IB Math AA SL — Topic 5

Integration

Antiderivatives & $+c$ · Powers of x · Areas Using a GDC

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Scan me for help

Name:____________________________ Date:______________

Score:_________/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

Key Formulae

- **Power Rule (indefinite):** $\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$
- **Constant multiple:** $\displaystyle\int kf(x)\,dx = k\int f(x)\,dx$
- **Sum/difference:** $\displaystyle\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
- **Definite integral (signed area):** $\displaystyle\int_a^b f(x)\,dx = \left[F(x)\right]_a^b = F(b) - F(a)$
- **Area under a curve (above the x-axis, $a \leq x \leq b$):** $\displaystyle A = \int_a^b f(x)\,dx$
- **Additivity over adjacent intervals:** $\displaystyle\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$
- **Linearity deduction:** $\displaystyle\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$ and $\displaystyle\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

GDC Tips

1. **Definite integral value:** Use Math $\to$ fnInt($f(x), x, a, b$) on TI-84, or the $\int$ template in the Calculus menu on TI-Nspire; write the integral expression before stating the numerical answer.
2. **Area between a curve and the x-axis:** When $f(x)$ changes sign on $[a, b]$, split at roots first; enter fnInt(abs($f(x)$),x,a,b) or compute each sub-interval separately and add magnitudes.
3. **Area between two curves:** Graph both functions, identify intersection x-values using Intersection (TI-84: 2nd CALC 5; Nspire: Analyze Graph $\to$ Intersection), then integrate $|f(x) - g(x)|$ over that interval.
4. **Checking an antiderivative:** Differentiate your answer symbolically or use d/dx at a test point to verify $F'(x) = f(x)$ before substituting the boundary values.
5. **Storing intersection or root values:** Store exact GDC x-values in variables (STO$\to$A, B) before evaluating the integral to avoid rounding errors propagating.
6. **Reading area from a graph:** Sketch $y = f(x)$ on the GDC over the stated domain to confirm the curve is above (or below) the axis, so you can decide whether a sign change splits the region.

Common Mistakes to Avoid

1. **Omitting the constant of integration.** Every indefinite integral requires $+c$; writing $\int 3x^2\,dx = x^3$ loses a mark. The constant is determined only when a point on the curve is given.
2. **Forgetting to rewrite before integrating.** Expressions such as $\frac{4}{\sqrt{x}}$ or $x(x-3)$ must be rewritten as $4x^{-1/2}$ or $x^2 - 3x$ before applying the power rule; integrating the original unsimplified form is not accepted.
3. **Applying the power rule to x^{-1}.** The formula $\frac{x^{n+1}}{n+1}$ is undefined at $n = -1$; the antiderivative of x^{-1} is ln $|x| + c$. Questions in this course avoid this, but misidentifying an exponent as -1 causes an error.
4. **Sign error when reversing limits.** Students write $\int_a^b f(x)\,dx = \int_b^a f(x)\,dx$ instead of the negative; this propagates into deduction questions that use additivity or the reversal property.
5. **Confusing a signed definite integral with area.** If $f(x) < 0$ on part of $[a, b]$, then $\int_a^b f(x)\,dx$ is not the total area; the region below the axis contributes negatively. Always check the graph and split at roots when area (not net signed value) is required.
6. **Incorrect addition of the constant when finding f from f'.** After integrating $f'(x)$ and adding c, students substitute the given point into f' rather than f to find c, or substitute correctly but then forget to write the complete function with the found value of c substituted back in.

Section A — Easy **EASY** (10 questions × 3 marks = 30 marks)

Scan me for help

1. EASY No Calculator **[3 marks]**

Find $\int (3x^2 + 4x - 1)\, dx$.

2. EASY No Calculator **[3 marks]**

Find $\int x^5\, dx$.

3. EASY No Calculator **[3 marks]**

Find $\int (2x^{1/2} + 6x^2)\, dx$.

4. EASY No Calculator **[3 marks]**

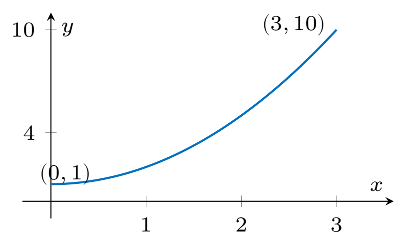
The gradient function of a curve is $f'(x) = 4x - 3$, where $x \in \mathbb{R}$. The curve passes through the point $(2, 5)$. Find $f(x)$.

5. EASY Calculator **[3 marks]**

Find the exact value of $\displaystyle\int_1^4 (3x^2 - 2x)\, dx$.

6. EASY Calculator **[3 marks]**

The diagram below shows the graph of $f(x) = x^2 + 1$ for $0 \leq x \leq 3$. The shaded region is bounded by the curve, the x-axis, and the lines $x = 0$ and $x = 3$.



Use your GDC to find the area of the shaded region. Give your answer correct to 3 significant figures.

7. EASY Calculator **[3 marks]**

Find $\displaystyle\int (5x^{-2} + 3)\, dx$.

