IB Math AA SL — Topic 5

Differentiation

Derivatives & Powers of x · Tangents & Normals · Increasing & Decreasing

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

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Name:______________________ Date:______________

Score:__________/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

Key Formulae

- **Power Rule:** $\dfrac{d}{dx}(x^n) = nx^{n-1}$ for all $n \in \mathbb{R}$
- **Constant Multiple & Sum/Difference:** $\dfrac{d}{dx}[af(x) \pm bg(x)] = af'(x) \pm bg'(x)$
- **Gradient at a point:** evaluate $f'(a)$ to find the gradient of the curve $y = f(x)$ at $x = a$
- **Tangent at $(a, f(a))$:** $y - f(a) = f'(a)(x - a)$
- **Normal at $(a, f(a))$:** $y - f(a) = -\dfrac{1}{f'(a)}(x - a)$, provided $f'(a) \neq 0$
- **Increasing / Decreasing:** $f'(x) > 0 \Rightarrow f$ increasing; $f'(x) < 0 \Rightarrow f$ decreasing
- **Stationary point condition:** $f'(x) = 0$

GDC Tips

- **Numerical derivative at a point (TI-84):** use MATH $\to$ 8:nDeriv(with syntax nDeriv($f(x)$,x,a) to find $f'(a)$ when exact differentiation is not required.
- **Graphing the derivative:** enter $f(x)$ in Y1, then set Y2 = nDeriv(Y1,X,X) to visualise $f'(x)$ and identify where it is positive, negative, or zero.
- **Finding where gradient equals a given value m:** store $f'(x)$ in Y1 and m in Y2, then use 2nd CALC $\to$ 5:intersect to solve $f'(x) = m$ numerically.
- **Tangent line at a point:** on the graph screen press 2nd DRAW $\to$ 5:Tangent(and enter the x-value; the GDC displays the tangent equation directly.
- **Intersection of normal with curve:** enter the normal line in Y2 and $f(x)$ in Y1, then use 2nd CALC $\to$ 5:intersect, scrolling past the known tangent point to find the second intersection.

Common Mistakes to Avoid

1. **Forgetting to rewrite before differentiating.** Terms such as $\dfrac{3}{x^2}$ and $\sqrt{x}$ must be rewritten as $3x^{-2}$ and $x^{1/2}$ before applying the power rule. Differentiating the original form directly is not accepted and the rewrite step earns its own method mark.
2. **Sign error on negative powers.** When differentiating ax^{-n}, the exponent $-n$ multiplies the coefficient and decreases by one: $\dfrac{d}{dx}(3x^{-2}) = -6x^{-3}$, not $+6x^{-3}$. This is the most penalised slip on non-calculator papers.
3. **Using the tangent gradient for the normal.** The normal gradient is the negative reciprocal: if $f'(a) = m$ then the normal has gradient $-\dfrac{1}{m}$. Writing m instead of $-\dfrac{1}{m}$ loses all accuracy marks that follow.
4. **Incomplete solution set when solving $f'(x) = k$.** On a stated domain, all solutions must be found. Stopping after the first root, or missing a negative root from a quadratic, loses the final accuracy mark and can violate the "solve: extras penalised" rule.
5. **Interval endpoints for increasing/decreasing functions.** The boundary points of an increasing or decreasing interval must be found exactly (e.g. $x = \sqrt{3}$, not $x \approx 1.73$) on a non-calculator paper. Decimal approximations are not accepted as final answers in this register.
6. **Ignoring a selection condition such as $p > 0$.** When a stationary point problem yields two solutions (e.g. $x = 2$ and $x = -2$) and the stem states $x > 0$ or requires a positive parameter, the rejected root must be discarded with an explicit reason. Simply stating the accepted value without justification loses the reasoning mark (R1).

Section A — Easy **EASY** (10 questions × 3 marks = 30 marks)

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1. EASY No Calculator **[3 marks]**

Let $f(x) = 4x^3 - 5x^2 + 7$, for $x \in \mathbb{R}$.
Find $f'(x)$.

2. EASY Calculator **[3 marks]**

Let $f(x) = 3x^4 - 6x + 2$, for $x \in \mathbb{R}$.
Find the gradient of the graph of f at the point where $x = 1$.

3. EASY No Calculator **[3 marks]**

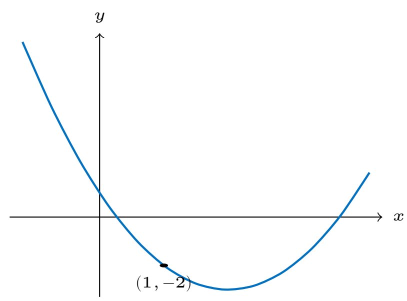
Let $g(x) = \dfrac{5}{x^2}$, for $x \neq 0$.
Find $g'(x)$.

