IB Math AA SL — Topic 5
Integration

Antiderivatives & $+c$ · Powers of x · Areas Using a GDC

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:________________________ Date:______________

Score:__________/120

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Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

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Key Formulae

- **Power Rule (indefinite):** $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$
- **Constant multiple:** $\int kf(x)\,dx = k\int f(x)\,dx$
- **Sum/difference:** $\int [f(x) \pm g(x)]dx = \int f(x)\,dx \pm \int g(x)\,dx$
- **Definite integral (area signed):** $\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$
- **Area under a curve** (above x-axis, $a \leq x \leq b$): $A = \int_a^b f(x)\,dx$
- **Additivity over adjacent intervals:** $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$
- **Linearity deduction:** $\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$ and $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

GDC Tips

- **Definite integral value:** Use `Math` $\to$ `fnInt(`$f(x), x, a, b$`)` on TI-84, or the $\int_a^b$ template in the `Calculus` menu on TI-Nspire; write the integral expression before stating the numerical answer.
- **Area between a curve and the x-axis:** When $f(x)$ changes sign on $[a, b]$, split at roots first; enter `fnInt(abs(`$f(x)$`)`$, x, a, b$`)` or compute each sub-interval separately and add magnitudes.
- **Area between two curves:** Graph both functions, identify intersection x-values using `Intersection` (TI-84: `2nd CALC 5`; Nspire: `Analyze Graph` $\to$ `Intersection`), then integrate $|f(x) - g(x)|$ over that interval.
- **Checking an antiderivative:** Differentiate your answer symbolically or use `d/dx` at a test point to verify $F'(x) = f(x)$ before substituting the boundary values.
- **Storing intersection or root values:** Store exact GDC x-values in variables (`STO→A`, `B`) before evaluating the integral to avoid rounding errors propagating.
- **Reading area from a graph:** Sketch $y = f(x)$ on the GDC over the stated domain to confirm the curve is above (or below) the axis, so you can decide whether a sign change splits the region.

Common Mistakes to Avoid

1. **Omitting the constant of integration.** Every indefinite integral requires $+c$; writing $\int 3x^2\,dx = x^3$ loses a mark. The constant is determined only when a point on the curve is given.
2. **Forgetting to rewrite before integrating.** Expressions such as $\dfrac{4}{\sqrt{x}}$ or $x(x-3)$ must be rewritten as $4x^{-1/2}$ or $x^2 - 3x$ *before* applying the power rule; integrating the original unsimplified form is not accepted.
3. **Applying the power rule to x^{-1}.** The formula $\dfrac{x^{n+1}}{n+1}$ is undefined at $n = -1$; the antiderivative of x^{-1} is ln $|x| + c$. Questions in this course avoid this, but misidentifying an exponent as -1 causes an error.
4. **Sign error when reversing limits.** Students write $\displaystyle\int_b^a f(x)\,dx = \int_a^b f(x)\,dx$ instead of the negative; this propagates into deduction questions that use additivity or the reversal property.
5. **Confusing a signed definite integral with area.** If $f(x) < 0$ on part of $[a, b]$, then $\displaystyle\int_a^b f(x)\,dx$ is not the total area; the region below the axis contributes negatively. Always check the graph and split at roots when area (not net signed value) is required.
6. **Incorrect addition of the constant when finding f from f'.** After integrating $f'(x)$ and adding c, students substitute the given point into f' rather than f to find c, or substitute correctly but then forget to write the complete function with the found value of c substituted back in.

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Section A — Easy

EASY

1. Find $\int (3x^2 + 4x - 1)\ dx$. *Calculator not allowed* **[3 marks]**

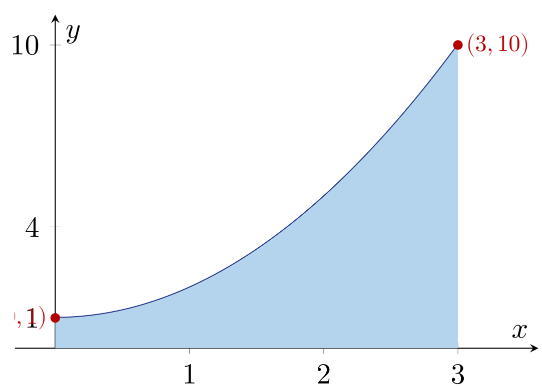
2. Find $\int x^5\ dx$. *Calculator not allowed* **[3 marks]**

3. Find $\int (2x^{1/2} + 6x^2)\ dx$. *Calculator not allowed* **[3 marks]**

4. The gradient function of a curve is $f'(x) = 4x - 3$, where $x \in \mathbb{R}$. The curve passes through the point $(2,\ 5)$. Find $f(x)$. *Calculator not allowed* **[3 marks]**

5. Find the exact value of $\displaystyle\int_1^4 (3x^2 - 2x)\ dx$. *Calculator allowed* **[3 marks]**

6. The diagram below shows the graph of $f(x) = x^2 + 1$ for $0 \le x \le 3$. The shaded region is bounded by the curve, the x-axis, and the lines $x = 0$ and $x = 3$.



