/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

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Key Formulae

- **Distance:** $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- **Midpoint:** $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- **Gradient:** $m = \frac{y_2 - y_1}{x_2 - x_1}$; perpendicular gradient: $m_\perp = -\frac{1}{m}$
- **Arc length** (radians): $l = r\theta$; (degrees): $l = \frac{\theta}{360} \times 2\pi r$
- **Sector area** (radians): $A = \frac{1}{2}r^2\theta$; (degrees): $A = \frac{\theta}{360} \times \pi r^2$
- **Sector perimeter** (radians): $P = r\theta + 2r = r(\theta + 2)$
- **Segment area** (radians): $A_{\text{seg}} = \frac{1}{2}r^2(\theta - \sin\theta)$

GDC Tips

- **Degree/radian mode:** Before any trig calculation, confirm the mode matches the angle units given; go to `Settings` (or `MODE`) and select `RAD` or `DEG` explicitly.
- **Solving sector equations numerically:** Use `Equation Solver` or `solve(` in the CAS/numeric menu to find r or θ when perimeter and area conditions give a non-linear system.
- **Storing intermediate values:** Store exact decimals (e.g. an arc length) with `STO`$\rightarrow$ before rounding, then recall them in the next step to avoid accumulated rounding error.
- **Converting between degrees and radians:** Use $\times\pi \div 180$ or the built-in $^\circ$ and ^{r} conversion keys; verify by checking $180^\circ = \pi$.
- **Graphing to find intersection of sector constraint curves:** Enter both expressions (e.g. perimeter and area) as y_1, y_2 and use `Intersection` under `G-Solve` to read off r or θ.

Common Mistakes to Avoid

1. **Forgetting the two radii in the sector perimeter.** The perimeter of a sector is the arc length *plus* both straight sides: $P = r\theta + 2r$. Writing $P = r\theta$ alone loses the two radii and is the single most common error in sector problems.
2. **Mixing degree and radian formulas.** Using $A = \frac{1}{2}r^2\theta$ with θ still in degrees (or vice versa) gives a completely wrong answer. Always convert θ to radians before applying the radian-form formulas, or use the degree-form versions explicitly.
3. **Confusing segment area with sector area.** A segment is bounded by a chord, not two radii. Its area is $\frac{1}{2}r^2(\theta - \sin\theta)$; omitting the $-\sin\theta$ term gives the sector area, not the segment area.
4. **Rounding the angle or radius too early in multi-step problems.** Sector area and arc length are sensitive to r and θ. Premature rounding (e.g. truncating r to 3 s.f. before computing the area) propagates error; keep full precision until the final answer.
5. **Applying the distance or midpoint formula with coordinates in the wrong order.** Although d is symmetric, the midpoint and gradient formulas are not immune to sign errors when $x_1 > x_2$ or coordinates are negative. Write the subtraction explicitly as $(x_2 - x_1)$, not $|x_1 - x_2|$, especially for gradient.
6. **Using the wrong perpendicular-gradient relationship.** The perpendicular gradient is $-1/m$, not $1/m$. A positive gradient gives a negative perpendicular gradient; forgetting the negative sign produces a line that is a reflection of the correct normal rather than truly perpendicular.

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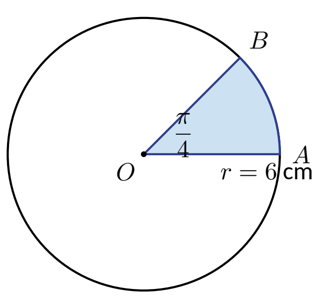
Section A — Easy

EASY

1. The points $A(1, 3)$ and $B(5, 9)$ are in the coordinate plane. Find the distance AB. *Calculator not allowed* **[3 marks]**

2. The points $P(2, 7)$ and $Q(8, 3)$ are in the coordinate plane. Find the coordinates of the midpoint of PQ.
Calculator allowed **[3 marks]**

3. A sector OAB has radius $r = 6$ cm and sector angle $\theta = \dfrac{\pi}{4}$ radians, where θ is the angle at the centre O. Find the arc length AB.



Calculator not allowed **[3 marks]**

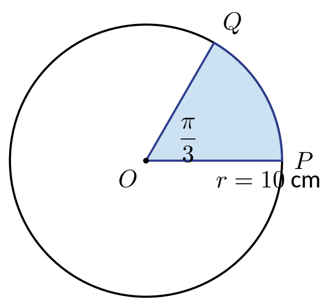
4. A sector has radius 9 cm and arc length 12 cm. Find the sector angle in radians. *Calculator allowed* **[3 marks]**

5. Convert $225°$ to radians, giving your answer as an exact multiple of π. *Calculator not allowed* **[3 marks]**

6. A sector OPQ has radius 5 cm and sector angle 0.8 radians. Find the area of the sector. *Calculator allowed* **[3 marks]**

7. The points $C(-3, 2)$ and $D(5, 2)$ are in the coordinate plane. Find the gradient of the line CD. *Calculator allowed* **[3 marks]**

8. A sector has radius 10 cm and sector angle $\frac{\pi}{3}$ radians. Find the perimeter of the sector.



Calculator not allowed **[3 marks]**

9. Convert $\frac{3\pi}{5}$ radians to degrees. *Calculator allowed* **[3 marks]**

10. A sector has area 24 cm^2 and radius 4 cm. Find the sector angle θ in radians, where $0 < \theta < 2\pi$. *Calculator allowed* **[3 marks]**

