IB Math AA SL — Topic 3
Trigonometry

Right-Angled Trigonometry · Sine & Cosine Rule · Elevation & Depression · Bearings

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:_____ Date:_____

Score:_____/120

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Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

Key Formulae

- Pythagorean theorem: $a^2 + b^2 = c^2$ (right-angled triangle, c the hypotenuse)
- Right-angle trigonometry: $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$, $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$, $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$
- Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$
- Cosine rule (side): $a^2 = b^2 + c^2 - 2bc\cos A$
- Cosine rule (angle): $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
- Area of a triangle: $\text{Area} = \dfrac{1}{2}ab\sin C$
- Pythagorean identity (for exact surd work): $\sin^2\theta + \cos^2\theta = 1$, so $\sin\theta = \sqrt{1 - \cos^2\theta}$ when θ is acute

GDC Tips

- Solving a triangle (sine/cosine rule): Store unrounded intermediate values in GDC memory (STO→) and recall them in the next calculation to avoid accumulation of rounding error.
- Angle mode: Check MODE before every trigonometric calculation; set Degrees for bearing and elevation problems (declared), Radians otherwise. A wrong mode gives no method credit.
- Inverse trig: Use 2nd $[\sin^{-1}]$, 2nd $[\cos^{-1}]$ to find angles; note the GDC only returns the principal value, so check whether the obtuse supplement is also valid (ambiguous case).
- Area from two sides and included angle: Enter $\frac{1}{2} \times a \times b \times \sin(C)$ directly; ensure the angle is in the correct mode.
- Bearings: Use the GDC to evaluate expressions of the form $90° - \theta$ or $180° + \theta$ when converting between interior angles and three-figure bearings; store north-line offsets to avoid arithmetic slips.
- Numeric solve / intersection: On Paper 2, use Solver or graph intersection (CALC → intersect) to find an unknown side that appears in both the sine and cosine rules simultaneously, rather than solving by hand.

Common Mistakes to Avoid

1. **Ignoring the ambiguous case of the sine rule.** When finding an angle using $\sin A = k$ with $0 < k < 1$, there are two candidates: A and $180° - A$. Both must be tested against the remaining angle sum; omitting the obtuse solution loses marks even if the acute answer is correct.
2. **Using premature rounding in chained triangles.** Rounding a side or angle to 3 s.f. midway through a two-triangle problem and feeding that value into the next step causes the final answer to fall outside the acceptable range. Always carry full GDC precision and round only in the final step.
3. **Confusing the sine rule direction when finding a side.** Writing the side under the wrong sine ratio inverts the formula. The side is always paired with the angle opposite it; re-write the rule explicitly before substituting.
4. **Wrong sign for $\cos A$ in the cosine rule.** When the included angle is obtuse, $\cos A$ is negative, so $-2bc\cos A$ becomes a positive addition to $b^2 + c^2$, giving a larger a^2. Forgetting the sign change produces a side that is too short.
5. **Incorrect three-figure bearing from a return journey.** A bearing from P to Q of $\theta°$ gives a bearing from Q back to P of $(\theta \pm 180)°$, not $360° - \theta$. Always draw a north line at the new point and derive the return bearing from first principles rather than guessing the supplement.
6. **Using a decimal approximation instead of an exact surd in non-calculator "show that" chains.** On Paper 1, once $\cos A = \frac{p}{q}$ is established exactly, $\sin A$ must be derived via $\sin A = \sqrt{1 - \cos^2 A}$ (arguing the positive root from the acute angle) and kept in surd form throughout. Substituting a decimal approximation breaks the chain and the final AG line scores nothing.

Section A — Easy EASY (10 questions × 3 marks = 30 marks)

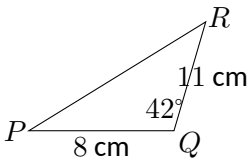
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1. EASY No Calculator [3 marks]

Triangle ABC has a right angle at C, with $AC = 5$ cm and $BC = 12$ cm. Find the length of AB.

2. EASY Calculator [3 marks]

In triangle PQR, $PQ = 8$ cm, $QR = 11$ cm, and $\hat{Q} = 42°$. The diagram is not to scale.
Find the area of triangle PQR.



3. EASY Calculator [3 marks]

In triangle ABC, $AB = 7$ cm, $AC = 9$ cm, and $\hat{A} = 55°$. Find the length of BC.

4. **EASY** **Calculator** **[3 marks]**

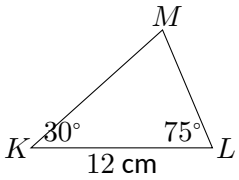
A vertical flagpole stands on horizontal ground. The angle of elevation of the top of the flagpole from a point 15 m away on the ground is $38°$. Find the height of the flagpole, giving your answer to 3 significant figures.

5. **EASY** **Calculator** **[3 marks]**

In triangle DEF, $DE = 10$ cm, $EF = 6$ cm, and $DF = 8$ cm. Find the size of angle $\hat{E}$, giving your answer to 3 significant figures.

6. **EASY** **No Calculator** **[3 marks]**

In triangle KLM, $\hat{K} = 30°$, $\hat{L} = 75°$, and $KL = 12$ cm. The diagram is not to scale.
Find the length of LM.



7. **EASY** **No Calculator** **[3 marks]**

A right-angled triangle has an angle of $\frac{\pi}{6}$ radians, with the side opposite this angle measuring 4 cm. Find the length of the hypotenuse.

8. **EASY** **Calculator** **[3 marks]**

Two ships leave port O at the same time. Ship A travels on a bearing of 050° for 30 km. Ship B travels on a bearing of 110° for 45 km. Find the distance between the two ships, giving your answer to 3 significant figures.

