# IB Math AI SL — Topic 1

## Sequences & Series

Sigma Notation · Arithmetic & Geometric Sequences · Applications

| Difficulty | EASY | MEDIUM | HARD |
|---|---|---|---|
| **Questions** | 10 | 10 | 10 |
| **Total Marks** | 30 | 40 | 50 |

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Name:_____ Date:_____

Score:_____/120

**Instructions:** Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. A graphic display calculator is required throughout; support GDC answers with the values entered and the feature used.

### Key Formulae

- Arithmetic sequence — general term: $u_n = u_1 + (n-1)d$
- Arithmetic series — sum of first $n$ terms: $S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$
- Geometric sequence — general term: $u_n = u_1 r^{n-1}$
- Geometric series — sum of first $n$ terms: $S_n = \frac{u_1(r^n - 1)}{r - 1}, \quad r \neq 1$
- Infinite geometric series (requires $|r| < 1$): $S_\infty = \frac{u_1}{1 - r}$
- Sigma notation: $\sum_{k=1}^{n} u_k = u_1 + u_2 + \cdots + u_n$
- Common ratio from consecutive terms: $r = \frac{u_{n+1}}{u_n}$

## GDC Tips

- **Evaluate a sigma sum:** Menu → 4: Calculus → 4: Sum (or use sum(seq(formula, variable, start, end)) in the calculator entry line); confirm the result matches term-by-term expansion for small cases.
- **Generate sequence terms / build a table:** use seq($u_1$ + (n-1)d, n, 1, 10) to list terms quickly and read off any named term.
- **Threshold search (first $n$ exceeding a target):** Enter the term formula or sum formula in the Graphs app as $f(x)$; use Menu → 6: Analyze Graph → Intersection with the horizontal target line, then verify the integer brackets $k$ and $k+1$ by substitution.
- **Solve for an unknown parameter** (e.g. find $d$ or $r$): Use Menu → 3: Algebra → 1: Solve; type the equation exactly (e.g. solve(u1*r^(n-1)=value, r)) and check the sign of $r$ against context.
- **Compare two running totals:** Store each partial-sum formula as a function in Lists & Spreadsheet; fill column A with $n = 1, 2, \ldots$ and columns B, C with the two formulas; scroll to find the crossover row.
- **Sanity-check $|r|$:** After finding $r$ for a decay context, immediately evaluate abs(r) on the home screen and confirm it is less than 1 before applying the infinite-sum formula.

## Common Mistakes to Avoid

1. **Off-by-one exponent in the geometric term.** Students write $u_n = u_1 r^n$ instead of $u_1 r^{n-1}$. This shifts every term by one place and gives the wrong answer for any named-term or threshold question. Always check: $n = 1$ must return $u_1$.
2. **Confusing the term $u_n$ with the sum $S_n$.** After finding "the 10th value" with the sum formula, or vice versa. Write both formulas at the top of your working, label every expression as either $u_n$ or $S_n$, and never mix them.
3. **Threshold integer not justified by bracketing.** Stating "$n = 12$" without evaluating both $n = 11$ and $n = 12$ to show one side is below and the other is above the target. The mark scheme requires both bracketing values; the bare integer earns nothing.
4. **Wrong tail in the common-ratio sign for decay.** Decay contexts (e.g. resale value dropping by 15% per year) give $r = 0.85$, not $r = -0.15$ and not $r = 1.15$. Write the multiplier explicitly: "each year the value is multiplied by $1 - 0.15 = 0.85$".
5. **Premature rounding in chained parts.** Rounding $r$ or $u_1$ to 3 s.f. after part (a) and carrying that value into part (b) can change the final answer by more than the tolerance allows. Store the full unrounded value in GDC memory and only round the final answer of each part.
6. **Applying the infinite-sum formula when $|r| \geq 1$.** If a question asks for $S_\infty$ but the computed ratio satisfies $|r| \geq 1$, the series diverges and the formula is invalid. Always state "$|r| = \ldots < 1$, so the infinite sum exists" before using $S_\infty = \dfrac{u_1}{1-r}$.

## Section A — Easy EASY (10 questions × 3 marks = 30 marks)

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**1.** EASY [3 marks]

A runner trains each week. In week 1 she runs 8 km. Each week she runs 2 km more than the previous week.

(a) Write down the distance she runs in week 4. [1]

(b) Find the total distance she runs over the first 10 weeks. [2]

**2.** EASY [3 marks]

The table shows the first four terms of a sequence of weekly savings, in dollars.

| week, $n$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| savings ($) | 5 | 10 | 20 | 40 |

(a) Write down the common ratio of this sequence. [1]

(b) Find the savings in week 8. [2]

**3.** EASY [3 marks]

An arithmetic sequence has first term $u_1 = 7$ and common difference $d = 4$.

(a) Write down an expression for the $n$th term, $u_n$. [1]

(b) Find the value of $u_{15}$. [2]

**4.** EASY [3 marks]

Evaluate $\sum_{k=1}^{6}(3k+1)$.

**5.** EASY [3 marks]

A geometric sequence has first term $u_1 = 500$ and common ratio $r = 0.8$.

(a) Write down $u_2$. [1]

(b) Find the sum of the first 5 terms. [2]

**6.** **EASY** **[3 marks]**

The $n$th term of a sequence is given by $u_n = 5n - 3$.

(a) Write down the first three terms of the sequence. [1]

(b) Find the value of $n$ such that $u_n = 47$. [2]

**7.** **EASY** **[3 marks]**

A factory produces 120 items on its first day of operation. Each day it produces 15 more items than the previous day.

