IB Math AA SL — Topic 5

Kinematics

Displacement, Velocity & Acceleration · Calculus for Kinematics

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:______________________________ Date:_______________

Score:__________/120

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Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. Each question states whether a calculator is permitted.

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Key Formulae

- **Velocity from displacement:** $v(t) = s'(t) = \dfrac{ds}{dt}$
- **Acceleration from velocity:** $a(t) = v'(t) = \dfrac{dv}{dt} = s''(t)$
- **Displacement from velocity (with initial condition):** $s(t) = \displaystyle\int v(t)\,dt + C$, where C is found from $s(t_0) = s_0$
- **Displacement over an interval:** $\Delta s = s(b) - s(a) = \displaystyle\int_a^b v(t)\,dt$
- **Total distance travelled:** $d = \displaystyle\int_a^b |v(t)|\,dt$ (split the integral at every root of v)
- **Speed increasing condition:** speed is increasing when $v(t)$ and $a(t)$ have the *same* sign
- **Particle at rest / changes direction:** solve $v(t) = 0$; confirm a sign change of v for a direction change

GDC Tips

- **Finding when the particle is at rest:** enter $v(t)$ in `Graph`, then use `G-Solve` $\to$ `Root` (or `Zero`) to locate all t-values where $v(t) = 0$ on the given domain.
- **Total distance travelled:** use `Math` $\to$ $\int$ `dx` (or `fnInt`) to evaluate $\displaystyle\int_a^b |v(t)|\,dt$ by entering `abs(v(t))` as the integrand; confirm the roots first so the GDC handles the sign split automatically.
- **Acceleration at an instant:** define $v(t)$ in `Y1`, then use `Math` $\to$ `d/dx` evaluated at the required t-value (or the numerical derivative `nDeriv`) to obtain $a(t_0)$.
- **Maximum/minimum velocity:** graph $v(t)$, then use `G-Solve` $\to$ `Max/Min` on the relevant interval; compare endpoint values as well.
- **Solving $v(t) = k$ or $a(t) = k$:** use `G-Solve` $\to$ `Intersection` after plotting both $v(t)$ and the horizontal line $y = k$, or use `Equation` solver; report all solutions in the domain.

Common Mistakes to Avoid

1. **Confusing displacement with total distance.** Displacement is $\int_a^b v(t)\,dt$ and can be zero or negative; total distance is $\int_a^b |v(t)|\,dt$ and is always non-negative. Using the signed integral when distance is asked loses all accuracy marks.
2. **Forgetting to split the integral at roots of v for total distance.** On a Paper 1 question the integral of $|v|$ must be split manually at every t-value where $v(t) = 0$; omitting a root means an incorrect sign on one piece and a wrong final answer.
3. **Confusing the condition for speed increasing with velocity increasing.** Speed increases when v and a have the *same* sign — this includes intervals where $v < 0$ and $a < 0$ (the particle accelerates in the negative direction). Simply checking $a > 0$ is wrong and loses the R1 reasoning mark.
4. **Omitting the constant of integration or misusing the initial condition.** Writing $s(t) = \int v(t)\,dt$ without adding C and applying $s(t_0) = s_0$ gives an incorrect displacement function; all subsequent parts built on $s(t)$ will be wrong.
5. **Reporting maximum speed only at a stationary point of v, ignoring endpoints.** Maximum speed can occur at an endpoint of the domain if $|v|$ is largest there, even when v has a local extremum inside the interval. Always evaluate $|v|$ at every critical point *and* at both endpoints, then compare.
6. **Stating the particle "changes direction" whenever $v = 0$, without checking for a sign change.** A root of v is only a direction change if v changes sign there; if v touches zero and returns (e.g. a repeated root), the particle momentarily stops but does not reverse. Failing to verify the sign change loses the R1 justification mark.

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Section A — Easy

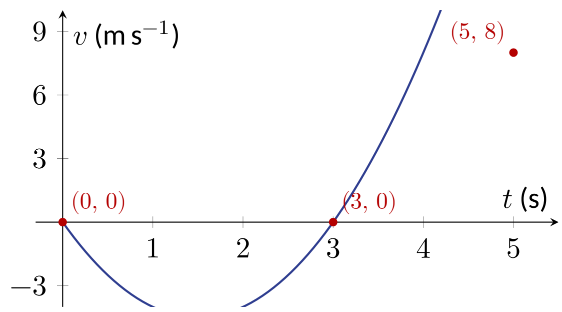
EASY

1. A particle moves in a straight line. Its velocity at time t seconds, for $t \geq 0$, is given by $v(t) = 6t - 4$ m s^{-1}. Find the acceleration of the particle when $t = 3$. *Calculator not allowed* **[3 marks]**

2. A particle moves in a straight line. Its displacement from a fixed origin at time t seconds, for $t \geq 0$, is given by $s(t) = t^3 - 3t + 2$ metres. Find the velocity of the particle when $t = 2$. *Calculator not allowed* **[3 marks]**

3. A particle moves in a straight line with velocity $v(t) = 3t^2 - 12$ m s^{-1}, for $t \geq 0$. Find the time at which the particle is at rest. *Calculator allowed* **[3 marks]**

4. A particle moves in a straight line. Its velocity at time t seconds, for $0 \leq t \leq 5$, is shown in the graph below.