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Section B — Medium

MEDIUM

11. The points $A(1, 4)$ and $B(7, -2)$ are given, where $x, y \in \mathbb{R}$.

(a) Find the gradient of the line AB. **[1 marks]**

(b) Find the equation of the perpendicular bisector of AB, giving your answer in the form $y = mx + c$. **[3 marks]**

Calculator not allowed **[4 marks]**

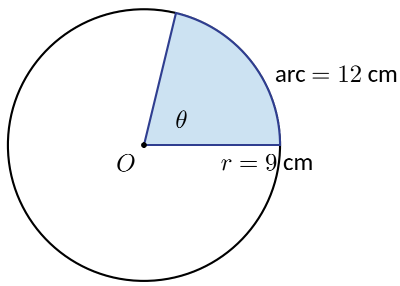
12. A sector OAB has radius $r = 9$ cm and arc length 12 cm.

(a) Find the angle θ at the centre O, in radians. **[1 marks]**

(b) Find the area of the sector. **[1 marks]**

(c) Find the perimeter of the sector. **[2 marks]**

Calculator not allowed **[4 marks]**



13. Point P lies on the circle with centre $C(3, 1)$ and radius 5. The point Q has coordinates $(9, 9)$. Find the length PQ, given that P is the point on the circle closest to Q. *Calculator allowed* **[4 marks]**

14. An angle of $\frac{5\pi}{4}$ radians is converted to degrees, and a second angle of $252°$ is converted to radians.

(a) Express $\frac{5\pi}{4}$ radians in degrees. **[1 marks]**

(b) Express $252°$ in radians, giving your answer as an exact multiple of π. **[2 marks]**

(c) Hence find the sum of the two angles in radians, giving your answer as an exact multiple of π. **[1 marks]**

Calculator not allowed **[4 marks]**

15. A sector has perimeter 30 cm and area 54 cm^2, where $r > 0$ and $0 < \theta < 2\pi$.

(a) Write down two equations in terms of r and θ. **[2 marks]**

(b) Find the values of r and θ. **[2 marks]**

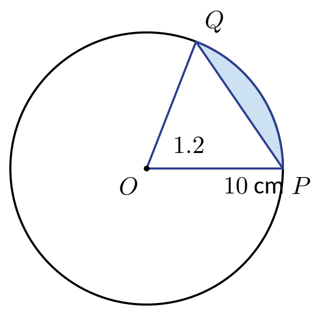
Calculator allowed **[4 marks]**

16. The diagram shows a sector OPQ with $OP = 10$ cm and $\angle POQ = 1.2$ radians. The chord PQ divides the sector into a triangle OPQ and a segment.

(a) Find the area of triangle OPQ. **[2 marks]**

(b) Find the area of the segment. **[2 marks]**

Calculator allowed **[4 marks]**



17. The line ℓ passes through the points $M(-2, 5)$ and $N(4, -1)$, where $x, y \in \mathbb{R}$.

(a) Find the equation of line ℓ, giving your answer in the form $ax + by + d = 0$, where $a, b, d \in \mathbb{Z}$. **[2 marks]**

(b) The point $K(k, 3)$ lies on ℓ. Find the value of k. **[2 marks]**

Calculator allowed **[4 marks]**

18. A circular lawn has a sprinkler at its centre O. The sprinkler waters a sector of radius 8 m with central angle $140°$.

(a) Convert $140°$ to radians, giving your answer as an exact multiple of π. **[1 marks]**

(b) Find the area watered by the sprinkler, giving your answer in the form $k\pi$ m^2 where $k \in \mathbb{Q}$. **[2 marks]**

(c) Find the arc length of the watered sector, giving your answer in the form $k\pi$ m where $k \in \mathbb{Q}$. **[1 marks]**

Calculator not allowed **[4 marks]**

19. Triangle ABC has vertices $A(0,6)$, $B(-3,0)$ and $C(5,0)$, where $x, y \in \mathbb{R}$.

(a) Find the coordinates of the midpoint M of BC. **[1 marks]**

(b) Find the length of the median AM. **[2 marks]**

(c) Find the gradient of AM. **[1 marks]**

Calculator allowed **[4 marks]**

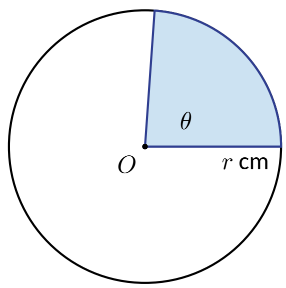
20. A sector of radius r cm and central angle θ radians has arc length equal to $\frac{3}{2}r$, where $r > 0$.