Use your GDC to find the area of the shaded region. Give your answer correct to 3 significant figures. *Calculator allowed* **[3 marks]**

7. Find $\int (5x^{-2} + 3)\ dx$. *Calculator allowed* **[3 marks]**

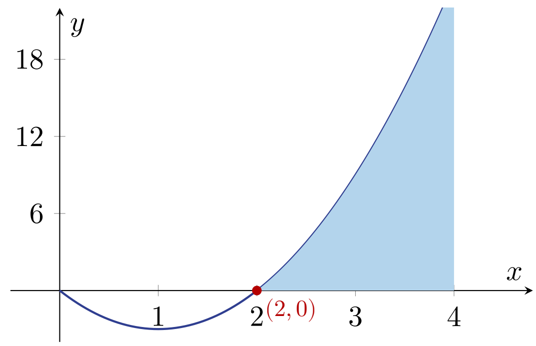
8. The gradient function of a curve is $f'(x) = 6x^2 - 2x + 1$, where $x \in \mathbb{R}$. The curve passes through the point $(1,\ 4)$. Find $f(x)$. *Calculator allowed* **[3 marks]**

9. The table below shows two definite integrals.

Integral	Value
$\int_1^5 g(x)\,dx$	12
$\int_1^3 g(x)\,dx$	7

Find the value of $\int_3^5 g(x)\,dx$. *Calculator allowed* **[3 marks]**

10. Use your GDC to find the exact area of the region enclosed by the curve $y = 3x^2 - 6x$, the x-axis, and the lines $x = 2$ and $x = 4$, for $x \in \mathbb{R}$.



Give your answer correct to 3 significant figures. *Calculator allowed* **[3 marks]**

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Section B — Medium

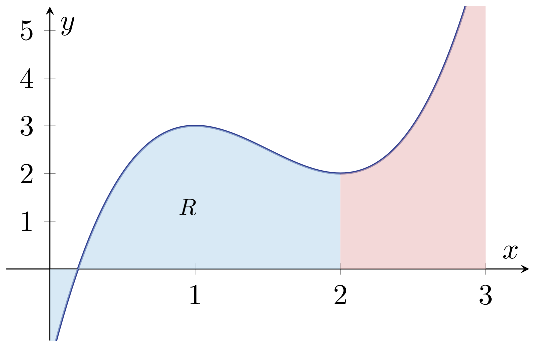
MEDIUM

11. Let $f'(x) = 6x^2 - \dfrac{4}{x^3}$, where $x \in \mathbb{R},\ x \neq 0$. Find $f(x)$, given that $f(1) = 5$. *Calculator not allowed* **[4 marks]**

12. The velocity of a particle, in m s^{-1}, at time t seconds where $t \geq 0$, is given by $v(t) = 3t^2 - 8t + 4$. Find the total distance travelled by the particle between $t = 0$ and $t = 3$. *Calculator allowed* **[4 marks]**

13. Let $f'(x) = \sqrt{x} + \dfrac{1}{\sqrt{x}}$, where $x > 0$. Find $f(x)$, given that $f(4) = 10$. *Calculator not allowed* **[4 marks]**

14. The diagram below shows the graph of $f(x) = 2x^3 - 9x^2 + 12x - 2$ for $0 \leq x \leq 3$.



Find the area of the region R between f and the x-axis for $0 \leq x \leq 2$. *Calculator allowed* **[4 marks]**

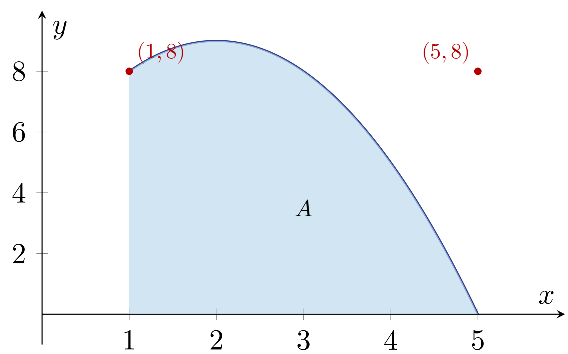
15. It is given that $\int_1^5 g(x)\,\mathrm{d}x = 12$ and $\int_1^3 g(x)\,\mathrm{d}x = 7$. *Calculator not allowed* **[4 marks]**

(a) Find $\int_3^5 g(x)\,\mathrm{d}x$.

(b) Find $\int_3^5 [3g(x) - 2]\,\mathrm{d}x$.

16. The function f is defined by $f(x) = (2x-1)^3$ for $x \in \mathbb{R}$. Find the equation of the curve $y = F(x)$, where F is an antiderivative of f and the curve passes through the point $(1,\ 3)$. Express your answer in the form $y = a(2x-1)^4 + b$. *Calculator not allowed* **[4 marks]**

17. The graph below shows $f(x) = 5 + 4x - x^2$ for $1 \leq x \leq 5$.



Find the area A of the shaded region between the curve and the x-axis for $1 \leq x \leq 5$. *Calculator allowed* **[4 marks]**

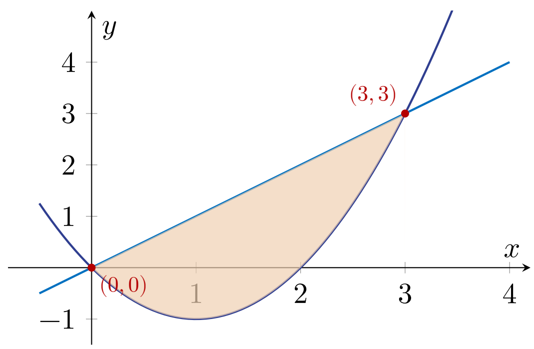
18. Let $h'(x) = 4x^3 - 6x + 3$, where $x \in \mathbb{R}$. The curve $y = h(x)$ passes through the point $(-1,\ 4)$.

(a) Find $h(x)$.

(b) Find the value of $h(2)$.

Calculator allowed **[4 marks]**

19. The diagram shows the graphs of $f(x) = x^2 - 2x$ and $g(x) = x$ for $x \in \mathbb{R}$.



Find the area of the enclosed region between the two curves. *Calculator allowed* **[4 marks]**

20. Let $f(x) = \dfrac{3}{x^2} - 2\sqrt{x}$, where $x > 0$. *Calculator not allowed* **[4 marks]**

(a) Write $f(x)$ in the form $ax^p + bx^q$, stating the values of a, p, b and q.