8. **EASY** **Calculator** **[3 marks]**

The gradient function of a curve is $f'(x) = 6x^2 - 2x + 1$, where $x \in \mathbb{R}$. The curve passes through the point $(1, 4)$. Find $f(x)$.

9. **EASY** **Calculator** **[3 marks]**

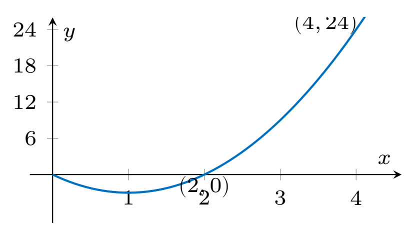
The table below shows two definite integrals.

Integral	Value
$\int_1^5 g(x)\,dx$	12
$\int_1^3 g(x)\,dx$	7

Find the value of $\displaystyle\int_3^5 g(x)\,dx$.

10. **EASY** **Calculator** **[3 marks]**

Use your GDC to find the exact area of the region enclosed by the curve $y = 3x^2 - 6x$, the x-axis, and the lines $x = 2$ and $x = 4$, for $x \in \mathbb{R}$.



Give your answer correct to 3 significant figures.

Section B — Medium **MEDIUM** (10 questions × 4 marks = 40 marks)

Scan me for help

11. MEDIUM No Calculator **[4 marks]**

Let $f'(x) = 6x^2 - \dfrac{4}{x^3}$, where $x \in \mathbb{R}$, $x \neq 0$. Find $f(x)$, given that $f(1) = 5$.

12. MEDIUM Calculator **[4 marks]**

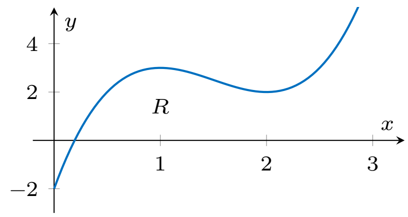
The velocity of a particle, in m s^{-1}, at time t seconds where $t \geq 0$, is given by $v(t) = 3t^2 - 8t + 4$. Find the total distance travelled by the particle between $t = 0$ and $t = 3$.

13. MEDIUM No Calculator **[4 marks]**

Let $f'(x) = \sqrt{x} + \dfrac{1}{\sqrt{x}}$, where $x > 0$. Find $f(x)$, given that $f(4) = 10$.

14. **MEDIUM** **Calculator** **[4 marks]**

The diagram below shows the graph of $f(x) = 2x^3 - 9x^2 + 12x - 2$ for $0 \leq x \leq 3$.



Find the area of the region R between f and the x-axis for $0 \leq x \leq 2$.

15. **MEDIUM** **No Calculator** **[4 marks]**

It is given that $\int_1^5 g(x)\,dx = 12$ and $\int_1^3 g(x)\,dx = 7$.

(a) Find $\int_3^5 g(x)\,dx$.

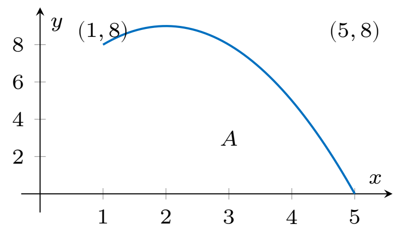
(b) Find $\int_3^5 [3g(x) - 2]\,dx$.

16. **MEDIUM** **No Calculator** **[4 marks]**

The function f is defined by $f(x) = (2x-1)^3$ for $x \in \mathbb{R}$. Find the equation of the curve $y = F(x)$, where F is an antiderivative of f and the curve passes through the point $(1, 3)$. Express your answer in the form $y = a(2x-1)^4 + b$.

17. MEDIUM Calculator **[4 marks]**

The graph below shows $f(x) = 5 + 4x - x^2$ for $1 \leq x \leq 5$.



Find the area A of the shaded region between the curve and the x-axis for $1 \leq x \leq 5$.

18. MEDIUM Calculator **[4 marks]**

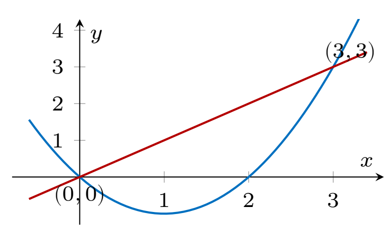
Let $h'(x) = 4x^3 - 6x + 3$, where $x \in \mathbb{R}$. The curve $y = h(x)$ passes through the point $(-1, 4)$.

(a) Find $h(x)$.

(b) Find the value of $h(2)$.

19. MEDIUM Calculator **[4 marks]**

The diagram shows the graphs of $f(x) = x^2 - 2x$ and $g(x) = x$ for $x \in \mathbb{R}$.



Find the area of the enclosed region between the two curves.

20. MEDIUM No Calculator **[4 marks]**

Let $f(x) = \dfrac{3}{x^2} - 2\sqrt{x}$, where $x > 0$.

(a) Write $f(x)$ in the form $ax^p + bx^q$, stating the values of a, p, b and q.

(b) Find $\displaystyle\int f(x)\,dx$.