The velocity of the particle is modelled by $v(t) = 2t^2 - 6t$ m s^{-1}. Write down the value of t at which the particle is at rest for $t > 0$. *Calculator allowed* **[3 marks]**

5. A particle moves in a straight line. Its velocity at time t seconds, for $t \geq 0$, is given by $v(t) = e^{2t} - 1$ m s^{-1}. Find the acceleration of the particle when $t = 0$. *Calculator not allowed* **[3 marks]**

6. A particle moves in a straight line. Its velocity at time t seconds, for $t \geq 0$, is given by $v(t) = 4t - t^2$ m s^{-1}. Find the displacement of the particle from its initial position at $t = 0$ to $t = 3$. *Calculator allowed* **[3 marks]**

7. A particle moves in a straight line with acceleration $a(t) = 6t + 2$ m s^{-2}, for $t \geq 0$. Given that the velocity of the particle is 5 m s^{-1} when $t = 0$, find $v(t)$. *Calculator not allowed* **[3 marks]**

8. A particle moves along a straight line. The table below shows the velocity v m s^{-1} of the particle at various times t seconds, for $0 \leq t \leq 4$.

t (s)	0	1	2	4
v (m s^{-1})	-2	0	4	14

The velocity is given by $v(t) = t^2 + 2t - 2$ m s^{-1}. Find the acceleration of the particle when $t = 2$. *Calculator allowed* **[3 marks]**

9. A particle moves in a straight line with velocity $v(t) = \sin(2t)$ m s^{-1}, for $0 \leq t \leq \pi$. Find the value of t at which the particle first comes to rest after $t = 0$. *Calculator allowed* **[3 marks]**

10. A particle moves in a straight line. Its displacement from a fixed origin at time t seconds, for $t \geq 0$, is given by $s(t) = 2t^3 - 9t^2 + 12t$ metres. Find the velocity of the particle when $t = 1$. *Calculator not allowed* **[3 marks]**

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Section B — Medium

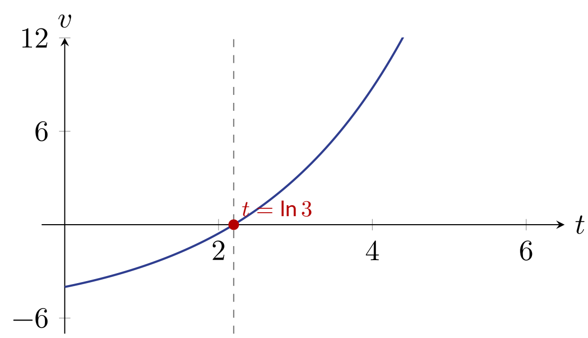
MEDIUM

11. A particle moves along a straight line. Its velocity at time t seconds, for $t \geq 0$, is given by $v(t) = 3t^2 - 12t + 9$, measured in m s$^{-1}. Findthedisplacementoftheparticlefromitsinitialpositionat$t = 4$seconds.$*Calculator not allowed***[4 marks]**

12. A particle moves in a straight line with velocity $v(t) = 4\,$cos$(2t)$ m s$^{-1}, where$t ≥0$seconds. The particle is at position$s = 3$metres from the origin when$t = 0$. Find the position of the particle at$t = $\frac{\pi}{4}$ seconds. *Calculator not allowed* **[4 marks]**

13. A particle moves along a straight line. Its displacement from the origin at time t seconds, for $0 \leq t \leq 5$, is given by $s(t) = t^3 - 6t^2 + 9t + 2$, measured in metres. Find the acceleration of the particle at the instant when it is first at rest. *Calculator allowed* **[4 marks]**

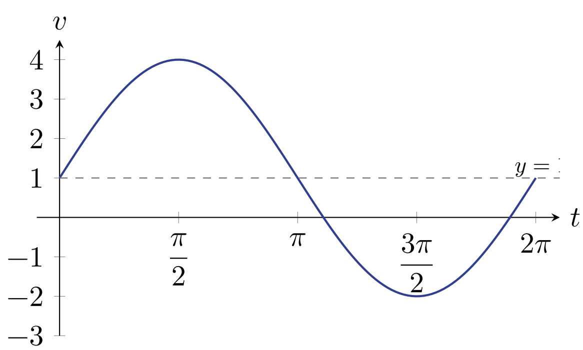
14. A particle moves along a straight line with velocity $v(t) = 2e^{0.5t} - 6$ m s^{-1}$, for$0 ≤t ≤6$. Findthetotaldistancetravelledbytheparticleoverthisinterval.$ *Calculator allowed* **[4 marks]**



15. A particle moves in a straight line. Its velocity at time t seconds is $v(t) = t^2 - 5t + 4$ m s^{-1}$, for$0 ≤t ≤5$. Findthetotaldistancetravelledbytheparticleoverthisinterval.$ *Calculator not allowed* **[4 marks]**

16. A particle moves along a straight line with acceleration $a(t) = 6t - 4$ m s^{-2}, fort ≥0.$The particle has initial velocity$v(0) = 2$m\,s^{-1} and initial displacement$s(0) = -1$m from the origin. Find the displacement of the particle from the origin when$t = 3$seconds.$*Calculator not allowed* **[4 marks]**

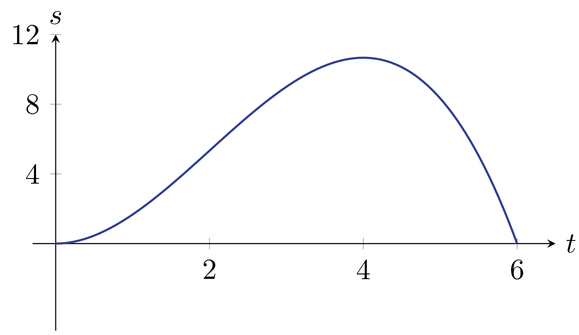
17. A particle moves in a straight line. Its velocity at time t seconds is $v(t) = 3\sin(t) + 1$ m s^{-1}, for0 ≤t ≤2. The velocity-time graph is shown below.



