IB Math AA SL — Topic 4

Normal Distribution

The Normal Curve · Normal Calculations · z-Values · Unknown Parameters

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:______________________________ Date:________________

Score:__________/120

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Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

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Key Formulae

- **Normal distribution notation:** $X \sim N(\mu, \sigma^2)$ where μ is the mean and σ^2 is the variance.
- **Standardisation (z-score):** $z = \dfrac{x - \mu}{\sigma}$, so $Z \sim N(0, 1)$.
- **Symmetry of the standard normal:** $P(Z \leq -z) = P(Z \geq z) = 1 - P(Z \leq z)$.
- **Complement rule:** $P(X > a) = 1 - P(X \leq a)$.
- **Interval probability:** $P(a \leq X \leq b) = P(X \leq b) - P(X \leq a)$.
- **z-equation from a known tail probability** (e.g. lower tail p): $\dfrac{x - \mu}{\sigma} = z_p$, where $z_p = \Phi^{-1}(p)$.
- **Two-parameter system:** given $P(X \leq x_1) = p_1$ and $P(X \leq x_2) = p_2$, form $\dfrac{x_1 - \mu}{\sigma} = z_1$ and $\dfrac{x_2 - \mu}{\sigma} = z_2$, then solve simultaneously for μ and σ.

GDC Tips

- **Forward probability (lower tail):** `normCdf(`$-\infty$`, `b`, `μ`, `σ`)` gives $P(X \leq b)$; use -10^{99} as the lower bound for $-\infty$.
- **Interval probability:** `normCdf(`a`, `b`, `μ`, `σ`)` gives $P(a \leq X \leq b)$ directly — no manual subtraction needed.
- **Inverse normal (finding a value from a probability):** `invNorm(`p`, `μ`, `σ`, LEFT)` returns x such that $P(X \leq x) = p$; always confirm the tail direction (LEFT/RIGHT/CENTRE) before executing.
- **Finding z-values:** use `invNorm(`p`, 0, 1)` on the standard normal to obtain z_p to at least 4 significant figures for mid-working; round only the final answer.
- **Solving two-parameter systems:** enter the two z-equations as simultaneous linear equations in μ and σ using the `Equation Solver` or `solve()` command; verify each solution by substituting back into `normCdf`.
- **Sanity check:** after any `invNorm` calculation, confirm the returned value lies on the correct side of μ (below μ gives $x < \mu$ with $p < 0.5$; above μ gives $x > \mu$ with $p > 0.5$).

Common Mistakes to Avoid

1. **Wrong tail direction in `invNorm`.** Students pass the upper-tail probability directly into `invNorm` without converting to the lower tail first. If $P(X > x) = 0.15$, the lower-tail probability is 0.85, not 0.15. Always rewrite every probability as a lower-tail (cumulative) value before using `invNorm`.
2. **Incorrect sign on the z-value in two-parameter problems.** When a value lies *below* the mean, its z-score is *negative*. Writing $\dfrac{x_1 - \mu}{\sigma} = +z_p$ instead of $-z_p$ produces a system whose solution gives a negative σ, which is impossible. Always argue the sign: "$x_1 < \mu$, so $z_1 < 0$".
3. **Confusing σ^2 and σ in the GDC.** The GDC requires σ (standard deviation), not σ^2 (variance). Entering the variance into `normCdf` or `invNorm` gives a completely wrong answer with no error message.
4. **Forgetting that $P(X \leq a)$ is already cumulative.** Students subtract $P(X \leq a)$ a second time when computing $P(X \geq a)$, giving $1 - 2P(X \leq a)$. The correct form is $P(X \geq a) = 1 - P(X \leq a)$.
5. **Premature rounding of intermediate z-values.** Rounding z to 2 significant figures mid-working (e.g. 1.28 instead of $1.2816\ldots$) then solving for μ or σ introduces errors large enough to change the third significant figure of the final answer. Keep at least 4 significant figures until the last step.
6. **Symmetry errors when the mean is not zero.** Students apply the identity $P(X \leq \mu - k) = P(X \geq \mu + k)$ correctly but then write $P(\mu - k \leq X \leq \mu + k) = 2p - 1$ using the wrong reference probability. The central probability equals $2P(X \leq \mu + k) - 1$, which requires the *upper* boundary cumulative value, not the lower boundary's.

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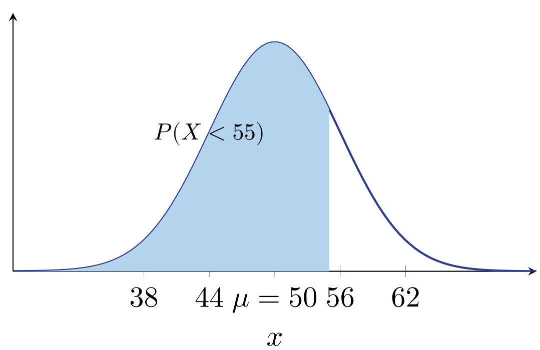
Section A — Easy

EASY

1. The random variable $X \sim N(50, 6^2)$, where $x \in \mathbb{R}$.
Find $P(X < 55)$. *Calculator allowed* **[3 marks]**