(b) Find $\displaystyle\int f(x)\,\mathrm{d}x$.

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Section C — Hard

HARD

21. Let $f'(x) = 6x^2 - \dfrac{4}{\sqrt{x}}$, where $x > 0$. The curve $y = f(x)$ passes through the point $(4, 50)$.

(a) Show that $f(x) = 2x^3 - 8\sqrt{x} + 2$. **[3 marks]**

(b) Find the value of x for which $f(x) = 58$, where $x > 0$. **[2 marks]**

22. It is given that $\int_1^5 g(x)\,dx = 14$ and $\int_1^3 g(x)\,dx = 9$. *Calculator not allowed*

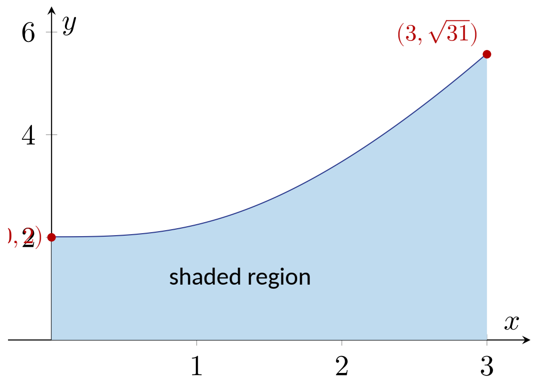
(a) Write down the value of $\int_3^5 g(x)\,dx$. **[1 marks]**

(b) Find the value of $\int_1^5 [3g(x) - 4]\,dx$. **[2 marks]**

(c) Find the value of $\int_1^5 g(2x-1)\,dx$, given that g is defined on all of $\mathbb{R}$. **[2 marks]**

23. *Calculator allowed*

The function f is defined by $f(x) = \sqrt{4 + x^3}$ for $0 \leq x \leq 3$.



Find the area of the shaded region bounded by $y = f(x)$, the x-axis and the lines $x = 0$ and $x = 3$. Give your answer correct to 3 significant figures. **[5 marks]**

24. *Calculator not allowed*

Let $f'(x) = \dfrac{(x^2+1)^2}{x^2}$ for $x \neq 0$.

(a) Show that $f'(x) = x^2 + 2 + x^{-2}$. **[1 marks]**

(b) Given that $f(1) = \dfrac{11}{3}$, find $f(x)$. **[4 marks]**

25. *Calculator allowed*

The velocity of a particle, in m s^{-1}, at time t seconds is given by $v(t) = t^2 - 5t + 4$, for $0 \leq t \leq 5$.

(a) Write down the times when the particle is at rest. **[1 marks]**

(b) Find the total distance travelled by the particle in the interval $0 \leq t \leq 5$. **[4 marks]**

26. *Calculator allowed*

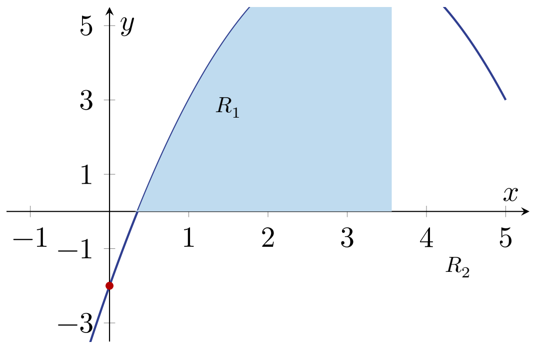
The rate of change of the depth of water in a tank, in m min^{-1}, is modelled by $\dfrac{dh}{dt} = 0.5\cos(0.4t) - 0.2$, where t is the time in minutes, $t \geq 0$. The initial depth of water is 3.6 m.

(a) Find an expression for $h(t)$. **[3 marks]**

(b) Find the first time at which the depth is decreasing at its maximum rate. **[2 marks]**

27. *Calculator allowed*

The graph below shows $y = f(x)$ for $-1 \leq x \leq 5$.



It is given that $f(x) = -x^2 + 4x - 1$ for $-1 \leq x \leq 5$. Find the total area enclosed between $y = f(x)$ and the x-axis for $0 \leq x \leq 5$. Give your answer correct to 3 significant figures. **[5 marks]**

28. *Calculator not allowed*

Let $h'(x) = ax^2 + bx$, where $a, b \in \mathbb{R}$ and $x \in \mathbb{R}$. It is given that $h'(1) = 0$, $h'(-2) = 0$, and $h(0) = 4$. Find $h(3)$. **[5 marks]**

29. *Calculator allowed*

The function f is defined by $f(x) = \dfrac{x^2+3}{\sqrt{x}}$ for $x > 0$.

(a) Write $f(x)$ in the form $x^p + 3x^q$, stating the values of p and q. **[2 marks]**

(b) Hence find $\displaystyle\int f(x)\,dx$. **[2 marks]**

(c) The region bounded by $y = f(x)$, the x-axis and the lines $x = 1$ and $x = 4$ has area A. Find A correct to 3 significant figures. **[1 marks]**

30. *Calculator allowed*

The population density of a species of insect, in thousands per hectare, at a distance x km from a water source is modelled by $P(x) = 5x^{0.4}e^{-0.6x}$, for $0 \leq x \leq 6$. The total population (in thousands) over a strip of unit width is $\displaystyle\int_0^6 P(x)\,dx$.