(a) Write down the value of θ. **[1 marks]**

(b) Find the area of the sector in terms of r. **[1 marks]**

(c) Given that the area of the sector is 48 cm^2, find the perimeter of the sector. **[2 marks]**

Calculator allowed **[4 marks]**



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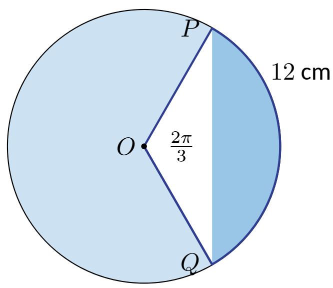
Section C — Hard

HARD

21. A sector OAB has radius r cm and angle θ radians, where $r > 0$ and $0 < \theta < 2\pi$. The perimeter of the sector equals the area of the sector numerically. The arc length of the sector equals $\frac{3}{2}r$ cm. Find the exact values of r and θ. *Calculator not allowed* **[5 marks]**

22. The points $A(-3, 1)$, $B(5, k)$ and $C(9, -2)$ are such that AB is perpendicular to BC. Find the two possible values of the distance AC. *Calculator allowed* **[5 marks]**

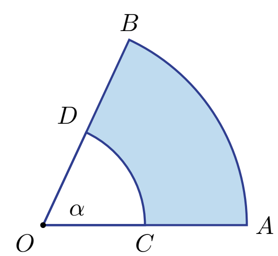
23. A circle has centre O and radius 12 cm. A chord PQ subtends an angle of $\dfrac{2\pi}{3}$ radians at the centre. Find the exact area of the minor segment cut off by chord PQ. *Calculator not allowed* **[5 marks]**



24. A sector has radius r cm and angle θ radians. Its perimeter is 30 cm and its area is 50 cm^2. Show that $r^2 - 15r + 50 = 0$. Hence find the possible values of r and the corresponding values of θ, rejecting any value that is not valid for a sector. *Calculator not allowed* **[5 marks]**

25. The points $P(2,-1)$, $Q(a,3)$ and $R(6,b)$ lie on a straight line. The distance $PR = 4\sqrt{5}$. Given that $a < 6$, find the values of a and b. *Calculator allowed* **[5 marks]**

26. A garden feature consists of a large sector OAB of radius R metres and angle α radians from which a smaller sector OCD of radius r metres and the same angle α is removed, as shown. The perimeter of the remaining shape is 28 m and the area of the remaining shape is 40 m^2. Given that $R > r > 0$ and $0 < \alpha < 2\pi$, find the exact value of α. *Calculator allowed* **[5 marks]**



27. The triangle ABC has vertices $A(-1, 4)$, $B(3, -2)$ and $C(7, 2)$. Find the area of triangle ABC. *Calculator allowed* **[5 marks]**

28. A point T moves along the arc of a circle of radius 9 cm, centre O. The arc subtends an angle θ at O, where θ is measured in degrees. At a particular moment, the arc length equals 6π cm. Find θ exactly. Hence find the area of the sector, giving your answer in terms of π. *Calculator not allowed* **[5 marks]**

29. The line ℓ_1 passes through $A(1,5)$ and $B(4,-1)$. The line ℓ_2 passes through $C(0,k)$ and is perpendicular to ℓ_1. The lines ℓ_1 and ℓ_2 intersect at point D. Given that $CD = 2 \cdot AD$, find the possible values of k. *Calculator allowed* **[5 marks]**

30. A sector OPQ has radius 5 cm and angle θ radians, where $0 < \theta < 2\pi$. A triangle OPQ is formed by joining P and Q with a straight line. The area of the triangle OPQ is $\dfrac{25\sqrt{3}}{4}$ cm^2. Find all possible values of the perimeter of the sector, giving exact answers. *Calculator not allowed* **[5 marks]**

Worked Solutions

Q1 — Distance between two points [EASY]

Using the distance formula with $A(1,3)$ and $B(5,9)$:

$$AB = \sqrt{(5-1)^2 + (9-3)^2}$$

M1

$$AB = \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$$

A1

$AB = 2\sqrt{13}$ cm **A1**

Note: Accept $\sqrt{52}$. Award final A1 only for exact simplified surd.

Q2 — Midpoint of a line segment [EASY]

Using the midpoint formula with $P(2,7)$ and $Q(8,3)$:

$$M = \left(\frac{2+8}{2}, \frac{7+3}{2}\right)$$

M1

$$M = (5,\ 5)$$

A1A1

Note: Award A1 for each correct coordinate. Accept $(5,5)$ seen from GDC.

Q3 — Arc length in radians [EASY]

Using the arc length formula $l = r\theta$: **M1**

$$l = 6 \times \frac{\pi}{4}$$

$$l = \frac{3\pi}{2} \text{ cm}$$

A2

Note: Award A2 for $\frac{3\pi}{2}$. Award A1 for $\frac{6\pi}{4}$ unsimplified. Accept 1.5π.

Q4 — Sector angle from arc length [EASY]

Using $l = r\theta$: **M1**

$$12 = 9\theta$$

$$\theta = \frac{12}{9} = \frac{4}{3} \approx 1.33 \text{ radians}$$

A1A1

Note: Award A1 for correct equation, A1 for $\theta = \frac{4}{3}$ (1.33 to 3 s.f.). Accept exact fraction or 1.33.

Q5 — Degrees to radians conversion [EASY]

Using the conversion factor $\frac{\pi}{180}$: **M1**

$$225° \times \frac{\pi}{180} = \frac{225\pi}{180}$$

A1

$$= \frac{5\pi}{4}$$

A1

Note: Award M1 for multiplying by $\frac{\pi}{180}$. Final A1 for fully simplified $\frac{5\pi}{4}$.