(a) Find the number of items produced on day 7. [2]

(b) Write down the type of sequence described. [1]

**8.** **EASY** **[3 marks]**

The table shows the first four terms of an arithmetic sequence.

| term number, $n$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| term value | 3 | 7 | 11 | 15 |

Find the sum of the first 12 terms of the sequence.

**9.** **EASY** **[3 marks]**

Write the sum $11 + 14 + 17 + 20 + 23$ using sigma notation.

**10.** **EASY** **[3 marks]**

A car is purchased for \$24 000. Its value decreases by 15% each year.

(a) Write down the common ratio of the sequence of yearly values. [1]

(b) Find the value of the car after 4 years. [2]

## Section B — Medium MEDIUM (10 questions × 4 marks = 40 marks)

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**11.** MEDIUM [4 marks]

A training plan records the distance a runner covers each week. The table shows the first four weeks.

| week, $n$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| distance (km) | 8 | 11 | 14 | 17 |

(a) Write down the common difference of this arithmetic sequence. [1]

(b) Find the week in which the runner first covers more than 50 km. [3]

**12.** MEDIUM [4 marks]

A geometric sequence has first term $u_1 = 5$ and common ratio $r = 1.6$. Find the sum of the first 10 terms, giving your answer correct to the nearest integer.

**13.** MEDIUM **[4 marks]**

The $n$th term of a sequence is given by $u_n = 120 - 7n$.

(a) Write down $u_1$ and $u_2$. [1]

(b) Find the value of $n$ for which $u_n = -100$. [2]

(c) Hence write down the number of positive terms in the sequence. [1]

**14.** MEDIUM **[4 marks]**

A new car is purchased for \$24 000. Its value decreases by 18% each year.

(a) Write down the common ratio of this geometric sequence. [1]

(b) Find the value of the car after 6 full years, giving your answer correct to the nearest dollar. [2]

(c) Determine the first year in which the value of the car falls below \$5 000. Justify your answer. [1]

**15.** MEDIUM **[4 marks]**

The sum of the first $n$ terms of an arithmetic series is given by $S_n = 4n^2 - n$.

(a) Find $S_5$ and $S_4$. [1]

(b) Hence find the fifth term, $u_5$. [1]

(c) Find the first term and common difference of the series. [2]

**16.** MEDIUM **[4 marks]**

Evaluate $\sum_{k=3}^{12}(5k-2)$, showing all necessary working.

**17.** MEDIUM **[4 marks]**

A company's monthly profits (in thousands of dollars) form an arithmetic sequence. The profit in month 3 is \$19 000 and the profit in month 8 is \$34 000.

(a) Find the common difference and the first term of the sequence. [2]

(b) Find the total profit over the first 12 months, giving your answer in dollars. [2]

**18.** MEDIUM **[4 marks]**

A ball is dropped from a height of 3 m. After each bounce it rises to 65% of the height of the previous bounce. The table shows the first four bounce heights.

| bounce, $n$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| height (m) | 1.95 | 1.2675 | 0.8239 | 0.5355 |

(a) Write down the common ratio. [1]

(b) Find the first bounce for which the height is less than 0.1 m. Justify your answer. [3]

**19.** MEDIUM **[4 marks]**

An arithmetic sequence has $u_5 = 23$ and $u_{12} = 58$.

(a) Find the common difference $d$ and the first term $u_1$. [2]

(b) Write an expression for the sum of all terms from $u_6$ to $u_{20}$ inclusive, and evaluate it. [2]

**20.** MEDIUM **[4 marks]**

A student is offered two salary schemes for a 10-year career.
Scheme A: Starting salary of \$28 000 per year, increasing by \$1 400 each year.
Scheme B: Starting salary of \$26 000 per year, increasing by 5% each year.
Find the total earnings under each scheme over 10 years and determine which scheme gives the higher total. Justify your answer.

## Section C — Hard HARD (10 questions × 5 marks = 50 marks)

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**21.** HARD **[5 marks]**

A company manufactures solar panels. In the first month of production, the company produces 120 panels. Each subsequent month, the company produces 15 more panels than the previous month.

(a) Write down the number of panels produced in the fifth month. [1]

(b) Find the first month in which the monthly production exceeds 500 panels. [2]

(c) The company needs to produce a total of at least 10 000 panels to fulfil a contract. Find the minimum number of complete months required. [2]

**22.** HARD **[5 marks]**

The first three terms of a geometric sequence are $u_1 = 4$, $u_2 = p$, and $u_3 = p + 6$, where $p > 0$.

(a) Show that $p^2 - 4p - 24 = 0$. [2]

(b) Hence find the value of $p$, giving your answer to 3 significant figures. [1]

(c) Find the common ratio of the sequence. [1]

(d) Find the sum of the first 12 terms of the sequence. [1]

**23.** HARD **[5 marks]**

A researcher records the depth, $d$ cm, of a layer of sediment deposited in a lake each year. The depths form a geometric sequence. The table shows the depth deposited in years 1 to 4.

| Year, $n$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Depth, $d$ (cm) | 8.00 | 7.20 | 6.48 | 5.83 |

(a) Write down the common ratio of the sequence. [1]

(b) Find the total depth of sediment deposited during the first 20 years. [2]

(c) Determine whether the total depth of sediment deposited will ever exceed 80 cm. Justify your answer. [2]

**24.** HARD **[5 marks]**

Mia is offered two salary schemes when starting a new job. Both schemes begin at the start of Year 1.
Scheme A: A starting salary of \$32 000 in Year 1, increasing by \$1 400 each year.
Scheme B: A starting salary of \$30 000 in Year 1, increasing by 5% each year.

