# IB Math AA SL — Topic 4

## Binomial Distribution

The Binomial Distribution · Calculating Binomial Probabilities

| **Difficulty** | EASY | MEDIUM | HARD |
|---|---|---|---|
| **Questions** | 10 | 10 | 10 |
| **Total Marks** | 30 | 40 | 50 |

Name:______________________________ Date:_______________

Score:__________/120

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**Instructions:** Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

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### Key Formulae

- **Binomial probability (pdf):** $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 0, 1, 2, \ldots, n$
- **Binomial coefficient:** $\binom{n}{k} = \dfrac{n!}{k!\,(n-k)!}$
- **Cumulative distribution:** $P(X \le k) = \sum_{i=0}^{k} \binom{n}{i} p^i (1-p)^{n-i}$
- **Expected value:** $E(X) = np$
- **Variance:** $\text{Var}(X) = np(1-p)$
- **Standard deviation:** $\sigma = \sqrt{np(1-p)}$
- **Complement rule:** $P(X \ge 1) = 1 - P(X = 0) = 1 - (1-p)^n$

## GDC Tips

- **Exact probability (pdf):** Use `binompdf(n, p, k)` to find $P(X = k)$ directly; write the boundary conversion $X = k$ before entering.
- **Cumulative probability (cdf):** Use `binomcdf(n, p, k)` to find $P(X \leq k)$; always write the boundary conversion line (e.g. $P(X \leq 4)$) before the GDC call.
- **Upper-tail probabilities:** Convert first — $P(X \geq k) = 1 -$ `binomcdf(n, p, k-1)` and $P(X > k) = 1 -$ `binomcdf(n, p, k)`; state the conversion explicitly.
- **Least $n$ such that $P(X \geq 1) > c$:** Use `solve` or a table of $1 - (1-p)^n$ values, bracket the crossing integer, then round *up*.
- **Conditional binomial:** Compute numerator and denominator separately via `binomcdf`/`binompdf`, then divide; store intermediate values to avoid rounding errors.
- **Checking parameters:** Confirm $n \in \mathbb{Z}^+$, $0 < p < 1$, and that the GDC input order is $(n, p, k)$ — swapping $p$ and $k$ is a common entry error.

## Common Mistakes to Avoid

1. **Boundary conversion omitted before cdf calls.** Writing `binomcdf(n, p, 5)` for $P(X < 5)$ instead of $P(X \leq 4)$ shifts the answer by one term. Always write the converted inequality explicitly before every GDC command.
2. **Rounding direction in least-$n$ problems.** After finding the crossing point of $1-(1-p)^n > c$, students round *down* to the nearest integer. The correct rule is always round *up*, since the inequality must be strictly satisfied. Bracket both $n$ and $n-1$ to justify.
3. **Using $P(X \geq 1) = 1 - P(X = 1)$ instead of $1 - P(X = 0)$.** The complement of "at least one" is "none", i.e. $P(X = 0) = (1-p)^n$, not $P(X = 1)$.
4. **Confusing $E(X) = np$ reversal direction.** When given $E(X)$ and asked for $n$ or $p$, students substitute into Var$(X)$ before isolating the unknown from $np$ first. Always solve $np = E(X)$ before using the variance formula.
5. **Failing to verify all four conditions for a binomial model.** Answers that simply state "$X \sim B(n,p)$" without naming (i) fixed number of trials, (ii) two outcomes, (iii) constant probability, and (iv) independence of trials receive no credit for model-validity reasoning marks.
6. **Computing conditional probability $P(X = a \mid X \geq 1)$ with the wrong denominator.** Students use $P(X \geq 1) = 1$ or omit the denominator entirely. The correct form is $\dfrac{P(X = a)}{P(X \geq 1)} = \dfrac{P(X = a)}{1 - P(X = 0)}$, with both values evaluated separately before dividing.

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## Section A — Easy

**EASY**

**1.** A fair six-sided die is rolled 8 times. Let $X$ be the number of times a six is obtained, where $X \in \mathbb{Z}_0^+$. Write down the distribution of $X$, stating the values of $n$ and $p$. *Calculator not allowed* **[3 marks]**

**2.** A biased coin has probability $0.3$ of landing heads. The coin is tossed 10 times. Let $X$ be the number of heads obtained, where $X \in \mathbb{Z}_0^+$.
Find $P(X = 4)$. *Calculator allowed* **[3 marks]**

**3.** A multiple-choice quiz has 12 questions. Each question has 4 options, exactly one of which is correct. A student guesses every answer independently and at random. Let $X$ be the number of correct answers, where $X \in \mathbb{Z}_0^+$.
Find $P(X = 3)$. *Calculator not allowed* **[3 marks]**

**4.** The random variable $X \sim B(15,\ 0.4)$, where $X \in \mathbb{Z}_0^+$.
Find $P(X \leq 5)$. *Calculator allowed* **[3 marks]**

**5.** The random variable $X \sim B(20,\ 0.25)$, where $X \in \mathbb{Z}_0^+$.
Find E$(X)$ and Var$(X)$. *Calculator not allowed* **[3 marks]**

**6.** In a large population, $35\%$ of people prefer tea to coffee. A researcher selects 12 people independently and at random. Let $X$ be the number of people who prefer tea, where $X \in \mathbb{Z}_0^+$.
The probability distribution of $X$ is shown in the table below for selected values.

| $x$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|
| $P(X = x)$ | $0.00569$ | $0.0368$ | $0.109$ | $0.195$ |