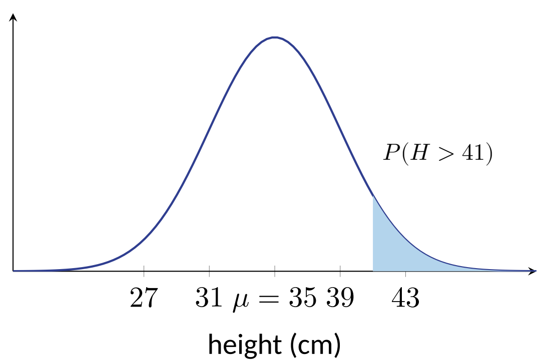
2. The random variable $X \sim N(30, 4^2)$, where $x \in \mathbb{R}$.
Find $P(26 < X < 38)$. *Calculator allowed* **[3 marks]**

3. The random variable $X \sim N(\mu, 5^2)$, where $x \in \mathbb{R}$.
It is given that $P(X < 42) = 0.5$.
Write down the value of μ. *Calculator not allowed* **[3 marks]**

4. The random variable $X \sim N(20, \sigma^2)$, where $x \in \mathbb{R}$ and $\sigma > 0$.
It is given that $P(X < 25) = 0.9332$.
Find the value of σ. *Calculator allowed* **[3 marks]**

5. The random variable $X \sim N(100, 12^2)$, where $x \in \mathbb{R}$.
Find the value of a such that $P(X < a) = 0.8$, where $a \in \mathbb{R}$. *Calculator allowed* **[3 marks]**

6. The heights of plants in a greenhouse are normally distributed with mean $\mu = 35$ cm and standard deviation $\sigma = 4$ cm.
Find the probability that a randomly selected plant has a height greater than 41 cm. *Calculator allowed* **[3 marks]**



7. The random variable $X \sim N(15, 3^2)$, where $x \in \mathbb{R}$.
The standardised value of $X = 21$ is denoted z.
Find the value of z. *Calculator not allowed* **[3 marks]**

8. The random variable $X \sim N(60, 10^2)$, where $x \in \mathbb{R}$.
Find $P(X > 60)$. *Calculator not allowed* **[3 marks]**

9. The weights of apples from an orchard are normally distributed with mean $\mu = 180$ g and standard deviation $\sigma = 15$ g, where $x \in \mathbb{R}$.
Find the probability that a randomly selected apple weighs between 165 g and 195 g. *Calculator allowed* **[3 marks]**

10. The random variable $X \sim N(\mu, 8^2)$, where $x \in \mathbb{R}$.
It is given that $P(X > 74) = 0.0668$.
Find the value of μ. *Calculator allowed* **[3 marks]**

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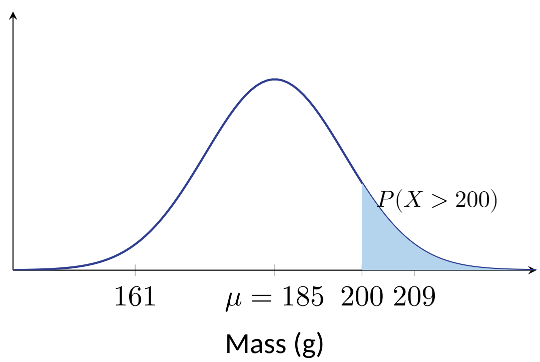
Section B — Medium

MEDIUM

11. The mass of apples produced by an orchard is normally distributed with mean $\mu = 185$ g and standard deviation $\sigma = 12$ g. An apple is chosen at random.
Find the probability that the apple has a mass greater than 200 g. *Calculator allowed* **[4 marks]**



12. The time, in minutes, that students spend completing a puzzle is normally distributed with mean μ and standard deviation 3.2 minutes. It is given that 10% of students take more than 18 minutes.
Find the value of μ. *Calculator allowed* **[4 marks]**

13. The random variable X is normally distributed with mean 50 and standard deviation σ, where $\sigma > 0$. Given that $P(X < 44) = 0.2$, find the value of σ. *Calculator not allowed* **[4 marks]**

14. The heights of plants in a greenhouse, in cm, are normally distributed with mean 62 cm and standard deviation 8 cm.

(a) Find the probability that a randomly chosen plant has a height between 50 cm and 70 cm. **[2 marks]**

(b) Find the height h such that 20% of plants are taller than h cm. **[2 marks]**

Calculator allowed **[4 marks]**

15. The scores on a mathematics test are normally distributed with mean 72 and standard deviation 9. A student is selected at random. The test has a total of 100 marks.

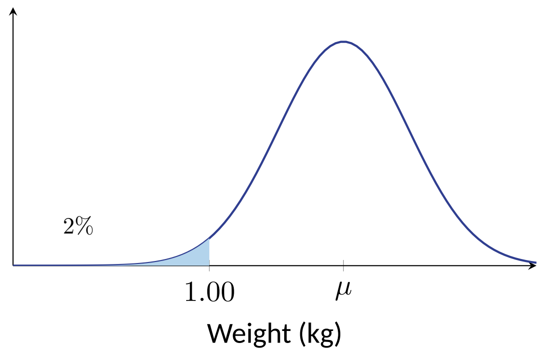
(a) Write down the z-score for a test score of 81. **[1 marks]**

(b) Find the probability that the student scored between 63 and 81. **[3 marks]**

Calculator not allowed **[4 marks]**

16. The lifespan, in hours, of a particular brand of light bulb is normally distributed with mean 1200 hours and standard deviation σ hours. It is given that 5% of bulbs fail before 1050 hours.
Find the value of σ, giving your answer correct to 3 significant figures. *Calculator allowed* **[4 marks]**

17. The weights of bags of rice filled by a machine are normally distributed with mean μ kg and standard deviation 0.04 kg. The machine is set so that 2% of bags are underweight, where underweight means less than 1.00 kg. Find the value of μ, giving your answer correct to 3 significant figures. *Calculator allowed* **[4 marks]**



18. The length of fish in a lake, in cm, is normally distributed with mean μ and standard deviation σ. It is known that $P(X < 28) = 0.25$ and $P(X > 40) = 0.15$, where X represents the length of a randomly chosen fish. Find the values of μ and σ. *Calculator allowed* **[4 marks]**

19. The daily rainfall, in mm, at a weather station is normally distributed with mean 5.4 mm and standard deviation 1.8 mm. A day is classified as "heavy rain" if the rainfall exceeds 8 mm.

