IB Math AA SL — Topic 4
Binomial Distribution

The Binomial Distribution · Calculating Binomial Probabilities

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:_____ Date:_____

Score:_____/120

Scan me for help

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

Key Formulae

Binomial probability (pdf): $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 0, 1, 2, \ldots, n$

Binomial coefficient: $\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$

Cumulative distribution: $P(X \leq k) = \sum_{i=0}^{k} \binom{n}{i} p^i (1-p)^{n-i}$

Expected value: $E(X) = np$

Variance: $\text{Var}(X) = np(1-p)$

Standard deviation: $\sigma = \sqrt{np(1-p)}$

Complement rule: $P(X \geq 1) = 1 - P(X = 0) = 1 - (1-p)^n$

GDC Tips

- **Exact probability (pdf):** Use `binompdf(n, p, k)` to find $P(X = k)$ directly; write the boundary conversion $X = k$ before entering.
- **Cumulative probability (cdf):** Use `binomcdf(n, p, k)` to find $P(X \leq k)$; always write the boundary conversion line (e.g. $P(X \leq 4)$) before the GDC call.
- **Upper-tail probabilities:** Convert first — $P(X \geq k) = 1 - \texttt{binomcdf}(n, p, k-1)$ and $P(X > k) = 1 - \texttt{binomcdf}(n, p, k)$; state the conversion explicitly.
- **Least n such that $P(X \geq 1) > c$:** Use solve or a table of $1 - (1-p)^n$ values, bracket the crossing integer, then round up.
- **Conditional binomial:** Compute numerator and denominator separately via `binomcdf/binompdf`, then divide; store intermediate values to avoid rounding errors.
- **Checking parameters:** Confirm $n \in \mathbb{Z}^+$, $0 < p < 1$, and that the GDC input order is (n, p, k) — swapping p and k is a common entry error.

Common Mistakes to Avoid

1. **Boundary conversion omitted before cdf calls.** Writing `binomcdf(n,p,5)` for $P(X < 5)$ instead of $P(X \leq 4)$ shifts the answer by one term. Always write the converted inequality explicitly before every GDC command.
2. **Rounding direction in least-n problems.** After finding the crossing point of $1-(1-p)^n > c$, students round down to the nearest integer. The correct rule is always round up, since the inequality must be strictly satisfied. Bracket both n and $n-1$ to justify.
3. **Using $P(X \geq 1) = 1 - P(X = 1)$ instead of $1 - P(X = 0)$.** The complement of "at least one" is "none", i.e. $P(X = 0) = (1-p)^n$, not $P(X = 1)$.
4. **Confusing $E(X) = np$ reversal direction.** When given $E(X)$ and asked for n or p, students substitute into Var(X) before isolating the unknown from np first. Always solve $np = E(X)$ before using the variance formula.
5. **Failing to verify all four conditions for a binomial model.** Answers that simply state "$X \sim B(n, p)$" without naming (i) fixed number of trials, (ii) two outcomes, (iii) constant probability, and (iv) independence of trials receive no credit for model-validity reasoning marks.
6. **Computing conditional probability $P(X = a \mid X \geq 1)$ with the wrong denominator.** Students use $P(X \geq 1) = 1$ or omit the denominator entirely. The correct form is $\dfrac{P(X = a)}{P(X \geq 1)} = \dfrac{P(X = a)}{1 - P(X = 0)}$, with both values evaluated separately before dividing.

Section A — Easy EASY (10 questions × 3 marks = 30 marks)

Scan me for help

1. EASY No Calculator [3 marks]

A fair six-sided die is rolled 8 times. Let X be the number of times a six is obtained, where $X \in \mathbb{Z}_0^+$. Write down the distribution of X, stating the values of n and p.

2. EASY Calculator [3 marks]

A biased coin has probability 0.3 of landing heads. The coin is tossed 10 times. Let X be the number of heads obtained, where $X \in \mathbb{Z}_0^+$.
Find $P(X = 4)$.

3. EASY No Calculator [3 marks]

A multiple-choice quiz has 12 questions. Each question has 4 options, exactly one of which is correct. A student guesses every answer independently and at random. Let X be the number of correct answers, where $X \in \mathbb{Z}_0^+$.
Find $P(X = 3)$.

4. EASY Calculator **[3 marks]**

The random variable $X \sim B(15, 0.4)$, where $X \in \mathbb{Z}_0^+$.
Find $P(X \leq 5)$.

5. EASY No Calculator **[3 marks]**

The random variable $X \sim B(20, 0.25)$, where $X \in \mathbb{Z}_0^+$.
Find $E(X)$ and Var(X).

6. EASY Calculator **[3 marks]**

In a large population, 35% of people prefer tea to coffee. A researcher selects 12 people independently and at random. Let X be the number of people who prefer tea, where $X \in \mathbb{Z}_0^+$.
The probability distribution of X is shown in the table below for selected values.

x	0	1	2	3
$P(X = x)$	0.00569	0.0368	0.109	0.195

Find $P(X \geq 2)$.