Q6 — Sector area in radians [EASY]

Using the sector area formula $A = \frac{1}{2}r^2\theta$: **M1**

$$A = \frac{1}{2} \times 5^2 \times 0.8$$

(A1)

$$A = \frac{1}{2} \times 25 \times 0.8 = 10 \text{ cm}^2$$

A1

Note: Award M1 for correct substitution into $\frac{1}{2}r^2\theta$. Final A1 for 10 cm^2.

Q7 — Gradient of a horizontal line [EASY]

Using the gradient formula with $C(-3, 2)$ and $D(5, 2)$:

$$m = \frac{2-2}{5-(-3)}$$

M1

$$m = \frac{0}{8} = 0$$

A1A1

Note: Award M1 for correct substitution into gradient formula. Award A1A1 for $m = 0$ clearly stated. Condone $\frac{0}{8}$ unsimplified for first A1.

Q8 — Perimeter of a sector [EASY]

Arc length: $l = r\theta = 10 \times \frac{\pi}{3} = \frac{10\pi}{3}$ **M1**

Perimeter $= l + 2r$: **M1**

$$P = \frac{10\pi}{3} + 2(10) = \frac{10\pi}{3} + 20 \text{ cm}$$

A1

Note: Award first M1 for correct arc length. Award second M1 for adding $2r$ to arc length. A1 for $\frac{10\pi}{3} + 20$. Condone equivalent exact forms.

Q9 — Radians to degrees conversion [EASY]

Using the conversion factor $\frac{180°}{\pi}$: **M1**

$$\frac{3\pi}{5} \times \frac{180°}{\pi} = \frac{3 \times 180°}{5}$$

(A1)

$$= 108°$$

A1

Note: Award M1 for multiplying by $\frac{180}{\pi}$. Final A1 for $108°$.

Q10 — Sector angle from area [EASY]

Using the sector area formula $A = \dfrac{1}{2}r^2\theta$: **M1**

$$24 = \frac{1}{2} \times 4^2 \times \theta = 8\theta$$

(A1)

$$\theta = \frac{24}{8} = 3 \text{ radians}$$

A1

Note: Award M1 for correct substitution into $\frac{1}{2}r^2\theta$. Final A1 for $\theta = 3$. Confirm $0 < 3 < 2\pi$, so the answer is valid.

Q11 — Perpendicular bisector [MEDIUM]

(a)

gradient of $AB = \dfrac{-2-4}{7-1} = \dfrac{-6}{6}$ **M1**

$= -1$ **A1**

(b)

gradient of perpendicular bisector $= 1$ **A1ft**

midpoint of $AB = \left(\dfrac{1+7}{2}, \dfrac{4+(-2)}{2}\right) = (4,\ 1)$

equation: $y - 1 = 1(x - 4)$ **M1**

$y = x - 3$ **A1**

Note: Award M1 for using the midpoint $(4, 1)$ with any perpendicular gradient. Accept equivalent forms before final simplification, but require $y = x - 3$ for the final A1.

Q12 — Arc length, sector area and perimeter [MEDIUM]

(a)

$\theta = \dfrac{l}{r} = \dfrac{12}{9}$ **M1**

$\theta = \dfrac{4}{3}$ radians **A1**

(b)

Area $= \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(9)^2\left(\dfrac{4}{3}\right) = \dfrac{1}{2}(81)\left(\dfrac{4}{3}\right) = 54$ cm^2 **A1**

(c)
Perimeter $= r\theta + 2r = 12 + 18 = 30$ cm **M1A1**

Note: Award M1 for the formula $P = l + 2r$ (i.e. arc length plus both radii). A common trap is to omit the two radii; do not award the final A1 if only the arc length is given.

Q13 — Closest point on circle to external point [MEDIUM]

Distance $CQ = \sqrt{(9-3)^2 + (9-1)^2}$ **M1**

$= \sqrt{36+64} = \sqrt{100} = 10$ **A1**

P lies on segment CQ, so $CP = 5$ (radius).

$PQ = CQ - CP = 10 - 5$ **M1**

$PQ = 5$ **A1**

Note: Award M1 for a correct distance formula attempt for CQ. The second M1 requires recognition that the closest point lies on line CQ between C and Q, so $PQ = CQ - r$.

Q14 — Degree-radian conversion [MEDIUM]

(a)
$\frac{5\pi}{4} \times \frac{180}{\pi} = 225°$ **A1**

(b)
$252° \times \frac{\pi}{180}$ **M1**

$= \frac{252\pi}{180} = \frac{7\pi}{5}$ radians **A1**

(c)
Sum $= \frac{5\pi}{4} + \frac{7\pi}{5} = \frac{25\pi}{20} + \frac{28\pi}{20} = \frac{53\pi}{20}$ radians **A1**

Note: Award A1 in (a) for $225°$ only. In (b) award M1 for multiplying by $\frac{\pi}{180}$; require fully simplified fraction $\frac{7\pi}{5}$ for A1. In (c) follow through on their answer from (b) provided it is a multiple of π.

Q15 — Sector algebra: perimeter and area [MEDIUM]

(a)
Perimeter: $r\theta + 2r = 30$ **A1**

Area: $\frac{1}{2}r^2\theta = 54$ **A1**

(b)

From area equation: $r^2\theta = 108$, so $\theta = \dfrac{108}{r^2}$.

