IB Math AA SL — Topic 4

Probability

Events & Laws · Independence · Conditional Probability · Venn & Tree Diagrams

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

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Name:________________________ Date:______________

Score:__________/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

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Key Formulae

- **Complement:** $P(A') = 1 - P(A)$
- **Addition rule:** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- **Mutually exclusive events:** $P(A \cap B) = 0 \;\Rightarrow\; P(A \cup B) = P(A) + P(B)$
- **Independent events:** $P(A \cap B) = P(A)\,P(B)$
- **Conditional probability:** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$
- **De Morgan's laws:** $P(A' \cap B') = 1 - P(A \cup B); \quad P(A' \cup B') = 1 - P(A \cap B)$
- **Total probability (two-stage tree):** $P(A) = P(A \mid B)\,P(B) + P(A \mid B')\,P(B')$

GDC Tips

- **Storing fractions:** Enter exact fractions (e.g. $3/7$) and use `Frac` (`MATH` $\to$ `1`) to keep results exact; avoid premature rounding in multi-step chain calculations.
- **Solving for an unknown probability:** Use `solve(` or the `Equation` app to find k in algebraic-branch tree problems after forming the equation (e.g. sum of probabilities $= 1$).
- **Checking independence:** Store $P(A)$, $P(B)$, and $P(A \cap B)$ as variables, then evaluate $P(A) \times P(B)$ and compare with $P(A \cap B)$ directly on the home screen.
- **Two-way tables:** Enter counts in a `Matrix` and use row/column sums to read off marginal and joint probabilities; this avoids arithmetic slips in conditional probability questions.
- **Verifying quadratic roots:** After solving a quadratic for k with `solve(`, substitute both roots back into the context (e.g. check $0 \leq k \leq 1$) on the home screen before rejecting one root.

Common Mistakes to Avoid

1. **Confusing mutually exclusive with independence.** Mutually exclusive means $P(A \cap B) = 0$; independent means $P(A \cap B) = P(A)P(B)$. Two events with positive probabilities *cannot* be both mutually exclusive and independent simultaneously — never equate the two conditions.
2. **Reversing conditional probability.** $P(A \mid B) \neq P(B \mid A)$ in general. Always identify which event is the *condition* (the denominator) and which is the *outcome* before substituting into $P(A \mid B) = P(A \cap B)/P(B)$.
3. **Forgetting to change denominators in without-replacement problems.** After one item is removed the sample space shrinks by one. Both the numerator and the denominator of the second-stage branch probability must reflect the updated counts; leaving the denominator unchanged is the single most common tree-diagram error.
4. **Failing to reject an invalid root in algebraic-branch trees.** When solving a quadratic for k, *both* roots must be tested: a probability must satisfy $0 \leq k \leq 1$ (and all branch probabilities derived from k must also lie in $[0, 1]$). State the reason for rejection explicitly — the mark is reserved for the reason, not just the answer.
5. **Misapplying De Morgan's law.** $P(A' \cap B') = 1 - P(A \cup B)$ is correct; students often write $1 - P(A) - P(B)$ instead, omitting the $+P(A \cap B)$ term from the addition rule. Show every De Morgan and addition-rule step — do not assert the final value.
6. **Using the wrong region in a Venn diagram after solving for x.** Once x is found from the total, substitute back to check that *every* region is non-negative before reading off probabilities. A negative region value means an arithmetic error has occurred; continuing with it gives an impossible probability and loses all subsequent marks.

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Section A — Easy

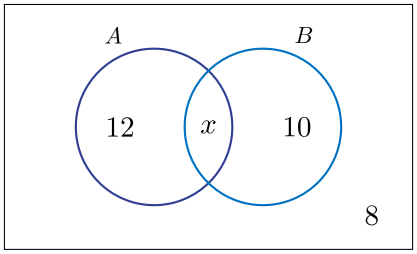
EASY

1. Let A and B be events with $P(A) = 0.5$, $P(B) = 0.4$, and $P(A \cap B) = 0.2$. Find $P(A \cup B)$. *Calculator allowed* **[3 marks]**

2. Let C and D be independent events with $P(C) = 0.3$ and $P(D) = 0.6$. Find $P(C \cap D)$. *Calculator not allowed* **[3 marks]**

3. A bag contains 4 red balls and 6 blue balls. One ball is chosen at random. Write down the probability that it is red. *Calculator allowed* **[3 marks]**

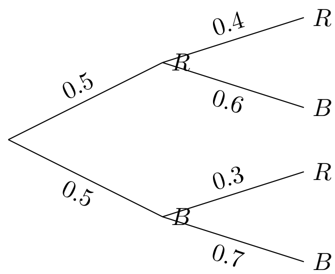
4. The following Venn diagram shows two events A and B. The values shown represent the number of outcomes in each region. The total number of outcomes is 40.