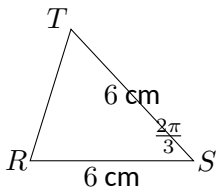
9. **EASY** **Calculator** **[3 marks]**

In triangle XYZ, $YZ = 14$ cm, $\hat{Y} = 62^\circ$, and $\hat{Z} = 48^\circ$. Find the length of XZ. Give your answer to 3 significant figures.

10. **EASY** **No Calculator** **[3 marks]**

Triangle RST has $RS = 6$ cm, $ST = 6$ cm, and $\hat{S} = \dfrac{2\pi}{3}$ radians. The diagram is not to scale.

Find the exact area of triangle RST.

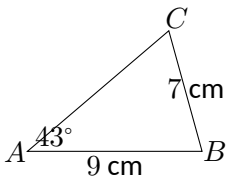


Section B — Medium MEDIUM (10 questions × 4 marks = 40 marks)

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11. MEDIUM Calculator [4 marks]

In triangle ABC, $AB = 9$ cm, $BC = 7$ cm, and $\angle BAC = 43°$. Find the two possible values of $\angle BCA$, giving your answers in degrees correct to one decimal place.
Diagram not to scale. Angles in degrees for this question.



12. MEDIUM Calculator [4 marks]

Triangle PQR has $PQ = 12$ cm, $QR = 8$ cm and $\angle PQR = \frac{2\pi}{5}$. Find the area of triangle PQR, giving your answer correct to 3 significant figures.

13. MEDIUM Calculator **[4 marks]**

A vertical tower CD stands on horizontal ground. From a point A on the ground, the angle of elevation of the top of the tower D is $28°$. From a point B, also on the ground and 15 m closer to the base of the tower along the same straight line as A, the angle of elevation of D is $41°$. Find the height of the tower CD.

14. MEDIUM No Calculator **[4 marks]**

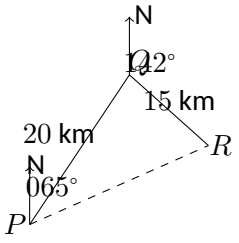
In triangle KLM, $KL = 10$ cm, $LM = 6$ cm, and $\cos(\angle KLM) = \frac{1}{4}$.

(a) Find the exact length of KM. [2]

(b) Find the exact area of triangle KLM, giving your answer in the form $p\sqrt{q}$ where $p, q \in \mathbb{Z}^+$. [2]

15. MEDIUM Calculator **[4 marks]**

A ship leaves port P and travels 20 km on a bearing of $065°$ to reach point Q. It then travels 15 km on a bearing of $142°$ to reach point R. Find the distance PR, giving your answer correct to 3 significant figures.
Diagram not to scale.



16. MEDIUM Calculator **[4 marks]**

In triangle ABC, $AC = 14$ cm, $BC = 11$ cm, and $AB = 9$ cm. Find $\angle BAC$, giving your answer in radians correct to 3 significant figures.

17. MEDIUM No Calculator **[4 marks]**

A right-angled triangle has legs of length $\sqrt{5}$ cm and $\sqrt{11}$ cm.

(a) Find the exact length of the hypotenuse. [1]

(b) Find the exact value of $\sin\theta$, where θ is the angle opposite the longer leg. [1]

(c) Hence find the exact value of $\cos 2\theta$. [2]

18. MEDIUM Calculator **[4 marks]**

From a harbour H, a lighthouse L is located 18 km away on a bearing of $310°$. A boat B is located 12 km from H on a bearing of $045°$. Find the distance BL, giving your answer correct to 3 significant figures.

19. MEDIUM Calculator **[4 marks]**

Triangle RST has $RS = 7$ cm, $RT = 5$ cm, and area $= 14$ cm^2. Given that $\angle SRT$ is obtuse, find $\angle SRT$ in radians, correct to 3 significant figures.

20. MEDIUM No Calculator **[4 marks]**

In triangle DEF, $DE = 13$ cm, $\angle DEF = \frac{\pi}{6}$, and $\angle EDF = \frac{\pi}{4}$. Find the exact length of EF.

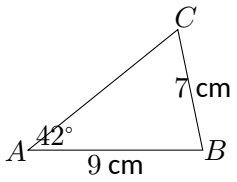
Section C — Hard HARD (10 questions × 5 marks = 50 marks)

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21. HARD Calculator [5 marks]

In triangle ABC, $AB = 9$ cm, $BC = 7$ cm and $\angle BAC = 42°$. Angles are in degrees. Find all possible values of $\angle ABC$.

Diagram not to scale.



22. HARD No Calculator [5 marks]

In triangle PQR, $PQ = 12$ cm, $QR = 8$ cm and $\cos(\angle PQR) = \frac{1}{6}$.

(a) Show that $PR = 4\sqrt{11}$ cm. [2]

(b) Hence find the exact value of $\sin(\angle PQR)$, giving your answer in the form $\frac{\sqrt{a}}{b}$ where $a, b \in \mathbb{Z}^+$. [1]

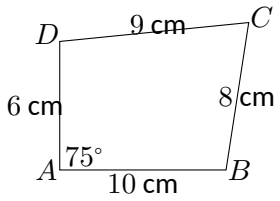
(c) Hence find the exact area of triangle PQR, giving your answer in the form $p\sqrt{q}$ where $p, q \in \mathbb{Z}^+$ and q is not divisible by the square of any integer greater than 1. [2]

23. **HARD** **Calculator** **[5 marks]**

A ship leaves port P and sails on a bearing of 065° for 28 km to reach point A. It then changes course and sails on a bearing of 152° for 43 km to reach point B. Find the bearing of B from P, giving your answer to the nearest degree.