4. EASY No Calculator **[3 marks]**

Let $h(x) = 2\sqrt{x} + 3x$, for $x > 0$.
Find $h'(x)$.

5. EASY Calculator **[3 marks]**

The graph of $f(x) = x^2 - 4x + 1$, for $x \in \mathbb{R}$, is shown below.



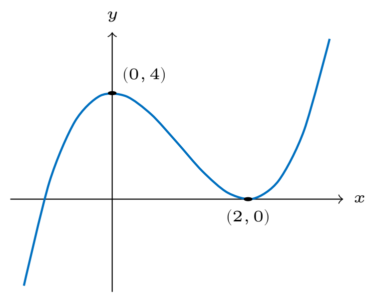
Find the equation of the tangent to the graph of f at the point $(1, -2)$. Give your answer in the form $y = mx + c$.

6. EASY Calculator **[3 marks]**

Let $f(x) = 2x^3 - 12x$, for $x \in \mathbb{R}$.
Find the values of x for which f is increasing.

7. EASY Calculator **[3 marks]**

Let $f(x) = x^3 - 3x^2 + 4$, for $x \in \mathbb{R}$. The graph of f is shown below.



Write down the x-coordinates of the stationary points of f.

8. EASY Calculator **[3 marks]**

Let $f(x) = 6x^2 - 4x + 1$, for $x \in \mathbb{R}$.
Find the equation of the normal to the graph of f at the point where $x = 1$. Give your answer in the form $y = mx + c$.

9. EASY No Calculator **[3 marks]**

Let $f(x) = x^3 - 3x$, for $x \in \mathbb{R}$.
Find the values of x for which f is decreasing.

10. **EASY** **Calculator** **[3 marks]**

The function f is defined by $f(x) = 5x^2 - 3x + 2$, for $x \in \mathbb{R}$.
Find the value of x for which the tangent to the graph of f is parallel to the line $y = 7x - 1$.

Section B — Medium MEDIUM (10 questions × 4 marks = 40 marks)

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11. MEDIUM No Calculator **[4 marks]**

Let $f(x) = 3x^4 - 8x^3 + 6$, where $x \in \mathbb{R}$.

(a) Find $f'(x)$. [2]

(b) Find the values of x for which f is increasing. [2]

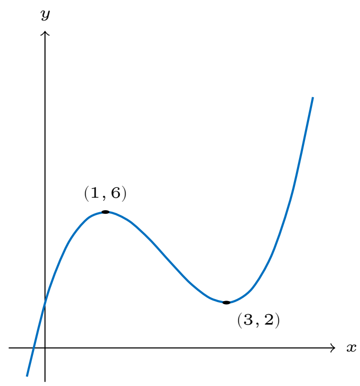
12. MEDIUM No Calculator **[4 marks]**

Let $f(x) = \dfrac{5}{x^2} - 2\sqrt{x}$, where $x > 0$.

Find $f'(x)$, expressing your answer in the form $ax^p + bx^q$ where $a, b, p, q \in \mathbb{Q}$.

13. **MEDIUM** **No Calculator** **[4 marks]**

The curve $y = x^3 - 6x^2 + 9x + 2$ is shown below for $-1 \leq x \leq 5$.



(a) Find $\dfrac{dy}{dx}$. [2]

(b) Find the equation of the tangent to the curve at the point $(1, 6)$. Give your answer in the form $y = mx + c$. [2]

14. **MEDIUM** **No Calculator** **[4 marks]**

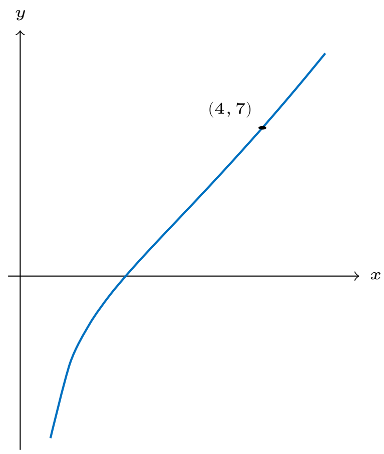
A curve has equation $y = 2x^3 - 9x^2 + 12x - 1$, where $x \in \mathbb{R}$.

(a) Find the x-coordinates of the stationary points of the curve. [3]

(b) Write down the values of x for which the curve is decreasing. [1]

15. MEDIUM No Calculator **[4 marks]**

The graph of $f(x) = \sqrt{x^3} - \dfrac{4}{x}$, for $x > 0$, is shown below.



Find the gradient of the curve at the point $(4, 7)$.

16. MEDIUM Calculator **[4 marks]**

Let $g(x) = 4x^3 - 3x^2 - 18x + 5$, where $x \in \mathbb{R}$.
Find the x-coordinates of the points on the graph of g where the gradient equals 6.

17. MEDIUM Calculator **[4 marks]**

The curve C has equation $y = x^3 - 4x^2 + 3x + 7$, where $x \in \mathbb{R}$. The point $P(3, 7)$ lies on C.