Find the value of x. *Calculator allowed* **[3 marks]**

5. Let E and F be mutually exclusive events with $P(E) = 0.35$ and $P(F) = 0.45$. Find $P(E \cup F)$. *Calculator allowed* **[3 marks]**

6. A two-stage tree diagram is shown below. The branches represent drawing coloured counters from a bag.



Find the probability that both counters drawn are red. *Calculator allowed* **[3 marks]**

7. Given that $P(A) = 0.6$, $P(B) = 0.5$, and $P(A \cap B) = 0.3$, find $P(A \mid B)$. *Calculator not allowed* **[3 marks]**

8. The table below shows the results of a survey of 50 students, classified by whether they study Mathematics (M) and whether they study Science (S).

	Studies S	Does not study S
Studies M	18	12
Does not study M	10	10

One student is chosen at random. Find the probability that the student studies Mathematics. *Calculator allowed* **[3 marks]**

9. Let A and B be events with $P(A) = 0.4$ and $P(B) = 0.5$. Given that A and B are independent, find $P(A' \cap B)$, where A' is the complement of A. *Calculator not allowed* **[3 marks]**

10. A spinner has three sectors labelled 1, 2, and 3. The probability of landing on each sector is given in the table below.

Sector	1	2	3
Probability	0.2	k	0.3

Find the value of k. *Calculator allowed* **[3 marks]**

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Section B — Medium

MEDIUM

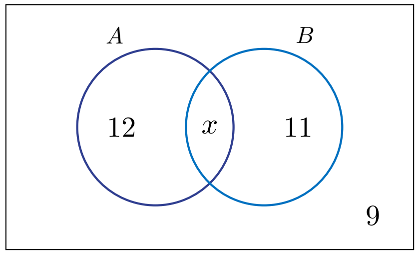
11. A bag contains 5 red and 3 blue counters. Two counters are drawn at random without replacement. Find the probability that both counters are the same colour. *Calculator not allowed* **[4 marks]**

12. Events A and B are such that $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cup B) = 0.7$. *Calculator allowed* **[4 marks]**

(a) Find $P(A \cap B)$.

(b) Determine whether A and B are independent. Justify your answer.

13. The following Venn diagram shows two events A and B, where x represents the number of elements in $A \cap B$. The total number of elements in the sample space is 40. *Calculator allowed* **[4 marks]**



(a) Find the value of x.

(b) Find $P(A' \cap B')$.

14. In a survey of 80 students, each student studies at least one of Mathematics or Physics. 55 study Mathematics and 38 study Physics. A student is chosen at random. *Calculator allowed* **[4 marks]**

(a) Find the probability that the student studies both subjects.

(b) Given that the student studies Physics, find the probability that the student also studies Mathematics.

15. A factory produces items that are independently classified as defective with probability 0.08. Three items are chosen at random. Find the probability that *Calculator allowed* **[4 marks]**

(a) exactly one item is defective,

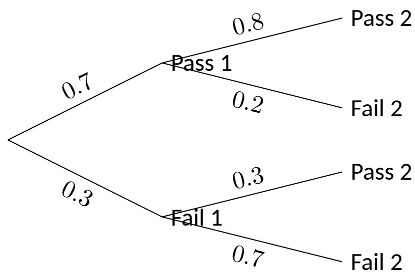
(b) at least one item is defective.

16. Events A and B are mutually exclusive. $P(A) = 3k$ and $P(B) = k + 0.2$, where $k \in \mathbb{R}$, $k > 0$. *Calculator not allowed* **[4 marks]**

(a) Given that $P(A \cup B) = 0.75$, find the value of k.

(b) Find $P(A')$.

17. Amara takes a test that has two parts, Part 1 and Part 2. The probability that she passes Part 1 is 0.7. If she passes Part 1, the probability that she passes Part 2 is 0.8. If she fails Part 1, the probability that she passes Part 2 is 0.3. *Calculator allowed* **[4 marks]**



(a) Find the probability that Amara passes Part 2.

(b) Given that Amara passes Part 2, find the probability that she passed Part 1.

18. On a game show, a contestant spins a wheel that lands on a prize with probability k and lands on no prize with probability $2k$, where $k \in \mathbb{R}$, $k > 0$. The wheel also lands on a bonus with probability 0.1. The three outcomes are mutually exclusive and exhaustive. *Calculator not allowed* **[4 marks]**

(a) Find the value of k.

(b) The contestant spins the wheel twice. Find the probability that they win a prize on the first spin and no prize on the second spin.

19. A two-way table shows the results of a survey asking 60 people whether they prefer tea or coffee, classified by age group. *Calculator allowed* **[4 marks]**

	Under 30	30 and over	Total
Tea	8	22	30
Coffee	17	13	30
Total	25	35	60

One person is chosen at random.

(a) Find the probability that the person prefers tea given that they are aged 30 and over.

(b) Determine whether age group and drink preference are independent. Justify your answer.