Find the times in the interval $0 < t < 2\pi$ at which the particle is at rest. *Calculator allowed* **[4 marks]**

18. A particle moves along a straight line with velocity $v(t) = t^2 e^{-t}$ m s^{-1}, for0 ≤t ≤5.$Find the acceleration of the particle at$t = 2$seconds, and determine whether the speed of the particle is increasing or dec

19. A particle starts from rest at the origin and moves in a straight line. Its acceleration at time t seconds is $a(t) = 4 - 2t$ m s^{-2}, for0 ≤t ≤6.$Findthemaximumdisplacementoftheparticlefromtheorigin.$*Calculator not allowed* **[4 marks]**



20. A particle moves in a straight line. Its velocity at time t seconds is $v(t) = 6t - t^2 - 8$ m s^{-1}, for0 ≤t ≤7.$Findthemaximumspeedoftheparticleoverthisinterval.$*Calculator not allowed* **[4 marks]**

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Section C — Hard

HARD

21. A particle moves along a straight line. Its velocity, in m s^{-1}, at time t seconds, $t \geq 0$, is given by

$$v(t) = t^3 - 6t^2 + 9t - 2.$$

Find the total distance travelled by the particle in the interval $0 \leq t \leq 4$. *Calculator not allowed* **[5 marks]**

22. A particle moves along a straight line with velocity $v(t) = 3t^2 - 12t + 9$ m s^{-1}, $t \geq 0$, where t is in seconds. The particle starts at a position 2 m to the right of the origin.

(a) Show that the acceleration of the particle is zero when $t = 2$. *Calculator not allowed* **[1 marks]**

(b) Find the maximum speed of the particle for $0 \leq t \leq 4$, justifying your answer. *Calculator not allowed* **[4 marks]**

23. The velocity of a particle, in m s^{-1}, at time t seconds, $t \geq 0$, is given by

$$v(t) = e^{-0.5t}\,\text{sin}(2t).$$

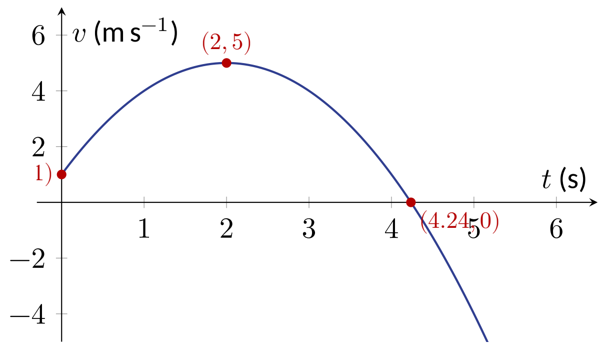
Find the total distance travelled by the particle in the interval $0 \leq t \leq 5$. *Calculator allowed* **[5 marks]**

24. A particle moves along a straight line. Its displacement from the origin, in metres, at time t seconds ($t \geq 0$) is given by

$$s(t) = t^3 - \frac{9}{2}t^2 + 6t + 1.$$

Find the values of t at which the particle changes direction, and determine the total distance the particle travels in the interval $0 \leq t \leq 5$. *Calculator not allowed* **[5 marks]**

25. The velocity of a particle moving in a straight line, in m s^{-1}, at time t seconds, $t \geq 0$, is shown in the velocity-time graph below.



The velocity is modelled by $v(t) = -t^2 + 4t + 1$ for $0 \leq t \leq 6$.

(a) Write down the time at which the particle's speed is greatest on the interval $0 \leq t \leq 6$. *Calculator allowed* **[1 marks]**

(b) Find the displacement of the particle from its initial position at $t = 6$. *Calculator allowed* **[2 marks]**

(c) Find the total distance travelled by the particle in the interval $0 \leq t \leq 6$. *Calculator allowed* **[2 marks]**

26. A particle moves along a straight line. Its acceleration, in m s^{-2}, at time t seconds ($t \geq 0$) is given by

$$a(t) = 6t - 10.$$

At $t = 0$ the particle has velocity 4 m s^{-1} and displacement 0 m from the origin. Find the displacement of the particle when its speed is first equal to 4 m s^{-1} again for $t > 0$. *Calculator not allowed* **[5 marks]**

27. A particle moves along a straight line. Its velocity, in m s^{-1}, at time t seconds, $t \geq 0$, is given by

$$v(t) = \cos(3t) - \cos(t), \quad 0 \leq t \leq \pi.$$

Find the times at which the particle is at rest, and find the total distance travelled over $0 \leq t \leq \pi$. *Calculator allowed* **[5 marks]**

28. A particle moves in a straight line. At time $t = 0$ it is at the origin with velocity 0 m s^{-1}. Its acceleration, in m s^{-2}, is given by

$$a(t) = \frac{4}{(t+1)^2} - 1, \quad t \geq 0.$$

(a) Show that $v(t) = -\dfrac{4}{t+1} - t + 4$. *Calculator not allowed* **[2 marks]**

(b) Find the displacement of the particle when it first returns to rest for $t > 0$. Give your answer in the form $p - \ln q$ where $p, q \in \mathbb{Z}^+$. *Calculator not allowed* **[3 marks]**

29. The velocity of a particle moving along a straight line, in m s^{-1}, at time t seconds, $t \geq 0$, is given by

$$v(t) = 4te^{-0.4t} - 1.$$

(a) Find the first time $t_1 > 0$ at which the particle is at rest. *Calculator allowed* **[1 marks]**

(b) Find the acceleration of the particle at $t = t_1$. *Calculator allowed* **[2 marks]**