24. **HARD** **Calculator** **[5 marks]**

The diagram below shows quadrilateral $ABCD$ where $AB = 10$ cm, $BC = 8$ cm, $CD = 9$ cm, $DA = 6$ cm and $\angle DAB = 75^\circ$. Angles are in degrees. Find the area of quadrilateral $ABCD$.
Diagram not to scale.



25. HARD Calculator [5 marks]

From the top of a vertical cliff 120 m high, Amara observes two boats, X and Y, both in the sea below. Boat X is due east of the cliff base and has an angle of depression of $35°$ from Amara. Boat Y is due south of the cliff base and has an angle of depression of $28°$ from Amara. Find the distance XY, giving your answer to 3 significant figures.

26. HARD No Calculator [5 marks]

In triangle KLM, $KL = (x + 2)$ cm, $LM = x$ cm, $KM = 5$ cm and $\angle KLM = 60°$. Angles are in degrees. Find the value of x, where $x \in \mathbb{R}^+$.

27. HARD Calculator [5 marks]

Two forest rangers, Bilal and Chloe, are positioned at points B and C respectively, where $BC = 3.6$ km. A fire is spotted at point F. Bilal measures the bearing of F from B as $038°$ and Chloe measures the bearing of F from C as $312°$. Given that C is due east of B, find the distance BF, giving your answer to 3 significant figures.

28. **HARD** **Calculator** **[5 marks]**

In triangle RST, $RS = 5$ cm, $ST = k$ cm and $RT = 8$ cm where $k \in \mathbb{R}^+$. The area of the triangle is 15 cm^2. Find all possible values of $\angle RST$ and, for each value, determine whether a valid triangle exists, clearly justifying your answer.

29. **HARD** **No Calculator** **[5 marks]**

Show that in triangle ABC with $a = BC$, $b = CA$, $c = AB$, if $b = \sqrt{5}$ cm, $c = 2$ cm and $\cos A = \dfrac{1}{2\sqrt{5}}$, then $a = \sqrt{7}$ cm. Hence find the exact area of triangle ABC in the form $\dfrac{p\sqrt{q}}{2}$ cm^2 where $p, q \in \mathbb{Z}^+$.

30. **HARD** **Calculator** **[5 marks]**

A vertical mast OT of height h metres stands at point O. Points A and B lie on horizontal ground. A is due north of O at a distance of 40 m, and B is due east of O at a distance of 55 m. The angle of elevation of T from A is α and the angle of elevation of T from B is β, where $\tan\alpha = \dfrac{h}{40}$ and $\tan\beta = \dfrac{h}{55}$. Given that $\angle ATB = 50°$, find the value of h, giving your answer to 3 significant figures.

Mark Schemes

Q1 — Pythagoras in a Right Triangle EASY

Applying Pythagoras' theorem **M1**

$AB^2 = 5^2 + 12^2 = 25 + 144 = 169$ **(A1)**

$AB = 13$ cm **A1**

Note: Award M1 for a correct statement of Pythagoras' theorem with their values substituted.

Q2 — Area Using One-Half ab sin C EASY

METHOD 1

Area $= \frac{1}{2} \times 8 \times 11 \times \sin 42^\circ$ **M1**

$= \frac{1}{2} \times 8 \times 11 \times 0.66913\ldots$ **(A1)**

$= 29.4\ \text{cm}^2$ **A1**

Note: Award M1 for substitution into $\frac{1}{2}ab\sin C$ with the included angle. Accept 29.4 to 3 s.f.

Q3 — Cosine Rule to Find a Side EASY

$BC^2 = 7^2 + 9^2 - 2(7)(9)\cos 55^\circ$ **M1**

$= 49 + 81 - 126 \times 0.57358\ldots = 130 - 72.27\ldots = 57.73\ldots$ **(A1)**

$BC = 7.60$ cm **A1**

Note: Award M1 for a correct substitution into the cosine rule. Accept answers rounding to 7.60.

Q4 — Angle of Elevation in a Right Triangle EASY

$\tan 38^\circ = \frac{h}{15}$ **M1**

$h = 15 \times \tan 38^\circ$ **(A1)**

$h = 11.7$ m **A1**

Note: Award M1 for setting up the correct trigonometric ratio. The answer 11.7 m must be to 3 significant figures for the final A1.

Q5 — Cosine Rule to Find an Angle EASY

$\cos \hat{E} = \dfrac{DE^2 + EF^2 - DF^2}{2 \cdot DE \cdot EF} = \dfrac{100 + 36 - 64}{2 \times 10 \times 6}$ **M1**

$= \dfrac{72}{120} = 0.6$ **(A1)**

$\hat{E} = \arccos(0.6) = 53.1^\circ$ **A1**

Note: Award M1 for a correct substitution into the cosine rule rearranged to find an angle. Accept 53.1° or 0.927 rad.

Q6 — Sine Rule to Find a Side EASY

Angle $\hat{M} = 180^\circ - 30^\circ - 75^\circ = 75^\circ$ **(A1)**

Applying the sine rule: $\dfrac{LM}{\sin 30^\circ} = \dfrac{12}{\sin 75^\circ}$ **M1**

$LM = \dfrac{12 \sin 30^\circ}{\sin 75^\circ} = \dfrac{12 \times \frac{1}{2}}{\sin 75^\circ} = \dfrac{6}{\sin 75^\circ} = 6\sqrt{6} - 6\sqrt{2}$ **A1**

Note: Accept the exact simplified form $\dfrac{6}{\sin 75^\circ}$ or equivalent exact expressions such as $3(\sqrt{6} - \sqrt{2})$. Award (A1) for finding angle $M = 75^\circ$.