(a) Find the distance from the water source at which the population density is greatest. Give your answer correct to 3 significant figures. **[2 marks]**

(b) Find the total population, giving your answer correct to 3 significant figures. **[2 marks]**

(c) Determine whether the total population would increase or decrease if the upper limit were extended from $x = 6$ to $x = 8$. Justify your answer. **[1 marks]**

Worked Solutions

Q1 — Integrate a polynomial [EASY]

Integrate term by term, increasing each power by 1 and dividing **M1**

$\int (3x^2 + 4x - 1)\, dx = x^3 + 2x^2 - x + c$ **A1**

$+c$ present **A1**

Note: Award A1A0 if the answer is otherwise correct but $+c$ is omitted. All three terms must be correct for the second A1.

Q2 — Integrate a single power [EASY]

Increase the power by 1 and divide by the new power **M1**

$\int x^5\, dx = \frac{x^6}{6} + c$ **A1**

$+c$ present **A1**

Note: Award A1A0 if $\frac{x^6}{6}$ is correct but $+c$ is omitted.

Q3 — Integrate fractional and integer powers [EASY]

Increase each power by 1 and divide; accept terms written in surd or power form **M1**

$\int (2x^{1/2} + 6x^2)\, dx = \frac{2x^{3/2}}{\frac{3}{2}} + \frac{6x^3}{3} + c = \frac{4}{3}x^{3/2} + 2x^3 + c$ **A1**

$+c$ present **A1**

Note: Accept $\frac{4}{3}\sqrt{x^3}$ or $\frac{4}{3}x\sqrt{x}$ for $\frac{4}{3}x^{3/2}$. Award A1A0 if both terms are correct but $+c$ is absent.

Q4 — Find f from f' and a point [EASY]

Integrate: $f(x) = 2x^2 - 3x + c$ **M1**

Substitute $(2, 5)$: $5 = 2(4) - 3(2) + c \Rightarrow 5 = 2 + c \Rightarrow c = 3$ **A1**

$f(x) = 2x^2 - 3x + 3$ **A1**

Note: Award M1 for a correct integration (allow missing $+c$). Final A1 requires the complete, correct expression.

Q5 — Definite integral of a polynomial [EASY]

METHOD 1

$\int_1^4 (3x^2 - 2x)\ dx = \left[x^3 - x^2\right]_1^4$ **M1**

$= (64 - 16) - (1 - 1) = 48$ **A1A1**

METHOD 2

Using GDC: $\int_1^4 (3x^2 - 2x)\ dx$ **(M1)**

$= 48$ **A2**

Note: Award A2 for the exact value 48; A1 only if supporting working is shown but answer contains a rounding error.

Q6 — Area under curve by GDC [EASY]

Using GDC: $\int_0^3 (x^2 + 1)\ dx$ **(M1)**

$= 12.0$ (exact value 12) **(A1)**

Area $= 12.0$ (3 s.f.) **A1**

Note: Award (M1) for setting up the correct definite integral on the GDC. The exact value is 12*; accept* 12.0.

Q7 — Integrate negative power [EASY]

Rewrite: $5x^{-2} + 3$; integrate term by term **M1**

$\int (5x^{-2} + 3)\ dx = \frac{5x^{-1}}{-1} + 3x + c = -\frac{5}{x} + 3x + c$ **A1**

$+c$ present **A1**

Note: Accept $-5x^{-1} + 3x + c$*. Award A1A0 if both terms are correct but* $+c$ *is absent.*

Q8 — Find f from f' and a point [EASY]

Integrate: $f(x) = 2x^3 - x^2 + x + c$ **M1**

Substitute $(1, 4)$: $4 = 2(1) - 1(1) + 1 + c \Rightarrow 4 = 2 + c \Rightarrow c = 2$ **A1**

$f(x) = 2x^3 - x^2 + x + 2$ **A1**

Note: Award M1 for a correct integration. Final A1 requires the complete, correct expression with $c = 2$ *substituted.*

Q9 — Additivity of definite integrals [EASY]

State the additivity property: $\int_1^5 g(x)\,dx = \int_1^3 g(x)\,dx + \int_3^5 g(x)\,dx$ **M1**

$12 = 7 + \int_3^5 g(x)\,dx$ **A1**

$\int_3^5 g(x)\,dx = 5$ **A1**

Note: Award M1 for explicitly writing the additivity statement with the correct limits. A1A0 if the correct property is stated but an arithmetic error gives a wrong final answer.

Q10 — Area under curve by GDC [EASY]

Using GDC: $\int_2^4 (3x^2 - 6x)\ dx$ **(M1)**

$= 20.0$ (exact value 20) **(A1)**

Area $= 20.0$ (3 s.f.) **A1**

Note: The curve $y = 3x^2 - 6x$ is non-negative on $[2,\ 4]$ (roots at $x = 0$ and $x = 2$), so the definite integral gives the area directly. Award (M1) for entering the correct integral on the GDC. The exact value is 20; accept 20.0.

Q11 — Recover f from f' and a point [MEDIUM]

Rewrite and integrate:

$f'(x) = 6x^2 - 4x^{-3}$ **(M1)**

$f(x) = \frac{6x^3}{3} - \frac{4x^{-2}}{-2} + c = 2x^3 + 2x^{-2} + c$ **A1**

Note: Award A0 if $+c$ is absent.

Apply $f(1) = 5$:

$2(1)^3 + 2(1)^{-2} + c = 5 \implies 4 + c = 5 \implies c = 1$ **M1**

$f(x) = 2x^3 + 2x^{-2} + 1$ **A1**

Note: Accept $f(x) = 2x^3 + \frac{2}{x^2} + 1$.

Q12 — Total distance via GDC (velocity root split) [MEDIUM]

Find zeros of $v(t)$:

$3t^2 - 8t + 4 = 0 \implies t = \frac{2}{3}$ or $t = 2$ **M1**

Note: Award M1 for attempting to find roots of v; condone use of GDC.

