IB Math AA SL — Topic 4
Probability

Events & Laws · Independence · Conditional Probability · Venn & Tree Diagrams

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:_____ Date:_____

Score:_____/120

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Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

Key Formulae

- Complement: $P(A') = 1 - P(A)$
- Addition rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Mutually exclusive events: $P(A \cap B) = 0 \Rightarrow P(A \cup B) = P(A) + P(B)$
- Independent events: $P(A \cap B) = P(A)P(B)$
- Conditional probability: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$
- De Morgan's laws: $P(A' \cap B') = 1 - P(A \cup B); \quad P(A' \cup B') = 1 - P(A \cap B)$
- Total probability (two-stage tree): $P(A) = P(A \mid B)P(B) + P(A \mid B')P(B')$

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GDC Tips

- **Storing fractions:** Enter exact fractions (e.g. $3/7$) and use Frac (MATH $\rightarrow$ 1) to keep results exact; avoid premature rounding in multi-step chain calculations.
- **Solving for an unknown probability:** Use solve(or the Equation app to find k in algebraic-branch tree problems after forming the equation (e.g. sum of probabilities $= 1$).
- **Checking independence:** Store $P(A)$, $P(B)$, and $P(A \cap B)$ as variables, then evaluate $P(A) \times P(B)$ and compare with $P(A \cap B)$ directly on the home screen.
- **Two-way tables:** Enter counts in a Matrix and use row/column sums to read off marginal and joint probabilities; this avoids arithmetic slips in conditional probability questions.
- **Verifying quadratic roots:** After solving a quadratic for k with solve(, substitute both roots back into the context (e.g. check $0 \leq k \leq 1$) on the home screen before rejecting one root.

Common Mistakes to Avoid

1. **Confusing mutually exclusive with independence.** Mutually exclusive means $P(A \cap B) = 0$; independent means $P(A \cap B) = P(A)P(B)$. Two events with positive probabilities cannot be both mutually exclusive and independent simultaneously — never equate the two conditions.
2. **Reversing conditional probability.** $P(A \mid B) \neq P(B \mid A)$ in general. Always identify which event is the condition (the denominator) and which is the outcome before substituting into $P(A \mid B) = P(A \cap B)/P(B)$.
3. **Forgetting to change denominators in without-replacement problems.** After one item is removed the sample space shrinks by one. Both the numerator and the denominator of the second-stage branch probability must reflect the updated counts; leaving the denominator unchanged is the single most common tree-diagram error.
4. **Failing to reject an invalid root in algebraic-branch trees.** When solving a quadratic for k, both roots must be tested: a probability must satisfy $0 \leq k \leq 1$ (and all branch probabilities derived from k must also lie in $[0, 1]$). State the reason for rejection explicitly — the mark is reserved for the reason, not just the answer.
5. **Misapplying De Morgan's law.** $P(A' \cap B') = 1 - P(A \cup B)$ is correct; students often write $1 - P(A) - P(B)$ instead, omitting the $+P(A \cap B)$ term from the addition rule. Show every De Morgan and addition-rule step — do not assert the final value.

6. **Using the wrong region in a Venn diagram after solving for x.** Once x is found from the total, substitute back to check that every region is non-negative (and, for count-based diagrams, an integer) before reading off probabilities. A negative or non-integer region value means an arithmetic error has occurred; continuing with it gives an impossible probability and loses all subsequent marks.

Section A — Easy EASY (10 questions × 3 marks = 30 marks)

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1. EASY Calculator **[3 marks]**

Let A and B be events with $P(A) = 0.5$, $P(B) = 0.4$, and $P(A \cap B) = 0.2$. Find $P(A \cup B)$.

2. EASY No Calculator **[3 marks]**

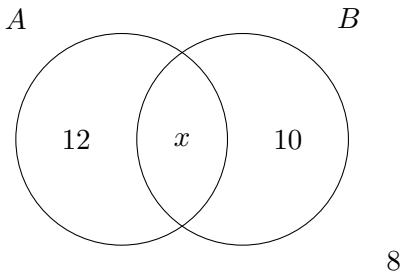
Let C and D be independent events with $P(C) = 0.3$ and $P(D) = 0.6$. Find $P(C \cap D)$.

3. EASY Calculator **[3 marks]**

A bag contains 4 red balls and 6 blue balls. One ball is chosen at random. Write down the probability that it is red.

