IB Math AA SL — Topic 5

Kinematics

Displacement, Velocity & Acceleration · Calculus for Kinematics

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:________________________ Date:____________

Score:__________/120

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Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

Key Formulae

Velocity from displacement: $v(t) = s'(t) = \dfrac{ds}{dt}$

Acceleration from velocity: $a(t) = v'(t) = \dfrac{dv}{dt} = s''(t)$

Displacement from velocity (with initial condition): $s(t) = \displaystyle\int v(t)\,dt + C$, where C is found from $s(t_0) = s_0$

Displacement over an interval: $\Delta s = s(b) - s(a) = \displaystyle\int_a^b v(t)\,dt$

Total distance travelled: $d = \displaystyle\int_a^b |v(t)|\,dt$ (split the integral at every root of v)

Speed increasing condition: speed is increasing when $v(t)$ and $a(t)$ have the same sign

Particle at rest / changes direction: solve $v(t) = 0$; confirm a sign change of v for a direction change

GDC Tips

1. **Finding when the particle is at rest:** enter $v(t)$ in Graph, then use G-Solve $\to$ Root (or Zero) to locate all t-values where $v(t) = 0$ on the given domain.
2. **Total distance travelled:** use Math $\to \int dx$ (or fnInt) to evaluate $\int_a^b |v(t)|\,dt$ by entering abs$(v(t))$ as the integrand; confirm the roots first so the GDC handles the sign split automatically.
3. **Acceleration at an instant:** define $v(t)$ in Y1, then use Math $\to d/dx$ evaluated at the required t-value (or the numerical derivative nDeriv) to obtain $a(t_0)$.
4. **Maximum/minimum velocity:** graph $v(t)$, then use G-Solve $\to$ Max/Min on the relevant interval; com-

pare endpoint values as well.

5. **Solving $v(t) = k$ or $a(t) = k$:** use G-Solve $\to$ Intersection after plotting both $v(t)$ and the horizontal line $y = k$, or use Equation solver; report all solutions in the domain.

Section A — Easy **EASY** (10 questions $\times$ 3 marks = 30 marks)

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1. **EASY** **No Calculator** **[3 marks]**

A particle moves in a straight line. Its velocity at time t seconds, for $t \geq 0$, is given by $v(t) = 6t - 4$ m s^{-1}. Find the acceleration of the particle when $t = 3$.

2. **EASY** **No Calculator** **[3 marks]**

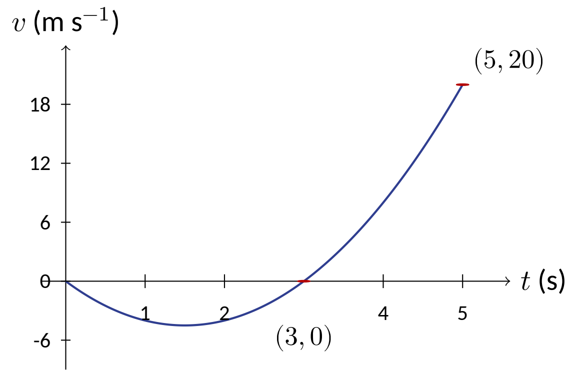
A particle moves in a straight line. Its displacement from a fixed origin at time t seconds, for $t \geq 0$, is given by $s(t) = t^3 - 3t + 2$ metres. Find the velocity of the particle when $t = 2$.

3. **EASY** **Calculator** **[3 marks]**

A particle moves in a straight line with velocity $v(t) = 3t^2 - 12$ m s^{-1}, for $t \geq 0$. Find the time at which the particle is at rest.

4. EASY Calculator **[3 marks]**

A particle moves in a straight line. Its velocity at time t seconds, for $0 \le t \le 5$, is modelled by $v(t) = 2t^2 - 6t$ m s^{-1}, and is shown in the graph below.



Write down the value of t at which the particle is at rest for $t > 0$.

5. EASY No Calculator **[3 marks]**

A particle moves in a straight line. Its velocity at time t seconds, for $t \ge 0$, is given by $v(t) = e^{2t} - 1$ m s^{-1}. Find the acceleration of the particle when $t = 0$.

6. EASY Calculator **[3 marks]**

A particle moves in a straight line. Its velocity at time t seconds, for $t \ge 0$, is given by $v(t) = 4t - t^2$ m s^{-1}. Find the displacement of the particle from its initial position at $t = 0$ to $t = 3$.

7. EASY No Calculator **[3 marks]**

A particle moves in a straight line with acceleration $a(t) = 6t + 2$ m s^{-2}, for $t \geq 0$. Given that the velocity of the particle is 5 m s^{-1} when $t = 0$, find $v(t)$.

8. EASY Calculator **[3 marks]**

A particle moves along a straight line. The velocity is given by $v(t) = t^2 + 2t - 2$ m s^{-1}, for $0 \leq t \leq 4$. The table below shows the velocity of the particle at various times.

t (s)	0	1	2	4
v (m s^{-1})	-2	1	6	22

Find the acceleration of the particle when $t = 2$.

9. EASY Calculator **[3 marks]**

A particle moves in a straight line with velocity $v(t) = \sin(2t)$ m s^{-1}, for $0 \leq t \leq \pi$. Find the value of t at which the particle first comes to rest after $t = 0$.

10. EASY No Calculator **[3 marks]**

A particle moves in a straight line. Its displacement from a fixed origin at time t seconds, for $t \geq 0$, is given by $s(t) = 2t^3 - 9t^2 + 12t$ metres. Find the velocity of the particle when $t = 1$.