7. EASY No Calculator **[3 marks]**

The random variable $X \sim B(n, 0.2)$, where $X \in \mathbb{Z}_0^+$, and $E(X) = 6$.
Find the value of n.

8. **EASY** **Calculator** **[3 marks]**

A factory produces light bulbs. The probability that any light bulb is defective is 0.05, independently of all others. A quality inspector selects 20 light bulbs at random. Let X be the number of defective bulbs, where $X \in \mathbb{Z}_0^+$.
Find $P(X = 0)$.

9. **EASY** **Calculator** **[3 marks]**

The random variable $X \sim B(18, 0.6)$, where $X \in \mathbb{Z}_0^+$.
Find $P(X \geq 12)$.

10. **EASY** **Calculator** **[3 marks]**

The table below shows the probability distribution for $X \sim B(5, p)$, where $X \in \mathbb{Z}_0^+$, for selected values.

x	0	1
$P(X = x)$	0.16807	0.36015

Find the value of p.

Section B — Medium MEDIUM (10 questions × 4 marks = 40 marks)

Scan me for help

11. MEDIUM Calculator [4 marks]

A biased coin has probability 0.35 of landing heads on any toss. The coin is tossed 12 times. Let X be the number of heads obtained, where $x \in \mathbb{Z}, 0 \leq x \leq 12$.

(a) Write down the distribution of X. [1]

(b) Find $P(X = 4)$. [1]

(c) Find $P(X \leq 4)$. [1]

(d) Find $P(2 \leq X \leq 5)$. [1]

12. MEDIUM No Calculator [4 marks]

The random variable $X \sim B(n, 0.4)$. Given that $E(X) = 6$, find the values of n, Var(X), and $E(2X - 1)$.

13. MEDIUM Calculator **[4 marks]**

A factory produces light bulbs, 8% of which are defective. Amara selects a random sample of 20 bulbs. Let D be the number of defective bulbs in the sample, where $d \in \mathbb{Z}, 0 \leq d \leq 20$.

(a) Find the probability that exactly 2 bulbs are defective. [2]

(b) Find the probability that at least 1 bulb is defective. [2]

14. MEDIUM Calculator **[4 marks]**

The probability distribution of a binomial random variable $X \sim B(10, p)$ is partially shown in the table below, where $p > 0$.

x	0	1	2	3
$P(X = x)$	0.1074	0.2684	k	0.2013

(a) Write down the value of p. [1]

(b) Find the value of k. [1]

(c) Find $P(X \geq 3)$. [2]

15. MEDIUM Calculator **[4 marks]**

In a large population, 30% of people are left-handed. A researcher selects a random sample of 15 people. Let L be the number of left-handed people in the sample, where $l \in \mathbb{Z}, 0 \leq l \leq 15$.

(a) Find $P(L = 5)$, giving your answer to four decimal places. [2]

(b) Find $P(L > 5)$. [1]

(c) Find the expected number of left-handed people in the sample. [1]

16. MEDIUM No Calculator **[4 marks]**

The random variable $X \sim B(8, p)$. Given that $\text{Var}(X) = 1.92$ and $p < 0.5$, find the value of p and hence find $P(X \geq 2)$.

17. MEDIUM Calculator **[4 marks]**

During a quiz, each question has 5 possible answers, exactly one of which is correct. Kofi randomly guesses the answer to each of 10 independent questions. Let C be the number of correct answers, where $c \in \mathbb{Z}, 0 \leq c \leq 10$.

(a) State the distribution of C, including the values of all parameters. [1]

(b) Find $P(C \geq 3)$. [2]

(c) Given that Kofi answers at least one question correctly, find the probability that he answers exactly two questions correctly. [1]

18. MEDIUM Calculator **[4 marks]**

A quality control inspector tests batches of n items, where 5% of all items are faulty. The inspector wants the probability of finding at least one faulty item in a batch to exceed 0.75. Find the least value of n.

19. MEDIUM Calculator **[4 marks]**

A spinner has three coloured sectors: red, blue and green. The probability of landing on red is 0.45. Priya spins the spinner 20 times. Let R be the number of times the spinner lands on red, where $r \in \mathbb{Z}$, $0 \leq r \leq 20$.

(a) Find $P(R = 9)$. [2]

(b) Find $P(7 \leq R \leq 11)$. [1]

(c) Find the expected number of times the spinner lands on red, and state the variance of R. [1]

20. MEDIUM No Calculator **[4 marks]**

The number of successes X in n independent trials satisfies $X \sim B(n, 0.4)$. It is given that $P(X = 0) = 0.07776$, where $n \in \mathbb{Z}^+$.