4. **EASY** **Calculator** **[3 marks]**

The following Venn diagram shows two events A and B. The values shown represent the number of outcomes in each region. The total number of outcomes is 40.



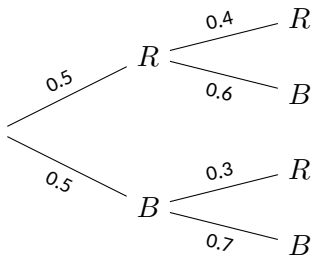
Find the value of x.

5. **EASY** **Calculator** **[3 marks]**

Let E and F be mutually exclusive events with $P(E) = 0.35$ and $P(F) = 0.45$. Find $P(E \cup F)$.

6. EASY Calculator **[3 marks]**

A two-stage tree diagram is shown below. The branches represent drawing coloured counters from a bag.



Find the probability that both counters drawn are red.

7. EASY No Calculator **[3 marks]**

Given that $P(A) = 0.6$, $P(B) = 0.5$, and $P(A \cap B) = 0.3$, find $P(A \mid B)$.

8. EASY Calculator **[3 marks]**

The table below shows the results of a survey of 50 students, classified by whether they study Mathematics (M) and whether they study Science (S).

	Studies S	Does not study S
Studies M	18	12
Does not study M	10	10

One student is chosen at random. Find the probability that the student studies Mathematics.

9. **EASY** **No Calculator** **[3 marks]**

Let A and B be events with $P(A) = 0.4$ and $P(B) = 0.5$. Given that A and B are independent, find $P(A' \cap B)$, where A' is the complement of A.

10. **EASY** **Calculator** **[3 marks]**

A spinner has three sectors labelled 1, 2, and 3. The probability of landing on each sector is given in the table below.

Sector	1	2	3
Probability	0.2	k	0.3

Find the value of k.

Section B — Medium MEDIUM (10 questions × 4 marks = 40 marks)

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11. MEDIUM No Calculator [4 marks]

A bag contains 5 red and 3 blue counters. Two counters are drawn at random without replacement. Find the probability that both counters are the same colour.

12. MEDIUM Calculator [4 marks]

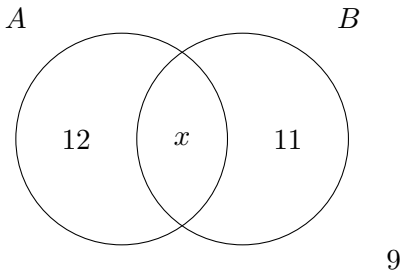
Events A and B are such that $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cup B) = 0.7$.

(a) Find $P(A \cap B)$.

(b) Determine whether A and B are independent. Justify your answer.

13. MEDIUM Calculator **[4 marks]**

The following Venn diagram shows two events A and B, where x represents the number of elements in $A \cap B$. The total number of elements in the sample space is 40.



(a) Find the value of x.

(b) Find $P(A' \cap B')$.

14. MEDIUM Calculator **[4 marks]**

In a survey of 80 students, each student studies at least one of Mathematics or Physics. 55 study Mathematics and 38 study Physics. A student is chosen at random.

(a) Find the probability that the student studies both subjects.

(b) Given that the student studies Physics, find the probability that the student also studies Mathematics.

15. MEDIUM Calculator [4 marks]

A factory produces items that are independently classified as defective with probability 0.08. Three items are chosen at random. Find the probability that

(a) exactly one item is defective,

(b) at least one item is defective.

16. MEDIUM No Calculator [4 marks]

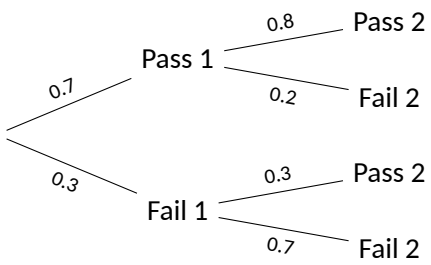
Events A and B are mutually exclusive. $P(A) = 3k$ and $P(B) = k + 0.2$, where $k \in \mathbb{R}$, $k > 0$.

(a) Given that $P(A \cup B) = 0.75$, find the value of k.

(b) Find $P(A')$.

17. MEDIUM Calculator [4 marks]

Amara takes a test that has two parts, Part 1 and Part 2. The probability that she passes Part 1 is 0.7. If she passes Part 1, the probability that she passes Part 2 is 0.8. If she fails Part 1, the probability that she passes Part 2 is 0.3.