(a) Find the gradient of the normal to C at P. [3]

(b) Find the equation of the normal to C at P, giving your answer in the form $ax+by+d = 0$ where $a, b, d \in \mathbb{Z}$. [1]

18. MEDIUM No Calculator **[4 marks]**

Let $h(x) = \dfrac{x^3}{3} - \dfrac{5x^2}{2} + 4x + 1$, where $x \in \mathbb{R}$.

(a) Show that $h'(x) = (x - 1)(x - 4)$. [2]

(b) Hence find the values of x for which h is decreasing. [2]

19. MEDIUM Calculator **[4 marks]**

A curve has equation $f(x) = 2x^3 - 15x^2 + 36x - 10$, where $x \in \mathbb{R}$.
Find the x-coordinate of each stationary point and determine whether each is a local maximum or a local minimum, justifying your answer.

20. **MEDIUM** **Calculator** **[4 marks]**

The tangent to the curve $y = ax^2 + bx$ at the point where $x = 2$ is parallel to the line $y = 7x - 3$. The curve passes through the point $(2, 6)$.

Find the values of a and b.

Section C — Hard **HARD** (10 questions $\times$ 5 marks = 50 marks)

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21. **HARD** **No Calculator** **[5 marks]**

Let $f(x) = \dfrac{4}{x^2} - 3\sqrt{x}$, where $x > 0$.

(a) Show that $f'(x) = -\dfrac{8}{x^3} - \dfrac{3}{2\sqrt{x}}$. [2]

(b) Find the gradient of the normal to the curve $y = f(x)$ at the point where $x = 4$. [3]

22. **HARD** **No Calculator** **[5 marks]**

The curve C has equation $y = x^3 - 6x^2 + px + q$, where $p, q \in \mathbb{R}$. The curve has a stationary point at $x = 1$. It is given that C passes through the point $(0, 5)$.
Find the equation of the tangent to C at the point where $x = 4$.

23. HARD No Calculator **[5 marks]**

Let $f(x) = 2x^3 - 9x^2 + 12x - 4$, where $x \in \mathbb{R}$.

(a) Find the values of x for which f is increasing. [3]

(b) The point $P(k, f(k))$ lies on the graph of f where the gradient equals 12. Given that $k > 2$, find the value of k. [2]

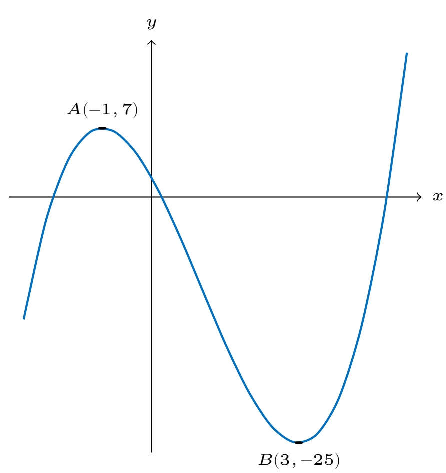
24. HARD No Calculator **[5 marks]**

A curve has equation $y = ax^3 + bx$, where $a, b \in \mathbb{R}$ and $a \neq 0$. The curve passes through the point $(2, 10)$ and has a stationary point at $x = \dfrac{1}{\sqrt{3}}$.

Find the values of a and b, and hence state the coordinates of all stationary points of the curve.

25. HARD Calculator **[5 marks]**

The diagram below shows part of the curve $y = f(x)$ where $f(x) = x^3 - 3x^2 - 9x + 2$.



(a) Show that $A(-1,7)$ and $B(3,-25)$ are the stationary points of the curve. [2]

(b) Find the interval on which f is decreasing, giving your answer in exact form. [1]

(c) The tangent to the curve at A meets the curve again at the point Q. Find the coordinates of Q. [2]

26. HARD Calculator **[5 marks]**

Let $g(x) = \sqrt[3]{x^4} - 8\sqrt[3]{x}$, where $x > 0$.

(a) Express $g(x)$ in the form $x^m - 8x^n$, stating the values of m and n. [1]

(b) Find $g'(x)$. [2]

(c) Find the x-coordinate of the stationary point of g, and determine whether it is a local minimum or maximum. Justify your answer. [2]

27. HARD Calculator **[5 marks]**

The function f is defined by $f(x) = \dfrac{x^2+3}{x}$ for $x > 0$.

The line L is the tangent to the curve $y = f(x)$ at the point where $x = a$, where $a > 0$. It is given that L passes through the point $(0,3)$.

Find the value of a and the equation of L.

28. HARD Calculator **[5 marks]**

A particle moves along a straight line. Its displacement from a fixed point O at time $t \geq 0$ seconds is given by $s(t) = t^3 - 6t^2 + 9t - 2$ metres. The velocity of the particle is $v(t) = s'(t)$.