(a) Show that $n = 5$. [2]

(b) Hence find $P(X \geq 2)$. [2]

Section C — Hard HARD (10 questions × 5 marks = 50 marks)

Scan me for help

21. HARD No Calculator [5 marks]

A biased coin has probability p of landing heads, where $0 < p < 1$. The coin is tossed n times and the number of heads, X, follows a binomial distribution. It is given that $E(X) = 3$ and $\text{Var}(X) = \frac{9}{4}$.
Find the values of n and p, and hence find $P(X \geq 2)$, giving your answer correct to 4 significant figures.

22. HARD Calculator [5 marks]

Let $X \sim B(12, p)$ where $p \in (0, 1)$. It is given that $P(X = 3) = P(X = 4)$.
Find p, and hence find $P(X \leq 3)$, giving your answer correct to 3 significant figures.

23. **HARD** **Calculator** **[5 marks]**

In a large factory, 8% of items produced are defective. A quality inspector selects items one at a time, independently and at random. Let N be the least number of items that must be selected so that the probability of finding at least one defective item exceeds 0.95.
Find N.

24. **HARD** **Calculator** **[5 marks]**

Each morning, Priya independently attempts to solve a puzzle. The probability that she solves it on any given morning is 0.35. In a period of 20 mornings, let X be the number of mornings on which she solves the puzzle. The following table shows some values of the probability distribution of X.

x	4	5	6	7
$P(X = x)$	a	0.1272	b	0.1844

(a) Find the values of a and b, correct to 4 decimal places. [2]

(b) Given that $X \geq 4$, find $P(X \leq 6)$. Give your answer correct to 3 significant figures. [3]

25. HARD No Calculator **[5 marks]**

Show that if $X \sim B(n, p)$ and $P(X = k+1) = P(X = k)$ for some integer k with $0 \leq k < n$, then $p = \dfrac{k+1}{n+1}$.

26. HARD Calculator **[5 marks]**

In a town, 15% of adults own an electric car. A random sample of n adults is selected. The random variable X represents the number of adults in the sample who own an electric car, where $X \sim B(n, 0.15)$.

(a) Find the least value of n such that $P(X \geq 2) > 0.7$. [3]

(b) For the value of n found in part (a), find $P(2 \leq X \leq 5)$. Give your answer correct to 3 significant figures. [2]

27. HARD Calculator **[5 marks]**

A factory machine produces bolts, each of which is independently defective with probability p, where $0 < p < 0.5$. In a batch of 10 bolts, the number of defective bolts is $D \sim B(10, p)$. It is given that $P(D = 2) = 3\,P(D = 1)$.

Find p, and hence find $P(D \leq 3)$, giving your answer correct to 3 decimal places.

28. HARD Calculator **[5 marks]**

Tomás plays a game in which he rolls a fair six-sided die repeatedly, stopping as soon as he rolls a 6. Let W be the event that Tomás rolls at most one non-6 before his first 6 in a sequence of exactly 5 rolls. However, suppose instead that Tomás rolls the die exactly 8 times regardless of outcome, and Y is the number of sixes obtained, where $Y \sim B\left(8, \frac{1}{6}\right)$.

(a) Find $P(Y = 2)$, showing your method clearly. [2]

(b) Find $P(Y \geq 3)$, giving your answer correct to 3 significant figures. [3]

29. HARD Calculator **[5 marks]**

A discrete random variable $X \sim B(n, 0.4)$, where $n \in \mathbb{Z}^+$. It is given that $P(X = 0) < 0.005$.
Find the least value of n satisfying this condition, justifying your answer with a bracketing check.

30. HARD Calculator **[5 marks]**

A school runs a multiple-choice quiz with 15 independent questions. Each question has 5 options, exactly one of which is correct. A student, Elif, claims she is not simply guessing; she believes she has some subject knowledge. Her teacher models Elif's score S as $B(15, p)$.

(a) State the value of p that would represent pure guessing. [1]

(b) The teacher decides to accept Elif's claim only if her score satisfies $P(S \geq s) < 0.05$ under the pure-guessing model. Find the least integer value of s for which the teacher accepts Elif's claim. [3]

(c) Elif scores exactly s (from part (b)) on the quiz. Determine whether the four conditions for $S \sim B(15, p)$ are satisfied in this context, identifying any condition that may be doubtful. [1]

Worked Solutions

Q1 — Binomial Distribution Identification EASY

Recognising $X \sim B(n, p)$ with n and p stated **M1**

$n = 8$ **A1**

$p = \dfrac{1}{6}$ **A1**

Note: Award M1 for identifying the binomial distribution; award both A1 marks for correct n and p. Accept $p = 0.16\overline{6}$.