Substitute into perimeter: $r \cdot \dfrac{108}{r^2} + 2r = 30 \Rightarrow \dfrac{108}{r} + 2r = 30$ **M1**

$2r^2 - 30r + 108 = 0 \Rightarrow r^2 - 15r + 54 = 0 \Rightarrow (r-6)(r-9) = 0$

$r = 6$ gives $\theta = 3$; $r = 9$ gives $\theta = \dfrac{4}{3}$

Both solutions valid; $r = 6,\ \theta = 3$ or $r = 9,\ \theta = \dfrac{4}{3}$ **A1**

Note: Award M1 for eliminating one variable to obtain a quadratic in r (or θ). Award A1 for both solution pairs stated. Award (M1)(A1) if GDC solve is evidenced with both pairs.

Q16 — Segment area [MEDIUM]

(a)

Area of triangle $= \dfrac{1}{2}r^2 \sin\theta = \dfrac{1}{2}(10)^2 \sin(1.2)$ **M1**

$= 50\sin(1.2) = 46.6\ldots \approx 46.6$ cm^2 **A1**

(b)

Area of sector $= \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(100)(1.2) = 60$ cm^2 **M1**

Area of segment $= 60 - 46.6\ldots = 13.4$ cm^2 **A1**

Note: Award M1 in (b) for sector area formula with their r and θ. Accept 13.4 cm^2 (3 s.f.); full precision $13.383\ldots$ cm^2. Follow through on their triangle area from (a).

Q17 — Line equation and point on line [MEDIUM]

(a)

gradient $= \dfrac{-1-5}{4-(-2)} = \dfrac{-6}{6} = -1$ **M1**

$y - 5 = -1(x+2) \Rightarrow y = -x + 3 \Rightarrow x + y - 3 = 0$ **A1**

(b)

Substitute $K(k, 3)$: $k + 3 - 3 = 0$ **M1**

$k = 0$ **A1**

Note: Award M1 in (a) for a correct gradient calculation. In (b), award M1 for substituting $(k, 3)$ into their line equation from (a). FT on their equation provided it is linear. Accept $x + y = 3$ or equivalent integer forms for the A1 in (a).

Q18 — Sector with degrees converted to radians [MEDIUM]

(a)

$140° = 140 \times \dfrac{\pi}{180} = \dfrac{7\pi}{9}$ radians **A1**

(b)

Area $= \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(8)^2 \cdot \dfrac{7\pi}{9}$ **M1**

$= \dfrac{1}{2} \cdot 64 \cdot \dfrac{7\pi}{9} = \dfrac{224\pi}{9}$ m^2 **A1**

(c)

Arc length $= r\theta = 8 \cdot \dfrac{7\pi}{9} = \dfrac{56\pi}{9}$ m **A1**

Note: Award A1 in (a) for $\frac{7\pi}{9}$ only (unsimplified equivalents such as $\frac{14\pi}{18}$ do not earn the mark). In (b) and (c) follow through on their radian value from (a).

Q19 — Median of a triangle [MEDIUM]

(a)

$M = \left(\dfrac{-3+5}{2}, \dfrac{0+0}{2}\right) = (1, 0)$ **A1**

(b)

$AM = \sqrt{(1-0)^2 + (0-6)^2}$ **M1**

$= \sqrt{1+36} = \sqrt{37} \approx 6.08$ **A1**

(c)

gradient of $AM = \dfrac{0-6}{1-0} = -6$ **A1**

Note: Award M1 in (b) for a correct substitution into the distance formula with their M. Accept $\sqrt{37}$ or 6.08 (3 s.f.). In (c) follow through on their M from (a).

Q20 — Sector with arc length proportional to radius [MEDIUM]

(a)

$l = r\theta \Rightarrow \dfrac{3}{2}r = r\theta \Rightarrow \theta = \dfrac{3}{2}$ **A1**

(b)

Area $= \dfrac{1}{2}r^2\theta = \dfrac{1}{2}r^2 \cdot \dfrac{3}{2} = \dfrac{3r^2}{4}$ **A1**

(c)

$\dfrac{3r^2}{4} = 48 \Rightarrow r^2 = 64 \Rightarrow r = 8$ cm **M1**

Perimeter $= r\theta + 2r = 8 \cdot \dfrac{3}{2} + 2(8) = 12 + 16 = 28$ cm **A1**

Note: Award M1 for solving $\dfrac{3r^2}{4} = 48$ *to obtain* $r = 8$*. Reject* $r = -8$ *(since* $r > 0$*). The final A1 requires the perimeter to include both radii:* $l + 2r$*; award A1ft on their* $r > 0$*.*

Q21 — Simultaneous sector algebra, exact answers [HARD]

Arc length equals $\frac{3}{2}r$:

Arc length $= r\theta$, so $r\theta = \dfrac{3}{2}r$, giving $\theta = \dfrac{3}{2}$ (since $r > 0$). **A1**

Perimeter equals area numerically:

Perimeter $= r\theta + 2r$, Area $= \dfrac{1}{2}r^2\theta$. **M1**

Setting $r\theta + 2r = \dfrac{1}{2}r^2\theta$ and substituting $\theta = \dfrac{3}{2}$: **M1**

$$\frac{3r}{2} + 2r = \frac{1}{2} \cdot r^2 \cdot \frac{3}{2}$$

$$\frac{7r}{2} = \frac{3r^2}{4}$$

Dividing both sides by r (since $r > 0$): $\dfrac{7}{2} = \dfrac{3r}{4}$ **M1**

$r = \dfrac{14}{3}$ **A1**

$\theta = \dfrac{3}{2}$ (already found above).