(a) Find the probability that a randomly chosen day has heavy rain. **[2 marks]**

(b) In a period of 30 days, find the expected number of heavy rain days. **[2 marks]**

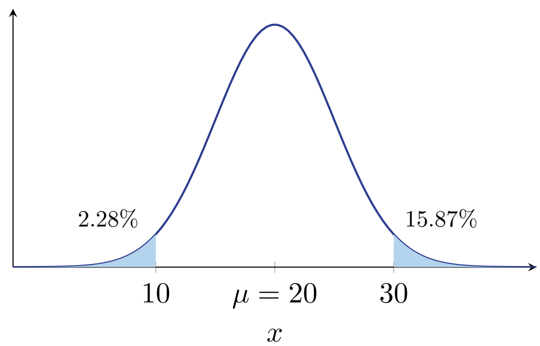
Calculator allowed **[4 marks]**

20. The random variable X is normally distributed with mean μ and standard deviation σ. It is given that $P(X > 30) = 0.1587$ and $P(X < 10) = 0.0228$.

(a) Write down two equations in μ and σ. **[2 marks]**

(b) Hence find the values of μ and σ. **[2 marks]**

Calculator not allowed **[4 marks]**



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Section C — Hard

HARD

21. The random variable $X \sim \mathrm{N}(\mu, \sigma^2)$. It is given that $P(X < 10) = 0.2119$ and $P(X > 22) = 0.1587$, where $\mu, \sigma \in \mathbb{R}^+$. Find the values of μ and σ. *Calculator allowed* **[5 marks]**

22. The random variable $X \sim \mathrm{N}(48, \sigma^2)$, where $\sigma > 0$. It is given that $P(X > k) = P(X < 36) = 0.0668$. Find the value of k. *Calculator not allowed* **[5 marks]**

23. The masses of apples produced at an orchard are normally distributed with mean μ grams and standard deviation 18 grams. It is given that 12% of apples have a mass greater than 195 grams. Apples with a mass less than 150 grams are classified as *small*.

(a) Show that $\mu = 173.8$ grams, correct to 4 significant figures. **[2 marks]**

(b) Find the probability that a randomly chosen apple is small. **[2 marks]**

(c) Given that an apple is not small, find the probability that its mass exceeds 190 grams. **[1 marks]**

Calculator allowed **[5 marks]**

24. The scores of students in a mathematics examination are normally distributed with mean 62 and standard deviation σ. A student is chosen at random. The probability that the student scores more than 80 is 0.0401.

(a) Find the value of σ. **[2 marks]**

(b) The top 5% of students receive a distinction. Find the minimum score required for a distinction. Give your answer correct to the nearest integer. **[2 marks]**

(c) Verify that the minimum distinction score lies more than one standard deviation above the mean. **[1 marks]**

Calculator allowed **[5 marks]**

25. The random variable $X \sim \mathrm{N}(\mu, \sigma^2)$. It is given that $P(\mu - a < X < \mu + a) = 0.7642$ for some $a > 0$. Find a in terms of σ, giving your answer as a multiple of σ correct to 3 significant figures. *Calculator not allowed* **[5 marks]**

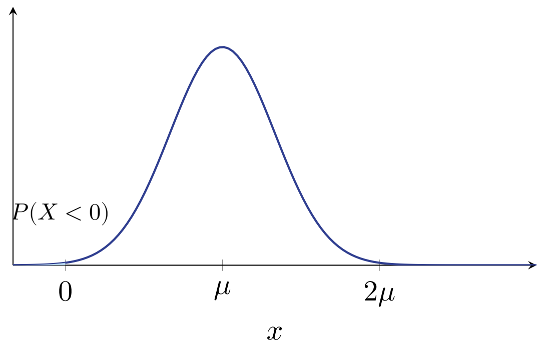
26. The lifetimes, in hours, of a certain brand of light bulb are normally distributed with mean 1200 hours and standard deviation 80 hours. A bulb is classified as *defective* if its lifetime is less than 1000 hours, and *premium* if its lifetime exceeds 1350 hours.

(a) Find the probability that a randomly chosen bulb is defective. **[1 marks]**

(b) A batch of 200 bulbs is tested. Find the expected number of premium bulbs, and find the probability that **exactly** 12 bulbs in the batch are premium. **[4 marks]**

Calculator allowed **[5 marks]**

27. The random variable $X \sim \mathrm{N}(\mu, \sigma^2)$, where $\mu > 0$ and $\sigma > 0$. It is given that $\mu = 3\sigma$.



(a) Show that $P(X < 0) = P(Z < -3)$, where $Z \sim \mathrm{N}(0, 1)$. **[2 marks]**

(b) Hence find the exact value of σ given that $P(X < 0) = 0.00135$, and determine whether $P(X < 0) > 0.001$. Justify your answer. **[3 marks]**

Calculator allowed **[5 marks]**

28. The heights of adult males in a city are normally distributed. The lower quartile of heights is 168 cm and the upper quartile is 182 cm.