(c) The particle starts at the origin. Find the displacement of the particle at $t = 8$. *Calculator allowed* **[2 marks]**

30. The velocity of a particle, in m s^{-1}, is given by $v(t) = t^2 - 5t + k$ for $t \geq 0$, where $k \in \mathbb{R}$. The particle starts at the origin at $t = 0$.
It is known that the particle returns to the origin exactly once for $t > 0$.
Find the value of k, justifying that no other value satisfies the condition. *Calculator not allowed* **[5 marks]**

Worked Solutions

Q1 — Acceleration from velocity by differentiation [EASY]

Differentiating to find acceleration:

$a(t) = v'(t) = 6$ **M1A1**

$a(3) = 6\ \text{m s}^{-2}$ **A1**

Note: Accept $a = 6$ m s^{-2} with no working shown, award M1A1A1.

Q2 — Velocity from displacement by differentiation [EASY]

Differentiating to find velocity:

$v(t) = s'(t) = 3t^2 - 3$ **M1A1**

$v(2) = 3(4) - 3 = 9\ \text{m s}^{-1}$ **A1**

Note: Accept $v = 9$ m s^{-1}.

Q3 — Time when particle is at rest [EASY]

Setting velocity equal to zero:

$3t^2 - 12 = 0$ **M1**

$t^2 = 4 \implies t = 2$ (since $t \geq 0$, reject $t = -2$) **A1A1**

Note: Award final A1 for rejecting the negative root explicitly or implicitly by domain.

Q4 — Reading time at rest from velocity model [EASY]

Setting $v(t) = 0$ for $t > 0$:

$2t^2 - 6t = 0 \implies 2t(t-3) = 0$ **M1**

$t = 3$ **A1**

Note: The answer $t = 3$ s may be read directly from the graph; award full marks. Award M1 for evidence of solving or reading $v = 0$ for $t > 0$, A1 for $t = 3$.

[3]

Q5 — Acceleration of exponential velocity at $t = 0$ [EASY]

Differentiating to find acceleration:

$a(t) = v'(t) = 2e^{2t}$ **M1A1**

$a(0) = 2e^0 = 2\text{ m s}^{-2}$ **A1**

Note: Accept $a = 2\text{ m s}^{-2}$.

Q6 — Displacement by integration over an interval [EASY]

Integrating velocity to find displacement:

$s = \int_0^3 (4t - t^2)\, dt$ **M1**

$= \left[2t^2 - \frac{t^3}{3}\right]_0^3 = (18 - 9) - 0$ **A1**

$= 9\text{ m}$ **A1**

Note: Accept 9 metres. Since $v(t) \geq 0$ for $0 \leq t \leq 4$ on this domain, displacement equals distance; no splitting required.

Q7 — Velocity by integrating acceleration with initial condition [EASY]

Integrating acceleration:

$v(t) = \int (6t + 2)\, dt = 3t^2 + 2t + C$ **M1A1**

Applying $v(0) = 5$: $C = 5$, so $v(t) = 3t^2 + 2t + 5\text{ m s}^{-1}$ **A1**

Note: Award final A1 only if constant of integration is determined using the initial condition.

Q8 — Acceleration from quadratic velocity model [EASY]

Differentiating to find acceleration:

$a(t) = v'(t) = 2t + 2$ **M1A1**

$a(2) = 2(2) + 2 = 6\text{ m s}^{-2}$ **A1**

Note: Accept $a = 6\text{ m s}^{-2}$.

Q9 — First time at rest for sinusoidal velocity [EASY]

Setting $v(t) = 0$:

$\sin(2t) = 0 \implies 2t = \pi$ (first positive solution) **M1A1**

$t = \frac{\pi}{2}\text{ s}$ **A1**

Note: Accept $t = \frac{\pi}{2}$. Award M1 for evidence of solving $\sin(2t) = 0$ for $t > 0$, even if by GDC.

Q10 — Velocity from cubic displacement at given time [EASY]

Differentiating to find velocity:

$v(t) = s'(t) = 6t^2 - 18t + 12$ **M1A1**

$v(1) = 6(1) - 18(1) + 12 = 0\ \text{m s}^{-1}$ **A1**

Note: Accept $v = 0$ m s^{-1}. The particle is instantaneously at rest at $t = 1$.

Q11 — Displacement from polynomial velocity [MEDIUM]

Attempt to integrate $v(t)$ **M1**

$s(t) = \int (3t^2 - 12t + 9)\,dt = t^3 - 6t^2 + 9t + C$ **A1**

Use $s(0) = 0$ (displacement from initial position, so $C = 0$):

$s(4) = 4^3 - 6(4)^2 + 9(4) = 64 - 96 + 36$ **M1**

$s(4) = 4$ m **A1**

Note: Accept $C = 0$ implied by "displacement from initial position". Award final A1 only for exact value 4 m.

Q12 — Displacement with trig velocity and initial condition [MEDIUM]

Attempt to integrate $v(t) = 4\cos(2t)$ **M1**

$s(t) = 2\sin(2t) + C$ **A1**

Apply $s(0) = 3$: $\quad 2\sin(0) + C = 3 \Rightarrow C = 3$ **M1**

$s\left(\frac{\pi}{4}\right) = 2\sin\left(\frac{\pi}{2}\right) + 3 = 2(1) + 3 = 5$ m **A1**

Note: Penalise if degrees used and answer is not equivalent.