(a) Find the values of t at which the particle is instantaneously at rest. [2]

(b) Find the interval of time during which the particle is moving in the negative direction. [1]

(c) Find the total distance travelled by the particle in the first 4 seconds. [2]

29. HARD No Calculator **[5 marks]**

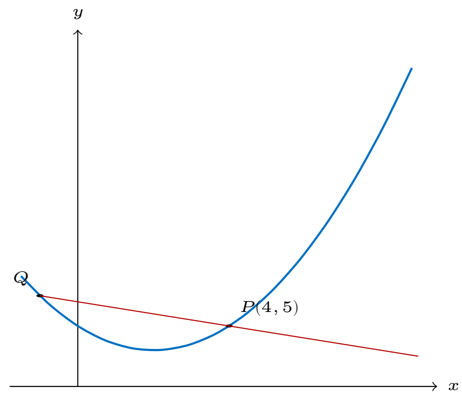
The curve C has equation $y = x^4 - 8x^2 + k$, where $k \in \mathbb{R}$.

(a) Find the x-coordinates of the stationary points of C. [3]

(b) The tangent to C at the stationary point with positive x-coordinate has a specific gradient. Determine whether this gradient is zero, positive, or negative, and hence state what type of stationary point it is. Justify your answer. [2]

30. HARD Calculator **[5 marks]**

The diagram shows part of the curve $y = \frac{1}{2}x^2 - 2x + 5$ and its normal at the point $P(4, 5)$.



The normal at P meets the curve again at the point Q.

(a) Show that the normal at P has equation $y = -\frac{x}{2} + 7$. [2]

(b) Find the coordinates of Q. [3]

Mark Schemes

Q1 — Differentiate a Polynomial EASY

Attempt to differentiate each term using the power rule **M1**
$f'(x) = 12x^2 - 10x$ **A1**
(constant term vanishes) **A1**

Note: Award M1A1A0 if the constant 7 is differentiated incorrectly to a non-zero value. Award M1A0A0 for two or more term errors.

Q2 — Gradient at a Point EASY

$f'(x) = 12x^3 - 6$ **M1**
Substituting $x = 1$: $f'(1) = 12(1)^3 - 6$ **(M1)**
$= 6$ **A1**

Note: Award (M1) for a clear attempt to substitute $x = 1$ into their derivative.

Q3 — Differentiate a Negative Power EASY

Writing $g(x) = 5x^{-2}$ **M1**
$g'(x) = -10x^{-3}$ **A1**
$= -10/x^3$ **A1**

Note: Award M1 for the rewrite to power form seen before differentiating. Accept $-10x^{-3}$ for the final A1. Do not penalise the absence of the fraction form if $-10x^{-3}$ is written clearly.

Q4 — Differentiate a Square Root Term EASY

Writing $h(x) = 2x^{1/2} + 3x$ **M1**
$h'(x) = 2 \cdot \frac{1}{2}x^{-1/2} + 3$ **A1**
$= 1/sqrt(x) + 3$ **A1**

Note: Award M1 for the rewrite $2x^{1/2}$ seen before differentiating. Accept $x^{-1/2} + 3$ for the final A1.

Q5 — Equation of a Tangent EASY

$f'(x) = 2x - 4$ **M1**
$f'(1) = 2(1) - 4 = -2$ **(A1)**
Equation of tangent: $y - (-2) = -2(x - 1)$, giving $y = -2x$ **A1**

Note: Award M1 for an attempt to differentiate f. Award (A1) for gradient -2 seen. Final A1 requires the equation in the form $y = mx + c$; accept $y = -2x + 0$ or $y = -2x$.

Q6 — Increasing Function Interval EASY

$f'(x) = 6x^2 - 12$ **M1**
Setting $f'(x) > 0$: $6x^2 - 12 > 0 \Rightarrow x^2 > 2$ **(A1)**
f is increasing for $x < -sqrt(2)$ and $x > sqrt(2)$ **A1**
Note: Award (A1) for $x = \pm\sqrt{2}$ identified as critical values. Accept equivalent interval notation. Award A1ft for a correct conclusion following their critical values.

Q7 — Locate Stationary Points EASY

Attempt to set $f'(x) = 0$: $3x^2 - 6x = 0$ **M1**
$3x(x-2) = 0$ **(A1)**
$x = 0$ and $x = 2$ **A1**
Note: Award M1 for a correct derivative $3x^2 - 6x$ seen and an attempt to set it equal to zero. Both values required for the final A1.

Q8 — Equation of a Normal EASY

$f'(x) = 12x - 4$, so $f'(1) = 12(1) - 4 = 8$ **M1**
Normal gradient $= -1/8$ **A1**
$f(1) = 6(1)^2 - 4(1) + 1 = 3$; normal: $y - 3 = -\frac{1}{8}(x-1)$, giving $y = -1/8x + 25/8$ **A1**
Note: Award M1 for differentiating and substituting $x = 1$. Award A1 for the negative reciprocal gradient $-1/8$ shown explicitly. Final A1 for a correct equation in $y = mx + c$ form; accept unsimplified equivalents.