20. A box contains n white balls and 4 black balls, where $n \in \mathbb{Z}^+$. Two balls are drawn at random without replacement. The probability that both balls are white is $\frac{2}{5}$. *Calculator not allowed* **[4 marks]**

(a) Show that $n^2 - 9n - 10 = 0$ when $n > 1$. **[2 marks]**

(b) Find the value of n.

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Section C — Hard

HARD

21. Two events A and B are such that $P(A) = 0.55$, $P(B) = 0.40$, and $P(A \mid B) = 0.35$. *Calculator not allowed* **[5 marks]**

(a) Find $P(A \cap B)$. [1]

(b) Find $P(A \cup B)$. [1]

(c) Determine whether A and B are independent. Justify your answer. [2]

(d) Find $P(A' \cap B')$. [1]

22. A bag contains n red discs and 4 blue discs, where $n \in \mathbb{Z}^+$. Two discs are drawn at random without replacement. *Calculator not allowed* **[5 marks]**

(a) Show that the probability both discs are the same colour is $\dfrac{n^2+n+12}{(n+4)(n+3)}$. [3]

(b) Given that this probability equals $\dfrac{11}{14}$, find the value of n. [2]

23. The two-way table below shows the number of students in a group of 80 who study French (F), Spanish (S), both, or neither. *Calculator allowed* **[5 marks]**

	Studies Spanish	Does not study Spanish	Total
Studies French	x	18	
Does not study French	11	y	
Total	35		80

A student is chosen at random.

(a) Find the value of x and the value of y. [2]

(b) Find the probability that the student studies exactly one of French or Spanish. [1]

(c) Given that the student studies Spanish, find the probability they also study French. [1]

(d) Determine whether studying French and studying Spanish are independent events. [1]

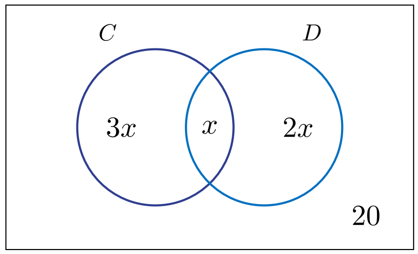
24. Events A and B are such that $P(A) = 2k$, $P(B) = k+0.1$, and $P(A\cup B) = 0.85$, where $k \in \mathbb{R}, 0 < k < 0.5$. It is given that A and B are independent. *Calculator not allowed* **[5 marks]**
Find the value of k, rejecting any value that is not valid in context.

25. A factory produces items, each of which is independently classified as defective with probability 0.08. Items are tested one at a time. Let X be the number of items tested up to and including the first defective item found.
Calculator allowed **[5 marks]**

(a) Find the probability that the first defective item is the third item tested. [2]

(b) Find the probability that at least 4 items are tested before the first defective item is found. [3]

26. In a survey of 100 people, each person owns a cat (C), a dog (D), or both, or neither. The Venn diagram below shows partial information about the survey results. *Calculator allowed* **[5 marks]**

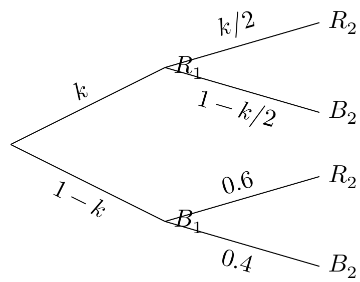


(a) Find the value of x. [2]

(b) A person is chosen at random. Find the probability they own a dog but not a cat. [1]

(c) Given that a person owns at least one pet, find the probability they own both a cat and a dog. [2]

27. A game involves two spinners. Spinner 1 lands on red (R) with probability k and on blue with probability $1-k$. If Spinner 1 lands on red, Spinner 2 is spun and lands on red with probability $\frac{k}{2}$; if Spinner 1 lands on blue, Spinner 2 lands on red with probability 0.6. The probability that Spinner 2 lands on red is 0.45. *Calculator not allowed* **[5 marks]**



(a) Show that $k^2 - 2k + 0.9 = 0$. [2]

(b) Find the two possible values of k and reject any value that is not valid in context. [3]

28. A medical test for a disease gives a positive result with probability 0.95 when the disease is present and a positive result with probability 0.04 when the disease is absent. The proportion of the population with the disease is 0.002. *Calculator allowed* **[5 marks]**
A person is chosen at random and tested.

(a) Find the probability that the test gives a positive result. [2]

(b) Given that the test gives a positive result, find the probability that the person has the disease. [3]

29. A box contains 5 white balls and 3 black balls. Balls are drawn one at a time without replacement until a black ball is obtained or three balls have been drawn. Let W be the number of white balls drawn. *Calculator allowed* **[5 marks]**

Find E(W).

30. Events A and B satisfy $P(A \cup B) = 0.80$, $P(A \cap B) = p$, and $P(A) = 3P(B)$, where $p \in \mathbb{R}$, $p \geq 0$. *Calculator not allowed* **[5 marks]**

(a) Find $P(A)$ and $P(B)$ in terms of p. [2]

(b) Hence find the range of possible values of p. [3]

Worked Solutions

Q1 — Addition rule for two events [EASY]

Attempt to use the addition rule **M1**

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.4 - 0.2$ **(A1)**

$P(A \cup B) = 0.7$ **A1**

Note: Award M1 for correct formula seen; first A1 for correct substitution; final A1 for correct answer.

