IB Math AI SL — Topic 1
Number Toolkit

Standard Form & Indices · Logarithms · Bounds & Percentage Error · GDC Equations

Scan me for help

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:_____ Date:_____

Score:_____/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. A graphic display calculator is required throughout; support GDC answers with the values entered and the feature used.

Scan me for help

Key Formulae

- Standard form: $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$
- Laws of indices: $a^m \times a^n = a^{m+n}$; $\quad a^m \div a^n = a^{m-n}$; $\quad (a^m)^n = a^{mn}$; $\quad a^0 = 1$; $\quad a^{-n} = \dfrac{1}{a^n}$
- Logarithm definition: $\log_{10} x = y \iff 10^y = x$
- Supplied scale formula (e.g. sound level): $L = 10 \log_{10}\left(\dfrac{I}{I_0}\right)$, where I_0 is a reference intensity
- Upper & lower bounds: for a value v measured to the nearest unit u, $\quad v - \dfrac{u}{2} \leq$ true value $< v + \dfrac{u}{2}$
- Bounds of a quotient: $\left(\dfrac{a}{b}\right)_{\max} = \dfrac{a_{\max}}{b_{\min}}$, $\quad \left(\dfrac{a}{b}\right)_{\min} = \dfrac{a_{\min}}{b_{\max}}$
- Percentage error: $\%\,\text{error} = \dfrac{|v_A - v_E|}{|v_E|} \times 100\%$, where v_E is the exact value

GDC Tips

- **Evaluating powers and standard form:** press [2nd][EE] (or $\times 10^x$) to enter $a \times 10^k$ directly; confirm the display shows the correct exponent before recording your answer.
- **Solving equations / finding intersections:** enter both sides as Y_1 and Y_2 in [Y=], then use [2nd][CALC] $\rightarrow$ 5: intersect; move the cursor near each root and press ENTER three times — repeat for every root.
- **Evaluating $\log_{10}$:** press [LOG] then type the argument and close the bracket; to invert (find x given L) evaluate $10^{(\text{value})}$.
- **Storing unrounded intermediates:** after a calculation press [STO$\rightarrow$][A] (or any letter) to save the full-precision value; recall it with [ALPHA][A] in subsequent steps to avoid premature rounding.
- **Checking bounds on a quotient:** store a_{max}, a_{min}, b_{max}, b_{min} in variables A–D, then compute A/D (upper bound) and C/B (lower bound) directly, reading each to 3 s.f. only at the final step.
- **Polynomial roots:** for a polynomial equation set equal to zero, enter the left-hand side as Y_1 and use [2nd][CALC] $\rightarrow$ 2: zero; bracket each root separately with left-bound and right-bound prompts.

Common Mistakes to Avoid

1. **Wrong denominator in percentage error.** Students divide by the approximate value v_A instead of the exact value v_E. Always place the exact (theoretical or given) value in the denominator: $\dfrac{|v_A - v_E|}{|v_E|} \times 100$.
2. **Standard form with a outside $[1, 10)$.** Writing 12.3×10^4 instead of 1.23×10^5 violates the definition. After any arithmetic on standard-form numbers, re-check that $1 \leq a < 10$ and adjust k accordingly.
3. **Pairing bounds incorrectly for a quotient.** To maximise a quotient, pair the upper bound of the numerator with the lower bound of the denominator — not the same-direction bounds. Swapping this pairing gives an answer that is actually the minimum.
4. **Premature rounding in chained calculations.** Rounding an intermediate result (e.g. a logarithm or a bound) and then using that rounded value in the next step accumulates error. Store the full GDC value in a variable and round only the final answer to the required accuracy.
5. **Wrong tail / direction when inverting a logarithmic scale.** When solving $L = 10 \log_{10}(I/I_0)$ for I, students sometimes apply $\log_{10}$ a second time instead of raising 10 to the power. Always isolate the $\log_{10}$ term first, then exponentiate: $I = I_0 \times 10^{L/10}$.
6. **Bounds certainty judgement without naming the worst case.** A conclusion such as "the value certainly exceeds the limit" is unsupported unless the student explicitly states which bound (upper or lower) represents the worst case and compares that bound to the threshold. An R1 mark requires the comparison to be written before the conclusion is stated. Remember also that the upper bound itself is *excluded* (true value $< v + u/2$ strictly) — a claim of the form "value $< v + u/2$" is therefore always guaranteed to hold.