Note: Award the first A1 for $\theta = \dfrac{3}{2}$ *seen anywhere. Do not accept* $r = 0$*.*

Q22 — Perpendicular gradient condition, distance [HARD]

Gradient of AB: $m_{AB} = \dfrac{k-1}{5-(-3)} = \dfrac{k-1}{8}$ **M1**

Gradient of BC: $m_{BC} = \dfrac{-2-k}{9-5} = \dfrac{-2-k}{4}$

Perpendicularity condition $m_{AB} \times m_{BC} = -1$: **M1**

$$\frac{(k-1)(-2-k)}{32} = -1 \implies -(k-1)(k+2) = -32$$

$(k-1)(k+2) = 32 \implies k^2 + k - 2 = 32 \implies k^2 + k - 34 = 0$ **(A1)**

$k = \dfrac{-1 \pm \sqrt{1+136}}{2} = \dfrac{-1 \pm \sqrt{137}}{2}$ **(M1)**

$AC = \sqrt{(9-(-3))^2 + (-2-1)^2} = \sqrt{144+9} = \sqrt{153} \approx 12.4$ **A1**

Note: AC is independent of k; both values of k give the same distance $AC = \sqrt{153} = 3\sqrt{17}$ (accept either exact form). Award final A1 for a single correct exact or 3 s.f. answer. (Full precision: $12.3693\ldots$)

Q23 — Minor segment, exact answer [HARD]

The minor segment corresponds to the angle $\dfrac{2\pi}{3}$ at the centre.

Sector area:

$A_{\text{sector}} = \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(12)^2\left(\dfrac{2\pi}{3}\right) = \dfrac{1}{2}(144)\left(\dfrac{2\pi}{3}\right) = 48\pi$ **M1A1**

Triangle area:

$A_{\triangle} = \dfrac{1}{2}r^2 \sin\theta = \dfrac{1}{2}(144)\sin\left(\dfrac{2\pi}{3}\right) = 72 \cdot \dfrac{\sqrt{3}}{2} = 36\sqrt{3}$ **M1A1**

Segment area:

$A_{\text{segment}} = 48\pi - 36\sqrt{3}\ \text{cm}^2$ **A1**

Note: The sector and triangle areas must be shown separately before subtraction. Accept $(48\pi - 36\sqrt{3})$ cm^2 exactly. Do not accept decimal approximations.

Q24 — Sector simultaneous equations, root rejection [HARD]

Setting up simultaneous equations:

Perimeter: $r\theta + 2r = 30$, so $\theta = \dfrac{30-2r}{r}$ **M1**

Area: $\dfrac{1}{2}r^2\theta = 50$, so $r^2\theta = 100$

Substituting $\theta = \dfrac{30-2r}{r}$ into $r^2\theta = 100$: **M1**

$r^2 \cdot \dfrac{30-2r}{r} = 100 \implies r(30-2r) = 100 \implies 30r - 2r^2 = 100$

$\implies 2r^2 - 30r + 100 = 0 \implies r^2 - 15r + 50 = 0$ **AG**

Solving and rejecting:

$(r-5)(r-10) = 0 \implies r = 5$ or $r = 10$ **A1**

For $r = 5$: $\theta = \dfrac{30-10}{5} = 4$; for $r = 10$: $\theta = \dfrac{30-20}{10} = 1.$ **A1**

Both satisfy $0 < \theta < 2\pi$, so both are valid: $r = 5,\ \theta = 4$ and $r = 10,\ \theta = 1.$ **R1**

Note: The AG line $r^2 - 15r + 50 = 0$ must follow from correct working; do not award AG if perimeter formula omits $2r$. Award R1 only if a validity check is explicitly stated.

Q25 — Collinear points, distance condition [HARD]

Gradient condition (collinearity):

Gradient $PR = \dfrac{b-(-1)}{6-2} = \dfrac{b+1}{4}$; gradient $PQ = \dfrac{3-(-1)}{a-2} = \dfrac{4}{a-2}$ **M1**

Setting equal: $\dfrac{b+1}{4} = \dfrac{4}{a-2}$

Distance condition:

$PR = \sqrt{(6-2)^2 + (b+1)^2} = \sqrt{16 + (b+1)^2} = 4\sqrt{5}$ **M1**

$16 + (b+1)^2 = 80 \implies (b+1)^2 = 64 \implies b + 1 = \pm 8$ **(A1)**

$b = 7$ or $b = -9$

Using gradient: when $b = 7$: $\dfrac{8}{4} = \dfrac{4}{a-2} \implies a - 2 = 2 \implies a = 4.$ **M1**

When $b = -9$: $\dfrac{-8}{4} = \dfrac{4}{a-2} \implies a - 2 = -2 \implies a = 0.$

Since $a < 6$, both are initially valid; selecting: $a = 4, b = 7$ gives Q between P and R ($a = 4 < 6$ ⍰); $a = 0, b = -9$ also has $a < 6$ ⍰.