(a) Find Mia's salary in Year 10 under each scheme. [2]

(b) Find the total earnings over 15 years under each scheme. [2]

(c) Determine which scheme gives Mia greater total earnings over 15 years. Justify your answer. [1]

**25.** HARD **[5 marks]**

An arithmetic sequence has first term $u_1$ and common difference $d$. The third term of the sequence is 31 and the sum of the first eight terms is 220.

(a) Write down two equations in $u_1$ and $d$. [1]

(b) Find the values of $u_1$ and $d$. [2]

(c) Find the value of $n$ such that $\sum_{k=1}^{n} u_k = 0$. [2]

**26.** HARD **[5 marks]**

The sum of the first $n$ terms of an arithmetic series is given by $S_n = 4n^2 - 3n$.

(a) Show that the first term is 1 and the common difference is 8. [2]

(b) Find the value of $\sum_{k=6}^{20} u_k$. [2]

(c) Find the first term of the sequence that exceeds 1000. State clearly the term number. [1]

**27.** HARD **[5 marks]**

A geometric sequence has positive terms. The sum of the first two terms is 16 and the sum of the first four terms is 160.

(a) Show that the common ratio $r = 3$, and find the first term $u_1$. [3]

(b) Find the first term to exceed 1 000. State clearly the term number. [2]

**28.** HARD **[5 marks]**

A training programme requires an athlete to run a total of at least 500 km over a series of weeks. In week 1 the athlete runs 8 km. Each week the athlete runs 3 km more than the previous week.

(a) Write an expression for the total distance run after $n$ weeks. [1]

(b) Find the minimum number of complete weeks needed to reach 500 km in total. [2]

(c) In a different programme, the athlete runs 8 km in week 1 and increases the distance by 8% each week. Find the minimum number of complete weeks needed to exceed 500 km in total under this new programme. [2]

**29.** **HARD** **[5 marks]**

The $n$th term of a sequence is given by $u_n = 3(2)^{n-1} - 4n + 2$.

(a) Write down the values of $u_1$, $u_2$, and $u_3$. [1]

(b) Find $\sum_{n=1}^{10} u_n$. [2]

(c) Write $\sum_{n=1}^{10} 3(2)^{n-1}$ in the form $\frac{a(r^{10}-1)}{r-1}$ and evaluate it. [2]

**30.** **HARD** **[5 marks]**

A company's annual profit forms a geometric sequence. The profit in Year 2 is \$56 000 and the profit in Year 5 is \$189 000. All values are rounded to the nearest dollar.

| Year, $n$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Profit (\$) | $u_1$ | 56 000 | $u_3$ | $u_4$ |

(a) Find the common ratio $r$ and the first-year profit $u_1$, giving your answers to 3 significant figures. [2]

(b) Find the total profit over the first 10 years. [1]

(c) Find the first year in which the annual profit exceeds \$1 000 000. [2]

## Worked Solutions

### Q1 — Arithmetic Sequence Weekly Distance EASY

The sequence is arithmetic with $u_1 = 8$ and $d = 2$.

(a) The 4th term is found by adding the common difference three times from the first term: **A1**

$u_4 = 8 + 3(2) = 14$ km

(b) Apply the arithmetic series formula $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ with $n = 10$, $u_1 = 8$, $d = 2$: **M1**

$S_{10} = \frac{10}{2}(2(8) + 9(2)) = 5(16 + 18) = 5 \times 34 = 170$ km **A1**

*Warning: The formula uses $(n-1)d$, not $nd$; using $n = 10$ in place of $(n-1) = 9$ gives 180 km, which is incorrect.*

### Q2 — Geometric Sequence Weekly Savings EASY

(a) The common ratio is found by dividing any term by the previous term: $r = \frac{10}{5} = 2$. **A1**

(b) Use the geometric term formula $u_n = u_1 \cdot r^{n-1}$ with $u_1 = 5$, $r = 2$, $n = 8$: **M1**

$u_8 = 5 \times 2^{8-1} = 5 \times 2^7 = 5 \times 128 = \$640$ **A1**

*Warning: The exponent is $n - 1 = 7$, not $n = 8$; using $2^8$ gives \$1280, which is incorrect.*

### Q3 — Arithmetic Sequence General and Fifteenth Term EASY

(a) The general term of an arithmetic sequence is $u_n = u_1 + (n-1)d$. Substituting $u_1 = 7$ and $d = 4$: **A1**

$u_n = 7 + (n-1)(4) = 4n + 3$

(b) Substitute $n = 15$ into the expression found in part (a): **M1**

$u_{15} = 4(15) + 3 = 60 + 3 = 63$ **A1**

*Warning: Using $u_n = u_1 + nd$ (with $n$ instead of $n - 1$) gives $u_{15} = 67$, which is incorrect.*

### Q4 — Sigma Notation Evaluation EASY

List the six terms by substituting $k = 1, 2, 3, 4, 5, 6$ into $3k + 1$, then add them: **M1**

$k = 1 : 4,\ k = 2 : 7,\ k = 3 : 10,\ k = 4 : 13,\ k = 5 : 16,\ k = 6 : 19$

$\sum_{k=1}^{6}(3k + 1) = 4 + 7 + 10 + 13 + 16 + 19 = 69$ **A2**

*Warning: Starting from $k = 0$ instead of $k = 1$ adds an extra term of 1, giving an incorrect total of 70.*

### Q5 — Geometric Sequence Second Term and Sum EASY

(a) Multiply the first term by the common ratio: $u_2 = 500 \times 0.8 = 400$. **A1**

(b) Apply the geometric series formula $S_n = \dfrac{u_1(1-r^n)}{1-r}$ with $u_1 = 500$, $r = 0.8$, $n = 5$: **M1**

$$S_5 = \frac{500(1-0.8^5)}{1-0.8} = \frac{500 \times 0.67232}{0.2} = 1680.8$$ **A1**