(a) Find the probability that Amara passes Part 2.

(b) Given that Amara passes Part 2, find the probability that she passed Part 1.

18. MEDIUM No Calculator [4 marks]

On a game show, a contestant spins a wheel that lands on a prize with probability k and lands on no prize with probability $2k$, where $k \in \mathbb{R}$, $k > 0$. The wheel also lands on a bonus with probability 0.1. The three outcomes are mutually exclusive and exhaustive.

(a) Find the value of k.

(b) The contestant spins the wheel twice. Find the probability that they win a prize on the first spin and no prize on the second spin.

19. MEDIUM Calculator [4 marks]

A two-way table shows the results of a survey asking 60 people whether they prefer tea or coffee, classified by age group.

	Under 30	30 and over	Total
Tea	8	22	30
Coffee	17	13	30
Total	25	35	60

One person is chosen at random.

(a) Find the probability that the person prefers tea given that they are aged 30 and over.

(b) Determine whether age group and drink preference are independent. Justify your answer.

20. MEDIUM No Calculator **[4 marks]**

A box contains n white balls and 4 black balls, where $n \in \mathbb{Z}^+$. Two balls are drawn at random without replacement. The probability that both balls are white is $\frac{1}{3}$.

(a) Show that $n^2 - 5n - 6 = 0$ when $n > 1$. [2]

(b) Find the value of n. [2]

Section C — Hard HARD (10 questions × 5 marks = 50 marks)

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21. HARD No Calculator [5 marks]

Two events A and B are such that $P(A) = 0.55$, $P(B) = 0.40$, and $P(A \mid B) = 0.35$.

(a) Find $P(A \cap B)$. [1]

(b) Find $P(A \cup B)$. [1]

(c) Determine whether A and B are independent. Justify your answer. [2]

(d) Find $P(A' \cap B')$. [1]

22. HARD No Calculator [5 marks]

A bag contains n red discs and 4 blue discs, where $n \in \mathbb{Z}^+$. Two discs are drawn at random without replacement.

(a) Show that the probability both discs are the same colour is $\dfrac{n^2 - n + 12}{(n+4)(n+3)}$. [3]

(b) Given that this probability equals $\dfrac{4}{9}$, find the value of n. [2]

23. **HARD** **Calculator** **[5 marks]**

The two-way table below shows the number of students in a group of 80 who study French (F), Spanish (S), both, or neither.

	Studies Spanish	Does not study Spanish	Total
Studies French	x	18	
Does not study French	11	y	
Total	35		80

A student is chosen at random.

(a) Find the value of x and the value of y. [2]

(b) Find the probability that the student studies exactly one of French or Spanish. [1]

(c) Given that the student studies Spanish, find the probability they also study French. [1]

(d) Determine whether studying French and studying Spanish are independent events. [1]

24. **HARD** **No Calculator** **[5 marks]**

Events A and B are such that $P(A) = 2k$, $P(B) = k + 0.1$, and $P(A \cup B) = 0.85$, where $k \in \mathbb{R}$, $0 < k < 0.5$. It is given that A and B are independent.

Find the value of k, rejecting any value that is not valid in context.

25. HARD Calculator **[5 marks]**

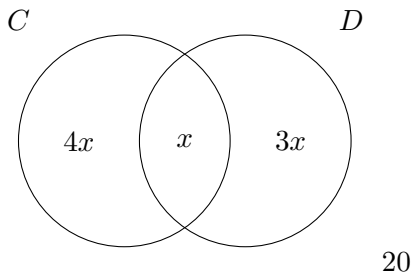
A factory produces items, each of which is independently classified as defective with probability 0.08. Items are tested one at a time. Let X be the number of items tested up to and including the first defective item found.

(a) Find the probability that the first defective item is the third item tested. [2]

(b) Find the probability that at least 4 items are tested before the first defective item is found. [3]

26. HARD Calculator **[5 marks]**

In a survey of 100 people, each person owns a cat (C), a dog (D), or both, or neither. The Venn diagram below shows partial information about the survey results.