Q2 — Binomial Pdf Calculation EASY

Recognising $X \sim B(10, 0.3)$ and attempting $P(X = 4) = \binom{10}{4}(0.3)^4(0.7)^6$ **M1**

Using `binompdf(10, 0.3, 4)` **(M1)**

$P(X = 4) = 0.200120\ldots \approx 0.200$ **A1**

Note: Award M1 for setting up the correct binomial expression; (M1) for GDC use; A1 for the correct answer to 3 significant figures.

Q3 — Exact Binomial Pdf by Hand EASY

Recognising $X \sim B(12, \frac{1}{4})$ and writing $P(X = 3) = \binom{12}{3}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^9$ **M1**

Evaluating $\binom{12}{3} = 220$ **A1**

$$P(X = 3) = 220 \times \frac{1}{64} \times \frac{19683}{262144} = \frac{220 \times 19683}{16777216} = \frac{4330260}{16777216}$$

$= 4330260/16777216 \approx 0.258$ **A1**

Note: Accept any exact equivalent fraction or 0.258 (3 s.f.). Award M1 for a correct formula with $n = 12$, $p = \frac{1}{4}$, $k = 3$; first A1 for $\binom{12}{3} = 220$ seen; second A1 for the correct exact or decimal answer.

Q4 — Binomial Cdf EASY

Recognising the need for `binomcdf` and writing $P(X \leq 5)$ with $X \sim B(15, 0.4)$ **M1**

Using `binomcdf(15, 0.4, 5)` **(M1)**

$P(X \leq 5) = 0.40322\ldots \approx 0.403$ **A1**

Note: Award M1 for identifying the correct distribution and boundary; (M1) for GDC use; A1 for the answer correct to 3 significant figures.

Q5 — Mean and Variance of Binomial EASY

Using $E(X) = np$ with $n = 20, p = 0.25$ **M1**

$E(X) = 20 \times 0.25 = 5$ **A1**

$\text{Var}(X) = np(1-p) = 20 \times 0.25 \times 0.75 = 3.75$ **A1**

Note: Award M1 for a correct formula for $E(X)$; first A1 for $E(X) = 5$; second A1 for $\text{Var}(X) = 3.75$. Accept $\frac{15}{4}$.

Q6 — Binomial Complement EASY

Writing $P(X \geq 2) = 1 - P(X \leq 1)$ or $1 - P(X = 0) - P(X = 1)$ **M1**

$P(X \leq 1) = 0.00569 + 0.0368 = 0.04249$ **A1**

$P(X \geq 2) = 1 - 0.04249 = 0.958$ **A1**

Note: Award M1 for a correct complement approach; first A1 for $P(X \leq 1) = 0.0425$ (3 s.f.); second A1 for 0.958 (3 s.f.). Values are taken directly from the given table.

Q7 — Reverse Binomial Mean EASY

Using $E(X) = np$ and setting $0.2n = 6$ **M1**

$n = \dfrac{6}{0.2}$ **A1**

$n = 30$ **A1**

Note: Award M1 for writing $np = 6$ with $p = 0.2$; first A1 for correct rearrangement; second A1 for $n = 30$.

Q8 — Binomial Zero Probability EASY

Recognising $X \sim B(20, 0.05)$ and writing $P(X = 0) = (0.95)^{20}$ **M1**

Using `binompdf(20, 0.05, 0)` **(M1)**

$P(X = 0) = 0.358485\ldots \approx 0.358$ **A1**

Note: Award M1 for correctly identifying $P(X = 0) = (1 - 0.05)^{20}$; (M1) for GDC use; A1 for answer correct to 3 significant figures.

Q9 — Binomial Upper Tail EASY

Writing $P(X \geq 12) = 1 - P(X \leq 11)$ with $X \sim B(18, 0.6)$ **M1**

Using $1 -$ `binomcdf(18, 0.6, 11)` **(M1)**

$P(X \geq 12) = 0.374277\ldots \approx 0.374$ **A1**

Note: Award M1 for a correct complement expression with boundary $X \leq 11$ stated; (M1) for GDC use; A1 for the answer correct to 3 significant figures.

Q10 — Finding Parameter p from Table EASY

Using $P(X = 0) = (1-p)^5 = 0.16807$ **M1**

$(1-p) = (0.16807)^{1/5} = 0.7$ **A1**

$p = 0.3$ **A1**

Note: Award M1 for writing a correct equation using $P(X = 0)$; first A1 for obtaining $1 - p = 0.7$; second A1 for $p = 0.3$. Candidates may verify using the $P(X = 1)$ entry: $5(0.3)(0.7)^4 = 0.36015$ — condone this as supporting work but it is not required.

Q11 — Binomial Probabilities with Cdf MEDIUM

(a) $X \sim B(12, 0.35)$ **A1**

(b) Using `binompdf(12, 0.35, 4)`
$P(X = 4) = 0.23669\ldots \approx 0.237$ **A1**

(c) Using `binomcdf(12, 0.35, 4)`
$P(X \leq 4) = 0.58335\ldots \approx 0.583$ **A1**

(d) $P(2 \leq X \leq 5) =$ `binomcdf(12, 0.35, 5)` $-$ `binomcdf(12, 0.35, 1)`
$= 0.82967\ldots - 0.08498\ldots = 0.74469\ldots \approx 0.745$ **A1**

Note: Accept answers rounding correctly to 3 significant figures.