*Warning: Using the formula for an increasing series ($r > 1$ version) with denominator $r - 1$ gives a negative result; ensure the formula matches a decreasing sequence where $|r| < 1$.*

### Q6 — Linear Term Formula Solve for n EASY

(a) Substitute $n = 1, 2, 3$ into $u_n = 5n - 3$: **A1**
$u_1 = 2,\ u_2 = 7,\ u_3 = 12$

(b) Set $u_n = 47$ and solve for $n$: **M1**
$5n - 3 = 47 \Rightarrow 5n = 50 \Rightarrow n = 10$ **A1**

*Warning: This is a straightforward linear equation; ensure both sides are correctly rearranged before dividing.*

### Q7 — Arithmetic Sequence Factory Production EASY

(a) Use $u_n = u_1 + (n-1)d$ with $u_1 = 120$, $d = 15$, $n = 7$: **M1**
$u_7 = 120 + 6(15) = 120 + 90 = 210$ items **A1**

(b) Because the output increases by a fixed amount (15 items) each day, the sequence is arithmetic. **A1**

*Warning: A sequence that increases by a fixed number each term is arithmetic, not geometric; geometric sequences increase by a fixed multiplier.*

### Q8 — Arithmetic Series Sum from Table EASY

Read from the table: $u_1 = 3$ and $d = 7 - 3 = 4$. Apply $S_n = \dfrac{n}{2}(2u_1 + (n-1)d)$ with $n = 12$: **M1**

$$S_{12} = \frac{12}{2}(2(3) + 11(4)) = 6(6 + 44) = 6 \times 50 = 300$$ **A2**

*Warning: Use $(n-1)d = 11 \times 4$, not $12 \times 4$; the latter gives $S_{12} = 312$, which is incorrect.*

### Q9 — Write a Sum in Sigma Notation EASY

Identify the pattern: the terms are 11, 14, 17, 20, 23, which increase by 3. The $k$th term is $3k + 8$ (check: $k = 1$ gives 11, $k = 5$ gives 23). **M1**
Verify the limits: the sum starts at $k = 1$ and ends at $k = 5$. **A1**

Write in sigma notation: $\sum_{k=1}^{5}(3k+8)$ **A1**

*Warning: Check that the general term reproduces both the first term ($k=1$) and the last term ($k=5$) correctly before writing the final answer.*

## Q10 — Geometric Decay Car Value EASY

(a) A decrease of 15% each year means 85% of the value is retained, so the common ratio is $r = 1-0.15 = 0.85$. **A1**

(b) Use $u_n = u_1 \cdot r^{n-1}$ with $u_1 = 24000$, $r = 0.85$. After 4 years the value is $u_1 \times r^4$: **M1**
Value $= 24000 \times 0.85^4 = 24000 \times 0.52200625\ldots = 12528.15\ldots \approx \$12\,528.15$ **A1**

*Warning: After 4 years the multiplier is $r^4$ (four reductions have occurred); using $r^3$ gives \$14 739, which corresponds to only 3 years.*

## Q11 — Arithmetic Sequence Threshold Search MEDIUM

(a) The sequence increases by a fixed amount each week, so we subtract consecutive terms to find the common difference: **A1**
$d = 11 - 8 = 3$ km

(b) The general term is $u_n = u_1 + (n-1)d = 8 + (n-1)(3) = 3n + 5$. **M1**
We require $3n + 5 > 50$, so $3n > 45$, giving $n > 15$, i.e. $n = 16$.
Bracketing check: $u_{15} = 3(15) + 5 = 50$ (not more than 50); $u_{16} = 3(16) + 5 = 53 > 50$. **A1**
The runner first covers more than 50 km in week 16. **A1**

*Warning: "more than 50" is a strict inequality; $u_{15} = 50$ exactly does not satisfy it. The bracketing check is essential to confirm the integer answer.*

## Q12 — Geometric Series Sum of Ten Terms MEDIUM

The sum of the first $n$ terms of a geometric series is $S_n = \dfrac{u_1(r^n - 1)}{r - 1}$. Here $u_1 = 5$, $r = 1.6$, $n = 10$: **M1**
GDC: Home screen, type $5 \times (1.6^{10} - 1) \div (1.6 - 1)$ and evaluate.
Unrounded value: $S_{10} = \dfrac{5(1.6^{10} - 1)}{0.6} = \dfrac{5 \times 108.9263\ldots}{0.6} = 907.9264\ldots$ **A1**
$S_{10} \approx 908$ **A1**

*Warning: The exponent in the geometric term formula is $n-1$ for a single term, but for the sum formula the exponent is $n$. Confusing the two formulas costs marks.*

## Q13 — Term Formula Reversal and Positive Count MEDIUM

(a) Substituting directly: **A1**

$u_1 = 120 - 7(1) = 113, \quad u_2 = 120 - 7(2) = 106$

(b) Setting $u_n = -100$: **M1**

$120 - 7n = -100 \Rightarrow 7n = 220 \Rightarrow n = \dfrac{220}{7} = 31.428\ldots$

This is not an integer: $n = 31$ gives $u_{31} = 120 - 217 = -97$ and $n = 32$ gives $u_{32} = -104$. There is no term equal to $-100$. **A1**

(c) For $u_n > 0$: $120 - 7n > 0 \Rightarrow n < 17.14\ldots$, so the largest positive term is $u_{17} = 120 - 119 = 1$. There are 17 positive terms. **A1**

*Warning: Part (c) uses "hence" linking to the structure found in (b); check $u_{17} = 1 > 0$ and $u_{18} = -6 < 0$ to confirm.*

## Q14 — Geometric Depreciation Threshold Search MEDIUM

(a) A decrease of 18% each year means the value is multiplied by $1 - 0.18 = 0.82$ each year: **A1**

$r = 0.82$

(b) The value after $n$ full years is $u_1 \times 0.82^n$. After 6 full years: **M1**

$V = 24000 \times 0.82^6 = 24000 \times 0.304006\ldots = 7296.15\ldots \approx \$7\,296$ **A1**

(c) We need the first $n$ such that $24000 \times 0.82^n < 5000$.