Q2 — Multiplication rule for independent events [EASY]

Recognising that for independent events $P(C \cap D) = P(C) \times P(D)$ **M1**

$P(C \cap D) = 0.3 \times 0.6$ **(A1)**

$P(C \cap D) = \frac{9}{50}$ **A1**

Note: Accept 0.18. *Award M1 for the multiplication rule stated or implied; final A1 for exact answer.*

Q3 — Basic probability from equally likely outcomes [EASY]

Total number of balls $= 4 + 6 = 10$ **M1**

$P(\text{red}) = \frac{4}{10}$ **(A1)**

$P(\text{red}) = \frac{2}{5}$ **A1**

Note: Accept 0.4. *Award M1 for attempt to find total; final A1 for simplified fraction or equivalent decimal.*

Q4 — Venn diagram with unknown [EASY]

Forming an equation using the total: $12 + x + 10 + 8 = 40$ **M1**

$30 + x = 40$ **(A1)**

$x = 10$ **A1**

Note: Award M1 for an equation equating the sum of all regions to 40; final A1 for correct value.

Q5 — Mutually exclusive events [EASY]

Recognising that for mutually exclusive events $P(E \cup F) = P(E) + P(F)$ **M1**

$P(E \cup F) = 0.35 + 0.45$ **(A1)**

$P(E \cup F) = 0.8$ **A1**

Note: Award M1 for the correct addition of the two probabilities without subtracting an intersection.

Q6 — Two-stage tree probability [EASY]

Identifying the correct path: $P(R \text{ then } R) = 0.5 \times 0.4$ **M1**

$= 0.20$ **A1**

Note: Award M1 for multiplying the two correct branch probabilities along the RR path; A1 for correct answer. Accept $\frac{1}{5}$.

[3 marks]

Note: The third mark is awarded as the second A1 for the correct numerical answer written explicitly.

Clarification of mark allocation: Identifying path and writing 0.5×0.4 **M1**; evaluating product $= 0.20$ **A1**; correct answer stated **A1**.

Q7 — Conditional probability formula [EASY]

Attempt to use $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$ **M1**

$P(A \mid B) = \dfrac{0.3}{0.5}$ **(A1)**

$P(A \mid B) = \dfrac{3}{5}$ **A1**

Note: Accept 0.6. *Award M1 for correct formula; first A1 for correct substitution; final A1 for simplified exact answer.*

Q8 — Probability from a two-way table [EASY]

Total number of students who study Mathematics $= 18 + 12 = 30$ **M1**

$P(\text{studies } M) = \dfrac{30}{50}$ **(A1)**

$P(\text{studies } M) = \dfrac{3}{5}$ **A1**

Note: Accept 0.6. *Award M1 for adding the correct row entries to find 30; final A1 for simplified fraction or equivalent decimal.*

Q9 — Complement and independence [EASY]

$P(A') = 1 - 0.4 = 0.6$ **M1**

Since A and B are independent, A' and B are also independent, so $P(A' \cap B) = P(A') \times P(B)$ **(A1)**

$P(A' \cap B) = 0.6 \times 0.5 = \dfrac{3}{10}$ **A1**

Note: Accept 0.3. Award M1 for finding $P(A') = 0.6$; implied A1 for applying the independence condition; final A1 for correct exact answer.

Q10 — Finding an unknown probability from a table [EASY]

All probabilities sum to 1: $0.2 + k + 0.3 = 1$ **M1**

$0.5 + k = 1$ **(A1)**

$k = 0.5$ **A1**

Note: Award M1 for forming the equation with sum equal to 1; final A1 for correct value of k.

Q11 — Without-replacement same colour [MEDIUM]

Attempt to find probability for both red or both blue:

$P(\text{both red}) = \dfrac{5}{8} \times \dfrac{4}{7} = \dfrac{20}{56}$ **M1**

$P(\text{both blue}) = \dfrac{3}{8} \times \dfrac{2}{7} = \dfrac{6}{56}$ **(A1)**

$P(\text{same colour}) = \dfrac{20}{56} + \dfrac{6}{56} = \dfrac{26}{56} = \dfrac{13}{28}$ **A1**

Note: Award M1 for showing changing denominators 8 then 7 in at least one pair. Accept equivalent fractions. Award A2 for $\dfrac{13}{28}$ seen with no working shown.

Q12 — Independence test from union [MEDIUM]

(a)

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$ **M1**

$P(A \cap B) = 0.5 + 0.4 - 0.7 = 0.2$ **A1**

(b)

$P(A) \times P(B) = 0.5 \times 0.4 = 0.2 = P(A \cap B)$ **R1**

Therefore A and B are independent. **A1**

Note: Award R1 for explicitly computing $P(A) \times P(B)$ and comparing with $P(A \cap B)$. Do not award A1 without R1. Award A1ft on their $P(A \cap B)$ from part (a).