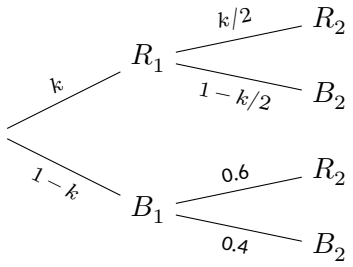
(a) Find the value of x. [2]

(b) A person is chosen at random. Find the probability they own a dog but not a cat. [1]

(c) Given that a person owns at least one pet, find the probability they own both a cat and a dog. [2]

27. **HARD** **No Calculator** **[5 marks]**

A game involves two spinners. Spinner 1 lands on red (R) with probability k and on blue with probability $1-k$. If Spinner 1 lands on red, Spinner 2 is spun and lands on red with probability $\frac{k}{2}$; if Spinner 1 lands on blue, Spinner 2 lands on red with probability 0.6. The probability that Spinner 2 lands on red is 0.45.



(a) Show that $k^2 - 1.2k + 0.3 = 0$. [2]

(b) Find the two possible values of k. [3]

28. **HARD** **Calculator** **[5 marks]**

A medical test for a disease gives a positive result with probability 0.95 when the disease is present and a positive result with probability 0.04 when the disease is absent. The proportion of the population with the disease is 0.002. A person is chosen at random and tested.

(a) Find the probability that the test gives a positive result. [2]

(b) Given that the test gives a positive result, find the probability that the person has the disease. [3]

29. HARD Calculator **[5 marks]**

A box contains 5 white balls and 3 black balls. Balls are drawn one at a time without replacement until a black ball is obtained or three balls have been drawn. Let W be the number of white balls drawn.

Find $E(W)$.

30. HARD No Calculator **[5 marks]**

Events A and B satisfy $P(A \cup B) = 0.80$, $P(A \cap B) = p$, and $P(A) = 3P(B)$, where $p \in \mathbb{R}$, $p \geq 0$.

(a) Find $P(A)$ and $P(B)$ in terms of p. [2]

(b) Hence find the range of possible values of p. [3]

Mark Schemes

Q1 — Addition rule for two events EASY

Attempt to use the addition rule **M1**

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.4 - 0.2$ **(A1)**

$P(A \cup B) = 0.7$ **A1**

Note: Award M1 for correct formula seen; first A1 for correct substitution; final A1 for correct answer.

Q2 — Multiplication rule for independent events EASY

Recognising that for independent events $P(C \cap D) = P(C) \times P(D)$ **M1**

$P(C \cap D) = 0.3 \times 0.6$ **(A1)**

$P(C \cap D) = \dfrac{9}{50}$ **A1**

Note: Accept 0.18 or equivalently 9/50. Award M1 for the multiplication rule stated or implied; final A1 for exact answer.

Q3 — Basic probability from equally likely outcomes EASY

Total number of balls $= 4 + 6 = 10$ **M1**

$P(\text{red}) = \dfrac{4}{10}$ **(A1)**

$P(\text{red}) = \dfrac{2}{5}$ **A1**

Note: Accept 0.4 or equivalently 2/5. Award M1 for attempt to find total; final A1 for simplified fraction or equivalent decimal.

Q4 — Venn diagram with unknown EASY

Forming an equation using the total: $12 + x + 10 + 8 = 40$ **M1**

$30 + x = 40$ **(A1)**

$x = 10$ **A1**

Note: Award M1 for an equation equating the sum of all regions to 40; final A1 for correct value.

Q5 — Mutually exclusive events EASY

Recognising that for mutually exclusive events $P(E \cup F) = P(E) + P(F)$ **M1**

$P(E \cup F) = 0.35 + 0.45$ **(A1)**

$P(E \cup F) = 0.8$ **A1**

Note: Award M1 for the correct addition of the two probabilities without subtracting an intersection.

Q6 — Two-stage tree probability EASY

Identifying the correct path: $P(R \text{ then } R) = 0.5 \times 0.4$ **M1**

$= 0.20$ **A1**

Note: Award M1 for multiplying the two correct branch probabilities along the RR path; A1 for correct answer. Accept $\frac{1}{5}$, i.e. 1/5.

Q7 — Conditional probability formula EASY

Attempt to use $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$ **M1**

$P(A \mid B) = \dfrac{0.3}{0.5}$ **(A1)**

$P(A \mid B) = \dfrac{3}{5}$ **A1**

Note: Accept 0.6 or equivalently 3/5. Award M1 for correct formula; first A1 for correct substitution; final A1 for simplified exact answer.