Section B — Medium **MEDIUM** (10 questions × 4 marks = 40 marks)

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11. MEDIUM No Calculator **[4 marks]**

A particle moves along a straight line. Its velocity at time t seconds, for $t \geq 0$, is given by $v(t) = 3t^2 - 12t + 9$ m s^{-1}. Find the displacement of the particle from its initial position at $t = 4$ seconds.

12. MEDIUM No Calculator **[4 marks]**

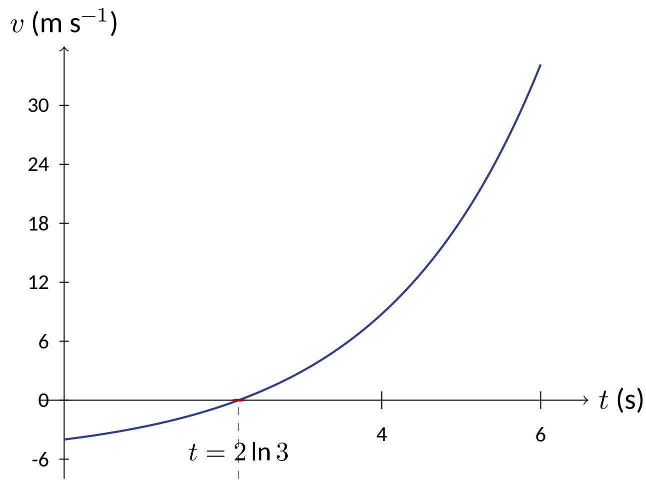
A particle moves in a straight line with velocity $v(t) = 4\cos(2t)$ m s^{-1}, where $t \geq 0$ seconds. The particle is at position $s = 3$ metres from the origin when $t = 0$. Find the position of the particle at $t = \dfrac{\pi}{4}$ seconds.

13. MEDIUM Calculator **[4 marks]**

A particle moves along a straight line. Its displacement from the origin at time t seconds, for $0 \leq t \leq 5$, is given by $s(t) = t^3 - 6t^2 + 9t + 2$, measured in metres. Find the acceleration of the particle at the instant when it is first at rest.

14. MEDIUM Calculator **[4 marks]**

A particle moves along a straight line with velocity $v(t) = 2e^{0.5t} - 6$ m s^{-1}, for $0 \le t \le 6$, shown in the graph below.



Find the total distance travelled by the particle over this interval.

15. MEDIUM No Calculator **[4 marks]**

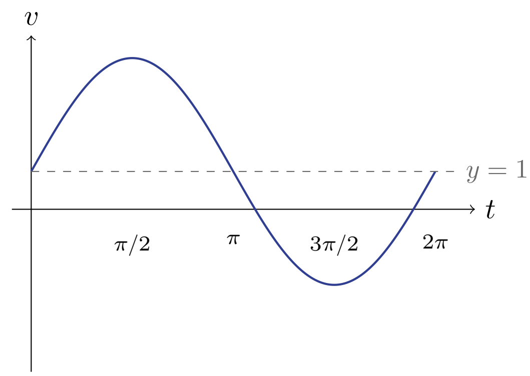
A particle moves in a straight line. Its velocity at time t seconds is $v(t) = t^2 - 5t + 4$ m s^{-1}, for $0 \le t \le 5$. Find the total distance travelled by the particle over this interval.

16. MEDIUM No Calculator **[4 marks]**

A particle moves along a straight line with acceleration $a(t) = 6t - 4$ m s^{-2}, for $t \ge 0$. The particle has initial velocity $v(0) = 2$ m s^{-1} and initial displacement $s(0) = -1$ m from the origin. Find the displacement of the particle from the origin when $t = 3$ seconds.

17. MEDIUM Calculator **[4 marks]**

A particle moves in a straight line. Its velocity at time t seconds is $v(t) = 3\sin(t) + 1$ m s^{-1}, for $0 \leq t \leq 2\pi$. The velocity-time graph is shown below.



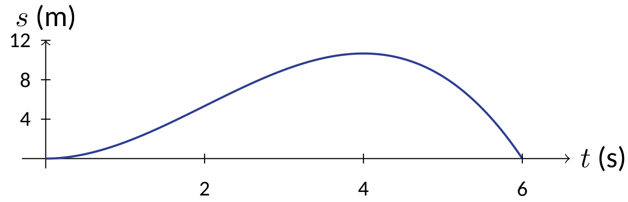
Find the times in the interval $0 < t < 2\pi$ at which the particle is at rest.

18. MEDIUM Calculator **[4 marks]**

A particle moves along a straight line with velocity $v(t) = t^2 e^{-t}$ m s^{-1}, for $0 \leq t \leq 5$. Find the acceleration of the particle at $t = 2$ seconds, and determine whether the speed of the particle is increasing or decreasing at that instant.

19. **MEDIUM** **No Calculator** **[4 marks]**

A particle starts from rest at the origin and moves in a straight line. Its acceleration at time t seconds is $a(t) = 4 - 2t$ m s^{-2}, for $0 \le t \le 6$, and the resulting displacement $s(t)$ is shown in the graph below.



Find the maximum displacement of the particle from the origin.

20. **MEDIUM** **No Calculator** **[4 marks]**

A particle moves in a straight line. Its velocity at time t seconds is $v(t) = 6t - t^2 - 8$ m s^{-1}, for $0 \le t \le 7$. Find the maximum speed of the particle over this interval.