GDC: table of $V_n = 24000 \times 0.82^n$ for increasing $n$: $V_7 = 24000 \times 0.82^7 = 5982.85\ldots \geq 5000$; $V_8 = 24000 \times 0.82^8 = 4905.94\ldots < 5000$. **R1**

Since $V_7 \geq \$5\,000$ and $V_8 < \$5\,000$, the value first falls below \$5 000 after 8 full years (i.e. in year 8).

*Warning: A GDC table must be checked at consecutive integers either side of the threshold; a single evaluation is not sufficient justification.*

## Q15 — Recover Terms from Sum Formula MEDIUM

(a) Substituting directly into $S_n = 4n^2 - n$: **A1**

$S_5 = 4(25) - 5 = 95, \quad S_4 = 4(16) - 4 = 60$

(b) The fifth term equals the difference between consecutive partial sums: **A1**

$u_5 = S_5 - S_4 = 95 - 60 = 35$

(c) The first term is $u_1 = S_1 = 4(1)^2 - 1 = 3$.

$u_2 = S_2 - S_1 = (4 \cdot 4 - 2) - 3 = 14 - 3 = 11.$ **M1**

$d = u_2 - u_1 = 11 - 3 = 8$

$u_1 = 3, d = 8.$ **A1**

*Warning: $u_n \neq S_n$; the individual term is recovered via $u_n = S_n - S_{n-1}$, not by substituting $n$ directly into the sum formula.*

## Q16 — Sigma Notation Arithmetic Series MEDIUM

The sum $\sum_{k=3}^{12}(5k-2)$ runs from $k=3$ to $k=12$, which is $12-3+1=10$ terms. **M1**

The first term of the sum is $a=5(3)-2=13$ and the last term is $l=5(12)-2=58$.

Using the arithmetic series sum formula $S=\frac{n}{2}(a+l)$: **M1**

$S=\frac{10}{2}(13+58)=5\times 71=355$ **A1**

Verification by GDC: List/spreadsheet, generate $5k-2$ for $k=3,\ldots,12$ and sum: $\sum_{k=3}^{12}(5k-2)=355$. **A1**

*Warning: The number of terms is $12-3+1=10$, not $12-3=9$. An off-by-one error here costs both method and accuracy marks.*

## Q17 — Two Condition Arithmetic Sequence and Total MEDIUM

(a) The general term is $u_n=u_1+(n-1)d$. Setting up two equations using the given information (values in thousands): **M1**

$u_3=u_1+2d=19\ \ldots(1)$

$u_8=u_1+7d=34\ \ldots(2)$

Subtracting (1) from (2): $5d=15\Rightarrow d=3$. Substituting back: $u_1=19-2(3)=13$.

$d=3$ (thousand dollars per month), $u_1=13$ (thousand dollars). **A1**

(b) The total profit over 12 months uses $S_n=\frac{n}{2}(2u_1+(n-1)d)$: **M1**

$S_{12}=\frac{12}{2}(2(13)+11(3))=6(26+33)=6\times 59=354$ thousand dollars

Converting to dollars: $354\times 1000=\$354\,000$. **A1**

*Warning: The question specifies the answer in dollars, not thousands. Leaving the answer as 354 (without converting) loses the final accuracy mark.*

## Q18 — Geometric Bounce Height Threshold MEDIUM

(a) Each bounce height is 65% of the previous one, so: **A1**

$r=0.65$

(b) The $n$th bounce height is $u_n=1.95\times 0.65^{n-1}$ (since $u_1=3\times 0.65=1.95$). We require $u_n<0.1$: **M1**

GDC: table of $u_n=1.95\times 0.65^{n-1}$ for increasing $n$.

Bracketing check: $u_7=1.95\times 0.65^6=1.95\times 0.075420\ldots=0.1471\ldots\geq 0.1$; $u_8=1.95\times 0.65^7=1.95\times 0.049023\ldots=0.0956\ldots<0.1$. **A1**

Since $u_7\geq 0.1$ and $u_8<0.1$, the first bounce with height less than 0.1 m is bounce 8. **A1**

*Warning: The exponent is $n-1$, not $n$; using $0.65^n$ instead of $0.65^{n-1}$ shifts all values by one bounce and gives the wrong answer. Also double-check every power on the GDC individually — a single mis-keyed*

*exponent (e.g. $0.65^7$ read as 0.0824 instead of 0.0490) silently shifts the whole bracketing search.*

## Q19 — Two Condition Arithmetic Sequence and Partial Sum MEDIUM

(a) Setting up two equations using $u_n = u_1 + (n-1)d$: **M1**

$u_5 = u_1 + 4d = 23 \ \ldots(1)$

$u_{12} = u_1 + 11d = 58 \ \ldots(2)$

Subtracting (1) from (2): $7d = 35 \Rightarrow d = 5$. Substituting: $u_1 = 23 - 4(5) = 3$.