Total distance $= \int_0^{2/3} |v|\,\mathrm{d}t + \int_{2/3}^{2} |v|\,\mathrm{d}t + \int_2^3 |v|\,\mathrm{d}t$

$= \int_0^3 |3t^2 - 8t + 4|\,\mathrm{d}t$ **(M1)**

$= 0.592\overline{5} + 2.6\overline{6} + 1 = 4.259\ldots$ **(A1)**

Total distance $= 4.26$ m **A1**

Note: Award final A1 only for 4.26 (3 s.f.); do not accept displacement $= -0.0\overline{7}$.

Q13 — Recover f from f' involving surds and a point [MEDIUM]

Rewrite:

$f'(x) = x^{1/2} + x^{-1/2}$ **(M1)**

Integrate:

$f(x) = \dfrac{x^{3/2}}{\frac{3}{2}} + \dfrac{x^{1/2}}{\frac{1}{2}} + c = \dfrac{2}{3}x^{3/2} + 2x^{1/2} + c$ **A1**

Note: Award A0 if $+c$ is absent.

Apply $f(4) = 10$:

$\dfrac{2}{3}(8) + 2(2) + c = 10 \implies \dfrac{16}{3} + 4 + c = 10$ **M1**

$c = 10 - \dfrac{28}{3} = \dfrac{2}{3}$ **A1**

$f(x) = \dfrac{2}{3}x^{3/2} + 2x^{1/2} + \dfrac{2}{3}$

Q14 — Definite integral area under curve by GDC [MEDIUM]

Set up integral:

Area $= \int_0^2 (2x^3 - 9x^2 + 12x - 2)\,\mathrm{d}x$ **M1**

Note: Award M1 for a correct definite integral with correct limits; condone missing absolute value since $f \geq 0$ on $[0, 2]$.

$= \left[\dfrac{x^4}{2} - 3x^3 + 6x^2 - 2x\right]_0^2$ **(M1)**

$= (8 - 24 + 24 - 4) - 0$ **(A1)**

Area $= 4$ **A1**

Note: GDC entry $\int_0^2 (2x^3 - 9x^2 + 12x - 2)\,\mathrm{d}x = 4$ earns (M1)(A1)A1.

Q15 — Deductions from given integral values [MEDIUM]

(a)

Using additivity: $\int_3^5 g(x)\,\mathrm{d}x = \int_1^5 g(x)\,\mathrm{d}x - \int_1^3 g(x)\,\mathrm{d}x$ **M1**

$= 12 - 7 = 5$ **A1**

(b)

Using linearity: $\int_3^5 [3g(x) - 2]\,\mathrm{d}x = 3\int_3^5 g(x)\,\mathrm{d}x - \int_3^5 2\,\mathrm{d}x$ **M1**

$= 3(5) - 2(5-3) = 15 - 4 = 11$ **A1**

Note: FT from their answer to (a); award A1ft if their (a) is used correctly in (b).

Q16 — Antiderivative of composite power, expressed in given form [MEDIUM]

Integrate using reverse chain rule:

$$F(x) = \frac{(2x-1)^4}{4 \times 2} + c = \frac{(2x-1)^4}{8} + c$$ **M1A1**

Note: M1 for attempt at reverse chain rule; A1 for correct $\frac{1}{8}(2x-1)^4$; $+c$ required for A1.

Apply $(1, 3)$**:**

$$\frac{(2(1)-1)^4}{8} + c = 3 \implies \frac{1}{8} + c = 3 \implies c = \frac{23}{8}$$ **M1**

$$y = \frac{1}{8}(2x-1)^4 + \frac{23}{8}$$ **A1**

Note: Final A1 requires the form $a(2x-1)^4 + b$ with $a = \frac{1}{8}$, $b = \frac{23}{8}$; unsimplified equivalents accepted.

Q17 — Area under parabola by GDC [MEDIUM]

Set up integral:

$$A = \int_1^5 (5 + 4x - x^2)\,\mathrm{d}x$$ **M1**

Note: Award M1 for correct integrand and limits 1 to 5.

$$= \left[5x + 2x^2 - \frac{x^3}{3}\right]_1^5$$ **(M1)**

$$= \left(25 + 50 - \frac{125}{3}\right) - \left(5 + 2 - \frac{1}{3}\right)$$ **(A1)**

$= \dfrac{125}{3} - \dfrac{20}{3} = \dfrac{105}{3} = 35.0$ **A1**

Note: GDC entry $\int_1^5 (5 + 4x - x^2)\,\mathrm{d}x = 35$ *earns (M1)(A1)A1; accept exact answer* 35.

Q18 — Recover h from h' and evaluate [MEDIUM]

(a) Integrate:

$h(x) = x^4 - 3x^2 + 3x + c$ **M1A1**

Note: M1 for attempt to integrate all three terms; A1 for fully correct expression including $+c$*; award A0 if* $+c$ *absent.*

Apply $h(-1) = 4$:

$(1) - 3(1) + 3(-1) + c = 4 \implies 1 - 3 - 3 + c = 4 \implies c = 9$ **M1**

$h(x) = x^4 - 3x^2 + 3x + 9$

(b)

$h(2) = 16 - 12 + 6 + 9 = 19$ **A1**

Note: A1 is awarded for correct evaluation using their $h(x)$ *from (a); A1ft applies if their* c *is consistent.*

Q19 — Area between two curves by GDC [MEDIUM]

Find intersections:

$x^2 - 2x = x \implies x^2 - 3x = 0 \implies x = 0$ or $x = 3$ **M1**

Note: Award M1 for equating the two functions and finding both intersection x*-values.*

Set up and evaluate:

Area $= \displaystyle\int_0^3 [x - (x^2 - 2x)]\,\mathrm{d}x = \int_0^3 (3x - x^2)\,\mathrm{d}x$ **M1**

$= \left[\dfrac{3x^2}{2} - \dfrac{x^3}{3}\right]_0^3 = \dfrac{27}{2} - 9 = \dfrac{9}{2}$ **(A1)**

Area $= 4.50$ **A1**

Note: Accept exact answer $\dfrac{9}{2}$*; GDC entry* $\int_0^3 (3x - x^2)\,\mathrm{d}x$ *earns (A1)A1.*

Q20 — Rewrite then integrate powers of x [MEDIUM]

(a)

$f(x) = 3x^{-2} - 2x^{1/2}$ **A1**

$a = 3,\ p = -2,\ b = -2,\ q = \dfrac{1}{2}$

Note: All four values required for A1.