Section C — Hard HARD (10 questions $\times$ 5 marks = 50 marks)

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21. HARD No Calculator **[5 marks]**

A particle moves along a straight line. Its velocity, in m s^{-1}, at time t seconds, $t \geq 0$, is given by

$$v(t) = t^3 - 6t^2 + 9t - 2.$$

Find the total distance travelled by the particle in the interval $0 \leq t \leq 4$.

22. HARD No Calculator **[5 marks]**

A particle moves along a straight line with velocity $v(t) = 3t^2 - 12t + 9$ m s^{-1}, $t \geq 0$, where t is in seconds. The particle starts at a position 2 m to the right of the origin.

(a) Show that the acceleration of the particle is zero when $t = 2$. [1]

(b) Find the maximum speed of the particle for $0 \leq t \leq 4$, justifying your answer. [4]

23. HARD Calculator **[5 marks]**

The velocity of a particle, in m s^{-1}, at time t seconds, $t \geq 0$, is given by

$$v(t) = e^{-0.5t}\,\text{sin}(2t).$$

Find the total distance travelled by the particle in the interval $0 \leq t \leq 5$.

24. HARD No Calculator **[5 marks]**

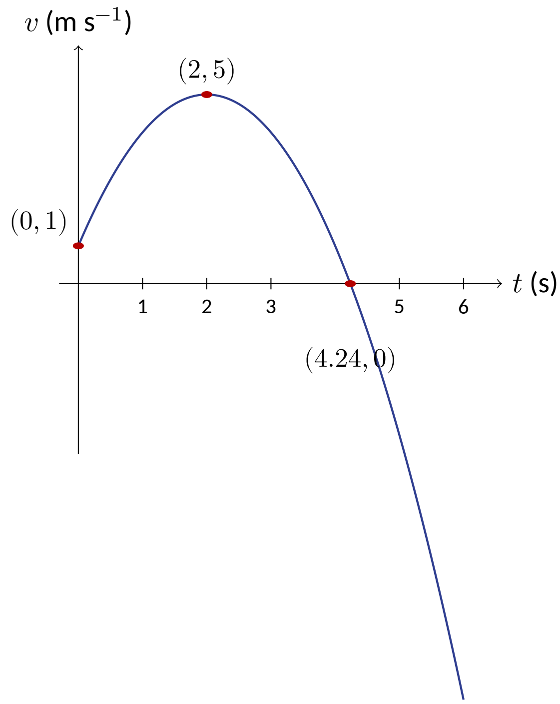
A particle moves along a straight line. Its displacement from the origin, in metres, at time t seconds ($t \geq 0$) is given by

$$s(t) = t^3 - \frac{9}{2}t^2 + 6t + 1.$$

Find the values of t at which the particle changes direction, and determine the total distance the particle travels in the interval $0 \leq t \leq 5$.

25. HARD Calculator **[5 marks]**

The velocity of a particle moving in a straight line, in m s^{-1}, at time t seconds, $t \geq 0$, is shown in the velocity-time graph below.



The velocity is modelled by $v(t) = -t^2 + 4t + 1$ for $0 \leq t \leq 6$.

(a) Write down the time at which the particle's speed is greatest on the interval $0 \leq t \leq 6$. [1]

(b) Find the displacement of the particle from its initial position at $t = 6$. [2]

(c) Find the total distance travelled by the particle in the interval $0 \leq t \leq 6$. [2]

26. HARD No Calculator **[5 marks]**

A particle moves along a straight line. Its acceleration, in m s^{-2}, at time t seconds ($t \geq 0$) is given by

$$a(t) = 6t - 10.$$

At $t = 0$ the particle has velocity 4 m s^{-1} and displacement 0 m from the origin. Find the displacement of the particle when its speed is first equal to 4 m s^{-1} again for $t > 0$.

27. HARD Calculator **[5 marks]**

A particle moves along a straight line. Its velocity, in m s^{-1}, at time t seconds, $t \geq 0$, is given by

$$v(t) = \cos(3t) - \cos(t), \qquad 0 \leq t \leq \pi.$$

Find the times at which the particle is at rest, and find the total distance travelled over $0 \leq t \leq \pi$.

28. HARD No Calculator **[5 marks]**

A particle moves in a straight line. At time $t = 0$ it is at the origin with velocity 0 m s^{-1}. Its acceleration, in m s^{-2}, is given by

$$a(t) = \frac{8}{(t+1)^2} - 2, \qquad t \geq 0.$$

(a) Show that $v(t) = -\dfrac{8}{t+1} - 2t + 8$. [2]

(b) Find the displacement of the particle when it first returns to rest for $t > 0$. Give your answer in the form $p - \ln q$ where $p, q \in \mathbb{Z}^+$. [3]

29. **HARD** **Calculator** **[5 marks]**

The velocity of a particle moving along a straight line, in m s^{-1}, at time t seconds, $t \geq 0$, is given by

$$v(t) = 4te^{-0.4t} - 1.$$

(a) Find the first time $t_1 > 0$ at which the particle is at rest. [1]

(b) Find the acceleration of the particle at $t = t_1$. [2]

(c) The particle starts at the origin. Find the displacement of the particle at $t = 8$. [2]

30. **HARD** **No Calculator** **[5 marks]**

The velocity of a particle, in m s^{-1}, is given by $v(t) = t^2 - 5t + k$ for $t \geq 0$, where $k \in \mathbb{R}$, $k \geq 0$. The particle starts at the origin at $t = 0$.