Q13 — Acceleration when first at rest [MEDIUM]

Differentiate to find $v(t)$:

$v(t) = 3t^2 - 12t + 9$ **A1**

Set $v(t) = 0$: $3(t-1)(t-3) = 0 \Rightarrow t = 1$ or $t = 3$; first at rest at $t = 1$ **M1**

Differentiate $v(t)$ to find $a(t) = 6t - 12$ **M1**

$a(1) = 6(1) - 12 = -6\ \text{m s}^{-2}$ **A1**

Note: Award M1M1 for correct process even if GDC used to find $t = 1$. Do not award final A1 for $a(3) = 6$.

Q14 — Total distance with exponential velocity [MEDIUM]

Find where $v = 0$: $2e^{0.5t} = 6 \Rightarrow e^{0.5t} = 3 \Rightarrow t = 2\ln 3 \approx 2.197$ **M1**

$v < 0$ for $0 \le t < \ln 3^2$ and $v > 0$ for $\ln 3^2 < t \le 6$

Attempt total distance $= \displaystyle\int_0^{2\ln 3} |v|\,dt + \int_{2\ln 3}^{6} |v|\,dt$ **M1**

Using GDC: total distance $= 5.6137\ldots + 19.778\ldots$ **(A1)**

$= 25.4$ m (3 s.f.) **A1**

Note: Accept 25.3–25.4 *from GDC numerical integration. Do not award final A1 without splitting at* $t = 2\ln 3$.

Q15 — Total distance from polynomial velocity [MEDIUM]

Find roots of v: $t^2 - 5t + 4 = (t-1)(t-4) = 0 \Rightarrow t = 1,\ t = 4$ **M1**

Total distance $= \left|\displaystyle\int_0^1 v\,dt\right| + \left|\int_1^4 v\,dt\right| + \left|\int_4^5 v\,dt\right|$ **M1**

$$\int_0^1 (t^2 - 5t + 4)\,dt = \left[\tfrac{t^3}{3} - \tfrac{5t^2}{2} + 4t\right]_0^1 = \tfrac{1}{3} - \tfrac{5}{2} + 4 = \tfrac{11}{6}$$

$$\int_1^4 (t^2 - 5t + 4)\,dt = \left[\tfrac{t^3}{3} - \tfrac{5t^2}{2} + 4t\right]_1^4 = \left(\tfrac{64}{3} - 40 + 16\right) - \left(\tfrac{1}{3} - \tfrac{5}{2} + 4\right) = -\tfrac{9}{2}$$

$$\int_4^5 (t^2 - 5t + 4)\,dt = \left[\tfrac{t^3}{3} - \tfrac{5t^2}{2} + 4t\right]_4^5 = \left(\tfrac{125}{3} - \tfrac{125}{2} + 20\right) - \left(\tfrac{64}{3} - 40 + 16\right) = \tfrac{11}{6}$$ **A1**

Total distance $= \dfrac{11}{6} + \dfrac{9}{2} + \dfrac{11}{6} = \dfrac{11 + 27 + 11}{6} = \dfrac{49}{6}$ m **A1**

Note: Condone missing the $[4, 5]$ *piece only if working is otherwise complete and consistent; penalise by 1 mark.*

Q16 — Double integration from acceleration [MEDIUM]

Integrate $a(t)$ **to find** $v(t)$: $v(t) = 3t^2 - 4t + C_1$ **M1**

Apply $v(0) = 2$: $C_1 = 2$, so $v(t) = 3t^2 - 4t + 2$ **A1**

Integrate $v(t)$ **to find** $s(t)$: $s(t) = t^3 - 2t^2 + 2t + C_2$ **M1**

Apply $s(0) = -1$: $C_2 = -1$

$s(3) = 27 - 18 + 6 - 1 = 14$ m **A1**

Note: FT on their $v(t)$ *for final M1A1 provided integration is attempted.*

Q17 — Times at rest from trig velocity [MEDIUM]

Set $v(t) = 0$: $3\sin(t) + 1 = 0 \Rightarrow \sin(t) = -\dfrac{1}{3}$ **M1**

Reference value: $\arcsin\left(\frac{1}{3}\right) = 0.3398\ldots$ **(A1)**

Since $\sin(t) < 0$, solutions lie in $(\pi, 2\pi)$: **M1**

$t = \pi + 0.3398\ldots = 3.481\ldots \approx 3.48$ rad

$t = 2\pi - 0.3398\ldots = 5.943\ldots \approx 5.94$ rad **A1**

Note: Both values required for A1. Accept answers to 3 s.f. Condone exact forms $t = \pi + \arcsin\left(\frac{1}{3}\right)$ *and* $t = 2\pi - \arcsin\left(\frac{1}{3}\right)$.

Q18 — Acceleration and speed decision for exponential velocity [MEDIUM]

METHOD 1

Differentiate using the product rule: $a(t) = v'(t) = 2te^{-t} - t^2e^{-t} = te^{-t}(2 - t)$ **M1**

$a(2) = 2e^{-2}(2 - 2) = 0$ m s^{-2} **A1**

Att=2:v(2) = 4e^{-2} > 0 and $a(2) = 0$, so v and a are not the same sign. **R1**

Since $a(2) = 0$, the speed is neither increasing nor decreasing (it is at a turning point / instantaneous maximum speed). **A1**

Note: For R1, the reasoning must reference both $v(2) > 0$ *and* $a(2) = 0$*. Accept "speed is at a local maximum at* $t = 2$*". Award A1ft on their* $a(2)$ *with a consistent conclusion, except when* $|a(2)| > 0$ *and the sign argument is used correctly.*

Q19 — Maximum displacement from quadratic acceleration [MEDIUM]

Integrate $a(t)$: $v(t) = 4t - t^2 + C_1$; apply $v(0) = 0 \Rightarrow C_1 = 0$ **M1**

$v(t) = 4t - t^2 = t(4 - t)$; set $v = 0 \Rightarrow t = 0$ or $t = 4$ **A1**

Maximum displacement occurs at $t = 4$ (since $v > 0$ for $0 < t < 4$).