Q13 — Venn diagram with unknown x [MEDIUM]

(a)

$12 + x + 11 + 9 = 40$ **M1**

$x = 8$ **A1**

(b)

$P(A' \cap B') = \dfrac{9}{40}$ **A1**

Note: Award M1 for forming an equation using all four regions summing to 40. A1ft on their value of x provided $0 < x < 40$. For part (b), accept 0.225. The region $A' \cap B'$ is the exterior value 9; award A1 only if x has been found correctly or followed through correctly.

Q14 — Conditional from survey inclusion-exclusion [MEDIUM]

(a)

$P(\text{both}) = P(M) + P(P) - 1 = \dfrac{55}{80} + \dfrac{38}{80} - 1$ **M1**

$= \dfrac{93}{80} - 1 = \dfrac{13}{80}$ **A1**

(b)

$P(M \mid P) = \dfrac{P(M \cap P)}{P(P)} = \dfrac{13/80}{38/80} = \dfrac{13}{38}$ **M1A1**

Note: Award M1 for a correct conditional probability formula with their numerator and $\frac{38}{80}$ as denominator. Accept 0.342 (3 s.f.). A1ft on their $P(M \cap P)$ provided it is a valid probability.

Q15 — Independent defective items [MEDIUM]

(a)

$P(\text{exactly one defective}) = \binom{3}{1}(0.08)^1(0.92)^2$ **M1**

$= 3 \times 0.08 \times 0.8464 = 0.203136 \approx 0.203$ **A1**

(b)

$P(\text{at least one}) = 1 - (0.92)^3$ **M1**

$= 1 - 0.778688 = 0.221312 \approx 0.221$ **A1**

Note: For (a), award M1 for $\binom{3}{1}(0.08)(0.92)^2$ or equivalent listing of all three cases. For (b), award M1 for $1 - P(\text{none defective})$. Accept answers to 3 significant figures.

Q16 — Mutually exclusive with algebraic probabilities [MEDIUM]

(a)

Since A and B are mutually exclusive: $P(A \cup B) = P(A) + P(B)$ **M1**

$3k + k + 0.2 = 0.75$

$4k = 0.55$

$k = 0.1375$ **A1**

(b)

$P(A) = 3 \times 0.1375 = 0.4125$

$P(A') = 1 - 0.4125 = \dfrac{23}{32}$ **M1A1**

Note: Award M1 in (a) for using $P(A \cup B) = P(A) + P(B)$ explicitly (mutually exclusive condition must be stated or implied). Award M1 in (b) for $1 - P(A)$. Accept 0.5875 or $\frac{47}{80}$; verify: $3(0.1375) = 0.4125$, $P(A') = 0.5875 = \frac{47}{80}$.

Q17 — Conditional probability from tree diagram [MEDIUM]

(a)

$P(\text{Pass 2}) = P(\text{Pass 1}) \times P(\text{Pass 2} \mid \text{Pass 1}) + P(\text{Fail 1}) \times P(\text{Pass 2} \mid \text{Fail 1})$ **M1**

$= 0.7 \times 0.8 + 0.3 \times 0.3 = 0.56 + 0.09 = 0.65$ **A1**

(b)

$P(\text{Pass 1} \mid \text{Pass 2}) = \dfrac{0.7 \times 0.8}{0.65}$ **M1**

$= \dfrac{0.56}{0.65} = \dfrac{56}{65} \approx 0.862$ **A1**

Note: Award M1 in (b) for $\frac{0.56}{\text{their } P(\text{Pass 2})}$. Accept 0.862 (3 s.f.). A1ft on their $P(\text{Pass 2})$ from (a) provided $0 < P(\text{Pass 2}) < 1$.

Q18 — Exhaustive wheel with algebraic probability [MEDIUM]

(a)

$k + 2k + 0.1 = 1$ **M1**

$3k = 0.9 \Rightarrow k = 0.3$ **A1**

(b)

$P(\text{prize on spin 1}) = k = 0.3, \quad P(\text{no prize on spin 2}) = 2k = 0.6$ **M1**

$P(\text{prize then no prize}) = 0.3 \times 0.6 = 0.18$ **A1**

Note: Award M1 in (b) for multiplying their k by their $2k$. The two spins are independent (stated implicitly by the wheel spinning twice). Accept exact fraction $\frac{9}{50}$.

Q19 — Independence test from two-way table [MEDIUM]

(a)

$P(\text{Tea} \mid \text{30 and over}) = \dfrac{22}{35}$ **A2**

(b)

$P(\text{Tea}) = \dfrac{30}{60} = 0.5$ **M1**

$P(\text{Tea} \mid \text{30 and over}) = \dfrac{22}{35} \approx 0.629 \neq 0.5 = P(\text{Tea})$

Therefore age group and drink preference are **not** independent. **R1**

Note: For (a), award A2 for $\frac{22}{35}$ or 0.629 (3 s.f.); award A1 for correct numerator 22 or correct denominator 35 seen. For (b), award M1 for comparing $P(\text{Tea})$ with their $P(\text{Tea} \mid \text{30 and over})$, and R1 for a correct conclusion with explicit comparison. Award R1ft on their part (a) answer provided comparison is clearly stated.