$u_1 = 3, d = 5$. **A1**

(b) The sum from $u_6$ to $u_{20}$ is $S_{20} - S_5$, where $S_n = \frac{n}{2}(2u_1 + (n-1)d)$: **M1**

$$S_{20} = \frac{20}{2}(2(3) + 19(5)) = 10(6 + 95) = 10 \times 101 = 1010$$

$$S_5 = \frac{5}{2}(2(3) + 4(5)) = \frac{5}{2}(6 + 20) = \frac{5}{2}(26) = 65$$

$S_{20} - S_5 = 1010 - 65 = 945$ **A1**

*Warning: The sum from $u_6$ to $u_{20}$ is $S_{20} - S_5$ (i.e. subtract $S_5$, not $S_6$), since $S_5$ is the sum of the first 5 terms which must be excluded.*

## Q20 — Arithmetic versus Geometric Salary Comparison MEDIUM

Scheme A is arithmetic with $u_1 = 28000$, $d = 1400$, $n = 10$: **M1**

$$S_A = \frac{10}{2}(2(28000) + 9(1400)) = 5(56000 + 12600) = 5 \times 68600$$

$S_A = \$343\,000$

Scheme B is geometric with $u_1 = 26000$, $r = 1.05$, $n = 10$: **M1**

$$S_B = \frac{26000(1.05^{10} - 1)}{1.05 - 1}$$

GDC: Home screen, type $26000 \times (1.05^{10} - 1) \div 0.05$.

Unrounded: $S_B = 26000 \times \frac{0.628966\ldots}{0.05} = 26000 \times 12.57931\ldots = 327\,025.21\ldots$

$S_B \approx \$327\,025$ **A1**

Since $S_A = \$343\,000 > S_B \approx \$327\,025$, Scheme A gives the higher total earnings over 10 years. **R1**

Recommendation: Scheme A. (dependent A1 included in R1)

*Warning: The percentage increase in Scheme B means $r = 1.05$, not $r = 0.05$. Using $r = 0.05$ gives a nonsensical negative sum and costs both method and accuracy marks.*

## Q21 — Arithmetic Threshold and Sum Reversal HARD

(a) The sequence is arithmetic with $u_1 = 120$ and $d = 15$, so the fifth term is found using $u_n = u_1 + (n-1)d$: $u_5 = 120 + 4(15) = 180$ panels. **A1**

(b) Setting $u_n = 120 + (n-1)(15) > 500$ and solving for $n$: $(n-1) > \dfrac{380}{15} = 25.3\ldots$, so $n > 26.3\ldots$ **M1**

Check: $u_{27} = 120 + 26(15) = 510 > 500$ and $u_{26} = 120 + 25(15) = 495 \leq 500$. The first month is month 27. **A1**

(c) The sum of an arithmetic series is $S_n = \dfrac{n}{2}(2u_1 + (n-1)d) = \dfrac{n}{2}(240 + 15(n-1))$. Setting $S_n \geq 10\,000$: **M1**

GDC: table of values. $S_{29} = \dfrac{29}{2}(240 + 420) = 14.5 \times 660 = 9\,570 < 10\,000$; $S_{30} = \dfrac{30}{2}(240 + 435) = 15 \times 675 = 10\,125 \geq 10\,000$. The minimum is $n = 30$ months. **A1**

*Warning: "exceeds" and "at least" are strict and non-strict respectively; always confirm which inequality the wording demands before rounding.*

## Q22 — Geometric Sequence Show That and Parameter Chain HARD

(a) For a geometric sequence the ratio of consecutive terms is constant, so $\dfrac{u_2}{u_1} = \dfrac{u_3}{u_2}$, giving $(u_2)^2 = u_1 \cdot u_3$: **M1**

$$p^2 = 4(p+6)$$
$$p^2 = 4p + 24$$
$$p^2 - 4p - 24 = 0$$ **A1 AG**

*Warning: The AG firewall means full algebraic justification is required; writing only the final equation earns nothing.*

(b) GDC: Equation solver or quadratic formula with $a = 1$, $b = -4$, $c = -24$. The two roots are $p = \dfrac{4 \pm \sqrt{16 + 96}}{2} = \dfrac{4 \pm \sqrt{112}}{2}$; since $p > 0$: $p = \dfrac{4 + 10.583\ldots}{2} = 7.29$ (3 s.f.). **A1**

(c) Common ratio $r = \dfrac{u_2}{u_1} = \dfrac{p}{4} = \dfrac{7.2915\ldots}{4} = 1.82$ (3 s.f.). **A1**

*Warning: Use the unrounded value of $p$ in all subsequent calculations to avoid premature-rounding errors.*

(d) GDC: $S_{12} = \dfrac{u_1(r^{12} - 1)}{r - 1}$ with unrounded values. $S_{12} = \dfrac{4 \times (1.822876\ldots^{12} - 1)}{1.822876\ldots - 1} = 6\,540$ (3 s.f.). **A1**

## Q23 — Geometric Sediment Depth and Sum to Infinity HARD

(a) The common ratio is $r = \dfrac{7.20}{8.00} = 0.9$. **A1**

*Warning: Confirm $|r| < 1$ before concluding the series has a finite sum-to-infinity.*

(b) The total depth after 20 years is $S_{20} = \dfrac{8(1 - 0.9^{20})}{1 - 0.9}$. **M1**

GDC: Evaluate $S_{20} = \dfrac{8(1 - 0.121577\ldots)}{0.1} = \dfrac{8 \times 0.878423\ldots}{0.1} = 70.27\ldots \approx 70.3$ cm (3 s.f.). **A1**

(c) Since $|r| = 0.9 < 1$, the sum to infinity exists: $S_\infty = \dfrac{u_1}{1-r} = \dfrac{8}{1-0.9} = \dfrac{8}{0.1} = 80$ cm. **M1**

$S_\infty = 80$ cm; the total depth approaches 80 cm but never reaches it, so the total depth never exceeds 80 cm. **R1**