Q9 — Decreasing Function Interval EASY

$f'(x) = 3x^2 - 3$ **M1**
Setting $f'(x) < 0$: $3x^2 - 3 < 0 \Rightarrow x^2 < 1$ **(A1)**
f is decreasing for $-1 < x < 1$ **A1**
Note: Award M1 for a correct derivative. Award (A1) for $x = \pm 1$ identified as critical values. Final A1 requires a correct interval with exact endpoints; accept $]-1, 1[$.

Q10 — Tangent Parallel to a Given Line EASY

$f'(x) = 10x - 3$ **M1**
Setting $f'(x) = 7$: $10x - 3 = 7 \Rightarrow 10x = 10$ **(A1)**
$x = 1$ **A1**

Note: Award M1 for differentiating f and equating their $f'(x)$ to the gradient of the given line. Award (A1) for the equation $10x - 3 = 7$ seen. Final A1 for $x = 1$.

Q11 — Increasing/Decreasing from Sign of f' MEDIUM

(a) $f'(x) = 12x^3 - 24x^2$ **A2**

Note: Award A1 for $12x^3$ correct and A1 for $-24x^2$ correct. Award A1A0 if an extra term appears.

(b) Setting $f'(x) > 0$: $12x^2(x - 2) > 0 \Rightarrow x > 2$ **M1A1**

Note: Accept $x > 2$ written as $(2, \infty)$. Do not accept $x \geq 2$.

Q12 — Differentiate Rational and Surd Terms MEDIUM

Rewrite $f(x)$ in power form: $f(x) = 5x^{-2} - 2x^{1/2}$ **M1**
Differentiate term by term: $f'(x) = -10x^{-3} - x^{-1/2}$, i.e. $-10x{-}3$ and $-x{-}1/2$ **M1A1A1**

Note: First M1 for rewriting both terms in power form (seen anywhere). Second M1 for applying the power rule to at least one term. Award A1 for $-10x^{-3}$ and A1 for $-x^{-1/2}$. Condone $-\frac{1}{2} \cdot 2 = -1$ shown explicitly. Accept equivalent unsimplified forms such as $-2 \cdot \frac{1}{2}x^{-1/2}$ for the second A1.

Q13 — Tangent at a Point on a Cubic MEDIUM

(a) $\dfrac{dy}{dx} = 3x^2 - 12x + 9$ **A2**

Note: Award A1 for $3x^2$ and A1 for $-12x + 9$.

(b) Substitute $x = 1$: $m = 3(1)^2 - 12(1) + 9 = 0$ **M1**
Tangent equation: $y = 6$ **A1**

Note: Accept $y = 0 \cdot x + 6$.

Q14 — Stationary Points and Decreasing Interval MEDIUM

(a) Differentiate: $\dfrac{dy}{dx} = 6x^2 - 18x + 12$ **M1**
Set equal to zero: $6(x^2 - 3x + 2) = 0 \Rightarrow 6(x - 1)(x - 2) = 0$ **M1**
$x = 1$ and $x = 2$ **A1**

Note: Award first M1 for correct derivative. Award second M1 for setting derivative equal to zero and *attempting to factorise or use the quadratic formula.*

(b) $1 < x < 2$ **A1**

Note: Accept $1 < x < 2$ or $(1, 2)$. *Do not accept* $1 \leq x \leq 2$.

Q15 — Gradient via Differentiation of Surd and Reciprocal MEDIUM

Rewrite in power form: $f(x) = x^{3/2} - 4x^{-1}$ **M1**
Differentiate: $f'(x) = \frac{3}{2}x^{1/2} + 4x^{-2}$ **M1A1**

Note: Award first M1 for rewriting both terms in power form. Award second M1 for applying the power rule to at least one term. A1 for fully correct derivative.

Substitute $x = 4$: $f'(4) = \frac{3}{2}(4)^{1/2} + 4(4)^{-2} = \frac{3}{2}(2) + \frac{4}{16} = 3 + \frac{1}{4} = 13/4$ **(M1)A1**

Note: The final A1 is the fourth mark. Award A1ft for correct substitution into their $f'(x)$ *provided the value is not impossible.*

Q16 — Points where Gradient Equals a Given Value MEDIUM

Differentiate: $g'(x) = 12x^2 - 6x - 18$ **M1A1**
Set $g'(x) = 6$: $12x^2 - 6x - 18 = 6 \Rightarrow 12x^2 - 6x - 24 = 0 \Rightarrow 2x^2 - x - 4 = 0$ **M1**
Using GDC to solve: $x = \dfrac{1 \pm \sqrt{33}}{4}$
$x = 1.686$ and $x = -1.186$ **A1**

Note: Award A1 for both values correct to 3 s.f. Accept exact forms $\dfrac{1 \pm \sqrt{33}}{4}$. *Do not penalise if both are given; however extra values not arising from the equation are penalised.*

Q17 — Normal to a Cubic at a Given Point MEDIUM

(a) Differentiate: $\dfrac{dy}{dx} = 3x^2 - 8x + 3$ **M1**
Substitute $x = 3$: $f'(3) = 3(9) - 8(3) + 3 = 27 - 24 + 3 = 6$ **(M1)**
Gradient of normal: $m_\perp = -1/6$ **A1**

Note: Award first M1 for correct derivative. Award (M1) for substituting $x = 3$. *A1 for* $-1/6$.