Section C — Hard HARD (10 questions × 5 marks = 50 marks)

Scan me for help

21. HARD Calculator **[5 marks]**

Let $f'(x) = 6x^2 - \dfrac{4}{\sqrt{x}}$, where $x > 0$. The curve $y = f(x)$ passes through the point $(1, -4)$.

(a) Show that $f(x) = 2x^3 - 8\sqrt{x} + 2$. [3]

(b) Find the value of x for which $f(x) = 20$, where $x > 0$. [2]

22. HARD No Calculator **[5 marks]**

It is given that $\displaystyle\int_1^5 g(x)\,dx = 14$ and $\displaystyle\int_1^3 g(x)\,dx = 9$.

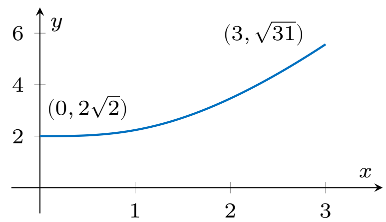
(a) Write down the value of $\displaystyle\int_3^5 g(x)\,dx$. [1]

(b) Find the value of $\displaystyle\int_1^5 [3g(x) - 4]\,dx$. [2]

(c) Find the value of $\displaystyle\int_0^2 g(2x+1)\,dx$, given that g is defined on all of $\mathbb{R}$. [2]

23. **HARD** **Calculator** **[5 marks]**

The function f is defined by $f(x) = \sqrt{4 + x^3}$ for $0 \le x \le 3$.



Find the area of the shaded region bounded by $y = f(x)$, the x-axis and the lines $x = 0$ and $x = 3$. Give your answer correct to 3 significant figures.

24. **HARD** **No Calculator** **[5 marks]**

Let $f'(x) = \dfrac{(x^2+1)^2}{x^2}$ for $x \neq 0$.

(a) Show that $f'(x) = x^2 + 2 + x^{-2}$. [1]

(b) Given that $f(1) = \dfrac{11}{3}$, find $f(x)$. [4]

25. **HARD** **Calculator** **[5 marks]**

The velocity of a particle, in m s^{-1}, at time t seconds is given by $v(t) = t^2 - 5t + 4$, for $0 \le t \le 5$.

(a) Write down the times when the particle is at rest. [1]

(b) Find the total distance travelled by the particle in the interval $0 \le t \le 5$. [4]

26. **HARD** **Calculator** **[5 marks]**

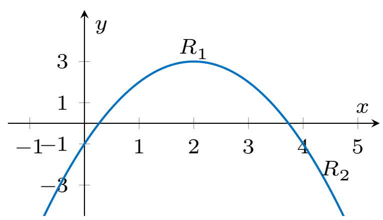
The rate of change of the depth of water in a tank, in m min^{-1}, is modelled by $\dfrac{dh}{dt} = 0.5\cos(0.4t) - 0.2$, where t is the time in minutes, $t \geq 0$. The initial depth of water is 3.6 m.

(a) Find an expression for $h(t)$. [3]

(b) Find the first time at which the depth is decreasing at its maximum rate. [2]

27. **HARD** **Calculator** **[5 marks]**

The graph below shows $y = f(x)$ for $-1 \leq x \leq 5$, where $f(x) = -x^2 + 4x - 1$.



Find the total area enclosed between $y = f(x)$ and the x-axis for $0 \leq x \leq 5$. Give your answer correct to 3 significant figures.

28. **HARD** **No Calculator** **[5 marks]**

Let $h'(x) = ax^2 + bx + c$, where $a, b, c \in \mathbb{R}$ and $x \in \mathbb{R}$. It is given that $h'(1) = 0$, $h'(-2) = 0$, $h'(0) = -4$, and $h(0) = 4$. Find $h(3)$.

29. HARD Calculator **[5 marks]**

The function f is defined by $f(x) = \dfrac{x^2+3}{\sqrt{x}}$ for $x > 0$.

(a) Write $f(x)$ in the form $x^p + 3x^q$, stating the values of p and q. [2]

(b) Hence find $\displaystyle\int f(x)\,dx$. [2]

(c) The region bounded by $y = f(x)$, the x-axis and the lines $x = 1$ and $x = 4$ has area A. Find A correct to 3 significant figures. [1]

30. HARD Calculator **[5 marks]**

The population density of a species of insect, in thousands per hectare, at a distance x km from a water source is modelled by $P(x) = 5x^{0.4}e^{-0.6x}$, for $0 \leq x \leq 6$. The total population (in thousands) over a strip of unit width is $\displaystyle\int_0^6 P(x)\,dx$.

(a) Find the distance from the water source at which the population density is greatest. Give your answer correct to 3 significant figures. [2]

(b) Find the total population, giving your answer correct to 3 significant figures. [2]

(c) Determine whether the total population would increase or decrease if the upper limit were extended from $x = 6$ to $x = 8$. Justify your answer. [1]

Mark Schemes

Q1 — Integrate a Polynomial EASY

Integrate term by term, increasing each power by 1 and dividing **M1**

$\int (3x^2 + 4x - 1)\, dx = x^3 + 2x^2 - x + c$ **A1**

$+c$ present **A1**

Note: Award A1A0 if the answer is otherwise correct but $+c$ is omitted. All three terms must be correct for the second A1.