Q7 — Right Triangle with Standard Angle pi/6 EASY

$\sin\left(\dfrac{\pi}{6}\right) = \dfrac{4}{h}$ **M1**

$\dfrac{1}{2} = \dfrac{4}{h}$ **(A1)**

$h = 8$ cm **A1**

Note: Award M1 for setting up the correct sine ratio. The exact value $\sin(\pi/6) = \dfrac{1}{2}$ must be used for full marks on a Calculator not allowed question.

Q8 — Bearings and Cosine Rule for Distance EASY

The angle between the two bearings is $110^\circ - 50^\circ = 60^\circ$ **(A1)**

$d^2 = 30^2 + 45^2 - 2(30)(45)\cos 60^\circ$ **M1**

$= 900 + 2025 - 2700 \times 0.5 = 2925 - 1350 = 1575$

$d = \sqrt{1575} = 39.686\ldots \approx 39.7$ km **A1**

Note: Award (A1) for identifying the angle between the paths as 60°. Award M1 for correct substitution into the cosine rule. Final answer must be to 3 s.f.

Q9 — Sine Rule to Find a Side EASY

Angle $\hat{X} = 180^\circ - 62^\circ - 48^\circ = 70^\circ$ **(A1)**

$\dfrac{XZ}{\sin 62^\circ} = \dfrac{14}{\sin 70^\circ}$ **M1**

$XZ = \dfrac{14 \sin 62^\circ}{\sin 70^\circ} = \dfrac{14 \times 0.88295\ldots}{0.93969\ldots} = 13.154\ldots \approx 13.2$ cm **A1**

Note: Award (A1) for finding $\hat{X} = 70^\circ$. Award M1 for a correct sine rule setup. Final answer must be to 3 s.f.

Q10 — Exact Area Using One-Half ab sin C with Radian Angle EASY

Area $= \dfrac{1}{2} \times 6 \times 6 \times \sin\left(\dfrac{2\pi}{3}\right)$ **M1**

$\sin\left(\dfrac{2\pi}{3}\right) = \dfrac{\sqrt{3}}{2}$ **(A1)**

Area $= \dfrac{1}{2} \times 36 \times \dfrac{\sqrt{3}}{2} = 9\sqrt{3}$ cm^2 **A1**

Note: Award M1 for substitution into the area formula. The exact value $\sin(2\pi/3) = \sqrt{3}/2$ must be stated or used for (A1). Final answer must be in exact form $9\sqrt{3}$ for the last A1.

Q11 — Ambiguous Sine Rule MEDIUM

Applying the sine rule:

$\dfrac{\sin(\angle BCA)}{AB} = \dfrac{\sin(\angle BAC)}{BC}$ **M1**

$\dfrac{\sin(\angle BCA)}{9} = \dfrac{\sin 43^\circ}{7}$

$\sin(\angle BCA) = \dfrac{9 \sin 43^\circ}{7} = 0.876855\ldots$ **(A1)**

First solution: $\angle BCA = \arcsin(0.876855\ldots) = 61.3^\circ$ **A1**

Second solution: $\angle BCA = 180^\circ - 61.3^\circ = 118.7^\circ$

Check: $43^\circ + 118.7^\circ = 161.7^\circ < 180^\circ$ (valid) **A1**

Note: Award final A1 only if both values are stated with the supplement recognised and validated. If only one value given, award at most 3 marks.

Q12 — Area of Triangle with Included Angle MEDIUM

Area $= \dfrac{1}{2} \times PQ \times QR \times \sin(\angle PQR)$ **M1**

$= \frac{1}{2} \times 12 \times 8 \times \sin\left(\frac{2\pi}{5}\right)$ **(A1)**

$= 48 \times \sin(72°) = 48 \times 0.95105\ldots = 45.650\ldots$ **(A1)**

Area $= 45.7$ cm^2 **A1**

Note: Accept full precision $45.650\ldots$ before rounding. Award final A1 for 45.7 only.

Q13 — Two-Angle Elevation Chain MEDIUM

Let h = height of tower, d = horizontal distance from B to base C.

From B: $\tan 41° = \frac{h}{d}$, so $h = d\tan 41°$ **M1**

From A: $\tan 28° = \frac{h}{d+15}$, so $h = (d+15)\tan 28°$ **(M1)**

Setting equal: $d\tan 41° = (d+15)\tan 28°$

$d(\tan 41° - \tan 28°) = 15\tan 28°$

$d = \frac{15\tan 28°}{\tan 41° - \tan 28°} = \frac{15 \times 0.53171}{0.86929 - 0.53171} = \frac{7.9756}{0.33758} = 23.624\ldots$ **(A1)**

$h = 23.624\ldots \times \tan 41° = 20.528\ldots$

Height of tower $= 20.5$ m **A1**

Note: Accept answers in the range 20.5 to 20.6 m due to intermediate rounding. Award A1ft for $h =$ their $d \times \tan 41°$.