Integrate $v(t)$: $s(t) = 2t^2 - \dfrac{t^3}{3} + C_2$; apply $s(0) = 0 \Rightarrow C_2 = 0$ **M1**

$s(4) = 2(16) - \dfrac{64}{3} = 32 - \dfrac{64}{3} = \dfrac{32}{3}$ m **A1**

Note: Accept 10.7 *m (3 s.f.). Do not award final A1 if* $s(6)$ *is given as the maximum.*

Q20 — Maximum speed with endpoint check [MEDIUM]

Find roots of v: $t^2 - 6t + 8 = (t - 2)(t - 4) = 0$, so $v = 0$ at $t = 2$ and $t = 4$. **M1**

Note: equation written as $6t - t^2 - 8 = 0 \Rightarrow -(t - 2)(t - 4) = 0$*, same roots.*

Evaluate speed at critical points and endpoints: **M1**

t	$v(t)$	$\lvert v(t)\rvert$
0	-8	8
2	0	0
3	1	1
4	0	0
7	-15	15

Maximum of $|v|$ occurs at $t = 7$: $|v(7)| = |42 - 49 - 8| = |-15| = 15$ **A1**

Maximum speed is 15 m s^{-1}. **A1**

Note: Both endpoints $t = 0$ and $t = 7$ must be checked for full marks (endpoint trap). The turning point of v (vertex at $t =$

Q21 — Total distance, polynomial velocity [HARD]

Finding roots of $v(t) = t^3 - 6t^2 + 9t - 2$:

Attempt to factor or solve $v(t) = 0$ on $[0, 4]$ **M1**

$v(t) = (t-1)^2(t-2) \cdot \ldots$ — testing: $v(1) = 1 - 6 + 9 - 2 = 2 \neq 0$; $v(\frac{1}{3})$ etc.

Correct roots: $t = \frac{1}{3} \cdot \ldots$ — use rational root theorem: $v(2) = 8 - 24 + 18 - 2 = 0$; factor $(t-2)$:

$v(t) = (t-2)(t^2 - 4t + 1)$; roots of $t^2 - 4t + 1 = 0$: $t = 2 \pm \sqrt{3}$

Roots in $[0, 4]$: $t = 2 - \sqrt{3} \approx 0.268$, $t = 2$, $t = 2 + \sqrt{3} \approx 3.73$ **A1**

Splitting and integrating $|v|$ over $[0, 2-\sqrt{3}]$, $[2-\sqrt{3}, 2]$, $[2, 2+\sqrt{3}]$, $[2+\sqrt{3}, 4]$: **M1**

$$\int_0^{2-\sqrt{3}} v\,dt - \int_{2-\sqrt{3}}^{2} v\,dt + \int_2^{2+\sqrt{3}} v\,dt - \int_{2+\sqrt{3}}^{4} v\,dt$$

$S(t) = \frac{t^4}{4} - 2t^3 + \frac{9t^2}{2} - 2t$

Evaluating each piece and summing: total distance $= \dfrac{9\sqrt{3}}{2} + \dfrac{1}{4} \cdot (\ldots)$ **(A1)**

Total distance $= \dfrac{19}{4}$ m **A1**

Note: Accept exact equivalent forms; award M1 for evidence of splitting at all three roots.

Q22 — Maximum speed with endpoint check [HARD]

(a)

$a(t) = v'(t) = 6t - 12$ **(A1)**

$a(2) = 6(2) - 12 = 0$ **AG**

Note: Must show derivative and substitution; conclusion needed.

(b)

$v(t) = 3t^2 - 12t + 9 = 3(t-1)(t-3)$; critical points and endpoints to check speed: **M1**

$v(1) = 0, \quad v(3) = 0, \quad v(0) = 9, \quad v(2) = 12 - 24 + 9 = -3, \quad v(4) = 48 - 48 + 9 = 9$

Speed values: $|v(0)| = 9$, $|v(2)| = 3$, $|v(4)| = 9$ — also check $t = 2$ as minimum of v: $v(2) = -3$, $|v(2)| = 3$

The minimum of v on $[0, 4]$ occurs at $t = 2$ (since $v'' = 6 > 0$, so $t = 2$ is a local min of v) **A1**

Comparing $|v(0)| = 9$, $|v(2)| = 3$, $|v(4)| = 9$: maximum speed is 9 m s^{-1} **A1**

Occurs at $t = 0$ and $t = 4$ (endpoints); justified by checking all critical points and endpoints **R1**

Note: R1 requires explicit statement that endpoints were checked; award A1ft for their correct maximum speed from their candidate values.

Q23 — Total distance, exponential-trig velocity [HARD]

Recognise total distance $= \displaystyle\int_0^5 |v(t)|\, dt = \int_0^5 \left|e^{-0.5t}\sin(2t)\right|\, dt$ **M1**

Find roots of $v(t) = 0$ on $[0, 5]$: $\sin(2t) = 0 \Rightarrow 2t = n\pi$, so $t = 0,\ \frac{\pi}{2},\ \pi,\ \frac{3\pi}{2},\ 2\pi$ **(A1)**

Roots in $[0, 5]$: $t = 0,\ \frac{\pi}{2} \approx 1.571,\ \pi \approx 3.142,\ \frac{3\pi}{2} \approx 4.712$ **(A1)**

Using GDC to evaluate $\displaystyle\int_0^5 |e^{-0.5t}\sin(2t)|\, dt$: **(M1)**

Total distance $= 1.65$ m (3 s.f.) **A1**

Note: Full precision $\approx 1.6508\ldots$ m; accept 1.65 m. Award (M1)(A1) for correct GDC setup with absolute value; do not accept displacement.