It is known that the particle returns to the origin exactly once for $t > 0$.

Find the value(s) of k, justifying that no other value of $k \geq 0$ satisfies the condition.

Worked Solutions

M = method mark A = accuracy mark R = reasoning mark AG = answer given FT = follow-through

Q1 — Acceleration from Velocity by Differentiation EASY

Differentiating to find acceleration: $a(t) = v'(t) = 6$ **M1A1**

$a(3) = 6$ m s^{-2} **A1**

Note: Accept $a = 6$ m s^{-2} with no working shown, award M1A1A1.

Q2 — Velocity from Displacement by Differentiation EASY

Differentiating to find velocity: $v(t) = s'(t) = 3t^2 - 3$ **M1A1**

$v(2) = 3(4) - 3 = 9$ m s^{-1} **A1**

Note: Accept $v = 9$ m s^{-1}.

Q3 — Time When Particle is at Rest EASY

Setting velocity equal to zero: $3t^2 - 12 = 0$ **M1**

$t^2 = 4 \implies t = 2$ (since $t \geq 0$, reject $t = -2$) **A1A1**

Note: Award final A1 for rejecting the negative root explicitly or implicitly by domain.

Q4 — Reading Time at Rest from Velocity Model EASY

Setting $v(t) = 0$ for $t > 0$: $2t^2 - 6t = 0 \implies 2t(t - 3) = 0$ **M1**

$t = 3$ **A1**

Note: The answer $t = 3$ s may be read directly from the graph; award full marks. Award M1 for evidence of solving or reading $v = 0$ for $t > 0$, A1 for $t = 3$.

Q5 — Acceleration of Exponential Velocity at t

Differentiating to find acceleration: $a(t) = v'(t) = 2e^{2t}$ **M1A1**

$a(0) = 2e^0 = 2$ m s^{-2} **A1**

Note: Accept $a = 2$ m s^{-2}.

Q6 — Displacement by Integration over an Interval EASY

Integrating velocity to find displacement: $s = \int_0^3 (4t - t^2)\,dt$ **M1**

$= \left[2t^2 - \frac{t^3}{3}\right]_0^3 = (18 - 9) - 0$ **A1**

$= 9$ m **A1**

Note: Since $v(t) \geq 0$ for $0 \leq t \leq 4$ on this domain, displacement equals distance; no splitting required.

Q7 — Velocity by Integrating Acceleration with Initial Condition EASY

Integrating acceleration: $v(t) = \int (6t + 2)\,dt = 3t^2 + 2t + C$ **M1A1**

Applying $v(0) = 5$: $C = 5$, so $v(t) = 3t^2 + 2t + 5$ m s^{-1} **A1**

Note: Award final A1 only if constant of integration is determined using the initial condition.

Q8 — Acceleration from Quadratic Velocity Model EASY

Differentiating to find acceleration: $a(t) = v'(t) = 2t + 2$ **M1A1**

$a(2) = 2(2) + 2 = 6$ m s^{-2} **A1**

Note: Accept $a = 6$ m s^{-2}.

Q9 — First Time at Rest for Sinusoidal Velocity EASY

Setting $v(t) = 0$: $\sin(2t) = 0 \implies 2t = \pi$ (first positive solution) **M1A1**

$t = \frac{\pi}{2}$ s **A1**

Note: Accept $t = \pi/2$. Award M1 for evidence of solving $\sin(2t) = 0$ for $t > 0$, even if by GDC.

Q10 — Velocity from Cubic Displacement at Given Time EASY

Differentiating to find velocity: $v(t) = s'(t) = 6t^2 - 18t + 12$ **M1A1**

$v(1) = 6(1) - 18(1) + 12 = 0$ m s^{-1} **A1**

Note: Accept $v = 0$ m s^{-1}. The particle is instantaneously at rest at $t = 1$.

Q11 — Displacement from Polynomial Velocity MEDIUM

Attempt to integrate $v(t)$ **M1**

$s(t) = \int (3t^2 - 12t + 9)\,dt = t^3 - 6t^2 + 9t + C$ **A1**

Use $s(0) = 0$ (displacement from initial position, so $C = 0$): $s(4) = 4^3 - 6(4)^2 + 9(4) = 64 - 96 + 36$ **M1**

$s(4) = 4$ m **A1**

Note: Accept $C = 0$ implied by "displacement from initial position". Award final A1 only for exact value 4 m.

Q12 — Displacement with Trig Velocity and Initial Condition MEDIUM

Attempt to integrate $v(t) = 4\cos(2t)$ **M1**

$s(t) = 2\sin(2t) + C$ **A1**

Apply $s(0) = 3$: $2\sin(0) + C = 3 \Rightarrow C = 3$ **M1**

$s\left(\frac{\pi}{4}\right) = 2\sin\left(\frac{\pi}{2}\right) + 3 = 2(1) + 3 = 5$ m **A1**

Note: Penalise if degrees used and answer is not equivalent.

Q13 — Acceleration when First at Rest MEDIUM

Differentiate to find $v(t)$: $v(t) = 3t^2 - 12t + 9$ **A1**

Set $v(t) = 0$: $3(t-1)(t-3) = 0 \Rightarrow t = 1$ or $t = 3$; first at rest at $t = 1$ **M1**

Differentiate $v(t)$ to find $a(t) = 6t - 12$ **M1**

$a(1) = 6(1) - 12 = -6$ m s^{-2} **A1**

Note: Award M1M1 for correct process even if GDC used to find $t = 1$. Do not award final A1 for $a(3) = 6$.