Q14 — Exact Cosine Rule and Surd Area MEDIUM

(a) Applying the cosine rule:

$KM^2 = KL^2 + LM^2 - 2 \cdot KL \cdot LM \cdot \cos(\angle KLM)$ **M1**

$= 100 + 36 - 2(10)(6)\left(\frac{1}{4}\right) = 136 - 30 = 106$

$KM = \sqrt{106}$ cm **A1**

(b) Since $\cos(\angle KLM) = \frac{1}{4}$ and the angle is in a triangle ($0 < \angle KLM < \pi$), $\sin(\angle KLM) > 0$:

$\sin(\angle KLM) = \sqrt{1 - \frac{1}{16}} = \sqrt{\frac{15}{16}} = \frac{\sqrt{15}}{4}$ **M1**

Area $= \frac{1}{2} \times 10 \times 6 \times \frac{\sqrt{15}}{4} = \frac{60\sqrt{15}}{8} = \frac{15\sqrt{15}}{2}$ cm^2 **A1**

Note: For (b), M1 requires use of $\sin^2\theta + \cos^2\theta = 1$ with the sign of $\sin\theta$ argued positive. Accept $\frac{15\sqrt{15}}{2}$ in exact form only.

Q15 — Bearings Cosine Rule MEDIUM

Finding angle $\angle PQR$:
Bearing from P to Q is $065°$, so the back-bearing is $065° + 180° = 245°$.
Bearing from Q to R is $142°$.
Interior angle $\angle PQR = 245° - 142° = 103°$ **M1**

Applying cosine rule in triangle PQR:
$PR^2 = PQ^2 + QR^2 - 2 \cdot PQ \cdot QR \cdot \cos(103°)$ **M1**
$= 400 + 225 - 2(20)(15)\cos(103°) = 625 - 600(-0.22495\ldots)$
$= 625 + 134.97\ldots = 759.97\ldots$ **(A1)**

$PR = \sqrt{759.97\ldots} = 27.568\ldots$
$PR = 27.6$ km **A1**

Note: Award M1 for a correct method to find angle $PQR = 103°$. Accept equivalent derivations of the interior angle. Award A1ft for correct use of their angle in the cosine rule.

Q16 — Cosine Rule for Angle in Radians MEDIUM

Applying the cosine rule to find $\angle BAC$:
$\cos(\angle BAC) = \dfrac{AB^2 + AC^2 - BC^2}{2 \cdot AB \cdot AC}$ **M1**
$= \dfrac{81 + 196 - 121}{2(9)(14)} = \dfrac{156}{252} = \dfrac{13}{21}$ **(A1)**

$\angle BAC = \arccos\left(\dfrac{13}{21}\right) = 0.90327\ldots$ rad **(A1)**

$\angle BAC = 0.903$ rad **A1**

Note: Award (A1) for the unsimplified or simplified exact fraction. Accept $\dfrac{13}{21}$ seen. Final A1 for 0.903 rad only; do not accept degrees unless converted correctly.

Q17 — Exact Right-Triangle Trig and Double Angle MEDIUM

(a) Hypotenuse $= \sqrt{(\sqrt{5})^2 + (\sqrt{11})^2} = \sqrt{5 + 11} = \sqrt{16} = 4$ cm **A1**

(b) θ is opposite the longer leg $\sqrt{11}$:
$\sin\theta = \dfrac{\sqrt{11}}{4}$ **A1**

(c) Using the double angle formula $\cos 2\theta = 1 - 2\sin^2\theta$: **M1**

$\cos 2\theta = 1 - 2\left(\dfrac{\sqrt{11}}{4}\right)^2 = 1 - 2 \times \dfrac{11}{16} = 1 - \dfrac{11}{8} = -\dfrac{3}{8}$ **A1**

Note: In (b) accept equivalent exact forms. In (c) also accept $\cos 2\theta = \cos^2\theta - \sin^2\theta = \dfrac{5}{16} - \dfrac{11}{16} = -\dfrac{3}{8}$

for full marks via METHOD 2.

METHOD 2

$\cos\theta = \frac{\sqrt{5}}{4}$, so $\cos 2\theta = \cos^2\theta - \sin^2\theta = \frac{5}{16} - \frac{11}{16} = -\frac{3}{8}$ **M1A1**

Q18 — Bearings Cosine Rule with Two Bearings MEDIUM

Finding angle $\angle LHB$:

Bearing of L from H is $310°$; bearing of B from H is $045°$.

$\angle LHB = 360° - 310° + 45° = 95°$ **M1**

Applying cosine rule in triangle BHL:

$BL^2 = HB^2 + HL^2 - 2 \cdot HB \cdot HL \cdot \cos(\angle LHB)$ **M1**

$= 144 + 324 - 2(12)(18)\cos(95°) = 468 - 432(-0.08716\ldots)$

$= 468 + 37.652\ldots = 505.65\ldots$ **(A1)**

$BL = \sqrt{505.65\ldots} = 22.487\ldots$

$BL = 22.5$ km **A1**

Note: Award M1 for a correct method to find $\angle LHB$. Accept $95°$ obtained by any valid reasoning. FT their angle into the cosine rule.

Q19 — Obtuse Angle from Area Formula MEDIUM

Using the area formula:

Area $= \frac{1}{2} \times RS \times RT \times \sin(\angle SRT)$ **M1**

$14 = \frac{1}{2} \times 7 \times 5 \times \sin(\angle SRT)$

$\sin(\angle SRT) = \frac{28}{35} = 0.8$ **(A1)**

First solution: $\arcsin(0.8) = 0.9272\ldots$ rad (acute — rejected since $\angle SRT$ is obtuse) **M1**

$\angle SRT = \pi - 0.9272\ldots = 2.2143\ldots$ rad

$\angle SRT = 2.21$ rad **A1**

Note: Award second M1 for recognising the obtuse solution as $\pi - \arcsin(0.8)$. Award A1 for 2.21 rad only. The trap is selecting the acute solution; this earns at most 2 marks.