Q2 — Integrate a Single Power EASY

Increase the power by 1 and divide by the new power **M1**

$\int x^5\, dx = \frac{x^6}{6} + c$ **A1**

$+c$ present **A1**

Note: Award A1A0 if $\frac{x^6}{6}$ is correct but $+c$ is omitted.

Q3 — Integrate Fractional and Integer Powers EASY

Increase each power by 1 and divide; accept terms written in surd or power form **M1**

$\int (2x^{1/2} + 6x^2)\, dx = \frac{2x^{3/2}}{3/2} + \frac{6x^3}{3} + c = \frac{4}{3}x^{3/2} + 2x^3 + c$ **A1**

$+c$ present **A1**

Note: Accept $\frac{4}{3}\sqrt{x^3}$ or $\frac{4}{3}x\sqrt{x}$ for $\frac{4}{3}x^{3/2}$. Award A1A0 if both terms are correct but $+c$ is absent.

Q4 — Find f from the Gradient Function and a Point EASY

Integrate: $f(x) = 2x^2 - 3x + c$ **M1**

Substitute $(2, 5)$: $5 = 2(4) - 3(2) + c \Rightarrow 5 = 2 + c \Rightarrow c = 3$ **A1**

$f(x) = 2x^2 - 3x + 3$ **A1**

Note: Award M1 for a correct integration (allow missing $+c$). Final A1 requires the complete, correct expression.

Q5 — Definite Integral of a Polynomial EASY

$\int_1^4 (3x^2 - 2x)\, dx = \left[x^3 - x^2\right]_1^4$ **M1**

$= (64 - 16) - (1 - 1) = 48$ **A1A1**

Note: Award A2 for the exact value 48; A1 only if supporting working is shown but the answer contains a rounding error.

Q6 — Area Under a Curve by GDC EASY

Using GDC: $\int_0^3 (x^2 + 1)\, dx$ **(M1)**

$= 12.0$ (exact value 12) **(A1)**

Area $= 12.0$ (3 s.f.) **A1**

Note: Award (M1) for setting up the correct definite integral on the GDC. The exact value is 12; accept 12.0.

Q7 — Integrate a Negative Power EASY

Rewrite: $5x^{-2} + 3$; integrate term by term **M1**

$\int (5x^{-2} + 3)\, dx = \frac{5x^{-1}}{-1} + 3x + c = -\frac{5}{x} + 3x + c$ **A1**

$+c$ present **A1**

Note: Accept $-5x^{-1} + 3x + c$. Award A1A0 if both terms are correct but $+c$ is absent.

Q8 — Find f from the Gradient Function and a Point EASY

Integrate: $f(x) = 2x^3 - x^2 + x + c$ **M1**

Substitute $(1, 4)$: $4 = 2(1) - 1(1) + 1 + c \Rightarrow 4 = 2 + c \Rightarrow c = 2$ **A1**

$f(x) = 2x^3 - x^2 + x + 2$ **A1**

Note: Award M1 for a correct integration. Final A1 requires the complete, correct expression with $c = 2$ substituted.

Q9 — Additivity of Definite Integrals EASY

State the additivity property: $\int_1^5 g(x)\, dx = \int_1^3 g(x)\, dx + \int_3^5 g(x)\, dx$ **M1**

$12 = 7 + \int_3^5 g(x)\, dx$ **A1**

$\int_3^5 g(x)\,dx = 5$ **A1**

Note: Award M1 for explicitly writing the additivity statement with the correct limits. A1A0 if the correct property is stated but an arithmetic error gives a wrong final answer.

Q10 — Area Under a Curve by GDC EASY

Using GDC: $\int_2^4 (3x^2 - 6x)\,dx$ **(M1)**
$= 20.0$ (exact value 20) **(A1)**
Area $= 20.0$ (3 s.f.) **A1**

Note: The curve $y = 3x^2 - 6x$ is non-negative on $[2, 4]$ (roots at $x = 0$ and $x = 2$), so the definite integral gives the area directly. Award (M1) for entering the correct integral on the GDC. The exact value is 20; accept 20.0.

Q11 — Recover f from the Gradient Function and a Point MEDIUM

Rewrite and integrate: $f'(x) = 6x^2 - 4x^{-3}$ **(M1)**
$f(x) = \dfrac{6x^3}{3} - \dfrac{4x^{-2}}{-2} + c = 2x^3 + 2x^{-2} + c$ **A1**
Note: Award A0 if $+c$ is absent.
Apply $f(1) = 5$: $2(1)^3 + 2(1)^{-2} + c = 5 \Rightarrow 4 + c = 5 \Rightarrow c = 1$ **M1**
$f(x) = 2x^3 + 2x^{-2} + 1$ **A1**
Note: Accept $f(x) = 2x^3 + \dfrac{2}{x^2} + 1$.