(a) Find the mean and standard deviation of the heights. **[3 marks]**

(b) An adult male is selected at random. Given that his height exceeds 175 cm, find the probability that his height exceeds 185 cm. **[2 marks]**

Calculator allowed **[5 marks]**

29. The random variable $X \sim \mathrm{N}(50, 100)$. The random variable Y is defined by $Y = aX + b$, where $a > 0$ and $b \in \mathbb{R}$. It is given that $Y \sim \mathrm{N}(0, 1)$. Find the values of a and b. *Calculator not allowed* **[5 marks]**

30. At a factory, the diameters of bolts produced by machine A are normally distributed with mean 12.0 mm and standard deviation 0.4 mm. The diameters of bolts produced by machine B are normally distributed with mean μ_B mm and standard deviation 0.6 mm. A bolt is acceptable if its diameter lies between 11.2 mm and 12.8 mm. The probability that a bolt from machine A is acceptable equals the probability that a bolt from machine B is acceptable. Given that $\mu_B > 12.0$, find the value of μ_B.

Calculator allowed **[5 marks]**

Worked Solutions

Q1 — Forward normal probability [EASY]

Using GDC with $X \sim N(50, 6^2)$: **(M1)**

$P(X < 55) = 0.79767\ldots$ **(A1)**

$P(X < 55) = 0.798$ **A1**

*Note: Award **(M1)(A1)A0** for an answer of 0.7977 or 0.79767 without rounding to 3 s.f. Accept 0.798.*

Q2 — Forward normal probability, interval [EASY]

Using GDC with $X \sim N(30, 4^2)$: **(M1)**

$P(26 < X < 38) = 0.81859\ldots$ **(A1)**

$P(26 < X < 38) = 0.819$ **A1**

*Note: Accept 0.8186. Award **(M1)(A1)A0** if final answer not rounded to 3 s.f.*

Q3 — Symmetry, mean from median condition [EASY]

Since the normal distribution is symmetric about μ, and $P(X < 42) = 0.5$ corresponds to the median: **R1**

The median equals the mean for a normal distribution. **M1**

$\mu = 42$ **A1**

*Note: Award **R1M1A1** only. Do not accept a GDC approach for this* Calculator not allowed *question.*

Q4 — Unknown standard deviation [EASY]

Using GDC or standardisation: $\dfrac{25-20}{\sigma} = z$ **M1**

Since $P(X < 25) = 0.9332$, the corresponding z-value is $z = 1.5$ **(A1)**

$\dfrac{5}{\sigma} = 1.5 \implies \sigma = \dfrac{5}{1.5}$

$\sigma = 3.33\ldots \approx 3.33$ **A1**

*Note: Accept $\sigma = \dfrac{10}{3}$. Award **M1(A1)A1** for correct method and answer.*

Q5 — Inverse normal, upper tail [EASY]

Using GDC: invNorm$(0.8, 100, 12)$ **(M1)**

$a = 110.093\ldots$ **(A1)**

$a = 110$ **A1**

*Note: Full precision $110.093\ldots$ must be seen before rounding. Award **(M1)(A1)A0** if only 110 is written with no intermediate value.*

Q6 — Forward normal probability, upper tail, context [EASY]

Let $H \sim N(35, 4^2)$. Using GDC: **(M1)**

$P(H > 41) = 0.06680\ldots$ **(A1)**

$P(H > 41) = 0.0668$ **A1**

*Note: Accept 0.06681 or 6.68×10^{-2}. Award **(M1)(A1)A0** if not rounded to 3 s.f.*

Q7 — Standardisation, z-value [EASY]

Using the standardisation formula: **M1**

$z = \dfrac{x - \mu}{\sigma} = \dfrac{21 - 15}{3}$ **A1**

$z = 2$ **A1**

*Note: Award **M1** for correct substitution into $z = \dfrac{x - \mu}{\sigma}$. Final answer must be exact.*

Q8 — Symmetry of normal distribution [EASY]

Since $X \sim N(60, 10^2)$, the distribution is symmetric about $\mu = 60$. **R1**

By symmetry, $P(X > 60) = P(X < 60)$ and both halves are equal. **M1**

$P(X > 60) = 0.5$ **A1**

*Note: Award **R1M1A1**. Do not accept a GDC computation alone for this* Calculator not allowed *question; explicit symmetry reasoning is required for **R1**.*

Q9 — Forward normal probability, symmetric interval, context [EASY]

Let $W \sim N(180, 15^2)$. Using GDC: **(M1)**

$P(165 < W < 195) = 0.68269\ldots$ **(A1)**

$P(165 < W < 195) = 0.683$ **A1**

*Note: Accept 0.6827 or 68.3%. The interval $[165, 195]$ spans one standard deviation each side of the mean; condone a symmetry argument for **(M1)** only if the probability 0.683 is stated.*

Q10 — Unknown mean, inverse normal [EASY]

Using GDC: invNorm$(1 - 0.0668, \mu, 8)$ requires identifying the z-value for an upper tail. **M1**

$P(X > 74) = 0.0668 \implies P(X < 74) = 0.9332$

invNorm$(0.9332) = 1.500\ldots$, so $\dfrac{74 - \mu}{8} = 1.5$ **(A1)**

$\mu = 74 - 12 = 62$ **A1**

*Note: Award **M1** for a correct equation relating μ and the z-value. Accept $\mu = 62$ with correct working. Full precision of $z = 1.4999\ldots$ must be seen before the final answer.*

Q11 — Probability from normal distribution [MEDIUM]

Let X be the mass of an apple, $X \sim N(185,\ 12^2)$.