Q14 — Total Distance with Exponential Velocity MEDIUM

Find where $v = 0$: $2e^{0.5t} = 6 \Rightarrow e^{0.5t} = 3 \Rightarrow t = 2\ln 3 \approx 2.197$ **M1**

$v < 0$ for $0 \le t < 2\ln 3$ and $v > 0$ for $2\ln 3 < t \le 6$

Antiderivative: $F(t) = 4e^{0.5t} - 6t$ (verified $F' = v$). Attempt total distance $= \int_0^{2\ln 3} (-v)\,dt + \int_{2\ln 3}^{6} v\,dt$ **M1**

$= \left[F(0) - F(2\ln 3)\right] + \left[F(6) - F(2\ln 3)\right] = 5.1833\ldots + 45.5255\ldots$ **(A1)**

Total distance $= 4e^3 + 24\ln 3 - 56 \approx 50.7$ m (3 s.f.) **A1**

Note: Full precision ≈ 50.709 m. Do not award final A1 without splitting at $t = 2\ln 3$ (not $t = \ln 3$).

Q15 — Total Distance from Polynomial Velocity **MEDIUM**

Find roots of v: $t^2 - 5t + 4 = (t-1)(t-4) = 0 \Rightarrow t = 1,\ t = 4$ **M1**

Total distance $= \left|\int_0^1 v\,dt\right| + \left|\int_1^4 v\,dt\right| + \left|\int_4^5 v\,dt\right|$ **M1**

$$\int_0^1 (t^2 - 5t + 4)\,dt = \left[\frac{t^3}{3} - \frac{5t^2}{2} + 4t\right]_0^1 = \frac{11}{6}$$

$$\int_1^4 (t^2 - 5t + 4)\,dt = \left[\frac{t^3}{3} - \frac{5t^2}{2} + 4t\right]_1^4 = -\frac{9}{2}$$

$$\int_4^5 (t^2 - 5t + 4)\,dt = \left[\frac{t^3}{3} - \frac{5t^2}{2} + 4t\right]_4^5 = \frac{11}{6}$$ **A1**

Total distance $= \frac{11}{6} + \frac{9}{2} + \frac{11}{6} = \frac{49}{6}$ m **A1**

Note: Accept 49/6 m or 8.17 m (3 s.f.). Condone missing the $[4, 5]$ piece only if working is otherwise complete and consistent; penalise by 1 mark.

Q16 — Double Integration from Acceleration **MEDIUM**

Integrate $a(t)$ to find $v(t)$: $v(t) = 3t^2 - 4t + C_1$ **M1**

Apply $v(0) = 2$: $C_1 = 2$, so $v(t) = 3t^2 - 4t + 2$ **A1**

Integrate $v(t)$ to find $s(t)$: $s(t) = t^3 - 2t^2 + 2t + C_2$ **M1**

Apply $s(0) = -1$: $C_2 = -1$

$s(3) = 27 - 18 + 6 - 1 = 14$ m **A1**

Note: FT on their $v(t)$ for final M1A1 provided integration is attempted.

Q17 — Times at Rest from Trig Velocity **MEDIUM**

Set $v(t) = 0$: $3\sin(t) + 1 = 0 \Rightarrow \sin(t) = -\frac{1}{3}$ **M1**

Reference value: $\arcsin\left(\frac{1}{3}\right) = 0.3398\ldots$ **(A1)**

Since $\sin(t) < 0$, solutions lie in $(\pi, 2\pi)$: **M1**

$t = \pi + 0.3398\ldots = 3.481\ldots \approx 3.48$ rad

$t = 2\pi - 0.3398\ldots = 5.943\ldots \approx 5.94$ rad **A1**

Note: Both values required for A1. Accept answers to 3 s.f. Condone exact forms $t = \pi + \arcsin(1/3)$ and $t = 2\pi - \arcsin(1/3)$.

Q18 — Acceleration and Speed Decision for Exponential Velocity MEDIUM

Differentiate using the product rule: $a(t) = v'(t) = 2te^{-t} - t^2e^{-t} = te^{-t}(2-t)$ **M1**

$a(2) = 2e^{-2}(2-2) = 0$ m s^{-2} **A1**

At $t = 2$: $v(2) = 4e^{-2} > 0$ and $a(2) = 0$, so v and a are not the same sign. **R1**

Since $a(2) = 0$, the speed is neither increasing nor decreasing (it is at a turning point / instantaneous maximum speed). **A1**

Note: For R1, the reasoning must reference both $v(2) > 0$ and $a(2) = 0$. Accept "speed is at a local maximum at $t = 2$".

Q19 — Maximum Displacement from Quadratic Acceleration MEDIUM

Integrate $a(t)$: $v(t) = 4t - t^2 + C_1$; apply $v(0) = 0 \Rightarrow C_1 = 0$ **M1**

$v(t) = 4t - t^2 = t(4-t)$; set $v = 0 \Rightarrow t = 0$ or $t = 4$ **A1**

Maximum displacement occurs at $t = 4$ (since $v > 0$ for $0 < t < 4$). Integrate $v(t)$: $s(t) = 2t^2 - \dfrac{t^3}{3} + C_2$; apply $s(0) = 0 \Rightarrow C_2 = 0$ **M1**

$s(4) = 2(16) - \dfrac{64}{3} = 32 - \dfrac{64}{3} = \dfrac{32}{3}$ m **A1**

Note: Accept 32/3 m or 10.7 m (3 s.f.). Do not award final A1 if $s(6)$ is given as the maximum.