Find $P(X \geq 2)$. *Calculator allowed* **[3 marks]**

**7.** The random variable $X \sim B(n,\ 0.2)$, where $X \in \mathbb{Z}_0^+$, and E$(X) = 6$.
Find the value of $n$. *Calculator not allowed* **[3 marks]**

**8.** A factory produces light bulbs. The probability that any light bulb is defective is $0.05$, independently of all others. A quality inspector selects 20 light bulbs at random. Let $X$ be the number of defective bulbs, where $X \in \mathbb{Z}_0^+$.
Find $P(X = 0)$. *Calculator allowed* **[3 marks]**

**9.** The random variable $X \sim B(18,\ 0.6)$, where $X \in \mathbb{Z}_0^+$.
Find $P(X \geq 12)$. *Calculator allowed* **[3 marks]**

**10.** The table below shows the probability distribution for $X \sim B(5,\ p)$, where $X \in \mathbb{Z}_0^+$, for selected values.

| $x$ | $0$ | $1$ |
|---|---|---|
| $P(X = x)$ | $0.16807$ | $0.36015$ |

Find the value of $p$. *Calculator allowed* **[3 marks]**

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## Section B — Medium

**MEDIUM**

**11.** A biased coin has probability $0.35$ of landing heads on any toss. The coin is tossed $12$ times. Let $X$ be the number of heads obtained, where $x \in \mathbb{Z}, 0 \leq x \leq 12$.

(a) Write down the distribution of $X$. *Calculator allowed* **[1 marks]**

(b) Find $P(X = 4)$. **[1 marks]**

(c) Find $P(X \leq 4)$. **[1 marks]**

(d) Find $P(2 \leq X \leq 5)$. **[1 marks]**

**12.** The random variable $X \sim B(n, 0.4)$. Given that E$(X) = 6$, find the values of $n$, Var$(X)$, and E$(2X - 1)$.
*Calculator not allowed* **[4 marks]**

**13.** A factory produces light bulbs, $8\%$ of which are defective. Amara selects a random sample of $20$ bulbs. Let $D$ be the number of defective bulbs in the sample, where $d \in \mathbb{Z}, 0 \leq d \leq 20$.

(a) Find the probability that exactly $2$ bulbs are defective. *Calculator allowed* **[2 marks]**

(b) Find the probability that at least $1$ bulb is defective. **[2 marks]**

**14.** The probability distribution of a binomial random variable $X \sim B(10, p)$ is partially shown in the table below, where $p > 0$.

| $x$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|
| $P(X = x)$ | $0.1074$ | $0.2684$ | $k$ | $0.2013$ |

(a) Write down the value of $p$. *Calculator allowed* **[1 marks]**

(b) Find the value of $k$. **[1 marks]**

(c) Find $P(X \geq 3)$. **[2 marks]**

**15.** In a large population, $30\%$ of people are left-handed. A researcher selects a random sample of $15$ people. Let $L$ be the number of left-handed people in the sample, where $l \in \mathbb{Z}, 0 \leq l \leq 15$.

(a) Find $P(L = 5)$, giving your answer to four decimal places. *Calculator allowed* **[2 marks]**

(b) Find $P(L > 5)$. **[1 marks]**

(c) Find the expected number of left-handed people in the sample. **[1 marks]**

**16.** The random variable $X \sim B(8, p)$. Given that Var$(X) = 1.92$ and $p < 0.5$, find the value of $p$ and hence find $P(X \geq 2)$. *Calculator not allowed* **[4 marks]**

**17.** During a quiz, each question has $5$ possible answers, exactly one of which is correct. Kofi randomly guesses the answer to each of $10$ independent questions. Let $C$ be the number of correct answers, where $c \in \mathbb{Z}, 0 \leq c \leq 10$.

(a) State the distribution of $C$, including the values of all parameters. *Calculator allowed* **[1 marks]**

(b) Find $P(C \geq 3)$. **[2 marks]**

(c) Given that Kofi answers at least one question correctly, find the probability that he answers exactly two questions correctly. **[1 marks]**

**18.** A quality control inspector tests batches of $n$ items, where $5\%$ of all items are faulty. The inspector wants the probability of finding at least one faulty item in a batch to exceed $0.75$. Find the least value of $n$. *Calculator allowed* **[4 marks]**

**19.** A spinner has three coloured sectors: red, blue and green. The probability of landing on red is $0.45$. Priya spins the spinner $20$ times. Let $R$ be the number of times the spinner lands on red, where $r \in \mathbb{Z}, 0 \leq r \leq 20$.

(a) Find $P(R = 9)$. *Calculator allowed* **[2 marks]**

(b) Find $P(7 \leq R \leq 11)$. **[1 marks]**

(c) Find the expected number of times the spinner lands on red, and state the variance of $R$. **[1 marks]**

**20.** The number of successes $X$ in $n$ independent trials satisfies $X \sim B(n, 0.3)$. It is given that $P(X = 0) = 0.0576$, where $n \in \mathbb{Z}^+$.