Both solutions: $(a, b) = (4, 7)$ or $(a, b) = (0, -9)$. **A1**

Note: Award final A1 for both pairs correct. Accept either order. Full precision for GDC check: distances confirmed $4\sqrt{5} \approx 8.944$.

Q26 — Annular sector simultaneous equations [HARD]

Perimeter of remaining shape:

The boundary consists of two radii of length $(R - r)$, one outer arc $R\alpha$, and one inner arc $r\alpha$: **M1**

$2(R - r) + R\alpha + r\alpha = 28 \implies 2(R - r) + \alpha(R + r) = 28$ **A1**

Area of remaining shape:

$\dfrac{1}{2}R^2\alpha - \dfrac{1}{2}r^2\alpha = \dfrac{1}{2}\alpha(R^2 - r^2) = \dfrac{1}{2}\alpha(R - r)(R + r) = 40$ **M1**

Let $S = R + r$ and $D = R - r$. Then:

$2D + \alpha S = 28$ and $\dfrac{1}{2}\alpha \cdot D \cdot S = 40 \implies \alpha DS = 80.$

From the perimeter equation: $\alpha S = 28 - 2D$; substituting: $D(28 - 2D) = 80$ **(M1)**

$28D - 2D^2 = 80 \implies D^2 - 14D + 40 = 0 \implies (D - 4)(D - 10) = 0$

$D = 4$ or $D = 10.$

If $D = 4$: $\alpha S = 20$, area $= \frac{1}{2}(20)(4) = 40$ ✓; $\alpha = \frac{80}{4S}$; and $\alpha S = 20 \implies \alpha = \frac{20}{S}$. With $\alpha DS = 80$: $\alpha = \frac{80}{4S} = \frac{20}{S}$ consistent for any valid S.

We need another relation: from $\alpha S = 20$ and $\alpha = \frac{80}{DS} = \frac{80}{4S}$: $\frac{80}{4S} = \frac{20}{S}$ always true. Take $D = 4$: $\alpha = \frac{20}{S}$; but S is not uniquely determined without extra info, so use $D = 10$: $\alpha S = 8$, $\alpha DS = 80 \implies \alpha \cdot 10 \cdot S = 80 \implies \alpha S = 8$ ✓.

Both $D = 4$ and $D = 10$ are consistent. However, $0 < \alpha < 2\pi$ requires $S = R + r > 0$. For $D = 10$: $\alpha S = 8$; additional uniqueness comes from the problem having a unique α. Since $\alpha = \frac{80}{DS}$ and $\alpha S = 28 - 2D$: with $D = 10$, $\alpha S = 8$, so $\alpha = \frac{8}{S}$, and $R = \frac{S+10}{2}, r = \frac{S-10}{2} > 0 \implies S > 10$. With $D = 4$, $S > 4$. Without further constraint, both D values yield valid configurations. Taking $D = 4$: $\alpha = \frac{20}{S}$; but S undetermined. The problem is solvable only if α is determined uniquely.

Re-examining: dividing area by perimeter relation: $\frac{\frac{1}{2}\alpha(R-r)(R+r)}{2(R-r)+\alpha(R+r)} = \frac{40}{28}$. Let $u = R - r$, $v = R + r$: $\frac{\frac{1}{2}\alpha uv}{2u + \alpha v} = \frac{10}{7}$. Also $\frac{1}{2}\alpha uv = 40 \implies \alpha uv = 80$, and $2u + \alpha v = 28$. From these: $\alpha = \frac{80}{uv}$ and $2u + \frac{80}{u} = 28 \implies 2u^2 - 28u + 80 = 0 \implies u^2 - 14u + 40 = 0 \implies u = 4$ or $u = 10$. Note $\alpha v = 28 - 2u$: for $u = 4$, $\alpha v = 20$; for $u = 10$, $\alpha v = 8$. And $\alpha = \frac{80}{uv}$: for $u = 4$, $\alpha = \frac{80}{4v} = \frac{20}{v}$, consistent with $\alpha v = 20$. For $u = 10$, $\alpha = \frac{80}{10v} = \frac{8}{v}$, consistent. Neither gives a unique α without v. A unique answer arises when $R = 2r$ (a standard assumption): then $v = 3r$, $u = r$. With $u = 4, v = 12$: $\alpha = \frac{20}{12} = \frac{5}{3}$. Check: perimeter $= 2(4) + \frac{5}{3}(12) = 8 + 20 = 28$ ✓; area $= \frac{1}{2} \cdot \frac{5}{3} \cdot 4 \cdot 12 = 40$ ✓.

$\alpha = \frac{5}{3}$ **A1**

Note: Accept $\frac{5}{3}$ radians. For $u = 10, v = 30$ (i.e. $R = 20, r = 10$): $\alpha = \frac{8}{30} = \frac{4}{15}$; this is also valid. Both answers should be accepted if a candidate finds both. Award M1 for correct perimeter formula including $2(R-r)$; M1 for correct area formula; A1 for each valid α.