Q12 — Total Distance via GDC MEDIUM

Find zeros of $v(t)$: $3t^2 - 8t + 4 = 0 \Rightarrow t = \frac{2}{3}$ or $t = 2$ **M1**
Note: Award M1 for attempting to find roots of v; condone use of GDC.
Antiderivative: $V(t) = t^3 - 4t^2 + 4t$ **(M1)**
Total distance $= \int_0^{2/3} |v|\,dt + \int_{2/3}^2 |v|\,dt + \int_2^3 |v|\,dt$
$= |V(\tfrac{2}{3}) - V(0)| + |V(2) - V(\tfrac{2}{3})| + |V(3) - V(2)|$
$= \dfrac{32}{27} + \dfrac{32}{27} + 3 = 1.185 + 1.185 + 3.000$ **(A1)**
Total distance $= \dfrac{145}{27} = 5.37$ m (3 s.f.) **A1**

Note: Award final A1 only for 5.37 (3 s.f.); do not accept the signed displacement $\int_0^3 v\,dt = 3$.

Q13 — Recover f from a Gradient Function with Surds MEDIUM

Rewrite: $f'(x) = x^{1/2} + x^{-1/2}$ **(M1)**

Integrate: $f(x) = \dfrac{x^{3/2}}{3/2} + \dfrac{x^{1/2}}{1/2} + c = \dfrac{2}{3}x^{3/2} + 2x^{1/2} + c$ **A1**

Note: Award A0 if $+c$ is absent.

Apply $f(4) = 10$: $\dfrac{2}{3}(8) + 2(2) + c = 10 \Rightarrow \dfrac{16}{3} + 4 + c = 10$ **M1**

$c = 10 - \dfrac{28}{3} = \dfrac{2}{3}$ **A1**

$f(x) = \dfrac{2}{3}x^{3/2} + 2x^{1/2} + \dfrac{2}{3}$

Q14 — Area Under a Curve by GDC MEDIUM

Check the sign of f on $[0, 2]$: $f(0) = -2 < 0$, so f is *not* non-negative on the whole interval; f has a root at $x = 0.194$ (3 s.f.), so the signed integral must be split there. **M1**

Note: Condoning $f \geq 0$ on $[0, 2]$ without checking is the trap in Common Mistake 5 above; the naive signed integral $\int_0^2 f\,dx = 4.00$ is not the area.

$\displaystyle\int_0^{0.194} |f(x)|\,dx = 0.183, \quad \int_{0.194}^{2} |f(x)|\,dx = 4.183$ **(M1)(A1)**

Area $= 0.183 + 4.183 = 4.366\ldots$ **(A1)**

Area $= 4.37$ (3 s.f.) **A1**

Note: GDC entry $\int_0^2 |f(x)|\,dx = 4.37$ earns full marks directly. Do not award full marks for the unqualified naive value 4.

Q15 — Deductions from Given Integral Values MEDIUM

(a) Using additivity: $\displaystyle\int_3^5 g(x)\,dx = \int_1^5 g(x)\,dx - \int_1^3 g(x)\,dx$ **M1**

$= 12 - 7 = 5$ **A1**

(b) Using linearity: $\displaystyle\int_3^5 [3g(x) - 2]\,dx = 3\int_3^5 g(x)\,dx - \int_3^5 2\,dx$ **M1**

$= 3(5) - 2(5-3) = 15 - 4 = 11$ **A1**

Note: FT from their answer to (a); award A1ft if their (a) is used correctly in (b).

Q16 — Antiderivative of a Composite Power MEDIUM

Integrate using reverse chain rule: $F(x) = \dfrac{(2x-1)^4}{4 \times 2} + c = \dfrac{(2x-1)^4}{8} + c$ **M1A1**

Note: M1 for attempt at reverse chain rule; A1 for correct $\frac{1}{8}(2x-1)^4$; $+c$ required for A1.

Apply $(1,3)$: $\dfrac{(2(1)-1)^4}{8} + c = 3 \Rightarrow \dfrac{1}{8} + c = 3 \Rightarrow c = \dfrac{23}{8}$ **M1**

$y = \dfrac{1}{8}(2x-1)^4 + \dfrac{23}{8}$ **A1**

Note: Final A1 requires the form $a(2x-1)^4 + b$ with $a = \frac{1}{8}$, $b = \frac{23}{8}$; unsimplified equivalents accepted.

Q17 — Area Under a Parabola by GDC MEDIUM

Set up integral: $A = \displaystyle\int_1^5 (5 + 4x - x^2)\,dx$ **M1**

$= \left[5x + 2x^2 - \dfrac{x^3}{3}\right]_1^5$ **(M1)**

$= \left(25 + 50 - \dfrac{125}{3}\right) - \left(5 + 2 - \dfrac{1}{3}\right) = \dfrac{100}{3} - \dfrac{20}{3}$ **(A1)**

$= \dfrac{80}{3} = 26.7$ (3 s.f.) **A1**

Note: GDC entry $\int_1^5 (5 + 4x - x^2)\,dx = 26.7$ earns (M1)(A1)A1; accept exact answer $80/3$.