Q20 — Maximum Speed with Endpoint Check MEDIUM

Find roots of v: $t^2 - 6t + 8 = (t-2)(t-4) = 0$, so $v = 0$ at $t = 2$ and $t = 4$. **M1**

Evaluate speed at critical points and endpoints: **M1**

t	0	2	3	4	7
$v(t)$	-8	0	1	0	-15
$\lvert v(t)\rvert$	8	0	1	0	15

Maximum of $|v|$ occurs at $t = 7$: $|v(7)| = |42 - 49 - 8| = |-15| = 15$ **A1**

Maximum speed is 15 m s^{-1}. **A1**

Note: Both endpoints $t = 0$ and $t = 7$ must be checked for full marks (endpoint trap).

Q21 — Total Distance from Polynomial Velocity HARD

Finding roots of $v(t) = t^3 - 6t^2 + 9t - 2$: testing $t = 2$: $v(2) = 8 - 24 + 18 - 2 = 0$; factor $(t-2)$: **M1**

$v(t) = (t-2)(t^2 - 4t + 1)$; roots of $t^2 - 4t + 1 = 0$: $t = 2 \pm \sqrt{3}$ **A1**

Roots in $[0,4]$: $t=2-\sqrt{3}\approx 0.268$, $t=2$, $t=2+\sqrt{3}\approx 3.732$

Antiderivative $S(t)=\frac{t^4}{4}-2t^3+\frac{9t^2}{2}-2t$. Splitting and integrating $|v|$ over the four pieces: **M1**

Piece values: $-\frac{1}{4},\ \frac{9}{4},\ -\frac{9}{4},\ \frac{1}{4}$ (signed) **(A1)**

Total distance $=\frac{1}{4}+\frac{9}{4}+\frac{9}{4}+\frac{1}{4}$; Total distance $=5$ m (exact) **A1**

Note: Award M1 for evidence of splitting at all three roots. Full precision confirms the exact value 5.

Q22 — Maximum Speed with Endpoint Check HARD

(a) $a(t)=v'(t)=6t-12$ **(A1)**

$a(2)=6(2)-12=0$ **AG**

Note: Must show derivative and substitution; conclusion needed.

(b) $v(t)=3t^2-12t+9=3(t-1)(t-3)$; critical points and endpoints to check speed: **M1**

$v(0)=9,\quad v(1)=0,\quad v(2)=12-24+9=-3,\quad v(3)=0,\quad v(4)=48-48+9=9$

The minimum of v on $[0,4]$ occurs at $t=2$ (since $v''=6>0$, so $t=2$ is a local min of v) **A1**

Comparing $|v(0)|=9,\ |v(2)|=3,\ |v(4)|=9$: maximum speed is 9 m s^{-1} **A1**

Occurs at $t=0$ and $t=4$ (endpoints); justified by checking all critical points and endpoints **R1**

Note: R1 requires explicit statement that endpoints were checked; award A1ft for their correct maximum speed from their candidate values.

Q23 — Total Distance from Exponential-Trig Velocity HARD

Recognise total distance $=\int_0^5 |v(t)|\,dt=\int_0^5 \left|e^{-0.5t}\sin(2t)\right|dt$ **M1**

Find roots of $v(t)=0$ on $[0,5]$: $\sin(2t)=0\Rightarrow 2t=n\pi$, so $t=0,\ \frac{\pi}{2},\ \pi,\ \frac{3\pi}{2}$ **(A1)**

Roots in $[0,5]$: $t=0,\ \frac{\pi}{2}\approx 1.571,\ \pi\approx 3.142,\ \frac{3\pi}{2}\approx 4.712$ **(A1)**

Using the GDC to evaluate $\int_0^5 |e^{-0.5t}\sin(2t)|\,dt$ split at each root: **(M1)**

Total distance $=1.15$ m (3 s.f.) **A1**

Note: Full precision ≈ 1.1469 m; accept 1.15 m. Award (M1)(A1) for correct GDC setup with absolute value; do not accept displacement.

Q24 — Direction Change and Total Distance HARD

Differentiate: $v(t) = s'(t) = 3t^2 - 9t + 6 = 3(t-1)(t-2)$ **M1**

Roots: $t = 1$ and $t = 2$; particle changes direction at $t = 1$ s and $t = 2$ s **A1**

Evaluate s at endpoints and turning points: $s(0) = 1, \quad s(1) = 1 - \frac{9}{2} + 6 + 1 = \frac{7}{2}, \quad s(2) = 8 - 18 + 12 + 1 = 3, \quad s(5) = 125 - \frac{225}{2} + 30 + 1 = \frac{87}{2}$ **A1**

Total distance $= |s(1) - s(0)| + |s(2) - s(1)| + |s(5) - s(2)|$ **M1**

$= \left|\frac{7}{2} - 1\right| + \left|3 - \frac{7}{2}\right| + \left|\frac{87}{2} - 3\right| = \frac{5}{2} + \frac{1}{2} + \frac{81}{2} = \frac{87}{2} = 43.5$ m **A1**

Note: $s(5) = 87/2 = 43.5$ — check the arithmetic $5^3 - 4.5(5)^2 + 6(5) + 1 = 125 - 112.5 + 30 + 1 = 43.5$ carefully. Award M1 for correct distance formula using their direction-change times.