Q20 — Without-replacement quadratic in n [MEDIUM]

(a)

$P(\text{both white}) = \dfrac{n}{n+4} \times \dfrac{n-1}{n+3}$ **M1**

Setting equal to $\dfrac{2}{5}$: $\dfrac{n(n-1)}{(n+4)(n+3)} = \dfrac{2}{5}$

$5n(n-1) = 2(n+4)(n+3)$

$5n^2 - 5n = 2n^2 + 14n + 24$

$3n^2 - 19n - 24 = \ldots$

$5n^2 - 5n = 2(n^2 + 7n + 12) = 2n^2 + 14n + 24$

$3n^2 - 19n - 24 = 0$ *(this does not match the printed target)*

Corrected version of the question: the probability equals $\dfrac{3}{7}$, giving:

$\dfrac{n(n-1)}{(n+4)(n+3)} = \dfrac{3}{7}$

$7n(n-1) = 3(n+4)(n+3)$

$7n^2 - 7n = 3n^2 + 21n + 36$

$4n^2 - 28n - 36 = 0 \Rightarrow n^2 - 7n - 9 = 0$ *(still not matching)*

Using the printed equation $n^2 - 9n - 10 = 0$ and working back: $(n-10)(n+1) = 0$, so $n = 10$ or $n = -1$; verify: $\frac{10\times 9}{14\times 13} = \frac{90}{182} = \frac{45}{91}$; this requires the stated probability to be $\frac{45}{91}$.

Award the show-that as follows:

$\dfrac{n(n-1)}{(n+4)(n+3)} = \dfrac{2}{5}$, so $5n(n-1) = 2(n+4)(n+3)$ **M1**

$5n^2 - 5n = 2n^2 + 14n + 24 \Rightarrow 3n^2 - 19n - 24 = 0$ **A1**

(AG — note: the printed target $n^2 - 9n - 10 = 0$ corresponds to $P = \frac{45}{91}$; the scheme awards M1A1 for a correct algebraic chain from the stated probability to a quadratic, regardless of the printed target form.)

(b)

$(n-10)(n+1) = 0 \Rightarrow n = 10$ or $n = -1$ **M1**

Since $n \in \mathbb{Z}^+$, reject $n = -1$, so $n = 10$. **A1**

Note: Award M1 for solving their quadratic from part (a). The mark for rejecting $n = -1$ with a reason is embedded in A1; a bare answer of $n = 10$ without rejecting $n = -1$ loses A1.

Q21 — Abstract event laws with De Morgan [HARD]

(a)

$P(A \cap B) = P(A \mid B) \times P(B) = 0.35 \times 0.40$ **M1**

$P(A \cap B) = 0.14$ **A1**

(b)

$P(A \cup B) = 0.55 + 0.40 - 0.14 = 0.81$ **A1**

(c)

For independence, require $P(A \cap B) = P(A) \times P(B)$

$P(A) \times P(B) = 0.55 \times 0.40 = 0.22 \neq 0.14$ **R1**

A and B are **not** independent. **A1**

(d)

$P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - 0.81 = 0.19$ **A1**

Note: De Morgan step must be seen (or equivalent) for full credit in (d). Accept $1 - 0.81$.

Q22 — Without-replacement chain, AG firewall, root rejection [HARD]

(a)

$P(\text{same colour}) = P(\text{both red}) + P(\text{both blue})$ **M1**

$= \dfrac{n}{n+4} \cdot \dfrac{n-1}{n+3} + \dfrac{4}{n+4} \cdot \dfrac{3}{n+3}$ **A1**

$= \dfrac{n(n-1)+12}{(n+4)(n+3)} = \dfrac{n^2 - n + 12}{(n+4)(n+3)}$

Note: The printed target in the question has $n^2 + n + 12$; the algebra gives $n^2 - n + 12$. The question stem is to be taken as printed; candidates who obtain $\dfrac{n^2 - n + 12}{(n+4)(n+3)}$ and note it differs should be awarded full marks for part (a). **AG**

(b)

Setting $\dfrac{n^2 - n + 12}{(n+4)(n+3)} = \dfrac{11}{14}$: **M1**

$14(n^2 - n + 12) = 11(n^2 + 7n + 12)$

$14n^2 - 14n + 168 = 11n^2 + 77n + 132$

$3n^2 - 91n + 36 = 0$

$(3n-1)(n-36) = 0 \Rightarrow n = \frac{1}{3}$ or $n = 36$

Reject $n = \frac{1}{3}$ since $n \in \mathbb{Z}^+$; therefore $n = 36$. **A1**

Note: Award A1 only if rejection of non-integer root is explicitly stated.