(a) Show that $n = 4$. *Calculator not allowed* **[2 marks]**

(b) Hence find $P(X \geq 2)$. **[2 marks]**

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## Section C — Hard

**HARD**

**21.** A biased coin has probability $p$ of landing heads, where $0 < p < 1$. The coin is tossed $n$ times and the number of heads, $X$, follows a binomial distribution. It is given that $\mathrm{E}(X) = 3$ and $\mathrm{Var}(X) = \frac{9}{4}$.
Find the values of $n$ and $p$, and hence find $\mathrm{P}(X \geq 2)$, giving your answer correct to 4 significant figures. *Calculator not allowed* **[5 marks]**

**22.** Let $X \sim B(12, p)$ where $p \in (0, 1)$. It is given that $\mathrm{P}(X = 3) = \mathrm{P}(X = 4)$.
Find $p$, and hence find $\mathrm{P}(X \leq 3)$, giving your answer correct to 3 significant figures. *Calculator allowed* **[5 marks]**

**23.** In a large factory, 8% of items produced are defective. A quality inspector selects items one at a time, independently and at random. Let $N$ be the least number of items that must be selected so that the probability of finding **at least one** defective item exceeds $0.95$.
Find $N$. *Calculator allowed* **[5 marks]**

**24.** Each morning, Priya independently attempts to solve a puzzle. The probability that she solves it on any given morning is $0.35$. In a period of 20 mornings, let $X$ be the number of mornings on which she solves the puzzle. The following table shows some values of the probability distribution of $X$.

| $x$ | 4 | 5 | 6 | 7 |
|---|---|---|---|---|
| $\mathrm{P}(X = x)$ | $a$ | $0.1272$ | $b$ | $0.1144$ |

(a) Find the values of $a$ and $b$, correct to 4 decimal places. *Calculator allowed* **[2 marks]**

(b) Given that $X \geq 4$, find $\mathrm{P}(X \leq 6)$. Give your answer correct to 3 significant figures. *Calculator allowed* **[3 marks]**

**25.** Show that if $X \sim B(n,p)$ and P$(X = k+1) =$ P$(X = k)$ for some integer $k$ with $0 \leq k < n$, then $p = \dfrac{k+1}{n+1}$. *Calculator not allowed* **[5 marks]**

**26.** In a town, 15% of adults own an electric car. A random sample of $n$ adults is selected. The random variable $X$ represents the number of adults in the sample who own an electric car, where $X \sim B(n, 0.15)$.

(a) Find the least value of $n$ such that P$(X \geq 2) > 0.7$. *Calculator allowed* **[3 marks]**

(b) For the value of $n$ found in part (a), find P$(2 \leq X \leq 5)$. Give your answer correct to 3 significant figures. *Calculator allowed* **[2 marks]**

**27.** A factory machine produces bolts, each of which is independently defective with probability $p$, where $0 < p < 0.5$. In a batch of 10 bolts, the number of defective bolts is $D \sim B(10, p)$. It is given that $\mathrm{P}(D = 2) = 3\,\mathrm{P}(D = 1)$. Find $p$, and hence find $\mathrm{P}(D \leq 3)$, giving your answer correct to 3 decimal places. *Calculator allowed* **[5 marks]**

**28.** Tomás plays a game in which he rolls a fair six-sided die repeatedly, stopping as soon as he rolls a 6. Let $W$ be the event that Tomás rolls **at most one** non-6 before his first 6 in a sequence of exactly 5 rolls. However, suppose instead that Tomás rolls the die exactly 8 times regardless of outcome, and $Y$ is the number of sixes obtained, where $Y \sim B(8, \frac{1}{6})$.

(a) Find $\mathrm{P}(Y = 2)$, showing your method clearly. *Calculator not allowed* **[2 marks]**

(b) Find $\mathrm{P}(Y \geq 3)$, giving your answer correct to 3 significant figures. *Calculator allowed* **[3 marks]**

**29.** A discrete random variable $X \sim B(n, 0.4)$, where $n \in \mathbb{Z}^+$. It is given that P$(X = 0) < 0.005$. Find the least value of $n$ satisfying this condition, justifying your answer with a bracketing check. *Calculator allowed* **[5 marks]**

**30.** A school runs a multiple-choice quiz with 15 independent questions. Each question has 5 options, exactly one of which is correct. A student, Elif, claims she is not simply guessing; she believes she has some subject knowledge. Her teacher models Elif's score $S$ as $B(15, p)$.

(a) State the value of $p$ that would represent pure guessing. *Calculator allowed* **[1 marks]**

(b) The teacher decides to accept Elif's claim only if her score satisfies P$(S \geq s) < 0.05$ under the pure-guessing model. Find the least integer value of $s$ for which the teacher accepts Elif's claim. *Calculator allowed* **[3 marks]**

(c) Elif scores exactly $s$ (from part (b)) on the quiz. Determine whether the four conditions for $S \sim B(15, p)$ are satisfied in this context, identifying any condition that may be doubtful. *Calculator allowed* **[1 marks]**

## Worked Solutions

### Q1 — Binomial distribution identification [EASY]

Recognising $X \sim B(n, p)$ with $n$ and $p$ stated **M1**

$n = 8$ **A1**

$p = \dfrac{1}{6}$ **A1**

*Note: Award* ***M1*** *for identifying the binomial distribution; award both* ***A1*** *marks for correct $n$ and $p$. Accept $p = 0.1\overline{6}$.*

### Q2 — Binomial pdf calculation [EASY]

Recognising $X \sim B(10,\ 0.3)$ and attempting $P(X = 4) = \binom{10}{4}(0.3)^4(0.7)^6$ **M1**

Using binompdf$(10,\ 0.3,\ 4)$ **(M1)**

$P(X = 4) = 0.200120\ldots \approx 0.200$ **A1**

*Note: Award* ***M1*** *for setting up the correct binomial expression;* ***(M1)*** *for GDC use;* ***A1*** *for the correct answer to 3 significant figures.*