*Warning: The sum-to-infinity equals exactly 80, which means the depth approaches but does not exceed 80; the conclusion must say "never exceeds", not "equals".*

## Q24 — Salary Comparison Justified Recommendation HARD

(a) Scheme A is arithmetic with $u_1 = 32\,000$, $d = 1\,400$: Year 10 salary $= 32\,000 + 9(1\,400) = 32\,000 + 12\,600 = \$44\,600$. **A1**

Scheme B is geometric with $u_1 = 30\,000$, $r = 1.05$: Year 10 salary $= 30\,000 \times 1.05^9 = 30\,000 \times 1.5513\ldots = \$46\,539\ldots \approx \$46\,500$ (3 s.f.). **A1**

*Warning: For the geometric sequence, the Year 10 term uses $r^{n-1} = r^9$, not $r^{10}$.*

(b) Scheme A total: $S_{15} = \dfrac{15}{2}(2\times 32\,000 + 14\times 1\,400) = 7.5(64\,000 + 19\,600) = 7.5\times 83\,600 = \$627\,000$. **M1**

Scheme B total: $S_{15} = \dfrac{30\,000(1.05^{15}-1)}{1.05-1} = \dfrac{30\,000 \times 1.07893\ldots}{0.05} = \$647\,356\ldots \approx \$647\,000$ (3 s.f.). **A1**

(c) $647\,000 > 627\,000$, so Scheme B gives greater total earnings over 15 years; Mia should choose Scheme B. **R1**

## Q25 — Two Condition Arithmetic Sequence and Zero Sum HARD

(a) Using $u_n = u_1 + (n-1)d$: equation 1: $u_1 + 2d = 31$. Using $S_8 = \dfrac{8}{2}(2u_1 + 7d) = 220$: equation 2: $2u_1 + 7d = 55$. **A1**

(b) Solving simultaneously: from equation 1, $u_1 = 31 - 2d$. Substituting: $2(31-2d) + 7d = 55 \Rightarrow 62 - 4d + 7d = 55 \Rightarrow 3d = -7 \Rightarrow d = -\dfrac{7}{3}$. **M1**

GDC: Simultaneous equations solver gives $d = -\dfrac{7}{3} \approx -2.33$ and $u_1 = 31 - 2\left(-\dfrac{7}{3}\right) = 31 + \dfrac{14}{3} = \dfrac{107}{3} \approx 35.7$ (3 s.f.). **A1**

*Warning: A negative common difference means the sequence is decreasing; the sum will eventually reach zero and become negative.*

(c) $S_n = \dfrac{n}{2}\left(2 \times \dfrac{107}{3} + (n-1)\left(-\dfrac{7}{3}\right)\right) = 0$. Since $n \neq 0$: $\dfrac{214}{3} - \dfrac{7(n-1)}{3} = 0 \Rightarrow 214 = 7(n-1) \Rightarrow n - 1 = \dfrac{214}{7} = 30.57\ldots \Rightarrow n = 31.57\ldots$ **M1**

Check $S_{31}$ and $S_{32}$ using GDC: solving $S_n = 0$ exactly gives $n = \dfrac{221}{7} = 31.57\ldots$, which is not an integer, so there is no integer $n$ with $S_n = 0$. **A1**

*Warning: Not all arithmetic sums attain exactly zero at an integer index; the question tests whether candidates check this.*

## Q26 — Arithmetic Series from Sum Formula HARD

(a) The first term is $u_1 = S_1 = 4(1)^2 - 3(1) = 4 - 3 = 1$. **M1**
The second term is $u_2 = S_2 - S_1 = (4(4) - 3(2)) - 1 = (16 - 6) - 1 = 9$, so $d = u_2 - u_1 = 9 - 1 = 8$. **A1 AG**
*Warning: Both $u_1 = 1$ and $d = 8$ must be derived from the given $S_n$ formula; simply stating them earns no marks.*

(b) $u_n = 1 + (n-1)(8) = 8n - 7$. The required sum is $\sum_{k=6}^{20} u_k = S_{20} - S_5$. **M1**

GDC: $S_{20} = 4(400) - 3(20) = 1600 - 60 = 1540$ and $S_5 = 4(25) - 3(5) = 100 - 15 = 85$. Sum $= 1540 - 85 = 1455$. **A1**

(c) Setting $u_n = 8n - 7 > 1000$: $n > \dfrac{1007}{8} = 125.875$, so the first term is $u_{126} = 8(126) - 7 = 1008 - 7 = 1001$, at term number 126. **A1**

## Q27 — Geometric Sequence Two Condition Show That HARD

(a) Let $u_1$ and $r$ be the first term and common ratio, with $S_2 = u_1(1+r) = 16$ and $S_4 = u_1(1+r)(1+r^2) = 160$. **M1**

Dividing $S_4$ by $S_2$: $1 + r^2 = \dfrac{160}{16} = 10 \Rightarrow r^2 = 9 \Rightarrow r = 3$ (since terms are positive, the negative root $r = -3$ is rejected). **A1**

Substituting back into $S_2 = u_1(1 + r) = 16$: $u_1(1 + 3) = 16 \Rightarrow u_1 = 4$. **A1 AG**

Check: terms are $4, 12, 36, 108$; $S_2 = 4 + 12 = 16$ and $S_4 = 4 + 12 + 36 + 108 = 160$, both matching the question.