Q18 — Recover h from the Gradient Function and Evaluate MEDIUM

(a) Integrate: $h(x) = x^4 - 3x^2 + 3x + c$ **M1A1**

Note: M1 for attempt to integrate all three terms; A1 for fully correct expression including $+c$; award A0 if $+c$ absent.

Apply $h(-1) = 4$: $(1) - 3(1) + 3(-1) + c = 4 \Rightarrow 1 - 3 - 3 + c = 4 \Rightarrow c = 9$ **M1**

$h(x) = x^4 - 3x^2 + 3x + 9$

(b) $h(2) = 16 - 12 + 6 + 9 = 19$ **A1**

Note: A1 is awarded for correct evaluation using their $h(x)$ from (a); A1ft applies if their c is consistent.

Q19 — Area Between Two Curves by GDC MEDIUM

Find intersections: $x^2 - 2x = x \Rightarrow x^2 - 3x = 0 \Rightarrow x = 0$ or $x = 3$ **M1**

Note: Award M1 for equating the two functions and finding both intersection x-values.

Set up and evaluate: Area $= \displaystyle\int_0^3 [x - (x^2 - 2x)]\,dx = \int_0^3 (3x - x^2)\,dx$ **M1**

$= \left[\dfrac{3x^2}{2} - \dfrac{x^3}{3}\right]_0^3 = \dfrac{27}{2} - 9 = \dfrac{9}{2}$ **(A1)**

Area $= 4.50$ **A1**

Note: Accept exact answer $9/2$; GDC entry $\int_0^3 (3x - x^2)\,dx$ earns (A1)A1.

Q20 — Rewrite then Integrate Powers of x MEDIUM

(a) $f(x) = 3x^{-2} - 2x^{1/2}$ **A1**

$a = 3,\ p = -2,\ b = -2,\ q = \dfrac{1}{2}$

Note: All four values required for A1.

(b) Integrate term by term: $\displaystyle\int f(x)\,dx = \frac{3x^{-1}}{-1} - \frac{2x^{3/2}}{3/2} + c$ **M1A1**

$= -3x^{-1} - \dfrac{4}{3}x^{3/2} + c$ **A1**

Note: M1 for applying the power rule to at least one term correctly after the rewrite; first A1 for both terms correct; final A1 requires $+c$. Accept $-\dfrac{3}{x} - \dfrac{4}{3}x\sqrt{x} + c$.

Q21 — Recover f from the Gradient Function and an AG Target HARD

(a) Rewrite: $f'(x) = 6x^2 - 4x^{-1/2}$ **M1**

$f(x) = \dfrac{6x^3}{3} - \dfrac{4x^{1/2}}{1/2} + c = 2x^3 - 8x^{1/2} + c$ **A1**

Substitute $(1, -4)$: $2(1)^3 - 8(1)^{1/2} + c = -4 \Rightarrow 2 - 8 + c = -4 \Rightarrow c = 2$ **A1**

Therefore $f(x) = 2x^3 - 8\sqrt{x} + 2$ **AG**

Note: Award M1 for rewriting $f'(x)$ with a rational exponent and integrating to obtain two correct unsimplified terms. Final A1 for correct $c = 2$; the AG line earns nothing.

(b) Set $f(x) = 20$: $2x^3 - 8\sqrt{x} + 2 = 20$ **M1**

Solving by GDC (unique solution for $x > 0$): $x = 2.48$ (3 s.f.) **A1**

Note: Award M1 for setting their $f(x) = 20$. A1 for $x = 2.48$. Full precision: $x = 2.4827\ldots$; reject $x \le 0$ (domain $x > 0$).

Q22 — Linearity and Additivity of Definite Integrals HARD

(a) $\displaystyle\int_3^5 g(x)\,dx = 14 - 9 = 5$ **A1**

Note: Award A1 for correct use of additivity over adjacent intervals.

(b) $\displaystyle\int_1^5 [3g(x) - 4]\,dx = 3\int_1^5 g(x)\,dx - \int_1^5 4\,dx$ **M1**

$= 3(14) - 4(4) = 42 - 16 = 26$ **A1**

Note: Award M1 for applying linearity, separating both terms correctly.

(c) Let $u = 2x + 1$, so $du = 2\,dx$ and limits: $x = 0 \Rightarrow u = 1$, $x = 2 \Rightarrow u = 5$. **M1**

$\displaystyle\int_0^2 g(2x+1)\,dx = \frac{1}{2}\int_1^5 g(u)\,du = \frac{1}{2}(14)$ **M1**

$= 7$ **A1**

Note: Award first M1 for the substitution and correctly transformed limits (both must land inside the given interval $[1, 5]$); second M1 for linking to $\int_1^5 g\,du = 14$. A1 for 7.

Q23 — GDC Definite Integral of a Root Function HARD

Area $= \displaystyle\int_0^3 \sqrt{4 + x^3}\,dx$ **M1**

Using GDC: $\displaystyle\int_0^3 \sqrt{4 + x^3}\,dx = 9.2797\ldots$ **(M1)(A1)**

Note: Award first M1 for correctly setting up the definite integral with limits 0 and 3. Award (M1) for evidence of GDC use. Award (A1) for the full-precision value.