Q8 — Probability from a two-way table EASY

Total number of students who study Mathematics $= 18 + 12 = 30$ **M1**

$P(\text{studies } M) = \dfrac{30}{50}$ **(A1)**

$P(\text{studies } M) = \dfrac{3}{5}$ **A1**

Note: Accept 0.6 or equivalently 3/5. Award M1 for adding the correct row entries to find 30; final A1 for simplified fraction or equivalent decimal.

Q9 — Complement and independence EASY

$P(A') = 1 - 0.4 = 0.6$ **M1**

Since A and B are independent, A' and B are also independent, so $P(A' \cap B) = P(A') \times P(B)$ **(A1)**

$P(A' \cap B) = 0.6 \times 0.5 = \frac{3}{10}$ **A1**

Note: Accept 0.3 or equivalently 3/10. Award M1 for finding $P(A') = 0.6$; implied A1 for applying the independence condition; final A1 for correct exact answer.

Q10 — Finding an unknown probability from a table EASY

All probabilities sum to 1: $0.2 + k + 0.3 = 1$ **M1**

$0.5 + k = 1$ **(A1)**

$k = 0.5$ **A1**

Note: Award M1 for forming the equation with sum equal to 1; final A1 for correct value of k.

Q11 — Without-replacement same colour MEDIUM

Attempt to find probability for both red or both blue:

$P(\text{both red}) = \frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$ **M1**

$P(\text{both blue}) = \frac{3}{8} \times \frac{2}{7} = \frac{6}{56}$ **(A1)**

$P(\text{same colour}) = \frac{20}{56} + \frac{6}{56} = \frac{26}{56} = \frac{13}{28}$ **A1**

Note: Award M1 for showing changing denominators 8 then 7 in at least one pair. Accept equivalent fractions, e.g. 13/28.

Q12 — Independence test from union MEDIUM

(a)

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$ **M1**

$P(A \cap B) = 0.5 + 0.4 - 0.7 = 0.2$ **A1**

(b)

$P(A) \times P(B) = 0.5 \times 0.4 = 0.2 = P(A \cap B)$ **R1**

Therefore A and B are independent. **A1**

Note: Award R1 for explicitly computing $P(A) \times P(B)$ and comparing with $P(A \cap B)$. Do not award A1 without R1.

Q13 — Venn diagram with unknown x MEDIUM

(a)

$12 + x + 11 + 9 = 40$ **M1**

$x = 8$ **A1**

(b)

$P(A' \cap B') = \frac{9}{40}$ **A1**

Note: Award M1 for forming an equation using all four regions summing to 40. For part (b), accept 0.225 or 9/40.

Q14 — Conditional from survey inclusion-exclusion MEDIUM

(a)

$P(\text{both}) = P(M) + P(P) - 1 = \frac{55}{80} + \frac{38}{80} - 1$ **M1**

$= \frac{93}{80} - 1 = \frac{13}{80}$ **A1**

(b)

$P(M \mid P) = \frac{P(M \cap P)}{P(P)} = \frac{13/80}{38/80} = \frac{13}{38}$ **M1A1**

Note: Accept 0.342 (3 s.f.); exact value 13/38.

Q15 — Independent defective items MEDIUM

(a)

$P(\text{exactly one defective}) = \binom{3}{1}(0.08)^1(0.92)^2$ **M1**

$= 3 \times 0.08 \times 0.8464 = 0.203136 \approx 0.203$ **A1**

(b)

$P(\text{at least one}) = 1 - (0.92)^3$ **M1**

$= 1 - 0.778688 = 0.221312 \approx 0.221$ **A1**

Note: Accept answers to 3 significant figures.

Q16 — Mutually exclusive with algebraic probabilities MEDIUM

(a)

Since A and B are mutually exclusive: $P(A \cup B) = P(A) + P(B)$ **M1**

$3k + k + 0.2 = 0.75$

$4k = 0.55$

$k = 0.1375$ **A1**

(b)

$P(A) = 3 \times 0.1375 = 0.4125$

$P(A') = 1 - 0.4125 = 0.5875 = \frac{47}{80}$ **M1A1**

Note: Award M1 in (a) for using $P(A \cup B) = P(A) + P(B)$ explicitly. Award M1 in (b) for $1 - P(A)$. Accept 0.5875 or 47/80; verify: $3(0.1375) = 0.4125$, $P(A') = 1 - 0.4125 = 0.5875$.