(b) Normal: $y - 7 = -\dfrac{1}{6}(x - 3)$, so $6y - 42 = -(x - 3)$, giving: $x + 6y - 45 = 0$ **A1**

Note: Accept any integer multiple, e.g. $-x - 6y + 45 = 0$. *The form* $ax + by + d = 0$ *with* $a, b, d \in \mathbb{Z}$ *must be achieved for this mark.*

Q18 — Show Derivative Factorises; Hence Decreasing Interval MEDIUM

(a) Differentiate: $h'(x) = x^2 - 5x + 4$ **M1**
Factorise: $x^2 - 5x + 4 = (x-1)(x-4)$ **A1**

Note: M1 for differentiating all three non-constant terms correctly. A1 for obtaining $x^2 - 5x + 4$ and showing the factorised form. The final line $(x-1)(x-4)$ is the AG; it earns nothing on its own but is required.

(b) h is decreasing when $h'(x) < 0$, i.e. $(x-1)(x-4) < 0$: **M1**
$1 < x < 4$ **A1**

Note: M1 for recognising the sign condition on their factorised derivative. Do not accept $1 \leq x \leq 4$.

Q19 — Stationary Points with Nature Justified MEDIUM

Differentiate: $f'(x) = 6x^2 - 30x + 36$ **M1**
Set $f'(x) = 0$: $6(x^2 - 5x + 6) = 0 \Rightarrow 6(x-2)(x-3) = 0$ **(M1)**
$x = 2$ and $x = 3$ **A1**
Sign analysis (or second derivative test): **R1**
$f''(x) = 12x - 30$; $f''(2) = -6 < 0 \Rightarrow$ local maximum at $x = 2$; $f''(3) = 6 > 0 \Rightarrow$ local minimum at $x = 3$.

Note: R1 requires an explicit justification — sign of f'' or sign change of f' stated for both points. Do not award R1 for conclusion alone.

Q20 — Find Parameters from Tangent and Point Conditions MEDIUM

The curve passes through $(2, 6)$: $4a + 2b = 6$ **M1**
Differentiate $y = ax^2 + bx$: $\dfrac{dy}{dx} = 2ax + b$ **M1**
At $x = 2$, gradient $= 7$: $4a + b = 7$ **A1**
Solve simultaneously: subtracting gives $b = -1$; substituting back, $4a = 8 \Rightarrow a = 2$.
$a = 2, b = -1$ **A1**

Note: Award first M1 for substituting $(2, 6)$ into the equation of the curve. Award second M1 for differentiating and equating the derivative at $x = 2$ to 7. A1 for a correct second equation. Final A1 for both $a = 2$ and $b = -1$ correct.

Q21 — Rewrite and Differentiate; Normal Gradient HARD

(a) Rewrite: $f(x) = 4x^{-2} - 3x^{1/2}$ **M1**
$f'(x) = 4 \cdot (-2)x^{-3} - 3 \cdot \frac{1}{2}x^{-1/2} = -\dfrac{8}{x^3} - \dfrac{3}{2\sqrt{x}}$ **AG**

Note: M1 requires both terms correctly rewritten as power form before differentiating. Award M1 even if one sign error, provided method is clear. Final line must match the printed target exactly for AG.

(b) $f'(4) = -\dfrac{8}{64} - \dfrac{3}{2\sqrt{4}} = -\dfrac{1}{8} - \dfrac{3}{4}$ **M1**
$= -\dfrac{1}{8} - \dfrac{6}{8} = -\dfrac{7}{8}$ **A1**
Gradient of normal $= 8/7$ **A1**
Note: Final A1 requires explicit use of negative reciprocal relationship. Condone $+8/7$ if negative reciprocal shown. Award A1ft on their $f'(4)$ provided it is non-zero and the reciprocal relationship is stated.

Q22 — Stationary Point Condition; Tangent from Parameters HARD

Stationary point at $x = 1$: $f'(x) = 3x^2 - 12x + p$ **M1**
$f'(1) = 0 \Rightarrow 3 - 12 + p = 0 \Rightarrow p = 9$ **A1**
Passes through $(0, 5)$: $q = 5$ **A1**
$f'(4) = 3(16) - 12(4) + 9 = 48 - 48 + 9 = 9$ **M1**
$f(4) = 64 - 96 + 36 + 5 = 9$
Tangent: $y - 9 = 9(x - 4)$, i.e. $y = 9x - 27$ **A1**
Note: Trap — students may forget to use the $q = 5$ condition and leave the tangent in terms of q; final A1 requires both parameters resolved. Accept $y = 9x - 27$ or equivalent.