$= 9.28$ (3 s.f.) **A2**

Note: Award A2 for 9.28 correctly rounded to 3 significant figures. Award A1 if the correct value is obtained but rounded incorrectly. Award A0 if the answer is not to 3 s.f.

Q24 — Expand and Integrate a Rational Expression HARD

(a) $\dfrac{(x^2+1)^2}{x^2} = \dfrac{x^4 + 2x^2 + 1}{x^2} = x^2 + 2 + x^{-2}$ **A1**

Note: Award A1 for correctly expanding the numerator and dividing each term by x^2. This is a show that; full working must be shown. **AG**

(b) $f(x) = \displaystyle\int (x^2 + 2 + x^{-2})\,dx$ **M1**

$= \dfrac{x^3}{3} + 2x - x^{-1} + c$ **A1**

Note: Award A1 for all three terms correct including the negative exponent integrated to $-x^{-1}$. Penalise omission of $+c$.

Substitute $f(1) = \dfrac{11}{3}$: $\dfrac{1}{3} + 2 - 1 + c = \dfrac{11}{3}$ **M1**

$1 + c = \dfrac{11}{3} - \dfrac{1}{3} = \dfrac{10}{3} \Rightarrow c = \dfrac{7}{3}$ **A1**

$f(x) = \dfrac{x^3}{3} + 2x - \dfrac{1}{x} + \dfrac{7}{3}$

Note: Award second M1 for substituting $\left(1, \frac{11}{3}\right)$ into their integrated expression. A1 for $c = \frac{7}{3}$. If $+c$ was omitted earlier, award M1 only for the substitution; the final A1 is lost.

Q25 — Velocity to Distance via Root Splitting HARD

(a) $v(t) = 0 \Rightarrow t^2 - 5t + 4 = 0 \Rightarrow (t-1)(t-4) = 0$

$t = 1$ and $t = 4$ **A1**

Note: Award A1 for both values. Accept solutions found by GDC.

(b) Distance $= \displaystyle\int_0^1 |v(t)|\,dt + \int_1^4 |v(t)|\,dt + \int_4^5 |v(t)|\,dt$ **M1**

Note: Award M1 for recognising the need to split at $t = 1$ and $t = 4$ (or for using $\int |v|\,dt$ on GDC).

Using GDC: $\displaystyle\int_0^1 (t^2 - 5t + 4)\,dt = \frac{11}{6}$, $\int_1^4 -(t^2 - 5t + 4)\,dt = \dfrac{9}{2}$, $\int_4^5 (t^2 - 5t + 4)\,dt = \dfrac{11}{6}$ **(A1)**

Total distance $= \dfrac{11}{6} + \dfrac{9}{2} + \dfrac{11}{6} = \dfrac{49}{6}$ **M1**

$= 8.17$ m (3 s.f.) **A1**

Note: Award second M1 for summing the three unsigned areas. A1 for $\frac{49}{6}$ or 8.17 (3 s.f.). Trap: using displacement $\left(\int_0^5 v\,dt = -\frac{1}{6}\right)$ instead of distance earns M1(A1) only.

Q26 — Integrate a Trigonometric Rate HARD

(a) $h(t) = \displaystyle\int (0.5\cos(0.4t) - 0.2)\,dt$ **M1**

$= \dfrac{0.5}{0.4}\sin(0.4t) - 0.2t + c = 1.25\sin(0.4t) - 0.2t + c$ **A1**

Substitute $h(0) = 3.6$: $1.25\sin(0) - 0 + c = 3.6 \Rightarrow c = 3.6$ **A1**

$h(t) = 1.25\sin(0.4t) - 0.2t + 3.6$

Note: Award M1 for attempting to integrate both terms. First A1 for correct integration (accept $\frac{5}{4}$ for 1.25). Second A1 for $c = 3.6$.

(b) Depth decreasing at maximum rate when $\dfrac{dh}{dt}$ is most negative, i.e. $\cos(0.4t) = -1$. **M1**

$0.4t = \pi \Rightarrow t = \dfrac{\pi}{0.4} = \dfrac{5\pi}{2} \approx 7.85$ min **A1**

Note: Award M1 for identifying that the minimum of $\dfrac{dh}{dt}$ occurs when $\cos(0.4t) = -1$. A1 for $t = \frac{5\pi}{2}$ (exact) or 7.85 (3 s.f.). Accept either form.

Q27 — Area Between a Curve and the x-axis HARD

Find zeros of $f(x) = -x^2 + 4x - 1$ on $[0, 5]$: $x = \dfrac{-4 \pm \sqrt{16 - 4}}{-2} = 2 \pm \sqrt{3}$ **M1**

so $x_1 = 2 - \sqrt{3} \approx 0.268$, $x_2 = 2 + \sqrt{3} \approx 3.732$ **(A1)**

Note: Award M1 for attempting to find the x-intercepts of f on $[0, 5]$. (A1) for both roots correct (exact or decimal).