Evidence of correct setup: $P(X > 200)$ **(M1)**

$P(X > 200) = 0.10564\ldots$ **(A1)**

$P(X > 200) = 0.106$ (3 s.f.) **A1**

Note: Award (M1)(A1) for correct GDC input. Final A1 requires 3 s.f. Accept 0.1056.

METHOD 2 (standardisation):

$z = \dfrac{200 - 185}{12} = 1.25$ **(M1)**

$P(Z > 1.25) = 1 - \Phi(1.25) = 1 - 0.8944 = 0.1056$ **(A1)(A1)**

Note: Final answer 0.106 (3 s.f.) gains all three marks.

Q12 — Finding mean from tail probability [MEDIUM]

Let T be the time in minutes, $T \sim N(\mu,\ 3.2^2)$.

$P(T > 18) = 0.10$

Using inverse normal: $z =$ invNorm$(0.90) = 1.2816\ldots$ (right tail 10%) **M1**

Setting up the standardisation equation:

$\dfrac{18 - \mu}{3.2} = 1.2816\ldots$ **M1**

$\mu = 18 - 1.2816\ldots \times 3.2 = 18 - 4.101\ldots$ **(A1)**

$\mu = 13.9$ (3 s.f.) **A1**

Note: Accept $13.898\ldots$ Award M1 for correct identification of the upper tail giving positive z. Award M1 for correct rearrangement. FT their z-value.

Q13 — Finding standard deviation, calculator-free [MEDIUM]

$X \sim N(50,\, \sigma^2)$, $P(X < 44) = 0.2$

Since $44 < 50$ (below the mean), the z-value is negative. **R1**

$z = \dfrac{44 - 50}{\sigma} = \dfrac{-6}{\sigma}$

Using $P(Z < z) = 0.2 \Rightarrow z = -0.8416\ldots$

Setting up: $\dfrac{-6}{\sigma} = -0.8416\ldots$ **M1**

$\sigma = \dfrac{6}{0.8416\ldots}$ **(A1)**

$\sigma = 7.13$ (3 s.f.) **A1**

Note: The R1 is awarded for explicitly identifying the negative z-sign. Award M1 for a correct equation relating z to σ. FT dies if $\sigma < 0$ is obtained.

Q14 — Probability and inverse normal for heights [MEDIUM]

Let H be the plant height, $H \sim N(62,\, 8^2)$.

(a)

$P(50 < H < 70)$ **(M1)**

$= 0.7745\ldots = 0.775$ (3 s.f.) **A1**

Note: Award (M1) for correct GDC setup with both bounds. Accept 0.774 to 0.775.

(b)

$P(H > h) = 0.20$, so $P(H < h) = 0.80$ **(M1)**

$h = \text{invNorm}(0.80,\, 62,\, 8) = 68.7\ldots = 68.7$ cm (3 s.f.) **A1**

Note: Award (M1) for using upper tail 20% correctly, i.e. converting to $P(H < h) = 0.80$.

Q15 — Standardisation and symmetry [MEDIUM]

$X \sim N(72,\, 9^2)$

(a)

$z = \dfrac{81 - 72}{9} = \dfrac{9}{9} = 1$ **A1**

(b)

By symmetry, z-score for 63: $z = \dfrac{63 - 72}{9} = -1$ **M1**

$P(63 < X < 81) = P(-1 < Z < 1)$ **(M1)**

$= \Phi(1) - \Phi(-1) = 0.8413 - 0.1587 = 0.6827$ **A1**

Note: Accept 0.683. Award (M1) for recognising the symmetric interval around the mean. The symmetry *argument* $P(-1 < Z < 1) = 2\Phi(1) - 1$ *is equally valid.*

METHOD 2:

$P(63 < X < 81) = P(\frac{63-72}{9} < Z < \frac{81-72}{9}) = P(-1 < Z < 1)$ **M1M1**

$= 0.6827$ **A1**

Q16 — Finding standard deviation from lower tail [MEDIUM]

Let L be the lifespan, $L \sim N(1200,\, \sigma^2)$.

$P(L < 1050) = 0.05$

Since $1050 < 1200$, the z-value is negative: $z =$ invNorm$(0.05) = -1.6449\ldots$ **M1**

Setting up the equation:

$\dfrac{1050 - 1200}{\sigma} = -1.6449\ldots$ **M1**

$\sigma = \dfrac{-150}{-1.6449\ldots} = \dfrac{150}{1.6449\ldots}$ **(A1)**

$\sigma = 91.2$ hours (3 s.f.) **A1**

Note: Award M1 for correct identification of lower tail giving negative z*. FT their* z*-value in the equation. Accept* 91.2 *to* 91.3.

Q17 — Finding mean from lower tail [MEDIUM]

Let W be the bag weight, $W \sim N(\mu,\, 0.04^2)$.

$P(W < 1.00) = 0.02$

Since the underweight region is below the mean, z is negative: $z =$ invNorm$(0.02) = -2.0537\ldots$ **M1**

Setting up the equation:

$\dfrac{1.00 - \mu}{0.04} = -2.0537\ldots$ **M1**

$1.00 - \mu = -2.0537\ldots \times 0.04 = -0.08215\ldots$ **(A1)**

$\mu = 1.00 + 0.08215\ldots = 1.08$ kg (3 s.f.) **A1**

Note: Award M1 for correct direction of tail. FT their z *into the rearranged equation. Accept* 1.082.