Q23 — Increasing Intervals; Reverse Gradient Solve with Domain Condition HARD

(a) $f'(x) = 6x^2 - 18x + 12$ **M1**
$f'(x) > 0 \Rightarrow 6(x^2 - 3x + 2) > 0 \Rightarrow 6(x - 1)(x - 2) > 0$ **A1**
f is increasing for $x < 1$ or $x > 2$ (i.e. $(-\infty, 1) \cup (2, \infty)$) **A1**
Note: Both intervals required for second A1. Accept strict or non-strict inequalities at boundary.

(b) $f'(k) = 12 \Rightarrow 6k^2 - 18k + 12 = 12 \Rightarrow 6k^2 - 18k = 0 \Rightarrow 6k(k - 3) = 0$ **M1**
$k = 0$ or $k = 3$; since $k > 2$, $k = 3$ **A1**
Note: Trap — both roots must be found; A1 requires explicit rejection of $k = 0$ by reference to $k > 2$. Do not award A1 if $k = 0$ is selected or if neither root is rejected.

Q24 — System of Equations; Stationary Points of a Cubic HARD

$y' = 3ax^2 + b$; stationary point at $x = 1/\sqrt{3}$: $3a \cdot \frac{1}{3} + b = 0 \Rightarrow a + b = 0 \Rightarrow b = -a$ **M1A1**
Passes through $(2, 10)$: $8a + 2b = 10 \Rightarrow 8a - 2a = 10 \Rightarrow 6a = 10 \Rightarrow a = 5/3$, $b = -5/3$ **A1**
Stationary points: $3 \cdot \frac{5}{3}x^2 - \frac{5}{3} = 0 \Rightarrow 5x^2 = \frac{5}{3} \Rightarrow x^2 = \frac{1}{3} \Rightarrow x = \pm 1/\sqrt{3}$ **M1**
Coordinates: $\left(\dfrac{1}{\sqrt{3}}, -\dfrac{10\sqrt{3}}{27}\right)$ and $\left(-\dfrac{1}{\sqrt{3}}, \dfrac{10\sqrt{3}}{27}\right)$ **A1**
Note: Accept rationalised forms. Trap — students may forget the negative root after computing a and b; both stationary points required for final A1. Condone equivalent exact forms.

Q25 — Stationary Points AG; Decreasing Interval; Tangent Meets Curve Again HARD

(a) $f'(x) = 3x^2 - 6x - 9 = 3(x^2 - 2x - 3) = 3(x-3)(x+1)$ **M1**
$f'(x) = 0 \Rightarrow x = -1$ or $x = 3$; $f(-1) = -1 - 3 + 9 + 2 = 7$, $f(3) = 27 - 27 - 27 + 2 = -25$ **A1**
Hence $A(-1, 7)$ and $B(3, -25)$ are stationary points. **AG**

(b) f is decreasing for $-1 < x < 3$ **A1**

(c) $f'(-1) = 0$, so the tangent to the curve at A is the horizontal line $y = 7$. **(M1)**
Using GDC to solve $x^3 - 3x^2 - 9x + 2 = 7$ for $x \neq -1$: $(x+1)^2(x-5) = 0$, so $x = -1$ (point of tangency, double root) or $x = 5$.
$f(5) = 125 - 75 - 45 + 2 = 7$; $Q = (5, 7)$ **A1**
Note: Award (M1) for recognising that the tangent at A is the horizontal line $y = 7$ because $f'(-1) = 0$. A1 requires both coordinates of Q correct.

Q26 — Rational Powers; Stationary Point Type with Justification HARD

(a) $m = 4/3$, $n = 1/3$ **A1**

(b) Rewrite: $g(x) = x^{4/3} - 8x^{1/3}$ **M1**
$g'(x) = \frac{4}{3}x^{1/3} - \frac{8}{3}x^{-2/3}$ **A1**
Note: M1 for attempt to differentiate both terms using power rule. A1 requires both coefficients and powers correct. Accept unsimplified.

(c) $g'(x) = 0$: $\frac{4}{3}x^{1/3} = \frac{8}{3}x^{-2/3}$; multiply both sides by $x^{2/3}$ (valid for $x > 0$): $\frac{4}{3}x = \frac{8}{3}$ **M1**
$x = 2$
$g''(x) = \frac{4}{9}x^{-2/3} + \frac{16}{9}x^{-5/3} > 0$ for $x > 0$, so $x = 2$ is a local minimum **A1R1**
Note: R1 for explicit statement that $g''(2) > 0$ (or sign of g' changes from negative to positive) confirming minimum. Accept sign-chart argument. Trap — students may not restrict to $x > 0$; no penalty if $x = 2$ identified correctly.