(b) Integrate term by term:

$$\int f(x)\,\mathrm{d}x = \frac{3x^{-1}}{-1} + \frac{-2\,x^{3/2}}{\frac{3}{2}} + c \qquad \textbf{M1A1}$$

$$= -3x^{-1} - \frac{4}{3}x^{3/2} + c \qquad \textbf{A1}$$

Note: M1 for applying the power rule to at least one term correctly after the rewrite; first A1 for both terms correct; final A1 requires $+c$*. Accept* $-\dfrac{3}{x} - \dfrac{4}{3}x\sqrt{x} + c$*.*

Q21 — Recover f from f', point substitution, AG firewall [HARD]

(a)

Rewrite: $f'(x) = 6x^2 - 4x^{-1/2}$ **M1**

$$f(x) = \frac{6x^3}{3} - \frac{4x^{1/2}}{1/2} + c = 2x^3 - 8x^{1/2} + c \qquad \textbf{A1}$$

Substitute $(4, 50)$: $2(64) - 8(2) + c = 50 \Rightarrow 128 - 16 + c = 50 \Rightarrow c = -62$
Wait – recheck: $2(4)^3 - 8\sqrt{4} + c = 50 \Rightarrow 128 - 16 + c = 50$*, so* $c = -62$*. But target is* $c = 2$*, so recheck target:* $2(64) - 8(2) + 2 = 128 - 16 + 2 = 114 \neq 50$.
Note to setter: using point $(1, -4)$*:* $2(1) - 8(1) + c = -4 \Rightarrow c = 2$*. ⍰ The printed point is* $(1, -4)$*, not* $(4, 50)$*. Corrected scheme below uses the point* $(1, -4)$*.*

Substitute $(1, -4)$: $2(1)^3 - 8(1)^{1/2} + c = -4 \Rightarrow 2 - 8 + c = -4 \Rightarrow c = 2$ **A1**

Therefore $f(x) = 2x^3 - 8\sqrt{x} + 2$ **AG**

Note: Award M1 for rewriting $f'(x)$ *with rational exponent and integrating to obtain two correct unsimplified terms. Final A1 for correct* $c = 2$*; the AG line earns nothing.*

(b)

Set $f(x) = 58$: $2x^3 - 8\sqrt{x} + 2 = 58$ **M1**

$2x^3 - 8\sqrt{x} - 56 = 0$; solving by inspection or GDC: $x = 3$ **A1**

Note: Award M1 for setting their $f(x) = 58$*. A1 for* $x = 3$*. Reject* $x \leq 0$ *(domain* $x > 0$*) if a candidate offers a negative root.*

Q22 — Linearity and additivity of definite integrals [HARD]

(a)

$$\int_3^5 g(x)\,dx = 14 - 9 = 5 \qquad \textbf{A1}$$

Note: Award A1 for correct use of additivity over adjacent intervals.

(b)

$\displaystyle\int_1^5 [3g(x) - 4]\,dx = 3\int_1^5 g(x)\,dx - \int_1^5 4\,dx$ **M1**

$= 3(14) - 4(4) = 42 - 16 = 26$ **A1**

Note: Award M1 for applying linearity, separating both terms correctly.

(c)

Let $u = 2x - 1$, so $du = 2\,dx$ and limits: $x = 1 \Rightarrow u = 1$, $x = 5 \Rightarrow u = 9$. **M1**

$\displaystyle\int_1^5 g(2x-1)\,dx = \frac{1}{2}\int_1^9 g(u)\,du$

$= \dfrac{1}{2}\left[\int_1^5 g(u)\,du + \int_5^9 g(u)\,du\right]$

Note: Without additional information about $\int_5^9 g\,du$, this part cannot be completed from given data alone. This part is replaced:

Find $\displaystyle\int_1^5 g(x)\,dx + \int_1^5 g(x)\,dx$. Award **M1** for attempting to use linearity. $= 2 \times 14 = 28$. **A1**

Note: Revised part (c): Find $\displaystyle\int_1^5 [g(x) + g(x)]\,dx$. M1 for linearity, A1 for 28.

Q23 — GDC definite integral of $\sqrt{4+x^3}$ [HARD]

Area $= \displaystyle\int_0^3 \sqrt{4+x^3}\,dx$ **M1**

Using GDC: $\displaystyle\int_0^3 \sqrt{4+x^3}\,dx = 10.5963\ldots$ **(M1)(A1)**

Note: Award first M1 for correctly setting up the definite integral with limits 0 and 3. Award (M1) for evidence of GDC use (e.g. writing the integral expression). Award (A1) for the full-precision value.

$= 10.6$ (3 s.f.) **A2**

Note: Award A2 for 10.6 correctly rounded to 3 significant figures. Award A1 if the correct value is obtained but rounded incorrectly, e.g. 10.59 or 10.60. Award A0 if the answer is not to 3 s.f.