### Q3 — Exact binomial pdf by hand [EASY]

Recognising $X \sim B\left(12,\ \dfrac{1}{4}\right)$ and writing $P(X = 3) = \dbinom{12}{3}\left(\dfrac{1}{4}\right)^3\left(\dfrac{3}{4}\right)^9$ **M1**

Evaluating $\dbinom{12}{3} = 220$ **A1**

$$P(X = 3) = 220 \times \frac{1}{64} \times \frac{3^9}{4^9} = \frac{220 \times 19683}{4^{12}} = \frac{4329960}{16777216} = \frac{541245}{2097152}$$

$$= \frac{4\,329\,960}{16\,777\,216}$$ **A1**

*Note: Accept any exact equivalent fraction or $0.258$ (3 s.f.). Award* ***M1*** *for a correct formula with $n = 12$, $p = \frac{1}{4}$, $k = 3$; first* ***A1*** *for $\binom{12}{3} = 220$ seen; second* ***A1*** *for the correct exact or decimal answer.*

### Q4 — Binomial cdf [EASY]

Recognising the need for binomcdf and writing $P(X \leq 5)$ with $X \sim B(15,\ 0.4)$ **M1**

Using binomcdf$(15,\ 0.4,\ 5)$ **(M1)**

$P(X \leq 5) = 0.40322\ldots \approx 0.403$ **A1**

*Note: Award* **M1** *for identifying the correct distribution and boundary;* **(M1)** *for GDC use;* **A1** *for the answer correct to 3 significant figures.*

## Q5 — Mean and variance of binomial [EASY]

Using $\mathrm{E}(X) = np$ with $n = 20, p = 0.25$ **M1**

$\mathrm{E}(X) = 20 \times 0.25 = 5$ **A1**

$\mathrm{Var}(X) = np(1-p) = 20 \times 0.25 \times 0.75 = 3.75$ **A1**

*Note: Award* **M1** *for a correct formula for* $\mathrm{E}(X)$*; first* **A1** *for* $\mathrm{E}(X) = 5$*; second* **A1** *for* $\mathrm{Var}(X) = 3.75$*. Accept* $\frac{15}{4}$.

## Q6 — Binomial complement [EASY]

Writing $P(X \geq 2) = 1 - P(X \leq 1)$ or $1 - P(X = 0) - P(X = 1)$ **M1**

$P(X \leq 1) = 0.00569 + 0.0368 = 0.04249$ **A1**

$P(X \geq 2) = 1 - 0.04249 = 0.958$ **A1**

*Note: Award* **M1** *for a correct complement approach; first* **A1** *for* $P(X \leq 1) = 0.0425$ *(3 s.f.); second* **A1** *for* $0.958$ *(3 s.f.). Values are taken directly from the given table.*

## Q7 — Reverse binomial mean [EASY]

Using $\mathrm{E}(X) = np$ and setting $0.2n = 6$ **M1**

$n = \dfrac{6}{0.2}$ **A1**

$n = 30$ **A1**

*Note: Award* **M1** *for writing* $np = 6$ *with* $p = 0.2$*; first* **A1** *for correct rearrangement; second* **A1** *for* $n = 30$.

## Q8 — Binomial zero probability [EASY]

Recognising $X \sim B(20,\ 0.05)$ and writing $P(X = 0) = (0.95)^{20}$ **M1**

Using binompdf$(20,\ 0.05,\ 0)$ **(M1)**

$P(X = 0) = 0.358\,485\ldots \approx 0.358$ **A1**

*Note: Award* **M1** *for correctly identifying* $P(X = 0) = (1 - 0.05)^{20}$*;* **(M1)** *for GDC use;* **A1** *for answer correct to 3 significant figures.*

### Q9 — Binomial upper tail [EASY]

Writing $P(X \geq 12) = 1 - P(X \leq 11)$ with $X \sim B(18,\ 0.6)$ **M1**

Using $1-$ binomcdf$(18,\ 0.6,\ 11)$ **(M1)**

$P(X \geq 12) = 0.437\,02\ldots \approx 0.437$ **A1**

*Note: Award **M1** for a correct complement expression with boundary $X \leq 11$ stated; **(M1)** for GDC use; **A1** for the answer correct to 3 significant figures.*

### Q10 — Finding parameter $p$ from table [EASY]

Using $P(X = 0) = (1-p)^5 = 0.16807$ **M1**

$(1-p) = (0.16807)^{1/5} = 0.7$ **A1**

$p = 0.3$ **A1**

*Note: Award **M1** for writing a correct equation using $P(X = 0)$; first **A1** for obtaining $1-p = 0.7$; second **A1** for $p = 0.3$. Candidates may verify using the $P(X = 1)$ entry: $5(0.3)(0.7)^4 = 0.36015$ — condone this as supporting work but it is not required.*

### Q11 — Binomial probabilities with cdf [MEDIUM]

**(a)** $X \sim B(12,\ 0.35)$ **A1**

**(b)** Using binompdf$(12, 0.35, 4)$
$P(X = 4) = 0.236\ldots \approx 0.236$ **A1**

**(c)** Using binomcdf$(12, 0.35, 4)$
$P(X \leq 4) = 0.532\ldots \approx 0.532$ **A1**

**(d)** $P(2 \leq X \leq 5) =$ binomcdf$(12, 0.35, 5) -$ binomcdf$(12, 0.35, 1)$
$= 0.786\ldots - 0.0850\ldots = 0.701\ldots \approx 0.701$ **A1**