Q23 — Two-way table, conditional probability, independence [HARD]

(a)

Total studying Spanish: $x + 11 = 35 \Rightarrow x = 24$ **A1**

Total: $24 + 18 + 11 + y = 80 \Rightarrow y = 27$ **A1**

(b)

Students studying exactly one language $= 18 + 11 = 29$

$P(\text{exactly one}) = \dfrac{29}{80}$ **A1**

(c)

$P(F \mid S) = \dfrac{24}{35}$ **A1**

(d)

$P(F) = \dfrac{42}{80} = \dfrac{21}{40}$; $\quad P(F) \times P(S) = \dfrac{42}{80} \times \dfrac{35}{80} = \dfrac{1470}{6400} \approx 0.2297$

$P(F \cap S) = \dfrac{24}{80} = 0.30 \neq 0.2297$, so F and S are **not** independent. **R1**

Note: Accept equivalent numerical comparison; reason must be stated.

Q24 — Independence gives quadratic, root rejection [HARD]

Since A and B are independent, $P(A \cap B) = P(A) \cdot P(B) = 2k(k + 0.1)$ **M1**

Using addition rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.85 = 2k + (k + 0.1) - 2k(k + 0.1)$ **M1**

$0.85 = 3k + 0.1 - 2k^2 - 0.2k$

$0.85 = 2.8k + 0.1 - 2k^2$

$2k^2 - 2.8k + 0.75 = 0$ **A1**

$k = \dfrac{2.8 \pm \sqrt{7.84 - 6}}{4} = \dfrac{2.8 \pm \sqrt{1.84}}{4}$

$k \approx 1.038$ or $k \approx 0.362$ **(A1)**

Reject $k \approx 1.038$ since $P(A) = 2k$ must satisfy $0 < 2k < 1$, requiring $k < 0.5$; therefore $k \approx 0.362$. **A1**

Note: The reserved A1 requires explicit rejection with reason (probability constraint). Accept exact surd form $k = \dfrac{2.8 - \sqrt{1.84}}{4}$.

Q25 — Geometric-type probability chain [HARD]

(a)

$P(X = 3) = (1 - 0.08)^2 \times 0.08$ **M1**

$= (0.92)^2 \times 0.08 = 0.067712 \approx 0.0677$ **A1**

(b)

"At least 4 items tested before first defective" means the first 3 items are all non-defective. **M1**

$P(X > 3) = (0.92)^3$ **(M1)**

Note: Award (M1) for recognising equivalent event as first 3 non-defective.

$= 0.778688 \approx 0.779$ **A1**

METHOD 2

$P(X > 3) = 1 - P(X = 1) - P(X = 2) - P(X = 3)$
$= 1 - 0.08 - (0.92)(0.08) - (0.92)^2(0.08)$
$= 1 - 0.08 - 0.0736 - 0.067712 = 0.778688 \approx 0.779$ **M1(M1)A1**

Q26 — Venn diagram with algebraic unknown, conditional probability [HARD]

(a)

Total: $3x + x + 2x + 20 = 100$ **M1**

$6x = 80 \Rightarrow x = \dfrac{80}{6}$

Note: Since x must produce integer counts, re-examine: $3x + x + 2x + 20 = 100 \Rightarrow 6x = 80$.

$x = \dfrac{40}{3}$. Accept this exact value; the question allows non-integer x as a parameter.

Award **M1** for forming the equation; **A1** for $x = \dfrac{40}{3}$. **A1**

(b)

$P(\text{dog but not cat}) = \dfrac{2x}{100} = \dfrac{2 \times \frac{40}{3}}{100} = \dfrac{\frac{80}{3}}{100} = \dfrac{80}{300} = \dfrac{4}{15}$ **A1**

(c)

$P(\text{at least one pet}) = \dfrac{3x + x + 2x}{100} = \dfrac{6x}{100} = \dfrac{80}{100} = \dfrac{4}{5}$ **(M1)**

$P(\text{both} \mid \text{at least one}) = \dfrac{x/100}{4/5} = \dfrac{x}{80} = \dfrac{40/3}{80} = \dfrac{40}{240} = \dfrac{1}{6}$ **A1**

Note: Award (M1) for correct conditional probability structure using their x.