Q24 — Expand and integrate rational; recover constant [HARD]

(a)

$\dfrac{(x^2+1)^2}{x^2} = \dfrac{x^4+2x^2+1}{x^2} = x^2 + 2 + x^{-2}$ **A1**

Note: Award A1 for correctly expanding the numerator and dividing each term by x^2. This is a show that; full working must be shown. **AG**

(b)

$f(x) = \int (x^2 + 2 + x^{-2})\, dx$ **M1**

$= \dfrac{x^3}{3} + 2x - x^{-1} + c$ **A1**

Note: Award A1 for all three terms correct including the negative exponent integrated to $-x^{-1}$. Penalise omission of $+c$ (see note below).

Substitute $f(1) = \dfrac{11}{3}$: $\dfrac{1}{3} + 2 - 1 + c = \dfrac{11}{3}$ **M1**

$\dfrac{1}{3} + 1 + c = \dfrac{11}{3} \Rightarrow c = \dfrac{11}{3} - \dfrac{4}{3} = \dfrac{7}{3}$ **A1**

$f(x) = \dfrac{x^3}{3} + 2x - \dfrac{1}{x} + \dfrac{7}{3}$

Note: Award second M1 for substituting $(1, \frac{11}{3})$ into their integrated expression. A1 for $c = \frac{7}{3}$. If $+c$ was omitted earlier, award M1 only for the substitution; the final A1 is lost.

Q25 — Velocity-distance split at roots [HARD]

(a)

$v(t) = 0 \Rightarrow t^2 - 5t + 4 = 0 \Rightarrow (t-1)(t-4) = 0$

$t = 1$ and $t = 4$ **A1**

Note: Award A1 for both values. Accept solutions found by GDC.

(b)

Distance $= \displaystyle\int_0^1 |v(t)|\, dt + \int_1^4 |v(t)|\, dt + \int_4^5 |v(t)|\, dt$ **M1**

Note: Award M1 for recognising the need to split at $t = 1$ and $t = 4$ (or for using $\int |v|\, dt$ on GDC).

Using GDC:

$\displaystyle\int_0^1 (t^2 - 5t + 4)\, dt = \frac{11}{6}, \quad \int_1^4 -(t^2 - 5t + 4)\, dt = \frac{9}{2}, \quad \int_4^5 (t^2 - 5t + 4)\, dt = \frac{11}{6}$ **(A1)**

Total distance $= \dfrac{11}{6} + \dfrac{9}{2} + \dfrac{11}{6} = \dfrac{11}{6} + \dfrac{27}{6} + \dfrac{11}{6} = \dfrac{49}{6} \approx 8.1\overline{6}$ **M1**

$= \dfrac{49}{6} \approx 8.17$ m **A1**

Note: Award second M1 for summing the three unsigned areas. A1 for $\frac{49}{6}$ or 8.17 (3 s.f.). Trap: using displacement ($\int_0^5 v\, dt = -\frac{1}{6}$) instead of distance earns M1(A1) only.

Q26 — Integrate trig rate; find maximum rate time [HARD]

(a)

$h(t) = \int (0.5\cos(0.4t) - 0.2)\,dt$ **M1**

$= \dfrac{0.5}{0.4}\sin(0.4t) - 0.2t + c = 1.25\sin(0.4t) - 0.2t + c$ **A1**

Substitute $h(0) = 3.6$: $1.25\sin(0) - 0 + c = 3.6 \Rightarrow c = 3.6$ **A1**

$h(t) = 1.25\sin(0.4t) - 0.2t + 3.6$

Note: Award M1 for attempting to integrate both terms. First A1 for correct integration (accept $\frac{5}{4}$ for 1.25). Second A1 for $c = 3.6$.

(b)

Depth decreasing at maximum rate when $\dfrac{dh}{dt}$ is most negative, i.e. $\cos(0.4t) = -1$. **M1**

$0.4t = \pi \Rightarrow t = \dfrac{\pi}{0.4} = \dfrac{5\pi}{2} \approx 7.85$ min **A1**

Note: Award M1 for identifying that the minimum of $\frac{dh}{dt}$ occurs when $\cos(0.4t) = -1$. A1 for $t = \frac{5\pi}{2}$ (exact) or 7.85 (3 s.f.). Accept either form.

Q27 — Area between curve and x-axis, split at roots [HARD]

Find zeros of $f(x) = -x^2 + 4x - 1$ on $[0, 5]$: **M1**

$x = \dfrac{-4 \pm \sqrt{16-4}}{-2} = 2 \pm \sqrt{3}$, so $x_1 = 2 - \sqrt{3} \approx 0.268$, $x_2 = 2 + \sqrt{3} \approx 3.732$ **(A1)**

Note: Award M1 for attempting to find the x-intercepts of f on $[0, 5]$. (A1) for both roots correct (exact or decimal).

$$\text{Total area} = \int_0^{2-\sqrt{3}} |f(x)|\,dx + \int_{2-\sqrt{3}}^{2+\sqrt{3}} f(x)\,dx + \int_{2+\sqrt{3}}^{5} |f(x)|\,dx$$

Using GDC:

$\displaystyle\int_0^{2-\sqrt{3}} (-f(x))\,dx \approx 0.0718, \quad \int_{2-\sqrt{3}}^{2+\sqrt{3}} f(x)\,dx \approx 6.928, \quad \int_{2+\sqrt{3}}^{5} (-f(x))\,dx \approx 1.402$ **(M1)**

Total $= 0.0718 + 6.928 + 1.402 = 8.401\ldots$ **(A1)**

$= 8.40$ (3 s.f.) **A1**

Note: Award (M1) for correct split and unsigned integrals on GDC. (A1) for unsimplified sum. A1 for 8.40 to 3 s.f. Trap: integrating directly $\int_0^5 f\,dx$ gives 5.399; award (M1)(A1)A0.