Q20 — Sine Rule with Angle Sum and Exact Answer MEDIUM

Finding $\angle DFE$:

$\angle DFE = \pi - \frac{\pi}{6} - \frac{\pi}{4} = \pi - \frac{2\pi}{12} - \frac{3\pi}{12} = \frac{7\pi}{12}$ **M1**

Applying the sine rule:

$$\frac{EF}{\sin(\angle EDF)} = \frac{DE}{\sin(\angle DFE)}$$ **M1**

$$\frac{EF}{\sin(\pi/4)} = \frac{13}{\sin(7\pi/12)}$$ **(A1)**

$$\sin\left(\frac{7\pi}{12}\right) = \sin\left(\pi - \frac{5\pi}{12}\right) = \sin\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{12}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$EF = \frac{13\sin(\pi/4)}{\sin(7\pi/12)} = \frac{13 \times \frac{\sqrt{2}}{2}}{\frac{\sqrt{6}+\sqrt{2}}{4}} = \frac{13\sqrt{2}}{2} \times \frac{4}{\sqrt{6}+\sqrt{2}} = \frac{26\sqrt{2}}{\sqrt{6}+\sqrt{2}}$$

Rationalising: $EF = \frac{26\sqrt{2}(\sqrt{6}-\sqrt{2})}{(\sqrt{6})^2-(\sqrt{2})^2} = \frac{26(2\sqrt{3}-2)}{4} = 13(\sqrt{3}-1)$

$EF = 13(\sqrt{3}-1)$ cm **A1**

Note: Award final A1 for the exact simplified form $13(\sqrt{3}-1)$. Accept equivalent exact forms. Do not accept decimal approximations.

Q21 — Ambiguous Sine Rule with Both Configurations HARD

Applying the sine rule to find $\angle BCA$ (the side AB is opposite $\angle BCA$):

$\frac{\sin(\angle BCA)}{AB} = \frac{\sin(\angle BAC)}{BC}$, so $\frac{\sin(\angle BCA)}{9} = \frac{\sin 42^\circ}{7}$ **M1**

$\sin(\angle BCA) = \frac{9\sin 42^\circ}{7} = \frac{9 \times 0.66913\ldots}{7} = 0.86031\ldots$ **(A1)**

First candidate: $\angle BCA = \arcsin(0.86031\ldots) = 59.35^\circ$
Second candidate: $\angle BCA = 180^\circ - 59.35^\circ = 120.65^\circ$
Check: $42^\circ + 120.65^\circ = 162.65^\circ < 180^\circ$, so both configurations are valid triangles. **A1**

Since $\angle ABC$ (not $\angle BCA$) is required, use $\angle ABC = 180^\circ - \angle BAC - \angle BCA$ for each case: **R1**

Case 1: $\angle ABC = 180^\circ - 42^\circ - 59.35^\circ = 78.65^\circ \approx 78.6^\circ$
Case 2: $\angle ABC = 180^\circ - 42^\circ - 120.65^\circ = 17.35^\circ \approx 17.4^\circ$
Therefore $\angle ABC = 78.6^\circ$ or $\angle ABC = 17.4^\circ$ (3 s.f.) **A1**

Note: The sine rule ratio $\sin(\angle BCA)/AB = \sin(\angle BAC)/BC$ solves for $\angle BCA$ (opposite AB), not $\angle ABC$ directly — the angle sum $\angle ABC = 180^\circ - \angle BAC - \angle BCA$ must be applied to each candidate to reach the required angle. Both values required for the final A1.

Q22 — Exact Cosine Rule Chain and Area as Surd HARD

(a) Using the cosine rule: $PR^2 = PQ^2 + QR^2 - 2(PQ)(QR)\cos(\angle PQR)$ **M1**

$$PR^2 = 12^2 + 8^2 - 2(12)(8)\left(\frac{1}{6}\right) = 144 + 64 - 32 = 176$$

$PR = \sqrt{176} = \sqrt{16 \times 11} = 4\sqrt{11}$ cm **A1 AG**

(b) Since $\cos(\angle PQR) = \frac{1}{6}$ and the angle is in a triangle (so sin > 0):

$\sin(\angle PQR) = \sqrt{1 - \frac{1}{36}} = \sqrt{\frac{35}{36}} = \frac{\sqrt{35}}{6}$ **A1**

(c) Area $= \frac{1}{2}(PQ)(QR)\sin(\angle PQR) = \frac{1}{2}(12)(8) \cdot \frac{\sqrt{35}}{6}$ **M1**

$= \frac{96\sqrt{35}}{12} = 8\sqrt{35}$ cm^2 **A1**

Note: 35 is not divisible by the square of any integer greater than 1 ($35 = 5 \times 7$). Award M1 for correct substitution into the area formula.

Q23 — Two-Triangle Bearing and Return Bearing HARD

Finding PB using the cosine rule. The interior angle at A between AP (produced, i.e. the back-bearing $065^\circ + 180^\circ = 245^\circ$) and AB (bearing 152°):

Interior angle at $A = 245^\circ - 152^\circ = 93^\circ$ **M1**

$PB^2 = 28^2 + 43^2 - 2(28)(43)\cos 93^\circ$

$= 784 + 1849 - 2408\cos 93^\circ = 2633 - 2408(-0.052336\ldots) = 2633 + 126.02\ldots = 2759.02\ldots$ **(A1)**

$PB = 52.526\ldots \approx 52.5$ km **A1**

Finding angle $\angle APB$ via the sine rule:

$\frac{\sin(\angle APB)}{43} = \frac{\sin 93^\circ}{52.526\ldots}$, so $\sin(\angle APB) = \frac{43\sin 93^\circ}{52.526\ldots} = 0.81758\ldots$ **M1**

$\angle APB = 54.84\ldots^\circ$

Bearing of B from $P = 065^\circ + 54.8^\circ = 119.8^\circ \approx 120^\circ$ (nearest degree) **A1**

Note: The interior angle at A is between AP produced backward (the reverse of the bearing $P \to A$) and AB, not the raw difference of the two forward bearings — it equals 180° minus that difference. Award final A1 only if a correct bearing formula is used. Accept 120°. FT from their PB and their angle APB provided the geometry is consistent.