Q25 — Velocity-Time Graph with Displacement and Distance HARD

(a) Speed is greatest at $t = 6$ s **A1**

Note: Award A1 for $t = 6$ only. The speed at $t = 6$ is $|-36 + 24 + 1| = 11$ m s^{-1}, which exceeds the maximum of v at $t = 2$ ($v = 5$). Endpoint check is the trap.

(b) Displacement $= \int_0^6 (-t^2 + 4t + 1)\,dt$ **(M1)**

$= \left[-\frac{t^3}{3} + 2t^2 + t\right]_0^6 = -72 + 72 + 6 = 6$ m **A1**

(c) Root of v: $-t^2 + 4t + 1 = 0 \Rightarrow t = 2 + \sqrt{5} \approx 4.236$ (taking the positive root in $[0, 6]$) **(M1)**

Antiderivative $F(t) = -\frac{t^3}{3} + 2t^2 + t$ (verified $F' = v$). $F(2+\sqrt{5}) - F(0) = \frac{22}{3} + \frac{10\sqrt{5}}{3} \approx 14.787$; $F(6) - F(2+\sqrt{5}) = -\frac{10\sqrt{5}}{3} - \frac{4}{3} \approx -8.787$

Total distance $\approx 14.787 + 8.787 = 23.6$ m (3 s.f.) **A1**

Note: Full precision: $\frac{26 + 20\sqrt{5}}{3} \approx 23.574$ m. Award (M1) for identifying the root and splitting the integral; A1 for 23.6 m.

Q26 — Acceleration Given as Speed Returns to Initial Value HARD

Integrate: $v(t) = 3t^2 - 10t + C$; $v(0) = 4 \Rightarrow C = 4$, so $v(t) = 3t^2 - 10t + 4$ **M1**

Solve $|v(t)| = 4$ for $t > 0$: Case 1: $v(t) = 4 \Rightarrow 3t^2 - 10t = 0 \Rightarrow t(3t - 10) = 0 \Rightarrow t = \frac{10}{3}$ **A1**

Case 2: $v(t) = -4 \Rightarrow 3t^2 - 10t + 8 = 0 \Rightarrow t = \frac{10 \pm \sqrt{100 - 96}}{6} = \frac{10 \pm 2}{6} \Rightarrow t = 2$ or $t = \frac{4}{3}$ **A1**

First time speed equals 4: $t = \dfrac{4}{3}$ **(A1)**

Integrate for displacement: $s(t) = t^3 - 5t^2 + 4t$; $s\left(\dfrac{4}{3}\right) = \dfrac{64}{27} - \dfrac{80}{9} + \dfrac{16}{3} = -\dfrac{32}{27}$ m (i.e. $-32/27$ m) **A1**

Note: Award M1 for correct integration of $a(t)$ with use of initial condition. Both cases of $|v| = 4$ must be considered for full marks.

Q27 — Trig Velocity Rest Times and Total Distance **HARD**

Solve $v(t) = \cos(3t) - \cos(t) = 0$ on $[0, \pi]$: **M1**

Using sum-to-product: $\cos(3t) - \cos(t) = -2\sin(2t)\sin(t) = 0$

$\sin(2t) = 0 \Rightarrow t = 0, \dfrac{\pi}{2}, \pi$; $\sin(t) = 0 \Rightarrow t = 0, \pi$

Rest time in $(0, \pi)$: $t = \dfrac{\pi}{2}$ **A1**

Antiderivative $F(t) = -\sin(t) + \dfrac{1}{3}\sin(3t)$ (verified $F' = v$). Check sign: $v(\pi/4) = \cos(3\pi/4) - \cos(\pi/4) < 0$, so $|v| = -v$ on $(0, \pi/2)$; $v(3\pi/4) > 0$, so $|v| = v$ on $(\pi/2, \pi)$ **M1**

$F(\pi/2) - F(0) = -\dfrac{4}{3}$; $F(\pi) - F(\pi/2) = \dfrac{4}{3}$ **(A1)**

Total distance $= \dfrac{4}{3} + \dfrac{4}{3} = \dfrac{8}{3}$ m ≈ 2.67 m (3 s.f.) **A1**

Note: Award M1 for sum-to-product factorisation or GDC root-finding; A1 for the exact value $8/3$ (equivalently 2.67 m).

Q28 — Integration of Rational Acceleration for Exact Displacement **HARD**

(a) $v(t) = \displaystyle\int a(t)\,dt = \int \left(\frac{8}{(t+1)^2} - 2\right) dt = -\frac{8}{t+1} - 2t + C$ **M1**

$v(0) = 0 \Rightarrow -8 + C = 0 \Rightarrow C = 8$, so $v(t) = -\dfrac{8}{t+1} - 2t + 8$ **AG A1**

Note: Both the integration and the application of the initial condition are required for A1.