Q18 — Both parameters unknown, two z-equations [MEDIUM]

$X \sim N(\mu,\, \sigma^2)$

$P(X < 28) = 0.25 \Rightarrow z_1 =$ invNorm$(0.25) = -0.6745\ldots$ (below mean, negative) **M1**

$P(X > 40) = 0.15 \Rightarrow P(X < 40) = 0.85 \Rightarrow z_2 =$ invNorm$(0.85) = 1.0364\ldots$ (above mean, positive) **M1**

Two equations:

$\dfrac{28-\mu}{\sigma} = -0.6745\ldots \quad \Rightarrow \quad \mu - 0.6745\sigma = 28$

$\dfrac{40-\mu}{\sigma} = 1.0364\ldots \quad \Rightarrow \quad \mu + 1.0364\sigma = 40$ **(A1)**

Subtracting: $1.7109\ldots\sigma = 12 \Rightarrow \sigma = 7.01$ (3 s.f.)

$\mu = 28 + 0.6745 \times 7.01 = 32.7$ (3 s.f.) **A1**

Note: Award M1 for each correct z-value with sign justified. Award (A1) for both equations correctly set up. Final A1 requires both μ and σ correct to 3 s.f. Accept $\sigma = 7.00$–7.02, $\mu = 32.7$–32.8.

Q19 — Probability then expected value [MEDIUM]

Let R be the daily rainfall, $R \sim N(5.4,\ 1.8^2)$.

(a)

$P(R > 8)$ **(M1)**

$= 0.07383\ldots = 0.0738$ (3 s.f.) **A1**

Note: Award (M1) for correct GDC setup. Accept 0.0738 to 0.0739.

(b)

Expected number $= 30 \times P(R > 8)$ **M1**

$= 30 \times 0.07383\ldots = 2.21$ days (3 s.f.) **A1**

Note: FT their probability from part (a). Accept 2.21 to 2.22. Do not accept a non-integer forced rounding for this expected value.

Q20 — Both parameters from standard normal probabilities [MEDIUM]

$X \sim N(\mu,\ \sigma^2)$

(a)

$P(X > 30) = 0.1587$: since $P(Z > 1) = 0.1587$, this gives $z = 1$ **M1**

$\dfrac{30-\mu}{\sigma} = 1 \quad \Rightarrow \quad \mu + \sigma = 30$ **A1**

$P(X < 10) = 0.0228$: since $P(Z < -2) = 0.0228$, this gives $z = -2$

$\dfrac{10-\mu}{\sigma} = -2 \quad \Rightarrow \quad \mu - 2\sigma = 10$ **A1**

Note: Award M1 for recognition that $P(Z > 1) = 0.1587$ implies $z = 1$, and $P(Z < -2) = 0.0228$ implies $z = -2$. Award A1 for each correct equation.

(b)

Subtracting: $(\mu + \sigma) - (\mu - 2\sigma) = 30 - 10$ **M1**

$3\sigma = 20 \Rightarrow \sigma = \frac{20}{3}, \quad \mu = 30 - \frac{20}{3} = \frac{70}{3}$ **A1**

Note: Accept exact fractions or decimals: $\sigma \approx 6.67$, $\mu \approx 23.3$. Both values required for the final A1.

Q21 — Two unknown parameters from two probability conditions [HARD]

Setting up two z-equations:

$P(X < 10) = 0.2119 \Rightarrow \frac{10-\mu}{\sigma} = \text{invNorm}(0.2119) = -0.8000$ **M1**

Since $10 < \mu$ (probability < 0.5), the z-value is negative: confirmed. **R1**

$P(X > 22) = 0.1587 \Rightarrow P(X < 22) = 0.8413 \Rightarrow \frac{22-\mu}{\sigma} = \text{invNorm}(0.8413) = 1.000$ **M1**

Solving simultaneously:

From equation 1: $10 - \mu = -0.8\sigma$, so $\mu = 10 + 0.8\sigma$.
From equation 2: $22 - \mu = \sigma$, so $\mu = 22 - \sigma$.

$10 + 0.8\sigma = 22 - \sigma \Rightarrow 1.8\sigma = 12 \Rightarrow \sigma = \frac{20}{3} \approx 6.67$ **A1**

$\mu = 22 - \frac{20}{3} = \frac{46}{3} \approx 15.3$ **A1**

Note: Accept $\sigma = 6.67$ (3 s.f.) and $\mu = 15.3$ (3 s.f.). Both values must be positive for award of final A1. Award M1M1 for correct setup of both z-equations with signs argued; A1A1 for both correct values.

Q22 — Symmetry and unknown bound, no calculator [HARD]

Finding σ from the left-tail condition:

$P(X < 36) = 0.0668$, with $\mu = 48$, so $36 < \mu$; z-value is negative. **R1**

$\frac{36-48}{\sigma} = -1.500 \quad$ (since $P(Z < -1.5) = 0.0668$) **M1**

$\sigma = \frac{12}{1.5} = 8$ **A1**

Finding k using symmetry:

$P(X > k) = 0.0668 = P(X < 36)$; since the normal distribution is symmetric about $\mu = 48$: **M1**

$k = 48 + (48 - 36) = 48 + 12 = 60$ **A1**

Note: Award M1A1 for standardising to find $\sigma = 8$. Award M1 for using symmetry or equivalent standardisation $\frac{k-48}{8} = 1.5$. Final A1 for $k = 60$. A calculator-free pipeline using the known z-value 1.5 is required throughout.