Q24 — Quadrilateral Split by Diagonal for Total Area HARD

Split quadrilateral along diagonal BD.

Step 1: Find diagonal BD using the cosine rule in $\triangle DAB$ ($DA = 6$, $AB = 10$, included angle $\angle DAB = 75^\circ$): **M1**

$BD^2 = DA^2 + AB^2 - 2(DA)(AB)\cos 75^\circ = 36 + 100 - 120\cos 75^\circ$

$= 136 - 120(0.25882\ldots) = 136 - 31.058\ldots = 104.94\ldots$ **(A1)**

$BD = 10.244\ldots$ cm

Area of $\triangle DAB = \frac{1}{2}(DA)(AB)\sin 75^\circ = 30\sin 75^\circ = 28.978\ldots$ cm^2 **A1**

Step 2: Check the triangle inequality for $\triangle DBC$ ($BD = 10.244\ldots$, $BC = 8$, $CD = 9$): $BC + CD =$

$17 > BD$, so $\triangle DBC$ is constructible. **M1**

Using Heron's formula: $s = \dfrac{10.244\ldots + 8 + 9}{2} = 13.622\ldots$

Area of $\triangle DBC = \sqrt{s(s-BD)(s-BC)(s-CD)} = \sqrt{13.622\ldots \times 3.378\ldots \times 5.622\ldots \times 4.622\ldots} = 34.579\ldots$ cm^2 **A1**

Total area $= 28.978\ldots + 34.579\ldots = 63.556\ldots \approx 63.6$ cm^2 (3 s.f.) **A1**

Note: Award M1 for a valid diagonal-split attempt with the triangle inequality confirmed for the second triangle, first A1 for correct BD, second A1 for correct area of the first triangle, third M1 for a valid second-triangle area method, final A1 for the total.

Q25 — Cliff Top with Two Boats in 3D HARD

Let O be the base of the cliff, height $= 120$ m.

Distance OX: $\tan 35^\circ = \dfrac{120}{OX}$, so $OX = \dfrac{120}{\tan 35^\circ} = \dfrac{120}{0.70021\ldots} = 171.36\ldots$ m **M1**

Distance OY: $\tan 28^\circ = \dfrac{120}{OY}$, so $OY = \dfrac{120}{\tan 28^\circ} = \dfrac{120}{0.53171\ldots} = 225.69\ldots$ m **(A1)**

Since X is due east and Y is due south of O, $\angle XOY = 90^\circ$. **R1**

$XY^2 = OX^2 + OY^2 = (171.36\ldots)^2 + (225.69\ldots)^2$ **M1**

$= 29364.4\ldots + 50936.1\ldots = 80300.5\ldots$

$XY = 283.37\ldots \approx 283$ m (3 s.f.) **A1**

Note: R1 awarded for explicit statement that $\angle XOY = 90^\circ$ (or equivalent). Final A1 requires 3 s.f. Accept 283 m. FT from their OX and OY provided both are positive.

Q26 — Cosine Rule with Algebraic Sides and Positive Root Selection HARD

Applying the cosine rule in $\triangle KLM$ with $\angle KLM = 60^\circ$: **M1**

$KM^2 = KL^2 + LM^2 - 2(KL)(LM)\cos 60^\circ$

$25 = (x+2)^2 + x^2 - 2(x+2)(x)\cdot\dfrac{1}{2}$ **A1**

$25 = x^2 + 4x + 4 + x^2 - x(x+2)$

$25 = x^2 + 4x + 4 + x^2 - x^2 - 2x$

$25 = x^2 + 2x + 4$ **(A1)**

$x^2 + 2x - 21 = 0$

$x = \dfrac{-2 \pm \sqrt{4+84}}{2} = \dfrac{-2 \pm \sqrt{88}}{2} = -1 \pm \sqrt{22}$ **M1**

Since $x \in \mathbb{R}^+$, reject $x = -1 - \sqrt{22} < 0$. **R1**

$x = -1 + \sqrt{22}$ **A1**

Note: R1 requires explicit rejection of the negative root with reference to $x \in \mathbb{R}^+$ or a side-length argument. Award R1 only if both roots are found. Accept $x = \sqrt{22} - 1$.

Q27 — Bearing Intersection and Sine Rule for Distance HARD

Let north be upward; C is due east of B, so BC lies along east–west, $BC = 3.6$ km.
Bearing of F from B is $038°$, so the angle between BF and BC (measured from east) is $\angle FBC = 90° - 38° = 52°$ **M1**

Bearing of F from C is $312°$, i.e. $48°$ west of north. The angle between CB (pointing west) and CF is $\angle FCB = 90° - 48° = 42°$ **(A1)**

Angle at F: $\angle BFC = 180° - 52° - 42° = 86°$ **A1**

Applying the sine rule: $\dfrac{BF}{\sin(\angle BCF)} = \dfrac{BC}{\sin(\angle BFC)}$ **M1**

$$BF = \frac{3.6\sin 42°}{\sin 86°} = \frac{3.6 \times 0.66913\ldots}{0.99756\ldots} = \frac{2.4089\ldots}{0.99756\ldots} = 2.414\ldots$$

$BF \approx 2.41$ km (3 s.f.) **A1**

Note: Award M1 for a correct method to find interior angles of $\triangle BCF$ from the bearings. Accept 2.41 km. FT on their angles provided $\angle BFC > 0$. Final A1 requires 3 s.f.