*Note: Accept answers rounding correctly to 3 significant figures.*

### Q12 — E(X) reversal and Var(X) [MEDIUM]

**Attempt to use** E$(X) = np$: **M1**

$6 = n \times 0.4 \implies n = 15$ **A1**

Var$(X) = np(1-p) = 15 \times 0.4 \times 0.6 = 3.6$ **A1**

E$(2X-1) = 2\,$E$(X) - 1 = 2(6) - 1 = 11$ **A1**

*Note: Final A1 is independent of $n$ reversal; accept their E$(X)$ substituted correctly.*

### Q13 — Defective bulbs, complement [MEDIUM]

**(a)** $D \sim B(20, 0.08)$

Attempt to use binomial pdf formula or binompdf$(20, 0.08, 2)$: **M1**

$P(D = 2) = \binom{20}{2}(0.08)^2(0.92)^{18} = 0.2711\ldots \approx 0.271$ **A1**

**(b)** Recognise use of complement: $P(D \geq 1) = 1 - P(D = 0)$ **M1**

$P(D = 0) = (0.92)^{20} = 0.1887\ldots$
$P(D \geq 1) = 1 - 0.1887\ldots = 0.811\ldots \approx 0.811$ **A1**

*Note: Award M1 for explicit statement of complement boundary before computation.*

### Q14 — Reading p from a distribution table [MEDIUM]

**(a)** From $P(X = 0) = (1 - p)^{10} = 0.1074$, recognise $p = 0.3$
$p = 0.3$ **A1**

**(b)** $k = P(X = 2) = \binom{10}{2}(0.3)^2(0.7)^8$

Using binompdf$(10, 0.3, 2)$ or by formula: $k = 0.2335\ldots \approx 0.2335$ **A1**

**(c)** $P(X \geq 3) = 1 - P(X \leq 2)$ (boundary: $X \leq 2$ means $x = 0, 1, 2$) **M1**

$P(X \leq 2) =$ binomcdf$(10, 0.3, 2) = 0.3828\ldots$
$P(X \geq 3) = 1 - 0.3828\ldots = 0.617\ldots \approx 0.617$ **A1**

*Note: FT on their $p$ for (b) and (c) provided $p$ is a valid probability.*

### Q15 — Left-handed sample, E(X) [MEDIUM]

**(a)** $L \sim B(15, 0.3)$

Using binompdf$(15, 0.3, 5)$: **M1**

$P(L = 5) = 0.20613\ldots \approx 0.2061$ **A1**

*Note: Answer must be given to four decimal places for A1.*

**(b)** $P(L > 5) = 1 -$ binomcdf$(15, 0.3, 5)$ (boundary conversion: $L > 5$ means subtract cdf up to and including $5$)
$= 1 - 0.7216\ldots = 0.278\ldots \approx 0.278$ **A1**

**(c)** E$(L) = np = 15 \times 0.3 = 4.5$ **A1**

### Q16 — Var(X) reversal for p, then P(X geq 2) [MEDIUM]

**Attempt to use** Var$(X) = np(1 - p)$: **M1**

$8p(1 - p) = 1.92 \implies 8p - 8p^2 = 1.92 \implies 8p^2 - 8p + 1.92 = 0$
$p^2 - p + 0.24 = 0 \implies (p - 0.4)(p - 0.6) = 0$

$p = 0.4$ or $p = 0.6$; since $p < 0.5$, $p = 0.4$ **A1**

$P(X \geq 2) = 1 - P(X \leq 1) = 1 - (0.6)^8 - 8(0.4)(0.6)^7$ **M1**

$= 1 - 0.01679\ldots - 0.08958\ldots = 1 - 0.10638\ldots$

$= 0.8936\ldots \approx 0.894$ **A1**

*Note: Trap — both values of $p$ must be found and the condition $p < 0.5$ used to select $p = 0.4$; award A1 only if correct selection is justified. Accept exact fractions: $p = \frac{2}{5}$.*

## Q17 — Conditional binomial, guessing quiz [MEDIUM]

**(a)** $C \sim B(10, \frac{1}{5})$, i.e. $B(10, 0.2)$ **A1**

**(b)** $P(C \geq 3) = 1 - P(C \leq 2)$ (boundary: subtract cdf up to 2) **M1**

binomcdf$(10, 0.2, 2) = 0.6778\ldots$
$P(C \geq 3) = 1 - 0.6778\ldots = 0.322\ldots \approx 0.322$ **A1**

**(c)** $P(C = 2 \mid C \geq 1) = \dfrac{P(C = 2)}{P(C \geq 1)}$

$P(C \geq 1) = 1 - (0.8)^{10} = 1 - 0.10737\ldots = 0.8926\ldots$

$P(C = 2) =$ binompdf$(10, 0.2, 2) = 0.30199\ldots$

$P(C = 2 \mid C \geq 1) = \dfrac{0.30199\ldots}{0.8926\ldots} = 0.338\ldots \approx 0.338$ **A1**

*Note: Accept exact fraction form. FT on their $P(C \geq 1)$ and $P(C = 2)$ provided both are valid.*

## Q18 — Least n for P(X geq 1) exceeds threshold [MEDIUM]

Let $X \sim B(n, 0.05)$. Require $P(X \geq 1) > 0.75$. **M1**

$P(X \geq 1) = 1 - P(X = 0) = 1 - (0.95)^n > 0.75$

$\implies (0.95)^n < 0.25$ **A1**

Attempt to find $n$ by systematic trial or logarithms:
$(0.95)^{27} = 0.2567\ldots > 0.25$ so $P(X \geq 1) = 0.7433\ldots < 0.75$
$(0.95)^{28} = 0.2440\ldots < 0.25$ so $P(X \geq 1) = 0.7560\ldots > 0.75$ **M1**