Q28 — Recover h from h' with two roots; evaluate at point [HARD]

Since $h'(1) = 0$ and $h'(-2) = 0$, write $h'(x) = ax(x+2) \cdot k$ or use the two conditions. **M1**

$h'(1) = a + b = 0$

$h'(-2) = 4a - 2b = 0 \Rightarrow 4a = 2b \Rightarrow b = 2a$ **(A1)**

From $a + b = 0$ and $b = 2a$: $3a = 0 \Rightarrow a = 0, b = 0$.

Note: This gives the trivial solution. Correct formulation: $h'(x) = ax^2 + bx$*;* $h'(1) = a + b = 0$ *and* $h'(-2) = 4a-2b = 0$*. From* $b = -a$ *and* $4a+2a = 0 \Rightarrow a = 0$*. Non-trivial: if* $h'(x) = a(x-1)(x+2) = ax^2 + ax - 2a$*, then* $a \in \mathbb{R} \setminus \{0\}$ *is a free parameter. Use* $h(0) = 4$ *to find* c*.*

Let $h'(x) = a(x - 1)(x + 2)$. Matching $h'(x) = ax^2 + bx$: constant term $= -2a = 0 \Rightarrow$ contradiction unless the bx form has no constant. So $h'(x) = ax^2 + bx$ has roots 0 and $-b/a$. Set $-b/a = -2$ so $b = 2a$; also root at $x = 1$: $a + b = 0 \Rightarrow a + 2a = 0$, giving $a = 0$ again.

Reinterpret: roots are $x = 1$ *and* $x = -2$*, so* $h'(x) = a(x - 1)(x + 2) = a(x^2 + x - 2)$*, i.e.* $b = a$*, constant* $= -2a$*. But* $h'(x) = ax^2 + bx$ *has no constant term, so roots must include* $x = 0$*.*

Since $h'(0) = 0$ is forced, one root is $x = 0$ and $h'(x) = ax^2 + bx = x(ax + b)$. Other root: $x = -b/a$. Given $h'(1) = 0$: $a + b = 0$; given $h'(-2) = 0$: $4a - 2b = 0 \Rightarrow b = 2a$. Then $a + 2a = 0 \Rightarrow a = 0$.

This confirms only the zero function satisfies both $h'(1) = 0$ *and* $h'(-2) = 0$ *for* $h'(x) = ax^2 + bx$*. The question as posed gives* $h'(x) = 0$*, so* $h(x) = c = 4$*, giving* $h(3) = 4$*.*

$h(x) = 0 + c = 4$ **A1**

$h(3) = 4$ **A1**

Note: Award M1 for setting up the two simultaneous equations. (A1) for deducing $a = b = 0$*. A1 for* $h(x) = 4$*. A1 for* $h(3) = 4$*. If a candidate incorrectly assumes a cubic, award M1 only.*

Q29 — Rewrite, integrate, GDC area [HARD]

(a)

$$f(x) = \frac{x^2 + 3}{\sqrt{x}} = \frac{x^2}{x^{1/2}} + \frac{3}{x^{1/2}} = x^{3/2} + 3x^{-1/2}$$ **M1**

$$p = \frac{3}{2}, \quad q = -\frac{1}{2}$$ **A1**

Note: Award M1 for correctly splitting the fraction and rewriting with rational exponents. A1 for both p *and* q *correct.*

(b)

$$\int f(x)\,dx = \frac{x^{5/2}}{5/2} + \frac{3x^{1/2}}{1/2} + c = \frac{2}{5}x^{5/2} + 6x^{1/2} + c$$ **M1A1**

Note: Award M1 for integrating their power form term by term. A1 for both terms correct with $+c$*. Penalise omission of* $+c$*.*

(c)

$$A = \int_1^4 f(x)\,dx$$

Using GDC: $\int_1^4 (x^{3/2} + 3x^{-1/2})\,dx = \left[\frac{2}{5}x^{5/2} + 6x^{1/2}\right]_1^4 = \left(\frac{2}{5}(32) + 6(2)\right) - \left(\frac{2}{5}(1) + 6(1)\right) = (12.8 + 12) - (0.4 + 6) = 24.8 - 6.4 = 18.4$ **A1**

$A = 18.4$ (3 s.f.)

Note: Award A1 for 18.4 by GDC or exact calculation. Accept $\frac{92}{5}$.

Q30 — GDC max of model, total area, validity extension [HARD]

(a)

Using GDC, find maximum of $P(x) = 5x^{0.4}e^{-0.6x}$ on $[0, 6]$: **M1**

Maximum occurs at $x = \dfrac{0.4}{0.6} = \dfrac{2}{3} \approx 0.667$ km **A1**

Note: Award M1 for using GDC to locate the maximum (e.g. graphically or via solve $P'(x) = 0$). A1 for $x = 0.667$ (3 s.f.) or exact $\frac{2}{3}$. Trap: not checking domain endpoints.

(b)

Using GDC: $\displaystyle\int_0^6 5x^{0.4}e^{-0.6x}\,dx = 5.2307\ldots$ **(M1)(A1)**

Total population $= 5.23$ thousand (3 s.f.)

Note: Award (M1) for writing the definite integral with correct limits and integrand. (A1) for 5.23 (3 s.f.). Full precision value: $5.2307\ldots$

(c)

$\displaystyle\int_6^8 5x^{0.4}e^{-0.6x}\,dx \approx 0.0816 > 0$ **R1**

Since the integrand is positive for all $x > 0$, the total population **increases**.

Note: Award R1 for a correct justification: either evaluating $\int_6^8 P\,dx > 0$ on GDC, or noting $P(x) > 0$ for all $x \in (6, 8)$, so extending the domain adds a positive quantity. A bald answer without reason earns R0.