Q17 — Conditional probability from tree diagram MEDIUM

(a)

$P(\text{Pass 2}) = P(\text{Pass 1}) \times P(\text{Pass 2} \mid \text{Pass 1}) + P(\text{Fail 1}) \times P(\text{Pass 2} \mid \text{Fail 1})$ **M1**

$= 0.7 \times 0.8 + 0.3 \times 0.3 = 0.56 + 0.09 = 0.65$ **A1**

(b)

$P(\text{Pass 1} \mid \text{Pass 2}) = \frac{0.7 \times 0.8}{0.65}$ **M1**

$= \frac{0.56}{0.65} = \frac{56}{65} \approx 0.862$ **A1**

Note: Award M1 in (b) for their $\frac{0.56}{P(\text{Pass 2})}$. Accept 0.862 (3 s.f.); exact value 56/65.

Q18 — Exhaustive wheel with algebraic probability MEDIUM

(a)

$k + 2k + 0.1 = 1$ **M1**

$3k = 0.9 \Rightarrow k = 0.3$ **A1**

(b)

$P(\text{prize on spin 1}) = k = 0.3, \quad P(\text{no prize on spin 2}) = 2k = 0.6$ **M1**

$P(\text{prize then no prize}) = 0.3 \times 0.6 = 0.18$ **A1**

Note: Award M1 in (b) for multiplying their k by their $2k$. Accept exact fraction $\frac{9}{50}$, i.e. 9/50.

Q19 — Independence test from two-way table MEDIUM

(a)

$P(\text{Tea} \mid 30 \text{ and over}) = \frac{22}{35}$, i.e. 22/35 **A2**

(b)

$P(\text{Tea}) = \dfrac{30}{60} = 0.5$ **M1**

$P(\text{Tea} \mid 30 \text{ and over}) = \dfrac{22}{35} \approx 0.629 \neq 0.5 = P(\text{Tea})$

Therefore age group and drink preference are not independent. **R1**

Note: For (a), award A2 for $\dfrac{22}{35}$ or 0.629 (3 s.f.). For (b), award M1 for comparing $P(\text{Tea})$ with their conditional probability, and R1 for a correct conclusion with explicit comparison.

Q20 — Without-replacement quadratic in n MEDIUM

(a)

$P(\text{both white}) = \dfrac{n}{n+4} \times \dfrac{n-1}{n+3}$ **M1**

Setting equal to $\dfrac{1}{3}$: $\dfrac{n(n-1)}{(n+4)(n+3)} = \dfrac{1}{3}$

$3n(n-1) = (n+4)(n+3)$
$3n^2 - 3n = n^2 + 7n + 12$
$2n^2 - 10n - 12 = 0$
$n^2 - 5n - 6 = 0$ **A1 AG**

(b)

$(n-6)(n+1) = 0 \Rightarrow n = 6$ or $n = -1$ **M1**

Since $n \in \mathbb{Z}^+$, reject $n = -1$, so $n = 6$. **A1**

Note: Award M1 for solving their quadratic from part (a). The mark for rejecting $n = -1$ with a reason is embedded in A1.

Q21 — Abstract event laws with De Morgan HARD

(a)

$P(A \cap B) = P(A \mid B) \times P(B) = 0.35 \times 0.40$ **M1**

$P(A \cap B) = 0.14$ **A1**

(b)

$P(A \cup B) = 0.55 + 0.40 - 0.14 = 0.81$ **A1**

(c)

For independence, require $P(A \cap B) = P(A) \times P(B)$

$P(A) \times P(B) = 0.55 \times 0.40 = 0.22 \neq 0.14$ **R1**

A and B are not independent. **A1**

(d)

$P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - 0.81 = 0.19$ **A1**

Note: De Morgan step must be seen (or equivalent) for full credit in (d). Accept $1 - 0.81$.