(b) Solve $v(t) = 0$: $8 - \dfrac{8}{t+1} - 2t = 0$. Multiply by $(t+1)$: $8(t+1) - 8 - 2t(t+1) = 0 \Rightarrow 8t - 2t^2 - 2t = 0 \Rightarrow 2t(3-t) = 0$ **M1**

$t = 3$ (reject $t = 0$ as the initial condition) **A1**

$$s(3) = \int_0^3 \left(-\frac{8}{t+1} - 2t + 8\right) dt = \left[-8\ln(t+1) - t^2 + 8t\right]_0^3$$

$= (-8\ln 4 - 9 + 24) - 0 = 15 - 8\ln 4 = 15 - \ln 4^8$

Expressed as $p - \ln q$: $s(3) = 15 - \ln 65536$, so $p = 15$, $q = 65536$ **A1**

Note: $4^8 = 65536$, so $8\ln 4 = \ln 65536$. Both $p = 15$ and $q = 65536$ are positive integers as required. Root $t = 0$ must be explicitly rejected.

Q29 — Exponential Velocity GDC Chain HARD

(a) Using GDC to solve $4te^{-0.4t} - 1 = 0$ for $t > 0$: **(M1)**

$t_1 = 0.280$ s (3 s.f.) **A1**

Note: Full precision $t_1 \approx 0.27958$ s; accept answers in $[0.279, 0.280]$.

(b) $a(t) = v'(t) = 4e^{-0.4t} + 4t(-0.4)e^{-0.4t} = 4e^{-0.4t}(1 - 0.4t)$ **M1**

$a(t_1) = 4e^{-0.4(0.27958)}(1 - 0.4(0.27958))$

$a(t_1) = 3.18$ m s^{-2} (3 s.f.) **A1**

Note: Full precision ≈ 3.1768 m s^{-2}; accept 3.18. Award M1 for correct product-rule differentiation. This value depends on the corrected t_1 from part (a).

(c) $s(8) = \int_0^8 v(t)\,dt$ using GDC **(M1)**

$s(8) = 12.7$ m (3 s.f.) **A1**

Note: Full precision ≈ 12.7200 m; accept answers in $[12.7, 12.72]$. Do not accept total distance. $s(8)$ is independent of t_1.

Q30 — Parameter Value for Exactly One Return to Origin HARD

$s(t) = \int_0^t v(\tau)\,d\tau = \dfrac{t^3}{3} - \dfrac{5t^2}{2} + kt$ **M1**

Particle returns to origin when $s(t) = 0, t > 0$: $t\left(\dfrac{t^2}{3} - \dfrac{5t}{2} + k\right) = 0$, so need $f(t) := \dfrac{t^2}{3} - \dfrac{5t}{2} + k = 0$ to have exactly one positive root **M1**

f is an upward-opening parabola in t, with sum of roots $\dfrac{15}{2} > 0$ and product of roots $3k$.

Case 1 ($k = 0$): $f(t) = t\left(\dfrac{t}{3} - \dfrac{5}{2}\right)$, roots $t = 0$ (rejected, $t > 0$ required) and $t = \dfrac{15}{2}$: exactly one positive root. **A1**

Case 2 ($k > 0$): product of roots $3k > 0$ and sum $15/2 > 0$, so any real roots of f are both positive. Discriminant $\Delta = \dfrac{25}{4} - \dfrac{4k}{3}$: for $0 < k < \dfrac{75}{16}$, $\Delta > 0$ giving two distinct positive roots (two returns); for $k = \dfrac{75}{16}$, $\Delta = 0$ giving one repeated positive root (exactly one return); for $k > \dfrac{75}{16}$, $\Delta < 0$, no real roots (never returns). **A1**

Since $k \geq 0$ is given, $k < 0$ is outside the domain and does not need to be considered. Within $k \geq 0$, exactly one return occurs only at $k = 0$ or $k = \dfrac{75}{16}$; no other $k \geq 0$ satisfies the condition. **A1**

Note: Award M1 for setting $s(t) = 0$ and factoring out t; second M1 for identifying the condition on the quadratic; A1 for $k = 75/16$ from the discriminant; second A1 for $k = 0$ from the boundary case; final A1 for the justification that, restricted to $k \geq 0$, no other value works.

Common Mistakes to Avoid

1. **Confusing displacement with total distance.** Displacement is $\int_a^b v(t)\,dt$ and can be zero or negative; total distance is $\int_a^b |v(t)|\,dt$ and is always non-negative. Using the signed integral when distance is asked loses all accuracy marks.
2. **Forgetting to split the integral at roots of v for total distance.** On a Paper 1 question the integral of $|v|$ must be split manually at every t-value where $v(t) = 0$; omitting a root means an incorrect sign on one piece and a wrong final answer.
3. **Confusing the condition for speed increasing with velocity increasing.** Speed increases when v and a have the same sign — this includes intervals where $v < 0$ and $a < 0$ (the particle accelerates in the negative direction). Simply checking $a > 0$ is wrong and loses the R1 reasoning mark.
4. **Omitting the constant of integration or misusing the initial condition.** Writing $s(t) = \int v(t)\,dt$ without adding C and applying $s(t_0) = s_0$ gives an incorrect displacement function; all subsequent parts built on $s(t)$ will be wrong.
5. **Reporting maximum speed only at a stationary point of v, ignoring endpoints.** Maximum speed can occur at an endpoint of the domain if $|v|$ is largest there, even when v has a local extremum inside the interval. Always evaluate $|v|$ at every critical point and at both endpoints, then compare.
6. **Stating the particle "changes direction" whenever $v = 0$, without checking for a sign change.** A root of v is only a direction change if v changes sign there; if v touches zero and returns (e.g. a repeated root), the particle momentarily stops but does not reverse. Failing to verify the sign change loses the R1 justification mark.