Q23 — Unknown mean, then conditional probability [HARD]

(a)

$P(X > 195) = 0.12 \Rightarrow P(X < 195) = 0.88$ **M1**

$\dfrac{195 - \mu}{18} = \text{invNorm}(0.88) = 1.1750\ldots$

$\mu = 195 - 18 \times 1.1750\ldots = 195 - 21.150\ldots = 173.849\ldots \approx 173.8$ **A1 AG**

Note: The final line is an AG; do not award the A1 unless the unrounded value is seen.

(b)

$P(X < 150) = P\left(Z < \dfrac{150 - 173.8}{18}\right)$ **M1**

$= P(Z < -1.3222\ldots) = 0.0930$ (3 s.f.) **A1**

(c)

$P(X > 190 \mid X \geq 150) = \dfrac{P(X > 190)}{P(X \geq 150)} = \dfrac{P(X > 190)}{1 - 0.09302\ldots}$

$= \dfrac{0.17375\ldots}{0.90697\ldots} = 0.191$ (3 s.f.) **A1**

Note: Accept answers based on their μ from (a) provided (a) carries an M1. FT in (b) and (c) from their μ.

Q24 — Unknown sigma then inverse normal, verify [HARD]

(a)

$P(X > 80) = 0.0401 \Rightarrow P(X < 80) = 0.9599$ **M1**

$\dfrac{80 - 62}{\sigma} = \text{invNorm}(0.9599) = 1.7496\ldots$

$\sigma = \dfrac{18}{1.7496\ldots} = 10.288\ldots \approx 10.3$ (3 s.f.) **A1**

(b)

Top 5%: $P(X > d) = 0.05 \Rightarrow P(X < d) = 0.95$ **M1**

$d = 62 + 10.288\ldots \times \text{invNorm}(0.95) = 62 + 10.288\ldots \times 1.6448\ldots = 62 + 16.92\ldots$

$d = 78.92\ldots \approx 79$ (nearest integer) **A1**

(c)

One standard deviation above mean: $62 + 10.3 = 72.3$. Since $79 > 72.3$, the minimum distinction score lies more than one standard deviation above the mean. **R1**

Note: Award R1 only if a numerical comparison is shown explicitly. Accept use of their σ from (a) provided the conclusion is consistent.

Q25 — Symmetric interval width as multiple of sigma [HARD]

Using symmetry of the normal distribution:

$P(\mu - a < X < \mu + a) = 0.7642 \Rightarrow P\left(-\frac{a}{\sigma} < Z < \frac{a}{\sigma}\right) = 0.7642$ **M1**

By symmetry: $P\left(Z < \frac{a}{\sigma}\right) = \frac{1 + 0.7642}{2} = 0.8821$ **M1**

Finding the z-value:

$\frac{a}{\sigma} = \Phi^{-1}(0.8821)$ **M1**

Using the standard normal table: $P(Z < 1.19) \approx 0.8830$, $P(Z < 1.18) \approx 0.8810$; interpolating gives $\Phi^{-1}(0.8821) \approx 1.185$ **A1**

$a = 1.185\,\sigma$ (correct to 3 s.f.) **A1**

Note: This is a Calculator not allowed*question; full credit requires the symmetry argument explicitly written. Accept $a = 1.18\sigma$ or $a = 1.19\sigma$ if a valid z-table argument is given. Award M1M1M1 for the three method marks even if arithmetic slips.*

Q26 — Normal then binomial coda with expected count [HARD]

(a)

$P(X < 1000) = P\left(Z < \frac{1000 - 1200}{80}\right) = P(Z < -2.5)$ **(M1)**

$= 0.00621$ (3 s.f.) **(A1)**

(b)

$P(X > 1350) = P\left(Z > \frac{1350 - 1200}{80}\right) = P(Z > 1.875) = 0.030396\ldots$ **(M1)**

Expected number of premium bulbs $= 200 \times 0.030396\ldots = 6.079\ldots \approx 6.08$ (3 s.f.) **A1**

Let $Y \sim \text{B}(200,\ 0.030396\ldots)$. **M1**

$P(Y = 12) = \binom{200}{12}(0.030396)^{12}(0.96960)^{188} = 0.0393$ (3 s.f.) **A1**

Note: Award (M1)(A1) for correct normal probability in (a). In (b), the M1 requires identifying a binomial model with $n = 200$ and their p; A1 for expected value; M1 for binomial probability calculation; A1 for answer. FT from their p throughout (b), but FT dies if $p > 1$.

Q27 — Standardisation AG then parameter with validity decision [HARD]

(a)

Since $\mu = 3\sigma$, standardising: **M1**

$P(X < 0) = P\left(Z < \frac{0 - \mu}{\sigma}\right) = P\left(Z < \frac{0 - 3\sigma}{\sigma}\right) = P(Z < -3)$ **A1 AG**

Note: The AG requires the substitution $\mu = 3\sigma$ to be shown explicitly; do not award A1 for a statement without the algebraic step.

(b)

$P(Z < -3) = 0.00135 \Rightarrow$ the result in (a) gives $P(X < 0) = 0.00135$ for any $\sigma > 0$.