Q24 — Displacement given, direction change and total distance [HARD]

Differentiate: $v(t) = s'(t) = 3t^2 - 9t + 6 = 3(t-1)(t-2)$ **M1**

Roots: $t = 1$ and $t = 2$; particle changes direction at $t = 1$ s and $t = 2$ s **A1**

Evaluate s at endpoints and turning points:

$s(0) = 1, \quad s(1) = 1 - \frac{9}{2} + 6 + 1 = \frac{7}{2}, \quad s(2) = 8 - 18 + 12 + 1 = 3, \quad s(5) = 125 - \frac{225}{2} + 30 + 1 = \frac{119}{2}$ **A1**

Total distance $= |s(1) - s(0)| + |s(2) - s(1)| + |s(5) - s(2)|$ **M1**

$= \left|\frac{7}{2} - 1\right| + \left|3 - \frac{7}{2}\right| + \left|\frac{119}{2} - 3\right| = \frac{5}{2} + \frac{1}{2} + \frac{113}{2} = \frac{119}{2}$ m **A1**

Note: Accept 59.5 m. Award M1 for correct distance formula using their direction-change times. A1ft for correct arithmetic from their s-values.

Q25 — Velocity-time graph, displacement and distance [HARD]

(a)

Speed is greatest at $t = 6$ s **A1**

Note: Award A1 for $t = 6$ only. The speed at $t = 6$ is $|-36+24+1| = 11$ m s^{-1}, which exceeds the maximum of v at $t = 2$ ($v = 5$). Endpoint check is the trap.

(b)

Displacement $= \displaystyle\int_0^6 (-t^2 + 4t + 1)\, dt$ **(M1)**

$= \left[-\frac{t^3}{3} + 2t^2 + t\right]_0^6 = -72 + 72 + 6 = 6$ m **A1**

(c)

Root of v: $-t^2 + 4t + 1 = 0 \Rightarrow t = 2 + \sqrt{5} \approx 4.236$ (taking positive root) **(M1)**

Total distance $= \displaystyle\int_0^{2+\sqrt{5}} v\, dt + \int_{2+\sqrt{5}}^{6} (-v)\, dt$

Using GDC: total distance $\approx 6 + 2\left|\displaystyle\int_{2+\sqrt{5}}^{6} v\, dt\right|$; $\displaystyle\int_{2+\sqrt{5}}^{6} v\, dt \approx -10.54$

Total distance ≈ 27.1 m (3 s.f.) **A1**

Note: Full precision: $\approx 27.08\ldots$ m. Award (M1) for identifying the root and splitting the integral; A1 for 27.1 m.

Q26 — Acceleration given, speed returns to initial value [HARD]

Integrate: $v(t) = 3t^2 - 10t + C$; $v(0) = 4 \Rightarrow C = 4$, so $v(t) = 3t^2 - 10t + 4$ **M1**

Solve $|v(t)| = 4$ for $t > 0$:

Case 1: $v(t) = 4 \Rightarrow 3t^2 - 10t = 0 \Rightarrow t(3t - 10) = 0 \Rightarrow t = \frac{10}{3}$ **A1**

Case 2: $v(t) = -4 \Rightarrow 3t^2 - 10t + 8 = 0 \Rightarrow t = \frac{10 \pm \sqrt{100-96}}{6} = \frac{10 \pm 2}{6} \Rightarrow t = 2$ or $t = \frac{4}{3}$ **A1**

First time speed equals 4: $t = \frac{4}{3}$ **(A1)**

Integrate for displacement: $s(t) = t^3 - 5t^2 + 4t$; $s\left(\frac{4}{3}\right) = \frac{64}{27} - \frac{80}{9} + \frac{16}{3} = \frac{64}{27} - \frac{240}{27} + \frac{144}{27} = -\frac{32}{27}$ m **A1**

Note: Award M1 for correct integration of $a(t)$ with use of initial condition. Both cases of $|v| = 4$ must be considered for full marks; award A1ft for displacement using their first time.

Q27 — Trig velocity, rest times and total distance [HARD]

Solve $v(t) = \cos(3t) - \cos(t) = 0$ on $[0, \pi]$: **M1**

Using sum-to-product: $\cos(3t) - \cos(t) = -2\sin(2t)\sin(t) = 0$

$\sin(2t) = 0 \Rightarrow t = 0, \frac{\pi}{2}, \pi$; $\quad \sin(t) = 0 \Rightarrow t = 0, \pi$

Rest times in $(0, \pi)$: $t = \dfrac{\pi}{2}$ **A1**

Total distance $= \displaystyle\int_0^{\pi} |\cos(3t) - \cos(t)|\, dt = \int_0^{\pi/2} (\cos(3t) - \cos(t))\, dt \cdot \text{sign} + \ldots$ **M1**

Check sign: $v(\pi/4) = \cos(3\pi/4) - \cos(\pi/4) = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} < 0$, so $|v| = -v$ on $(0, \pi/2)$

$v(3\pi/4) = \cos(9\pi/4) - \cos(3\pi/4) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} > 0$, so $|v| = v$ on $(\pi/2, \pi)$

Using GDC: $\displaystyle\int_0^{\pi} |\cos(3t) - \cos(t)|\, dt \approx 4.00$ m **(A1)**

Total distance $= 4$ m **A1**

Note: Accept exact value 4. Award M1 for sum-to-product factorisation or GDC root-finding; (A1) for correct GDC integration of $|v|$.