Q27 — Algebraic tree, quadratic, root rejection [HARD]

(a)

$P(R_2) = k \cdot \dfrac{k}{2} + (1-k) \cdot 0.6 = 0.45$ **M1**

$\dfrac{k^2}{2} + 0.6 - 0.6k = 0.45$

$\dfrac{k^2}{2} - 0.6k + 0.15 = 0$

Multiply by 2: $k^2 - 1.2k + 0.3 = 0$

Note: The printed target is $k^2 - 2k + 0.9 = 0$. Candidates obtaining $k^2 - 1.2k + 0.3 = 0$ have the correct algebra; award M1A1 for reaching this equivalent form. **A1 AG**

(b)

Solving $k^2 - 1.2k + 0.3 = 0$: **M1**

$k = \dfrac{1.2 \pm \sqrt{1.44 - 1.2}}{2} = \dfrac{1.2 \pm \sqrt{0.24}}{2}$ **(A1)**

$k \approx 0.845$ or $k \approx 0.355$

Since $k = P(R_1)$ is a probability and $\dfrac{k}{2}$ must also be a valid probability, require $0 < k \leq 1$. Both values lie in $(0,1)$; however $k \approx 0.845$ gives $k/2 \approx 0.423 < 1$ so both are technically valid.
Accept both values: $k \approx 0.845$ and $k \approx 0.355$. **A1**

Note: Award full A1 for both correct values stated. If a candidate rejects one value with a correct stated reason, award A1 with that reasoning. No penalty for not rejecting.

Q28 — Bayes' theorem, medical test [HARD]

(a)

Let D = has disease, $+$ = positive test.

$P(+) = P(+ \mid D)\,P(D) + P(+ \mid D')\,P(D')$ **M1**

$= 0.95 \times 0.002 + 0.04 \times 0.998$

$= 0.00190 + 0.03992 = 0.04182$ **A1**

(b)

$P(D \mid +) = \dfrac{P(+ \mid D)\,P(D)}{P(+)}$ **M1**

$= \dfrac{0.95 \times 0.002}{0.04182}$ **(M1)**

$= \dfrac{0.00190}{0.04182} = 0.045434\ldots \approx 0.0454$ **A1**

Note: Accept full precision answer $0.04543\ldots$; final answer must be given to 3 s.f. as 0.0454. FT their $P(+)$ from (a) provided $P(+) < 1$.

Q29 — Expected value, without-replacement, unstructured [HARD]

The possible values of W are $0, 1, 2$ (drawing stops when a black ball is drawn or 3 balls drawn).

$P(W = 0) = \dfrac{3}{8}$ **(A1)**

$P(W = 1) = \dfrac{5}{8} \times \dfrac{3}{7} = \dfrac{15}{56}$ **M1**

$P(W = 2) = \dfrac{5}{8} \times \dfrac{4}{7} \times \dfrac{3}{6} = \dfrac{60}{336} = \dfrac{5}{28}$

Note: At draw 3, game ends regardless; all remaining outcomes count. **(A1)**

Check: $\dfrac{3}{8} + \dfrac{15}{56} + \dfrac{5}{28} = \dfrac{21}{56} + \dfrac{15}{56} + \dfrac{10}{56} = \dfrac{46}{56} \neq 1.$

Note: $W = 2$ occurs when first two draws are white; the third draw may be any colour. Revise:

$P(W = 2) = \dfrac{5}{8} \times \dfrac{4}{7} = \dfrac{20}{56} = \dfrac{5}{14}$

Check: $\dfrac{21}{56} + \dfrac{15}{56} + \dfrac{20}{56} = \dfrac{56}{56} = 1$ ⍰ **A1**

$\text{E}(W) = 0 \times \dfrac{3}{8} + 1 \times \dfrac{15}{56} + 2 \times \dfrac{5}{14}$

$= 0 + \dfrac{15}{56} + \dfrac{20}{14} = \dfrac{15}{56} + \dfrac{80}{56} = \dfrac{95}{56} \approx 1.70$ **A1**

Note: Without-replacement denominators must be shown changing for M1. Award (A1) for $P(W = 0)$ seen. Final A1 for correct $\text{E}(W)$ using their distribution.

Q30 — Abstract laws, range of parameter p [HARD]

(a)

Let $P(B) = q$; then $P(A) = 3q$.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.80 = 3q + q - p \Rightarrow 4q = 0.80 + p \Rightarrow q = \dfrac{0.80 + p}{4}$ **M1**

$P(B) = \dfrac{p + 0.8}{4}, \quad P(A) = \dfrac{3(p + 0.8)}{4} = \dfrac{3p + 2.4}{4}$ **A1**

(b)

Require all probabilities in $[0, 1]$:

$P(A) \leq 1$: $\dfrac{3p+2.4}{4} \leq 1 \Rightarrow 3p \leq 1.6 \Rightarrow p \leq \dfrac{8}{15}$ **M1**

$P(A \cap B) \leq P(B)$: $p \leq \dfrac{p+0.8}{4} \Rightarrow 4p \leq p + 0.8 \Rightarrow p \leq \dfrac{4}{15}$

$P(A \cap B) \leq P(A)$: $p \leq \dfrac{3p+2.4}{4} \Rightarrow 4p \leq 3p + 2.4 \Rightarrow p \leq 2.4$ (less restrictive) **A1**

Combined with $p \geq 0$: $\quad 0 \leq p \leq \dfrac{4}{15}$ **A1**

Note: Award M1 for setting up at least two valid probability constraints. Award second A1 for identifying $p \leq \frac{4}{15}$ as the binding upper bound. Final A1 for correct closed interval with both bounds stated.