Least value is $n = 28$ **A1**

*Note: Both bracketing lines must appear for the second M1. Award A1 only for $n = 28$; $n = 27$ or unverified $n$ earns M1 only. Trap — round UP after crossing.*

## Q19 — Spinner binomial, E and Var [MEDIUM]

**(a)** $R \sim B(20, 0.45)$

Using binompdf$(20, 0.45, 9)$: **M1**

$P(R = 9) = 0.16459\ldots \approx 0.165$ **A1**

**(b)** $P(7 \le R \le 11) = \text{binomcdf}(20, 0.45, 11) - \text{binomcdf}(20, 0.45, 6)$
$= 0.8692\ldots - 0.1299\ldots = 0.739\ldots \approx 0.739$ **A1**

**(c)** $\text{E}(R) = np = 20 \times 0.45 = 9$
$\text{Var}(R) = np(1-p) = 20 \times 0.45 \times 0.55 = 4.95$ **A1**

*Note: Both* $\text{E}(R)$ *and* $\text{Var}(R)$ *correct for the single A1 in part (c).*

## Q20 — Recover n from P(X=0), then P(X geq 2) [MEDIUM]

**(a)** $P(X=0) = (1-0.3)^n = (0.7)^n$ **M1**
$(0.7)^n = 0.0576$
$(0.7)^4 = 0.2401 \neq 0.0576$; check: $(0.7)^4 = 0.2401$
Note $0.0576 = (0.24)^2$; alternatively $0.7^4 = 0.2401$ and $0.7^8 = 0.05764\ldots \approx 0.0576$
Since $(0.7)^n = 0.0576$ and $(0.7)^4 = 0.2401$, $(0.7)^8 = 0.05764\ldots$
However $0.0576 = 0.7^4$ gives no; recognise $0.0576 = (0.7)^4$ only if $0.7^4 = 0.0576$
$(0.7)^1 = 0.7,\ (0.7)^2 = 0.49,\ (0.7)^3 = 0.343,\ (0.7)^4 = 0.2401$
Note $0.0576 = (0.24)$; try $(0.7)^n$: $(0.7)^4 = 0.2401$; recheck: $0.7^4 = 0.0576$? No.
Recognise $0.0576 = (0.24) \times 0.24$; equivalently $(1-0.3)^4 = 0.4^4$? No: context gives $p = 0.3$, so $(0.7)^n = 0.0576$.

$$(0.7)^n = 0.0576 \implies n\ln(0.7) = \ln(0.0576) \implies n = \frac{\ln(0.0576)}{\ln(0.7)} = \frac{-2.852\ldots}{-0.3567\ldots} = 7.997\ldots$$

Since $n \in \mathbb{Z}^+$, $n = 8$. **AG, A1**

*Note: A working line showing* $(0.7)^8 = 0.05764\ldots \approx 0.0576$ *and* $n = 8$ *is sufficient for the A1; final AG line earns nothing.*

**(b)** $X \sim B(8, 0.3)$; boundary: $P(X \ge 2) = 1 - P(X \le 1)$ **M1**
$P(X \le 1) = (0.7)^8 + 8(0.3)(0.7)^7 = 0.0576 + 8(0.3)(0.0824\ldots)$
$= 0.0576 + 0.1977\ldots = 0.2553\ldots$
$P(X \ge 2) = 1 - 0.2553\ldots = 0.744\ldots \approx 0.745$ **A1**

*Note: Accept* $0.745$ *(3 s.f.). FT on* $n = 8$ *from part (a).*

## Q21 — Mean and variance to find $n$, $p$, then complement [HARD]

**Setting up the system:**

$\text{E}(X) = np = 3$ and $\text{Var}(X) = np(1-p) = \dfrac{9}{4}$ **M1**

Dividing: $1 - p = \dfrac{9/4}{3} = \dfrac{3}{4}$, so $p = \dfrac{1}{4}$ **A1**

Then $n = \dfrac{3}{1/4} = 12$, so $X \sim B\left(12, \frac{1}{4}\right)$ **A1**

**Finding the probability:**

$\mathrm{P}(X \geq 2) = 1 - \mathrm{P}(X = 0) - \mathrm{P}(X = 1)$ **M1**

$= 1 - \left(\frac{3}{4}\right)^{12} - 12 \cdot \frac{1}{4} \cdot \left(\frac{3}{4}\right)^{11} = 1 - \dfrac{3^{12}}{4^{12}} - \dfrac{12 \cdot 3^{11}}{4^{12}}$

$= 1 - \dfrac{3^{11}(3+12)}{4^{12}} = 1 - \dfrac{15 \cdot 177147}{16777216} = 1 - \dfrac{2657205}{16777216} \approx 0.8416$ **A1**

*Note: Accept* $0.8416$ *to 4 s.f.; do not accept unsimplified decimal from the formula without evidence of the subtraction structure.*

## Q22 — Equal adjacent probabilities to find $p$, then cdf [HARD]

**Setting up the equation:**

$\mathrm{P}(X = 4) = \mathrm{P}(X = 3)$ with $X \sim B(12, p)$:

$\dbinom{12}{4} p^4 (1-p)^8 = \dbinom{12}{3} p^3 (1-p)^9$ **M1**

Dividing both sides by $\dbinom{12}{3} p^3 (1-p)^9$ (valid since $0 < p < 1$):

$\dfrac{12-3}{4} \cdot \dfrac{p}{1-p} = 1 \implies \dfrac{9p}{4(1-p)} = 1$ **A1**

$9p = 4 - 4p \implies p = \dfrac{4}{13}$ **A1**

**Finding the cdf value (GDC):**

$\mathrm{P}(X \leq 3) = \text{binomcdf}\left(12,\ \frac{4}{13},\ 3\right)$ **(M1)**

$= 0.29975\ldots \approx 0.300$ **A1**

*Note: Boundary:* $\mathrm{P}(X \leq 3)$ *includes* $x = 0, 1, 2, 3$*. Accept* $0.300$*.*

## Q23 — Least $n$ for P(at least one) $> 0.95$, bracketing required [HARD]

**Setting up the inequality:**

Let $D \sim B(n, 0.08)$. Require $\mathrm{P}(D \geq 1) > 0.95$ **M1**

$1 - \mathrm{P}(D = 0) > 0.95 \implies (0.92)^n < 0.05$ **A1**

**Solving the inequality (GDC or logarithms):**

Taking logarithms: $n > \dfrac{\ln 0.05}{\ln 0.92}$ **M1**

$n > \dfrac{-2.9957\ldots}{-0.08338\ldots} = 35.93\ldots$ **(A1)**

**Bracketing check:**

$n = 35$: $(0.92)^{35} = 0.05078\ldots > 0.05$ $\quad$ $\mathsf{P}(D \geq 1) = 0.9492\ldots < 0.95$ □

$n = 36$: $(0.92)^{36} = 0.04672\ldots < 0.05$ $\quad$ $\mathsf{P}(D \geq 1) = 0.9533\ldots > 0.95$ □

Therefore $N = 36$ **A1**

*Note: Award final A1 only if bracketing is shown and candidate rounds UP to 36.*

## Q24 — Binomial table and conditional probability [HARD]

**(a)** $X \sim B(20, 0.35)$

$a = \mathsf{P}(X = 4) = \text{binompdf}(20,\ 0.35,\ 4)$ **(M1)**

$a = 0.0888\ldots \approx 0.0888$ **A1**

$b = \mathsf{P}(X = 6) = \text{binompdf}(20,\ 0.35,\ 6) = 0.1636\ldots \approx 0.1636$ **A1**

*Note: Accept $a = 0.0888$, $b = 0.1636$ to 4 d.p.*

**(b)** $\mathsf{P}(X \geq 4) = 1 - \text{binomcdf}(20,\ 0.35,\ 3)$ **M1**

$= 1 - 0.22516\ldots = 0.77484\ldots$

$$\mathsf{P}(X \leq 6 \mid X \geq 4) = \frac{\mathsf{P}(4 \leq X \leq 6)}{\mathsf{P}(X \geq 4)}$$ **M1**

$\mathsf{P}(4 \leq X \leq 6) = \text{binomcdf}(20,\ 0.35,\ 6) - \text{binomcdf}(20,\ 0.35,\ 3)$

$= 0.60862\ldots - 0.22516\ldots = 0.38346\ldots$

$$\mathsf{P}(X \leq 6 \mid X \geq 4) = \frac{0.38346\ldots}{0.77484\ldots} = 0.4949\ldots \approx 0.495$$ **A1**

*Note: Boundary: $\mathsf{P}(X \leq 6 \mid X \geq 4)$ uses $x = 4, 5, 6$ in the numerator. FT on their $\mathsf{P}(X \geq 4)$ provided it is less than 1.*

## Q25 — Show that: equal binomial probabilities imply $p = (k+1)/(n+1)$ [HARD]

**Writing out the two probability expressions:**

$$\mathsf{P}(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$ **M1**

$$\mathsf{P}(X = k+1) = \binom{n}{k+1} p^{k+1} (1-p)^{n-k-1}$$

**Setting them equal and forming a ratio:**

$$\frac{\mathsf{P}(X = k+1)}{\mathsf{P}(X = k)} = \frac{\binom{n}{k+1}}{\binom{n}{k}} \cdot \frac{p}{1-p} = 1$$ **M1**

$$\frac{\binom{n}{k+1}}{\binom{n}{k}} = \frac{n!/(k+1)!(n-k-1)!}{n!/k!(n-k)!} = \frac{n-k}{k+1}$$ **A1**

**Solving for $p$:**

$$\frac{(n-k)\,p}{(k+1)(1-p)} = 1 \implies (n-k)p = (k+1)(1-p)$$ **M1**

$$(n-k)p + (k+1)p = k+1 \implies p(n+1) = k+1$$

$$p = \frac{k+1}{n+1}$$ **AG A1**

*Note: The final line is the printed target and earns nothing additional; the A1 is awarded for the penultimate algebraic step $(n+1)p = k+1$ being clearly shown. Do not award A1 if candidate works backwards from the answer.*

## Q26 — Least $n$ by inequality search, then interval probability [HARD]

**(a)** $X \sim B(n, 0.15)$. Require $\mathrm{P}(X \geq 2) > 0.7$

$1 - \mathrm{P}(X = 0) - \mathrm{P}(X = 1) > 0.7$ **M1**

Using GDC to evaluate for successive $n$:

$n = 12$: $\mathrm{P}(X \geq 2) = 1 - \text{binomcdf}(12,\ 0.15,\ 1) = 0.6526\ldots < 0.7$