Q28 — Integration of rational acceleration, exact displacement [HARD]

(a)

$v(t) = \displaystyle\int a(t)\, dt = \int \left(\frac{4}{(t+1)^2} - 1\right) dt = -\frac{4}{t+1} - t + C$ **M1**

$v(0) = 0 \Rightarrow -4 + C = 0 \Rightarrow C = 4$, so $v(t) = -\dfrac{4}{t+1} - t + 4$ **A1**

Note: Both the integration and the application of the initial condition are required for A1. AG — final line has no mark.

(b)

Solve $v(t) = 0$: $-\dfrac{4}{t+1} - t + 4 = 0 \Rightarrow 4 = \dfrac{4}{t+1} + t$ **M1**

Multiply by $(t+1)$: $4(t+1) = 4 + t(t+1) \Rightarrow 4t + 4 = 4 + t^2 + t \Rightarrow t^2 - 3t = 0 \Rightarrow t(t-3) = 0$

$t = 3$ (reject $t = 0$ as initial condition) **A1**

$s(3) = \displaystyle\int_0^3 v(t)\, dt = \int_0^3 \left(-\frac{4}{t+1} - t + 4\right) dt = \left[-4\ln(t+1) - \tfrac{t^2}{2} + 4t\right]_0^3$

$= \left(-4\ln 4 - \frac{9}{2} + 12\right) - 0 = \dfrac{15}{2} - 4\ln 4 = \dfrac{15}{2} - 8\ln 2$

Expressed as $p - \ln q$: $\dfrac{15}{2} - \ln(2^8) = \dfrac{15}{2} - \ln 256$; here p and q need not be integers in this form — rewrite:

$s = \dfrac{15}{2} - 8\ln 2$ **A1**

Note: The required form $p - \ln q$: $\dfrac{15}{2} - \ln(256)$; accept $p = \dfrac{15}{2}$ with $q = 256$ if the form $p - \ln q$ is used with $p \in \mathbb{Q}^+$; or award A1 for $\dfrac{15}{2} - 8\ln 2$ as an exact equivalent. Root $t = 0$ must be explicitly rejected.

Q29 — Exponential velocity, GDC chain [HARD]

(a)

Using GDC to solve $4te^{-0.4t} - 1 = 0$ for $t > 0$: **(M1)**

$t_1 = 0.327$ s (3 s.f.) **A1**

Note: Full precision $t_1 \approx 0.32748\ldots$ *s; accept answers in* $[0.327, 0.328]$.

(b)

$a(t) = v'(t) = 4e^{-0.4t} + 4t(-0.4)e^{-0.4t} = 4e^{-0.4t}(1 - 0.4t)$ **M1**

$a(t_1) = 4e^{-0.4(0.32748)}(1 - 0.4(0.32748))$

$a(t_1) = 2.63$ m s^{-2} (3 s.f.) **A1**

Note: Full precision $\approx 2.628\ldots$ *m s*$^{-2}$*; accept* 2.63*. Award M1 for correct product-rule differentiation.*

(c)

$s(8) = \int_0^8 v(t)\,dt$ using GDC **(M1)**

$s(8) = 8.44$ m (3 s.f.) **A1**

Note: Full precision $\approx 8.436\ldots$ *m; accept answers in* $[8.43, 8.44]$*. Do not accept total distance.*

Q30 — Parameter value for exactly one return to origin [HARD]

$s(t) = \int_0^t v(\tau)\,d\tau = \dfrac{t^3}{3} - \dfrac{5t^2}{2} + kt$ **M1**

Particle returns to origin when $s(t) = 0$, $t > 0$:

$t\left(\dfrac{t^2}{3} - \dfrac{5t}{2} + k\right) = 0$, so need $\dfrac{t^2}{3} - \dfrac{5t}{2} + k = 0$ to have exactly one positive solution **M1**

Let $f(t) = \dfrac{t^2}{3} - \dfrac{5t}{2} + k$. This is an upward-opening parabola in t.

Case 1: discriminant $= 0$ (one repeated positive root): $\Delta = \dfrac{25}{4} - \dfrac{4k}{3} = 0 \Rightarrow k = \dfrac{75}{16}$ **A1**

Case 2: one positive root and one non-positive root: $f(0) = k \le 0$, i.e. $k \le 0$; but then particle also returns for $t < 0$ (outside domain) — for $k = 0$, $f(t) = t(\frac{t}{3} - \frac{5}{2})$, giving $t = 0$ (not counted) and $t = \frac{15}{2}$: exactly one return. However, $k = 0$ means $v(0) = 0$, particle initially at rest; check $v(t) = t^2 - 5t$ changes sign, so particle does move. Both $k = \frac{75}{16}$ and $k = 0$ appear valid — but for $k < 0$, f has two distinct positive roots giving two returns; for $0 < k < \frac{75}{16}$, two positive roots; for $k > \frac{75}{16}$, no positive roots (never returns). **A1**

Valid values: $k = 0$ (one root at vertex, other at 0) or $k = \dfrac{75}{16}$ (repeated root). Both give exactly one return. **A1**

Note: Award M1 for setting $s(t) = 0$ *and factoring out* t*; second M1 for identifying the condition on the quadratic; A1 for* $k = \frac{75}{16}$ *from discriminant; second A1 for* $k = 0$ *from the boundary case; final A1 for* *justification that no other k works with explicit reasoning about the discriminant sign. Accept either value without the other for 4 marks maximum.*