Q12 — Reversing the Mean to Find n and Variance MEDIUM

Attempt to use $E(X) = np$: **M1**

$6 = n \times 0.4 \implies n = 15$ **A1**

$\text{Var}(X) = np(1-p) = 15 \times 0.4 \times 0.6 = 3.6$ **A1**

$E(2X - 1) = 2E(X) - 1 = 2(6) - 1 = 11$ **A1**

Note: Final A1 is independent of n reversal; accept their $E(X)$ substituted correctly.

Q13 — Defective Bulbs Complement MEDIUM

(a) $D \sim B(20, 0.08)$

Attempt to use binomial pdf formula or `binompdf(20, 0.08, 2)`: **M1**

$P(D = 2) = \binom{20}{2}(0.08)^2(0.92)^{18} = 0.2711\ldots \approx 0.271$ **A1**

(b) Recognise use of complement: $P(D \geq 1) = 1 - P(D = 0)$ **M1**

$P(D = 0) = (0.92)^{20} = 0.1887\ldots$

$P(D \geq 1) = 1 - 0.1887\ldots = 0.811\ldots \approx 0.811$ **A1**

Note: Award M1 for explicit statement of complement boundary before computation.

Q14 — Reading p from a Distribution Table MEDIUM

(a) From $P(X = 0) = (1-p)^{10} = 0.1074$: testing $(1-p) = 0.8$ gives $(0.8)^{10} = 0.10737\ldots \approx 0.1074$ ✓, so $p = 0.2$

$p = 0.2$ **A1**

(b) $k = P(X = 2) = \binom{10}{2}(0.2)^2(0.8)^8$

Using `binompdf(10, 0.2, 2)`: $k = 0.3020\ldots \approx 0.3020$ **A1**

(c) $P(X \geq 3) = 1 - P(X \leq 2)$ (boundary: $X \leq 2$ means $x = 0, 1, 2$) **M1**

$P(X \leq 2) =$ `binomcdf(10, 0.2, 2)` $= 0.6778\ldots$

$P(X \geq 3) = 1 - 0.6778\ldots = 0.322\ldots \approx 0.322$ **A1**

Note: FT on their p for (b) and (c) provided p is a valid probability.

Q15 — Left-Handed Sample and E(X) MEDIUM

(a) $L \sim B(15, 0.3)$

Using `binompdf(15, 0.3, 5)`: **M1**

$P(L = 5) = 0.20613\ldots \approx 0.2061$ **A1**

Note: Answer must be given to four decimal places for A1.

(b) $P(L > 5) = 1 -$ `binomcdf(15, 0.3, 5)` (boundary conversion: $L > 5$ means subtract cdf up to and including 5)

$= 1 - 0.7216\ldots = 0.278\ldots \approx 0.278$ **A1**

(c) $E(L) = np = 15 \times 0.3 = 4.5$ **A1**

Q16 — Variance Reversal for p then Upper Tail Probability MEDIUM

Attempt to use Var$(X) = np(1-p)$: **M1**

$8p(1-p) = 1.92 \implies 8p - 8p^2 = 1.92 \implies 8p^2 - 8p + 1.92 = 0$
$p^2 - p + 0.24 = 0 \implies (p-0.4)(p-0.6) = 0$
$p = 0.4$ or $p = 0.6$; since $p < 0.5$, $p = 0.4$ **A1**

$P(X \geq 2) = 1 - P(X \leq 1) = 1 - (0.6)^8 - 8(0.4)(0.6)^7$ **M1**
$= 1 - 0.01679\ldots - 0.08958\ldots = 1 - 0.10638\ldots$
$= 0.8936\ldots \approx 0.894$ **A1**

Note: Trap — both values of p must be found and the condition $p < 0.5$ used to select $p = 0.4$; award A1 only if correct selection is justified. Accept exact fractions: $p = \frac{2}{5}$.

Q17 — Conditional Binomial Guessing Quiz MEDIUM

(a) $C \sim B(10, \frac{1}{5})$, i.e. $B(10, 0.2)$ **A1**

(b) $P(C \geq 3) = 1 - P(C \leq 2)$ (boundary: subtract cdf up to 2) **M1**

`binomcdf(10, 0.2, 2)` $= 0.6778\ldots$
$P(C \geq 3) = 1 - 0.6778\ldots = 0.322\ldots \approx 0.322$ **A1**

(c) $P(C = 2 \mid C \geq 1) = \dfrac{P(C=2)}{P(C \geq 1)}$
$P(C \geq 1) = 1 - (0.8)^{10} = 1 - 0.10737\ldots = 0.8926\ldots$
$P(C = 2) =$ `binompdf(10, 0.2, 2)` $= 0.30199\ldots$
$P(C = 2 \mid C \geq 1) = \dfrac{0.30199\ldots}{0.8926\ldots} = 0.338\ldots \approx 0.338$ **A1**

Note: Accept exact fraction form. FT on their $P(C \geq 1)$ and $P(C = 2)$ provided both are valid.