*Warning: The $n$ vs $n-1$ exponent trap: $u_n = 4 \times 3^{n-1}$, not $4 \times 3^n$.*

(b) Setting $u_n = 4 \times 3^{n-1} > 1000$: $3^{n-1} > 250$. GDC table: $3^5 = 243 < 250$; $3^6 = 729 > 250$, so $n - 1 = 6 \Rightarrow n = 7$. Check: $u_6 = 4 \times 243 = 972 < 1000$ and $u_7 = 4 \times 729 = 2916 > 1000$. First term to exceed 1000 is $u_7 = 2916$, at term number 7. **A1**

## Q28 — Arithmetic and Geometric Threshold Fusion HARD

(a) The programme is arithmetic with $u_1 = 8$ km and $d = 3$ km. Total distance after $n$ weeks: $S_n = \dfrac{n}{2}(2 \times 8 + (n-1) \times 3) = \dfrac{n}{2}(16 + 3n - 3) = \dfrac{n(3n+13)}{2}$ km. **A1**

(b) Setting $S_n \geq 500$: $\dfrac{n(3n+13)}{2} \geq 500 \Rightarrow 3n^2 + 13n - 1000 \geq 0$. **M1**

GDC: Equation solver or table. Positive root: $n = \dfrac{-13+\sqrt{169+12000}}{6} = \dfrac{-13+109.4\ldots}{6} = 16.07\ldots$

Check: $S_{16} = \dfrac{16(48+13)}{2} = \dfrac{16\times 61}{2} = 488 < 500$ and $S_{17} = \dfrac{17(51+13)}{2} = \dfrac{17\times 64}{2} = 544 \geq 500$.

Minimum is $n = 17$ weeks. **A1**

(c) For the new programme, $u_1 = 8$ km and $r = 1.08$. Total after $n$ weeks: $S_n = \dfrac{8(1.08^n - 1)}{0.08}$. Setting $S_n > 500$: **M1**

GDC: Table or solver. $S_{23} = \dfrac{8(1.08^{23}-1)}{0.08} = 487.15\ldots < 500$; $S_{24} = \dfrac{8(1.08^{24}-1)}{0.08} = 534.12\ldots > 500$.

Minimum is $n = 24$ weeks. **A1**

*Warning: Do not confuse the term $u_n$ with the sum $S_n$; the question asks when the total exceeds 500, so use the sum formula throughout.*

## Q29 — Hybrid Sequence Terms and Sigma Sums HARD

(a) Substituting $n = 1, 2, 3$ into $u_n = 3(2)^{n-1} - 4n + 2$: $u_1 = 3(1) - 4 + 2 = 1$; $u_2 = 3(2) - 8 + 2 = 0$; $u_3 = 3(4) - 12 + 2 = 2$. **A1**

(b) $\displaystyle\sum_{n=1}^{10} u_n = \sum_{n=1}^{10} 3(2)^{n-1} - \sum_{n=1}^{10}(4n-2)$. **M1**

GDC: List/Seq function or direct summation. The terms are $u_1, \ldots, u_{10} = 1, 0, 2, 10, 30, 74, 166, 354, 734, 1498$, summing to $\displaystyle\sum_{n=1}^{10} u_n = 2\,869$. **A1**

(c) The geometric part has $a = 3$, $r = 2$, so $\displaystyle\sum_{n=1}^{10} 3(2)^{n-1} = \frac{3(2^{10}-1)}{2-1} = \frac{3\times 1023}{1} = 3\,069$. **M1**

This is written as $\dfrac{3(2^{10}-1)}{2-1}$ with $a = 3$, $r = 2$. **A1**

*Warning: The exponent in the GS sum formula is $r^{10}$ (the number of terms), while the last term uses $r^{n-1} = r^9$; do not mix these. Also verify every term of a hybrid sequence individually on the GDC before summing — a single mis-evaluated term (e.g. $u_4$) silently corrupts the whole running total.*

## Q30 — Geometric Profit Sequence and Threshold HARD

(a) Using $u_n = u_1 r^{n-1}$: $u_2 = u_1 r = 56\,000$ and $u_5 = u_1 r^4 = 189\,000$. **M1**

Dividing: $r^3 = \dfrac{189\,000}{56\,000} = 3.375 \Rightarrow r = \sqrt[3]{3.375} = 1.5$ (exact). Then $u_1 = \dfrac{56\,000}{1.5} = 37\,333.3\ldots \approx \$37\,300$ (3 s.f.). So $r = 1.50$ and $u_1 \approx \$37\,300$. **A1**

*Warning: Use unrounded values of $u_1$ and $r$ in all subsequent parts to avoid premature-rounding errors.*

(b) GDC: $S_{10} = \dfrac{u_1(r^{10}-1)}{r-1} = \dfrac{37\,333.3\ldots \times (1.5^{10}-1)}{0.5} = \dfrac{37\,333.3\ldots \times 56.665\ldots}{0.5} = \$4\,230\,000$ (3 s.f.). **A1**

(c) Setting $u_n = u_1 \times 1.5^{n-1} > 1\,000\,000$: $1.5^{n-1} > \dfrac{1\,000\,000}{37\,333.3\ldots} = 26.786\ldots$ **M1**

GDC table: $1.5^8 = 25.63\ldots < 26.786$ and $1.5^9 = 38.44\ldots > 26.786$, so $n - 1 = 9 \Rightarrow n = 10$. Check: $u_9 = 37\,333.3\ldots \times 1.5^8 = 37\,333.3\ldots \times 25.63\ldots = 956\,813\ldots < 1\,000\,000$ and $u_{10} = 37\,333.3\ldots \times 1.5^9 = 37\,333.3\ldots \times 38.44\ldots = 1\,435\,219\ldots > 1\,000\,000$. First year is Year 10. **A1**

*Warning: The bracketing check is mandatory; every power of $r$ used in the search must be evaluated directly on the GDC and re-checked, since a single mis-keyed exponent (e.g. reading $1.5^{21}$ as 26.18 instead of the true 4988) silently derails the entire threshold search.*