Q22 — Without-replacement chain with AG target and root rejection HARD

(a)

$P(\text{same colour}) = P(\text{both red}) + P(\text{both blue})$ **M1**

$= \dfrac{n}{n+4} \cdot \dfrac{n-1}{n+3} + \dfrac{4}{n+4} \cdot \dfrac{3}{n+3}$ **A1**

$= \dfrac{n(n-1)+12}{(n+4)(n+3)} = \dfrac{n^2-n+12}{(n+4)(n+3)}$ **A1 AG**

(b)

Setting $\dfrac{n^2-n+12}{(n+4)(n+3)} = \dfrac{4}{9}$: **M1**

$9(n^2-n+12) = 4(n+4)(n+3)$
$9n^2-9n+108 = 4n^2+28n+48$
$5n^2-37n+60 = 0$
$(n-5)(5n-12) = 0 \Rightarrow n = 5$ or $n = \dfrac{12}{5}$

Reject $n = \dfrac{12}{5}$ since $n \in \mathbb{Z}^+$; therefore $n = 5$. **A1**

Note: Award A1 only if rejection of the non-integer root is explicitly stated.

Q23 — Two-way table with conditional probability and independence HARD

(a)

Total studying Spanish: $x + 11 = 35 \Rightarrow x = 24$ **A1**

Total: $24 + 18 + 11 + y = 80 \Rightarrow y = 27$ **A1**

(b)

Students studying exactly one language $= 18 + 11 = 29$

$P(\text{exactly one}) = \dfrac{29}{80}$, i.e. 29/80 **A1**

(c)

$P(F \mid S) = \dfrac{24}{35}$, i.e. 24/35 **A1**

(d)

$P(F) = \dfrac{42}{80} = \dfrac{21}{40}$; $\quad P(F) \times P(S) = \dfrac{42}{80} \times \dfrac{35}{80} = \dfrac{1470}{6400} \approx 0.2297$

$P(F \cap S) = \dfrac{24}{80} = 0.30 \neq 0.2297$, so F and S are not independent. **R1**

Note: Accept equivalent numerical comparison; reason must be stated.

Q24 — Independence gives quadratic with root rejection HARD

Since A and B are independent, $P(A \cap B) = P(A) \cdot P(B) = 2k(k+0.1)$ **M1**

Using addition rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.85 = 2k + (k+0.1) - 2k(k+0.1)$ **M1**

$0.85 = 3k + 0.1 - 2k^2 - 0.2k$

$0.85 = 2.8k + 0.1 - 2k^2$

$2k^2 - 2.8k + 0.75 = 0$ **A1**

$$k = \frac{2.8 \pm \sqrt{7.84 - 6}}{4} = \frac{2.8 \pm \sqrt{1.84}}{4}$$

$k \approx 1.038$ or $k \approx 0.361$ **(A1)**

Reject $k \approx 1.038$ since $P(A) = 2k$ must satisfy $0 < 2k < 1$, requiring $k < 0.5$; therefore $k \approx 0.361$. **A1**

Note: The reserved A1 requires explicit rejection with reason (probability constraint). Accept exact surd form $k = \frac{2.8 - \sqrt{1.84}}{4}$.

Q25 — Geometric-type probability chain HARD

(a)

$P(X = 3) = (1 - 0.08)^2 \times 0.08$ **M1**

$= (0.92)^2 \times 0.08 = 0.067712 \approx 0.0677$ **A1**

(b)

"At least 4 items tested before first defective" means the first 3 items are all non-defective. **M1**

$P(X > 3) = (0.92)^3$ **(M1)**

$= 0.778688 \approx 0.779$ **A1**

Q26 — Venn diagram with algebraic unknown and conditional probability HARD

(a)

Total: $4x + x + 3x + 20 = 100$ **M1**

$8x = 80 \Rightarrow x = 10$ **A1**

(b)

$P(\text{dog but not cat}) = \frac{3x}{100} = \frac{30}{100} = \frac{3}{10}$ **A1**

(c)

$P(\text{at least one pet}) = \frac{4x + x + 3x}{100} = \frac{8x}{100} = \frac{80}{100} = \frac{4}{5}$ **(M1)**

$P(\text{both} \mid \text{at least one}) = \frac{x/100}{4/5} = \frac{10/100}{4/5} = \frac{1}{8}$ **A1**

Note: Award (M1) for correct conditional probability structure using their x.