Q18 — Least n for At Least One Faulty Item MEDIUM

Let $X \sim B(n, 0.05)$. Require $P(X \geq 1) > 0.75$. **M1**

$P(X \geq 1) = 1 - P(X = 0) = 1 - (0.95)^n > 0.75 \implies (0.95)^n < 0.25$ **A1**

Attempt to find n by systematic trial or logarithms:
$(0.95)^{27} = 0.2567\ldots > 0.25$ so $P(X \geq 1) = 0.7433\ldots < 0.75$
$(0.95)^{28} = 0.2440\ldots < 0.25$ so $P(X \geq 1) = 0.7560\ldots > 0.75$ **M1**

Least value is $n = 28$ **A1**

Note: Both bracketing lines must appear for the second M1. Award A1 only for $n = 28$; $n = 27$ or unverified n earns M1 only. Trap — round UP after crossing.

Q19 — Spinner Binomial E and Var MEDIUM

(a) $R \sim B(20, 0.45)$
Using `binompdf(20, 0.45, 9)`: **M1**

$P(R = 9) = 0.17706\ldots \approx 0.177$ **A1**

(b) $P(7 \leq R \leq 11) =$ `binomcdf(20, 0.45, 11)` $-$ `binomcdf(20, 0.45, 6)`
$= 0.8692\ldots - 0.1299\ldots = 0.739\ldots \approx 0.739$ **A1**

(c) $E(R) = np = 20 \times 0.45 = 9$
$\text{Var}(R) = np(1-p) = 20 \times 0.45 \times 0.55 = 4.95$ **A1**

Note: Both $E(R)$ and $\text{Var}(R)$ correct for the single A1 in part (c).

Q20 — Recover n from a Given Probability then Upper Tail MEDIUM

(a) $P(X = 0) = (1 - 0.4)^n = (0.6)^n$ **M1**

$(0.6)^1 = 0.6,\ (0.6)^2 = 0.36,\ (0.6)^3 = 0.216,\ (0.6)^4 = 0.1296,\ (0.6)^5 = 0.07776$
Since $(0.6)^5 = 0.07776$ matches the given value exactly, $n = 5$. **AG, A1**

Note: A working line showing $(0.6)^5 = 0.07776$ is sufficient for the A1; final AG line earns nothing.

(b) $X \sim B(5, 0.4)$; boundary: $P(X \geq 2) = 1 - P(X \leq 1)$ **M1**

$P(X \leq 1) = (0.6)^5 + 5(0.4)(0.6)^4 = 0.07776 + 5(0.4)(0.1296)$
$= 0.07776 + 0.2592 = 0.33696$
$P(X \geq 2) = 1 - 0.33696 = 0.66304 \approx 0.663$ **A1**

Note: FT on $n = 5$ from part (a).

Q21 — Mean and Variance to Find n p then Complement HARD

Setting up the system:
$E(X) = np = 3$ and $\text{Var}(X) = np(1-p) = \dfrac{9}{4}$ **M1**

Dividing: $1 - p = \dfrac{9/4}{3} = \dfrac{3}{4}$, so $p = \dfrac{1}{4}$ **A1**

Then $n = \dfrac{3}{1/4} = 12$, so $X \sim B(12, \frac{1}{4})$ **A1**

Finding the probability:
$P(X \geq 2) = 1 - P(X = 0) - P(X = 1)$ **M1**

$= 1 - \left(\frac{3}{4}\right)^{12} - 12 \cdot \frac{1}{4} \cdot \left(\frac{3}{4}\right)^{11}$
$= 1 - \dfrac{2657205}{16777216} \approx 0.8416$ **A1**

Note: Accept 0.8416 to 4 s.f.; do not accept unsimplified decimal from the formula without evidence of the subtraction structure.

Q22 — Equal Adjacent Probabilities to Find p then Cdf HARD

Setting up the equation:

$P(X=4)=P(X=3)$ with $X\sim B(12,p)$:

$$\binom{12}{4}p^4(1-p)^8=\binom{12}{3}p^3(1-p)^9 \qquad \textbf{M1}$$

Dividing both sides by $\binom{12}{3}p^3(1-p)^9$ (valid since $0<p<1$):

$$\frac{12-3}{4}\cdot\frac{p}{1-p}=1 \implies \frac{9p}{4(1-p)}=1 \qquad \textbf{A1}$$

$$9p=4-4p \implies p=\frac{4}{13} \qquad \textbf{A1}$$

Finding the cdf value (GDC):

$P(X\leq 3)=\texttt{binomcdf}\left(12,\frac{4}{13},3\right)$ **(M1)**

$=0.46895\ldots\approx 0.469$ **A1**

Note: Boundary: $P(X\leq 3)$ *includes* $x=0,1,2,3$. *Accept* 0.469.