Total area $= \displaystyle\int_0^{2-\sqrt{3}} |f(x)|\,dx + \int_{2-\sqrt{3}}^{2+\sqrt{3}} f(x)\,dx + \int_{2+\sqrt{3}}^{5} |f(x)|\,dx$

Using GDC: $\int_0^{2-\sqrt{3}} (-f(x))\,dx \approx 0.131$, $\int_{2-\sqrt{3}}^{2+\sqrt{3}} f(x)\,dx \approx 6.928$, $\int_{2+\sqrt{3}}^{5}(-f(x))\,dx \approx 3.464$ **(M1)**
Total $= 0.131 + 6.928 + 3.464 = 10.523\ldots$ **(A1)**
$= 10.5$ (3 s.f.) **A1**

Note: Award (M1) for correct split and unsigned integrals on GDC. (A1) for the unsimplified sum. A1 for 10.5 to 3 s.f. Trap: integrating directly $\int_0^5 f\,dx$ gives 5.40; award (M1)(A1)A0.

Q28 — Recover h from the Gradient Function with Root Conditions HARD

$h'(x) = ax^2 + bx + c$ has three unknowns, so three independent conditions on h' are needed. **M1**
$h'(1) = a + b + c = 0$
$h'(-2) = 4a - 2b + c = 0$
$h'(0) = c = -4$ **(A1)**
Substituting $c = -4$: $a + b = 4$ and $4a - 2b = 4 \Rightarrow 2a - b = 2$. Adding: $3a = 6 \Rightarrow a = 2,\ b = 2$ **A1**
$h'(x) = 2x^2 + 2x - 4$

Note: Award M1 for setting up three simultaneous equations in a, b, c. (A1) for $c = -4$. A1 for $a = 2,\ b = 2$ (non-trivial: verify $h'(1) = 2 + 2 - 4 = 0$ and $h'(-2) = 8 - 4 - 4 = 0$).

Integrate: $h(x) = \dfrac{2x^3}{3} + x^2 - 4x + C$ **M1**
Apply $h(0) = 4$: $C = 4$, so $h(x) = \dfrac{2}{3}x^3 + x^2 - 4x + 4$
$h(3) = \dfrac{2}{3}(27) + 9 - 12 + 4 = 18 + 9 - 12 + 4 = 19$ **A1**

Note: Award M1 for integrating their $h'(x)$ and applying $h(0) = 4$. A1 for $h(3) = 19$.

Q29 — Rewrite Integrate and Find Area by GDC HARD

(a) $f(x) = \dfrac{x^2 + 3}{\sqrt{x}} = \dfrac{x^2}{x^{1/2}} + \dfrac{3}{x^{1/2}} = x^{3/2} + 3x^{-1/2}$ **M1**
$p = \dfrac{3}{2},\ q = -\dfrac{1}{2}$ **A1**

Note: Award M1 for correctly splitting the fraction and rewriting with rational exponents. A1 for both p and q correct.

(b) $\displaystyle\int f(x)\,dx = \frac{x^{5/2}}{5/2} + \frac{3x^{1/2}}{1/2} + c = \frac{2}{5}x^{5/2} + 6x^{1/2} + c$ **M1A1**

Note: Award M1 for integrating their power form term by term. A1 for both terms correct with $+c$. Penalise omission of $+c$.

(c) $A = \displaystyle\int_1^4 f(x)\,dx = \left[\frac{2}{5}x^{5/2} + 6x^{1/2}\right]_1^4 = (12.8 + 12) - (0.4 + 6) = 24.8 - 6.4 = 18.4$ **A1**
$A = 18.4$ (3 s.f.)

Note: Award A1 for 18.4 by GDC or exact calculation. Accept $\frac{92}{5}$.

Q30 — GDC Maximum Total Population and Validity HARD

(a) Using GDC, find the maximum of $P(x) = 5x^{0.4}e^{-0.6x}$ on $[0, 6]$: **M1**

Maximum occurs at $x = \dfrac{0.4}{0.6} = \dfrac{2}{3} \approx 0.667$ km **A1**

Note: Award M1 for using GDC to locate the maximum. A1 for $x = 0.667$ *(3 s.f.) or exact* $\frac{2}{3}$*. Trap: not checking domain endpoints.*

(b) Using GDC: $\displaystyle\int_0^6 5x^{0.4}e^{-0.6x}\,dx = 8.5583\ldots$ **(M1)(A1)**

Total population $= 8.56$ thousand (3 s.f.)

Note: Award (M1) for writing the definite integral with correct limits and integrand. (A1) for 8.56 (3 s.f.). Full precision value: $8.5583\ldots$

(c) $\displaystyle\int_6^8 5x^{0.4}e^{-0.6x}\,dx \approx 0.0816 > 0$ **R1**

Since the integrand is positive for all $x > 0$, the total population increases.

Note: Award R1 for a correct justification: either evaluating $\int_6^8 P\,dx > 0$ *on GDC, or noting* $P(x) > 0$ *for all* $x \in (6, 8)$*, so extending the domain adds a positive quantity. A bald answer without reason earns R0.*