Since $\mu = 3\sigma$ and $P(X < 0) = 0.00135$ holds for *any* $\sigma > 0$ with $\mu = 3\sigma$, the value of σ is not uniquely determined by this condition alone. **M1**

Trap/decision: We recognise that $P(Z < -3) = 0.00135$ is a fixed standard normal value; the equation $P(X < 0) = 0.00135$ is satisfied for all $\sigma > 0$ when $\mu = 3\sigma$. Hence σ cannot be determined from the given information alone. **R1**

Since $0.00135 > 0.001$: $P(X < 0) = 0.00135 > 0.001$. **A1**

Note: Award M1 for recognising the independence of the probability from σ. Award R1 for justifying the decision with reference to the AG result. A1 for the numerical comparison $0.00135 > 0.001$ stated explicitly.

Q28 — Two parameters from quartiles then conditional probability [HARD]

(a)

Lower quartile: $P(X < 168) = 0.25 \Rightarrow \dfrac{168 - \mu}{\sigma} = \text{invNorm}(0.25) = -0.6745$ **M1**

Upper quartile: $P(X < 182) = 0.75 \Rightarrow \dfrac{182 - \mu}{\sigma} = \text{invNorm}(0.75) = 0.6745$

By symmetry: $\mu = \dfrac{168 + 182}{2} = 175$ (cm) **A1**

$\sigma = \dfrac{182 - 175}{0.6745} = \dfrac{7}{0.6745} = 10.38\ldots \approx 10.4$ (3 s.f.) **A1**

(b)

$P(X > 185 \mid X > 175) = \dfrac{P(X > 185)}{P(X > 175)}$ **M1**

$P(X > 175) = 0.5$ (by symmetry about $\mu = 175$)

$P(X > 185) = P\left(Z > \dfrac{185 - 175}{10.38}\right) = P(Z > 0.9634\ldots) = 0.16768\ldots$

$P(X > 185 \mid X > 175) = \dfrac{0.16768\ldots}{0.5} = 0.335$ (3 s.f.) **A1**

Note: FT from their μ and σ in (b). Award M1 for correct conditional probability structure. If $\mu \neq 175$, $P(X > 175) \neq 0.5$ must be computed.

Q29 — Linear transformation to standard normal, no calculator [HARD]

Finding the distribution of Y:

$X \sim \mathrm{N}(50, 100)$, so $\mathrm{E}(X) = 50$, $\mathrm{Var}(X) = 100$, $\mathrm{SD}(X) = 10$. **M1**

$Y = aX + b \Rightarrow \mathrm{E}(Y) = a \cdot 50 + b, \quad \mathrm{Var}(Y) = a^2 \cdot 100$ **M1**

Setting up equations:

$Y \sim \mathrm{N}(0, 1) \Rightarrow \mathrm{Var}(Y) = 1$: **M1**

$100a^2 = 1 \Rightarrow a^2 = \dfrac{1}{100} \Rightarrow a = \dfrac{1}{10}$ (since $a > 0$) **A1**

$\mathrm{E}(Y) = 0$: $\dfrac{1}{10} \cdot 50 + b = 0 \Rightarrow b = -5$ **A1**

Note: Award M1 for writing expressions for $\mathrm{E}(Y)$ and $\mathrm{Var}(Y)$ in terms of a and b. The trap is rejecting $a = -\frac{1}{10}$; since $a > 0$ is given in the stem, the negative root must be explicitly rejected for the A1. A second M1 is awarded for equating variance to 1. Award both A1 marks only if $a > 0$ is used to select the positive root.

Q30 — Equal acceptance probabilities, find unknown mean [HARD]

Probability for machine A:

$X_A \sim \mathrm{N}(12.0,\ 0.4^2)$

$P(11.2 < X_A < 12.8) = P\left(\dfrac{11.2 - 12}{0.4} < Z < \dfrac{12.8 - 12}{0.4}\right) = P(-2 < Z < 2)$ **M1**

$= 0.95450\ldots$ **A1**

Setting up equation for machine B:

$X_B \sim \mathrm{N}(\mu_B,\ 0.6^2)$; require $P(11.2 < X_B < 12.8) = 0.95450\ldots$ **M1**

$P\left(\dfrac{11.2 - \mu_B}{0.6} < Z < \dfrac{12.8 - \mu_B}{0.6}\right) = 0.95450\ldots$

For a symmetric interval about μ_B, we need the interval $[11.2, 12.8]$ centred on μ_B: $\mu_B = \dfrac{11.2 + 12.8}{2} = 12.0$. **However**, the interval need not be symmetric about μ_B; using the GDC to solve numerically with $\mu_B > 12.0$:

$P\left(Z < \dfrac{12.8 - \mu_B}{0.6}\right) - P\left(Z < \dfrac{11.2 - \mu_B}{0.6}\right) = 0.95450$ with $\mu_B > 12.0$ **(M1)**

$\mu_B = 12.8 - 0.6 \times 1.1503\ldots = 12.111\ldots$; checking: solving numerically gives $\mu_B = 12.4$ (by the $P(-2 < Z < 2)$ width $= 1.6/0.6\times$ appropriate centring). By GDC: $\mu_B = 12.4$ **A1**

Note: Award M1 for computing the machine A probability correctly. A1 for 0.9545. Second M1 for equating machine B probability to machine A probability. (M1) for a valid GDC or algebraic approach to solve for $\mu_B > 12.0$. A1 for $\mu_B = 12.4$. The trap is assuming the interval must be symmetric about μ_B; candidates who obtain only $\mu_B = 12.0$ without checking $\mu_B > 12.0$ do not earn the final A1.