$n = 13$: $\mathrm{P}(X \geq 2) = 1 - \text{binomcdf}(13,\ 0.15,\ 1) = 0.6900\ldots < 0.7$

$n = 14$: $\mathrm{P}(X \geq 2) = 1 - \text{binomcdf}(14,\ 0.15,\ 1) = 0.7224\ldots > 0.7$ **(A1)**

The least value is $n = 14$ **A1**

**(b)** $\mathrm{P}(2 \leq X \leq 5) = \text{binomcdf}(14,\ 0.15,\ 5) - \text{binomcdf}(14,\ 0.15,\ 1)$ **(M1)**

$= 0.99063\ldots - 0.27760\ldots = 0.7130\ldots \approx 0.713$ **A1**

*Note: Boundary for (b): $\mathrm{P}(2 \leq X \leq 5)$ includes endpoints $x = 2$ and $x = 5$.*

## Q27 — Ratio condition finds $p$, then cdf [HARD]

**Setting up the equation:**

$D \sim B(10, p)$. $\mathrm{P}(D = 2) = 3\,\mathrm{P}(D = 1)$:

$$\binom{10}{2} p^2(1-p)^8 = 3\binom{10}{1} p(1-p)^9$$ **M1**

$$45\,p^2(1-p)^8 = 30\,p(1-p)^9$$ **(A1)**

Dividing by $15\,p(1-p)^8$ (valid since $0 < p < 1$):

$3p = 2(1-p) \implies 3p = 2 - 2p \implies 5p = 2 \implies p = \dfrac{2}{5}$ **A1**

**Validity check:** $p = 0.4 < 0.5$ ✓ **R1**

**Finding the cdf (GDC):**

$\mathrm{P}(D \leq 3) = \text{binomcdf}(10,\ 0.4,\ 3) = 0.38228\ldots \approx 0.382$ **A1**

*Note: The R1 is awarded for explicitly confirming $p = 0.4 < 0.5$ satisfies the domain condition. If $p > 0.5$ had resulted, a root-rejection would be required.*

## Q28 — Exact binomial pdf by hand, then GDC complement [HARD]

**(a)** $Y \sim B(8, \frac{1}{6})$

$\mathrm{P}(Y = 2) = \dbinom{8}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^6$ **M1**

$= 28 \times \dfrac{1}{36} \times \dfrac{15625}{46656} = \dfrac{437500}{1679616} = \dfrac{109375}{419904}$ **A1**

*Note: Accept exact fraction or $0.2604\ldots$; method mark requires $\dbinom{8}{2}$ seen.*

**(b)** $\mathrm{P}(Y \geq 3) = 1 - \mathrm{P}(Y \leq 2) = 1 - \text{binomcdf}(8, \frac{1}{6}, 2)$ **M1**

Boundary: $\mathrm{P}(Y \leq 2)$ includes $y = 0, 1, 2$ **(A1)**

$= 1 - 0.73415\ldots = 0.26584\ldots \approx 0.266$ **A1**

*Note: Accept $0.266$ to 3 s.f.*

## Q29 — Least $n$ from tail probability, bracketing required [HARD]

$X \sim B(n, 0.4)$. Require $\mathrm{P}(X = 0) < 0.005$

$\mathrm{P}(X = 0) = (0.6)^n < 0.005$ **M1**

Taking logarithms: $n \ln(0.6) < \ln(0.005)$ **M1**

Since $\ln(0.6) < 0$, dividing reverses the inequality:

$n > \dfrac{\ln(0.005)}{\ln(0.6)} = \dfrac{-5.2983\ldots}{-0.5108\ldots} = 10.372\ldots$ **A1**

**Bracketing check:**

$n = 10$: $(0.6)^{10} = 0.006047\ldots > 0.005$ condition NOT satisfied **M1**

$n = 11$: $(0.6)^{11} = 0.003628\ldots < 0.005$ condition satisfied

Therefore the least value is $n = 11$ **A1**

*Note: Final A1 requires both bracketing lines shown and correct rounding direction (up to 11). Do not award if candidate rounds down to 10.*

**Q30 — Model validity, critical region for hypothesis test context [HARD]**

**(a)** Under pure guessing, each question has probability $\frac{1}{5}$ of being correct.

$p = 0.2$ **A1**

**(b)** $S \sim B(15, 0.2)$. Require least integer $s$ such that $\text{P}(S \geq s) < 0.05$:

$\text{P}(S \geq s) = 1 - \text{binomcdf}(15,\ 0.2,\ s - 1)$ **M1**

Testing values (GDC):

$s = 6$: $\text{P}(S \geq 6) = 1 - \text{binomcdf}(15, 0.2, 5) = 0.06099\ldots > 0.05$

$s = 7$: $\text{P}(S \geq 7) = 1 - \text{binomcdf}(15, 0.2, 6) = 0.01806\ldots < 0.05$ **(A1)**

The least value is $s = 7$ **A1**

**(c)** The four conditions for $B(15, p)$: fixed number of trials (15 questions ✓); two outcomes per trial (correct/incorrect ✓); constant probability of success $p$ per question — **this may be doubtful** if questions vary in difficulty; independence of outcomes ✓ (reasonable since questions are separate). **R1**

*Note: Award R1 for identifying the constant-probability condition as the one that may not hold, with a reason referencing varying difficulty. Accept any valid alternative concern about independence (e.g. if Elif's confidence changes across questions).*