Q28 — Area Reversal with Both Angle Values and Validity Check HARD

Using Area $= \dfrac{1}{2}(RS)(ST)\sin(\angle RST)$: **M1**

$$15 = \frac{1}{2}(5)(k)\sin(\angle RST)$$
$$\sin(\angle RST) = \frac{6}{k}$$

Also, by the cosine rule: $RT^2 = RS^2 + ST^2 - 2(RS)(ST)\cos(\angle RST)$
$64 = 25 + k^2 - 10k\cos(\angle RST)$, i.e. $k^2 - 10k\cos(\angle RST) = 39$ **(A1)**

Using $\sin(\angle RST) = 6/k$ requires $6/k \leq 1$, i.e. $k \geq 6$.
Substituting $\cos(\angle RST) = \pm\sqrt{1 - 36/k^2}$ into $k^2 - 10k\cos(\angle RST) = 39$ and squaring:

$$(k^2 - 39)^2 = 100k^2\left(1 - \frac{36}{k^2}\right) = 100k^2 - 3600$$ **M1**

$$k^4 - 78k^2 + 1521 = 100k^2 - 3600$$
$$k^4 - 178k^2 + 5121 = 0$$
$$k^2 = \frac{178 \pm \sqrt{178^2 - 4(5121)}}{2} = \frac{178 \pm \sqrt{11200}}{2} = 89 \pm 20\sqrt{7}$$ **A1**

Acute case ($\cos(\angle RST) > 0$): $k^2 = 89 + 20\sqrt{7} \implies k = 11.913\ldots \approx 11.9$ cm, $\angle RST = \arcsin(6/11.913\ldots) = 30.2°$.
Obtuse case ($\cos(\angle RST) < 0$): $k^2 = 89 - 20\sqrt{7} \implies k = 6.007\ldots \approx 6.01$ cm, $\angle RST = 180° - \arcsin(6/6.007\ldots) = 92.8°$.
Both values satisfy $k \geq 6$, so both are valid triangles. **R1**

Therefore $\angle RST = 30.2°$ (with $k = 11.9$ cm) or $\angle RST = 92.8°$ (with $k = 6.01$ cm); both configurations give valid triangles. **A1**

Note: R1 is reserved for the explicit check $k \geq 6$ against both roots. Both cases must be considered to earn

R1. Award final A1 only if both values are retained with validity confirmed.

Q29 — Show-That Cosine Rule and Area as Surd HARD

Show that $a = \sqrt{7}$: Using the cosine rule, $a^2 = b^2 + c^2 - 2bc\cos A$: **M1**

$a^2 = (\sqrt{5})^2 + 2^2 - 2(\sqrt{5})(2) \cdot \dfrac{1}{2\sqrt{5}}$ **A1**

$= 5 + 4 - \dfrac{4\sqrt{5}}{2\sqrt{5}} = 9 - 2 = 7$

$a = \sqrt{7}$ cm **A1 AG**

Exact area: $\sin A = \sqrt{1 - \cos^2 A} = \sqrt{1 - \dfrac{1}{20}} = \sqrt{\dfrac{19}{20}} = \dfrac{\sqrt{95}}{10}$ (since $A \in (0°, 180°)$) **M1**

Area $= \dfrac{1}{2}bc\sin A = \dfrac{1}{2} \cdot \sqrt{5} \cdot 2 \cdot \dfrac{\sqrt{95}}{10} = \sqrt{5} \cdot \dfrac{\sqrt{95}}{10} = \dfrac{\sqrt{475}}{10} = \dfrac{5\sqrt{19}}{10} = \dfrac{\sqrt{19}}{2}$ cm^2 **A1**

Note: Accept $\dfrac{\sqrt{19}}{2}$ *cm*^{2}*, i.e.* $p = 1$, $q = 19$*. AG carries no mark.*

Q30 — Mast Elevation and Cosine Rule in 3D HARD

Express AT and BT in terms of h:

$AT^2 = h^2 + 40^2$ (since $\tan\alpha = h/40$, so $OA = 40$) **M1**

$BT^2 = h^2 + 55^2$ (since $OB = 55$)

$AT = \sqrt{h^2 + 1600}$, $BT = \sqrt{h^2 + 3025}$ **(A1)**

Find AB: since A is due north and B is due east of O, $\angle AOB = 90°$:

$AB^2 = 40^2 + 55^2 = 1600 + 3025 = 4625$, $AB = \sqrt{4625} = 5\sqrt{185}$ m **A1**

Apply the cosine rule in $\triangle ATB$ with $\angle ATB = 50°$: **M1**

$AB^2 = AT^2 + BT^2 - 2(AT)(BT)\cos 50°$

$4625 = (h^2 + 1600) + (h^2 + 3025) - 2\sqrt{(h^2 + 1600)(h^2 + 3025)}\cos 50°$

$0 = 2h^2 - 2\cos 50°\sqrt{(h^2 + 1600)(h^2 + 3025)}$

$h^2 = \cos 50°\sqrt{(h^2 + 1600)(h^2 + 3025)}$

GDC numerical solve: $h = 63.95\ldots \approx 64.0$ m (3 s.f.) **A1**

Note: Award M1 for setting up the cosine rule in $\triangle ATB$ *with correct expressions for all three sides in terms of* h*. Award final A1 for* $h = 64.0$ *m (3 s.f.). FT from their* AB *provided the equation is dimensionally consistent.*