Q27 — Algebraic tree with quadratic and root rejection HARD

(a)

$P(R_2) = k \cdot \frac{k}{2} + (1-k) \cdot 0.6 = 0.45$ **M1**

$\frac{k^2}{2} + 0.6 - 0.6k = 0.45$

$\frac{k^2}{2} - 0.6k + 0.15 = 0$

Multiply by 2: $k^2 - 1.2k + 0.3 = 0$ **A1 AG**

(b)

Solving $k^2 - 1.2k + 0.3 = 0$: **M1**

$k = \frac{1.2 \pm \sqrt{1.44 - 1.2}}{2} = \frac{1.2 \pm \sqrt{0.24}}{2}$ **(A1)**

$k \approx 0.845$ or $k \approx 0.355$ **A1**

Note: Both roots satisfy $0 < k < 1$ and $0 < k/2 < 1$, so both are valid in context; accept both values.

Q28 — Bayes' theorem for a medical test HARD

(a)

Let D = has disease, $+$ = positive test.

$P(+) = P(+ \mid D)P(D) + P(+ \mid D')P(D')$ **M1**

$= 0.95 \times 0.002 + 0.04 \times 0.998$

$= 0.00190 + 0.03992 = 0.04182$ **A1**

(b)

$P(D \mid +) = \frac{P(+ \mid D)P(D)}{P(+)}$ **M1**

$= \frac{0.95 \times 0.002}{0.04182}$ **(M1)**

$= \frac{0.00190}{0.04182} = 0.045434\ldots \approx 0.0454$ **A1**

Note: Accept full precision answer 0.04543...; final answer must be given to 3 s.f. as 0.0454.

Q29 — Expected value from an unstructured without-replacement process HARD

The possible values of W are $0, 1, 2, 3$ (drawing stops when a black ball is drawn or 3 balls have been drawn).

$P(W=0)=\frac{3}{8}$ **(A1)**

$P(W=1)=\frac{5}{8}\times\frac{3}{7}=\frac{15}{56}$ **M1**

After two white draws (5/8 × 4/7), the third ball is drawn regardless of colour — this contributes to both $W=2$ (third ball black) and $W=3$ (third ball white):

$P(W=2)=\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}=\frac{60}{336}=\frac{5}{28}$ **A1**

$P(W=3)=\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}=\frac{60}{336}=\frac{5}{28}$ **A1**

Note: $W=3$ (all three draws white) is easily missed — drawing stops after the third ball regardless of its colour, so WWW is a valid outcome with $W=3$, not $W=2$.

Check: $\frac{3}{8}+\frac{15}{56}+\frac{5}{28}+\frac{5}{28}=\frac{21}{56}+\frac{15}{56}+\frac{10}{56}+\frac{10}{56}=\frac{56}{56}=1$ ✓

$E(W)=0\times\frac{3}{8}+1\times\frac{15}{56}+2\times\frac{5}{28}+3\times\frac{5}{28}$

$=\frac{15}{56}+\frac{20}{56}+\frac{30}{56}=\frac{65}{56}\approx 1.16$ **A1**

Note: Without-replacement denominators must be shown changing for M1. Full distribution must sum to 1 for full credit. Final A1 for correct $E(W)=65/56\approx 1.16$ (3 s.f.) using the complete distribution.

Q30 — Abstract laws and the range of parameter p HARD

(a)

Let $P(B)=q$; then $P(A)=3q$.

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$0.80=3q+q-p\Rightarrow 4q=0.80+p\Rightarrow q=\frac{0.80+p}{4}$ **M1**

$P(B)=\frac{p+0.8}{4}=(p+0.8)/4,\quad P(A)=\frac{3(p+0.8)}{4}=\frac{3p+2.4}{4}=(3p+2.4)/4$ **A1**

(b)

Require all probabilities in $[0,1]$:

$P(A)\le 1$: $\frac{3p+2.4}{4}\le 1\Rightarrow 3p\le 1.6\Rightarrow p\le\frac{8}{15}$ **M1**

$P(A\cap B)\le P(B)$: $p\le\frac{p+0.8}{4}\Rightarrow 4p\le p+0.8\Rightarrow p\le\frac{4}{15}$

$P(A\cap B)\le P(A)$: $p\le\frac{3p+2.4}{4}\Rightarrow 4p\le 3p+2.4\Rightarrow p\le 2.4$ (less restrictive) **A1**

Combined with $p\ge 0$: $0\le p\le\frac{4}{15}$, i.e. $p\in[0,4/15]$ **A1**

Note: Award M1 for setting up at least two valid probability constraints. Award second A1 for identifying

$p \leq \frac{4}{15}$ *as the binding upper bound. Final A1 for correct closed interval with both bounds stated.*