Q27 — Tangent Through a Given Point HARD

Rewrite: $f(x) = x + 3/x$ **M1**
$f'(x) = 1 - 3x^{-2}$ **A1**
Tangent at $x = a$: gradient $= f'(a) = 1 - \frac{3}{a^2}$; point $\left(a, a + \frac{3}{a}\right)$
L passes through $(0, 3)$, so the tangent's y-value at $x = 0$ equals 3:
$f(a) - af'(a) = 3$ **M1**
$\left(a + \frac{3}{a}\right) - a\left(1 - \frac{3}{a^2}\right) = 3 \Rightarrow \frac{6}{a} = 3$ **A1**
$a = 2$ **A1**
Gradient of L: $f'(2) = 1 - 3/4 = 1/4$; $f(2) = 2 + 3/2 = 3.5$

Equation of L: $y = x/4 + 3$ **A1**

Note: Method marks awarded for correct use of the tangent-through-a-point condition $f(a) - af'(a) = c$, where c is the given y-intercept. FT on their a for the equation of L.

Q28 — Velocity Zeros; Negative Direction; Total Distance by GDC HARD

(a) $v(t) = 3t^2 - 12t + 9 = 3(t-1)(t-3)$ **M1**
$v(t) = 0 \Rightarrow t = 1$ or $t = 3$ **A1**

(b) $v(t) < 0$ for $1 < t < 3$ **A1**

(c) Total distance $= \left|\int_0^1 v\,dt\right| + \left|\int_1^3 v\,dt\right| + \left|\int_3^4 v\,dt\right|$ **M1**
$s(0) = -2$, $s(1) = 1 - 6 + 9 - 2 = 2$, $s(3) = 27 - 54 + 27 - 2 = -2$, $s(4) = 64 - 96 + 36 - 2 = 2$
Distance $= |s(1) - s(0)| + |s(3) - s(1)| + |s(4) - s(3)| = |4| + |-4| + |4| = 12$ metres **A1**

Note: Trap — award M1 for recognising the need to split the integral at the roots $t = 1$ and $t = 3$; final A1 requires correct total of 12 m. Do not accept $s(4) - s(0) = 4$ (net displacement). GDC integration evidences (M1).

Q29 — Quartic Stationary Points; Classification with Justification HARD

(a) $\dfrac{dy}{dx} = 4x^3 - 16x$ **M1**
$4x(x^2 - 4) = 0 \Rightarrow x(x-2)(x+2) = 0$ **A1**
$x = 0$, $x = 2$, $x = -2$ **A1**

Note: All three roots required for second A1.

(b) At the stationary point with positive x-coordinate, i.e. $x = 2$: gradient of the tangent is zero by definition. **R1**
$\dfrac{d^2y}{dx^2} = 12x^2 - 16$; at $x = 2$: $12(4) - 16 = 32 > 0$, so it is a local minimum. **A1**

Note: R1 for explicit statement that the tangent gradient at a stationary point is zero by definition, and A1 for correct second-derivative test confirming minimum. Accept sign-chart of f' near $x = 2$ for A1. Trap — question asks about the gradient of the tangent (which is zero), testing whether students conflate this with classification.

Q30 — Normal Equation AG; Normal Meets Curve Again HARD

(a) $\dfrac{dy}{dx} = x - 2$; at $x = 4$: gradient of tangent $= 2$ **M1**
Gradient of normal $= -1/2$
Normal: $y - 5 = -\dfrac{1}{2}(x - 4)$, i.e. $y = -x/2 + 7$ **AG**

Note: M1 for differentiating and evaluating at $x = 4$. Must show gradient of normal as negative reciprocal. Final line must exactly match $y = -x/2 + 7$ for AG; do not award if verified numerically only.

(b) Substitute $y = -x/2 + 7$ into $y = \frac{1}{2}x^2 - 2x + 5$: **M1**
$\frac{1}{2}x^2 - 2x + 5 = -\frac{x}{2} + 7 \Rightarrow \frac{1}{2}x^2 - \frac{3}{2}x - 2 = 0 \Rightarrow x^2 - 3x - 4 = 0$ **A1**
$(x-4)(x+1) = 0 \Rightarrow x = 4$ (point P) or $x = -1$ **(M1)**
$y = -\dfrac{(-1)}{2} + 7 = 15/2$; $Q = (-1, 15/2)$ **A1**

Note: (M1) for solving the quadratic (by GDC or factorisation). A1 requires both coordinates of Q correct. Trap — students may select $x = 4$ as Q; the problem states Q is the other intersection, so $x = 4$ must be rejected with reason. FT on their normal equation for M1 and subsequent A marks provided the quadratic is formed correctly.