Q23 — Least n for P(at least one) Exceeds 0.95 HARD

Setting up the inequality:

Let $D\sim B(n,0.08)$. Require $P(D\geq 1)>0.95$ **M1**

$$1-P(D=0)>0.95 \implies (0.92)^n<0.05 \qquad \textbf{A1}$$

Solving the inequality (GDC or logarithms):

Taking logarithms: $n>\dfrac{\ln 0.05}{\ln 0.92}$ **M1**

$$n>\frac{-2.9957\ldots}{-0.08338\ldots}=35.93\ldots \qquad \textbf{(A1)}$$

Bracketing check:

$n=35$: $(0.92)^{35}=0.05078\ldots>0.05$ $\quad P(D\geq 1)=0.9492\ldots<0.95$ ×

$n=36$: $(0.92)^{36}=0.04672\ldots<0.05$ $\quad P(D\geq 1)=0.9533\ldots>0.95$ ✓

Therefore $N=36$ **A1**

Note: Award final A1 only if bracketing is shown and candidate rounds UP to 36.

Q24 — Binomial Table and Conditional Probability HARD

(a) $X\sim B(20,0.35)$

$a=P(X=4)=\texttt{binompdf(20, 0.35, 4)}$ **(M1)**

$a=0.0738\ldots\approx 0.0738$ **A1**

$b=P(X=6)=\texttt{binompdf(20, 0.35, 6)}=0.1712\ldots\approx 0.1712$ **A1**

Note: Accept $a=0.0738$, $b=0.1712$ *to 4 d.p.*

(b) $P(X \geq 4) = 1 - \texttt{binomcdf(20, 0.35, 3)}$ **M1**

$= 1 - 0.04438\ldots = 0.95562\ldots$

$P(X \leq 6 \mid X \geq 4) = \dfrac{P(4 \leq X \leq 6)}{P(X \geq 4)}$ **M1**

$P(4 \leq X \leq 6) = \texttt{binomcdf(20, 0.35, 6)} - \texttt{binomcdf(20, 0.35, 3)}$

$= 0.41663\ldots - 0.04438\ldots = 0.37225\ldots$

$P(X \leq 6 \mid X \geq 4) = \dfrac{0.37225\ldots}{0.95562\ldots} = 0.3895\ldots \approx 0.390$ **A1**

Note: Boundary: $P(X \leq 6 \mid X \geq 4)$ uses $x = 4, 5, 6$ in the numerator. FT on their $P(X \geq 4)$ provided it is less than 1.

Q25 — Show that Equal Binomial Probabilities Imply p Formula HARD

Writing out the two probability expressions:

$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$ **M1**

$P(X = k+1) = \binom{n}{k+1} p^{k+1} (1-p)^{n-k-1}$

Setting them equal and forming a ratio:

$\dfrac{P(X = k+1)}{P(X = k)} = \dfrac{\binom{n}{k+1}}{\binom{n}{k}} \cdot \dfrac{p}{1-p} = 1$ **M1**

$\dfrac{\binom{n}{k+1}}{\binom{n}{k}} = \dfrac{n!/(k+1)!(n-k-1)!}{n!/k!(n-k)!} = \dfrac{n-k}{k+1}$ **A1**

Solving for p:

$\dfrac{(n-k)p}{(k+1)(1-p)} = 1 \implies (n-k)p = (k+1)(1-p)$ **M1**

$(n-k)p + (k+1)p = k+1 \implies p(n+1) = k+1$

$p = \dfrac{k+1}{n+1}$ **AG A1**

Note: The final line is the printed target and earns nothing additional; the A1 is awarded for the penultimate algebraic step $(n+1)p = k+1$ being clearly shown. Do not award A1 if candidate works backwards from the answer.

Q26 — Least n by Inequality Search then Interval Probability HARD

(a) $X \sim B(n, 0.15)$. Require $P(X \geq 2) > 0.7$

$1 - P(X = 0) - P(X = 1) > 0.7$ **M1**

Using GDC to evaluate for successive n:
$n = 14$: $P(X \geq 2) = 1 - \texttt{binomcdf(14, 0.15, 1)} = 0.6433\ldots < 0.7$
$n = 15$: $P(X \geq 2) = 1 - \texttt{binomcdf(15, 0.15, 1)} = 0.6814\ldots < 0.7$
$n = 16$: $P(X \geq 2) = 1 - \texttt{binomcdf(16, 0.15, 1)} = 0.7161\ldots > 0.7$ **(A1)**

The least value is $n = 16$ **A1**

Note: Both bracketing lines around the crossing point must be shown for the (A1).