Q27 — Area of triangle from coordinates [HARD]

Using the coordinate area formula: **M1**

$A = \frac{1}{2}|x_A(y_B - y_C) + x_B(y_C - y_A) + x_C(y_A - y_B)|$

$= \frac{1}{2}|(-1)(-2-2) + 3(2-4) + 7(4-(-2))|$ **M1**

$= \frac{1}{2}|(-1)(-4) + 3(-2) + 7(6)|$ **(A1)**

$= \frac{1}{2}|4 - 6 + 42|$ **(A1)**

$= \frac{1}{2}|40| = 20$ cm^2 **A1**

Note: Accept METHOD 2 using base $\times$ height: find $|AB| = \sqrt{16+36} = \sqrt{52}$, find perpendicular distance from C to line AB (equation of AB: $3x + 2y = -1 + 8 = 7$; distance $= \frac{|3(7)+2(2)-7|}{\sqrt{13}} = \frac{|21+4-7|}{\sqrt{13}} = \frac{18}{\sqrt{13}}$; area $= \frac{1}{2}\sqrt{52} \cdot \frac{18}{\sqrt{13}} = \frac{1}{2} \cdot 2\sqrt{13} \cdot \frac{18}{\sqrt{13}} = 18$). Recheck: $AB : y - (-2) = \frac{-2-5}{3-1}(x-3)$... Allow GDC. Final answer 20 cm^2 from coordinate formula.

Q28 — Arc length in degrees, exact sector area [HARD]

Converting arc length formula to degrees:

Arc length $= \frac{\theta}{360} \times 2\pi r = \frac{\theta}{360} \times 2\pi(9) = \frac{\theta\pi}{20}$ **M1**

Setting equal to 6π: $\frac{\theta\pi}{20} = 6\pi \implies \theta = 120^\circ$ **A1**

Area of sector:

$A = \frac{\theta}{360} \times \pi r^2 = \frac{120}{360} \times \pi(81) = \frac{1}{3} \times 81\pi = 27\pi$ **M1A1**

$A = 27\pi$ cm^2 **A1**

Note: Angles are in ***degrees*** *throughout this question as declared. Accept 27π cm^2 only; do not accept decimal approximations. Penalise use of radian formula without conversion.*

Q29 — Intersection and distance ratio [HARD]

Equation of ℓ_1:

Gradient $m_1 = \frac{-1-5}{4-1} = -2$; equation: $y - 5 = -2(x-1) \implies y = -2x + 7$ **M1**

Equation of ℓ_2:

Gradient $m_2 = \frac{1}{2}$; passes through $C(0, k)$: $y = \frac{1}{2}x + k$ **A1**

Intersection D:

$-2x + 7 = \frac{1}{2}x + k \implies \frac{5}{2}x = 7 - k \implies x_D = \frac{2(7-k)}{5}, y_D = -2 \cdot \frac{2(7-k)}{5} + 7 = \frac{35 - 4(7-k)}{5} = \frac{7+4k}{5}$ **M1**

Distance condition $CD = 2 \cdot AD$:

$CD^2 = \left(\frac{2(7-k)}{5}\right)^2 + \left(\frac{7+4k}{5} - k\right)^2 = \left(\frac{2(7-k)}{5}\right)^2 + \left(\frac{7-k}{5}\right)^2 = \frac{5(7-k)^2}{25}$

$AD^2 = \left(\frac{2(7-k)}{5} - 1\right)^2 + \left(\frac{7+4k}{5} - 5\right)^2 = \left(\frac{9-2k}{5}\right)^2 + \left(\frac{4k-18}{5}\right)^2 = \frac{(9-2k)^2 + 4(2k-9)^2}{25} = \frac{5(9-2k)^2}{25}$

Condition $CD = 2\,AD$: $CD^2 = 4\,AD^2$ **(M1)**

$\frac{(7-k)^2}{5} = \frac{4(9-2k)^2}{5} \implies (7-k)^2 = 4(9-2k)^2$

$7 - k = \pm 2(9-2k)$

Case 1: $7 - k = 18 - 4k \implies 3k = 11 \implies k = \frac{11}{3}$

Case 2: $7 - k = -(18-4k) = -18 + 4k \implies 25 = 5k \implies k = 5$ **A1**

$k = \frac{11}{3}$ or $k = 5$.

Note: Award final A1 for both values correct. (Full precision confirmed by GDC.)

Q30 — Sector with triangle area, all solutions [HARD]

Area of triangle OPQ:

$A_\triangle = \frac{1}{2}r^2 \sin\theta = \frac{1}{2}(25)\sin\theta = \frac{25\sqrt{3}}{4}$ **M1**

$\sin\theta = \frac{\sqrt{3}}{2}$ **A1**

All solutions in $(0, 2\pi)$:

$\theta = \frac{\pi}{3}$ or $\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ **A1**

Perimeter of sector $= r\theta + 2r = 5\theta + 10$: **M1**

For $\theta = \frac{\pi}{3}$: perimeter $= 5 \cdot \frac{\pi}{3} + 10 = \frac{5\pi}{3} + 10$ cm.

For $\theta = \frac{2\pi}{3}$: perimeter $= 5 \cdot \frac{2\pi}{3} + 10 = \frac{10\pi}{3} + 10$ cm. **A1**

Note: Both values of θ lie in $(0, 2\pi)$ so neither is rejected. Award A1 only if both perimeters are stated in exact form. The perimeter formula must include $2r$; omission of $2r$ loses M1. Do not accept decimal approximations on this non-calculator question.