(b) $P(2 \leq X \leq 5) = \texttt{binomcdf(16, 0.15, 5)} - \texttt{binomcdf(16, 0.15, 1)}$ **(M1)**

$= 0.98180\ldots - 0.28853\ldots = 0.6933\ldots \approx 0.693$ **A1**

Note: Boundary for (b): $P(2 \leq X \leq 5)$ includes endpoints $x = 2$ and $x = 5$. Evaluated at the corrected $n = 16$.

Q27 — Ratio Condition Finds p then Cdf HARD

Setting up the equation:
$D \sim B(10, p)$. $P(D = 2) = 3\,P(D = 1)$:
$\binom{10}{2} p^2 (1-p)^8 = 3\binom{10}{1} p(1-p)^9$ **M1**

$45\,p^2(1-p)^8 = 30\,p(1-p)^9$ **(A1)**

Dividing by $15\,p(1-p)^8$ (valid since $0 < p < 1$):
$3p = 2(1-p) \implies 3p = 2 - 2p \implies 5p = 2 \implies p = \dfrac{2}{5}$ **A1**

Validity check: $p = 0.4 < 0.5$ ✓ **R1**

Finding the cdf (GDC):
$P(D \leq 3) = \texttt{binomcdf(10, 0.4, 3)} = 0.38228\ldots \approx 0.382$ **A1**

Note: The R1 is awarded for explicitly confirming $p = 0.4 < 0.5$ satisfies the domain condition. If $p > 0.5$ had resulted, a root-rejection would be required.

Q28 — Exact Binomial Pdf by Hand then GDC Complement HARD

(a) $Y \sim B(8, \frac{1}{6})$
$P(Y = 2) = \binom{8}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^6$ **M1**

$= 28 \times \dfrac{1}{36} \times \dfrac{15625}{46656} = \dfrac{437500}{1679616} = \dfrac{109375}{419904}$ **A1**

Note: Accept exact fraction or $0.2604\ldots$; method mark requires $\binom{8}{2}$ seen.

(b) $P(Y \geq 3) = 1 - P(Y \leq 2) = 1 - \texttt{binomcdf}\left(8, \frac{1}{6}, 2\right)$ **M1**

Boundary: $P(Y \leq 2)$ includes $y = 0, 1, 2$ **(A1)**

$= 1 - 0.73415\ldots = 0.26584\ldots \approx 0.266$ **A1**

Note: Accept 0.266 *to 3 s.f.*

Q29 — Least n from Tail Probability Bracketing Required HARD

$X \sim B(n, 0.4)$. Require $P(X = 0) < 0.005$

$P(X = 0) = (0.6)^n < 0.005$ **M1**

Taking logarithms: $n \ln(0.6) < \ln(0.005)$ **M1**

Since $\ln(0.6) < 0$, dividing reverses the inequality:

$$n > \frac{\ln(0.005)}{\ln(0.6)} = \frac{-5.2983\ldots}{-0.5108\ldots} = 10.372\ldots$$ **A1**

Bracketing check:

$n = 10$: $(0.6)^{10} = 0.006047\ldots > 0.005$ condition NOT satisfied **M1**

$n = 11$: $(0.6)^{11} = 0.003628\ldots < 0.005$ condition satisfied

Therefore the least value is $n = 11$ **A1**

Note: Final A1 requires both bracketing lines shown and correct rounding direction (up to 11). Do not award if candidate rounds down to 10.

Q30 — Model Validity Critical Region for Hypothesis Test Context HARD

(a) Under pure guessing, each question has probability $\frac{1}{5}$ of being correct.

$p = 0.2$ **A1**

(b) $S \sim B(15, 0.2)$. Require least integer s such that $P(S \geq s) < 0.05$:

$P(S \geq s) = 1 - \texttt{binomcdf(15, 0.2, s-1)}$ **M1**

Testing values (GDC):

$s = 6$: $P(S \geq 6) = 1 - \texttt{binomcdf(15, 0.2, 5)} = 0.06099\ldots > 0.05$

$s = 7$: $P(S \geq 7) = 1 - \texttt{binomcdf(15, 0.2, 6)} = 0.01806\ldots < 0.05$ **(A1)**

The least value is $s = 7$ **A1**

(c) The four conditions for $B(15, p)$: fixed number of trials (15 questions ✓); two outcomes per trial (correct/incorrect ✓); constant probability of success p per question — this may be doubtful if questions vary in difficulty; independence of outcomes ✓ (reasonable since questions are separate). **R1**

Note: Award R1 for identifying the constant-probability condition as the one that may not hold, with a reason referencing varying difficulty. Accept any valid alternative concern about independence (e.g. if Elif's confidence changes across questions).