Section A — Easy EASY (10 questions × 3 marks = 30 marks)

Scan me for help

1. EASY [3 marks]

The mass of a grain of sand is approximately 0.0000647 grams.
Write this number in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

2. EASY [3 marks]

The distance from Earth to the Sun is approximately 1.496×10^8 km.
Write this distance in full, as an ordinary number. [1]
A spacecraft travels at 4.3×10^4 km per hour. Calculate how many hours it would take to travel from Earth to the Sun. Give your answer in standard form. [2]

3. EASY [3 marks]

Simplify the following expression, giving your answer as a single power of 5.

$$\frac{5^6 \times 5^3}{5^4}$$

4. **EASY** **[3 marks]**

The loudness of a sound, L decibels (dB), is given by the formula

$$L = 10 \log_{10}\left(\frac{I}{I_0}\right)$$

where I is the sound intensity in watts per square metre (W m^{-2}) and $I_0 = 1 \times 10^{-12}$ W m^{-2}. A classroom has a sound intensity of $I = 1 \times 10^{-5}$ W m^{-2}.

Calculate the loudness of the sound in the classroom.

5. **EASY** **[3 marks]**

A farmer measures the length of a field as 148 m, rounded to the nearest metre.

Write down the upper bound and the lower bound of the length of the field. [2]

Hence, write down the upper bound for the perimeter of a square field with this measured side length. [1]

6. **EASY** **[3 marks]**

A student estimates the area of a circular pond by rounding the radius to 1 significant figure before calculating. The actual radius of the pond is $r = 6.3$ m. The student uses $r = 6$ m in their calculation. The area of a circle is $A = \pi r^2$.

Calculate the percentage error in the student's estimate of the area. Give your answer correct to 2 decimal places.

7. **EASY** **[3 marks]**

The table shows the results of measuring the wingspan, w cm, of five butterflies.

Butterfly	1	2	3	4	5
Wingspan, w (cm)	4.2	3.8	5.1	4.6	3.9

Each wingspan is measured to the nearest 0.1 cm.

Write down the lower bound of the wingspan of butterfly 3. [1]

Write down the upper bound of the total wingspan of all five butterflies. [2]

8. **EASY** **[3 marks]**

Use a GDC to solve the equation

$$3x^2 - 5x - 7 = 0.$$

Give both solutions correct to 3 significant figures.

9. **EASY** **[3 marks]**

Evaluate $5^0 + 4^{-2} + 2^3$.

Give your answer as an exact fraction.

10. **EASY** **[3 marks]**

The approximate population of a city is given as $2\,400\,000$.

Write this number in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$. [1]

A second city has a population of 8.1×10^5. Calculate the combined population of the two cities, giving your answer in standard form. [2]

Section B — Medium MEDIUM (10 questions × 4 marks = 40 marks)

Scan me for help

11. MEDIUM [4 marks]

The distance from Earth to the star Proxima Centauri is approximately 4.02×10^{13} km. The distance from Earth to the star Sirius is approximately 8.13×10^{13} km.

(a) Write down the combined distance from Earth to Proxima Centauri and from Earth to Sirius, giving your answer in standard form. [2]

(b) The speed of light is 9.461×10^{12} km per year. Find how many times greater the distance to Sirius is compared to the distance light travels in one year, giving your answer to 3 significant figures. [2]

12. MEDIUM [4 marks]

A biologist measures the mass of a bacterium as 3.8×10^{-12} g, rounded to 2 significant figures.

(a) Write down the upper and lower bounds of this measurement. [2]

(b) The biologist claims the mass is less than 3.85×10^{-12} g. Determine whether this claim is certainly true. Justify your answer. [2]

13. MEDIUM **[4 marks]**

A scientist uses the formula for sound intensity level:

$$L = 10 \log_{10}\left(\frac{I}{I_0}\right)$$

where L is the sound level in decibels (dB), I is the intensity in W m^{-2}, and $I_0 = 1 \times 10^{-12}$ W m^{-2} is the reference intensity.

A vacuum cleaner produces a sound level of 75 dB. Find the intensity I of the vacuum cleaner, giving your answer in standard form to 3 significant figures.

14. MEDIUM **[4 marks]**

A rectangular field is measured as 84 m long and 37 m wide, each measurement rounded to the nearest metre.

(a) Write down the upper bound of the length and the lower bound of the width. [2]

(b) Find the upper bound of the area of the field, giving your answer in m^2. [2]

15. MEDIUM **[4 marks]**

A storage tank contains 2.56×10^4 litres of water. A pump removes water at a rate of 3.20×10^2 litres per hour.

quantity	value
initial volume (litres)	2.56×10^4
pump rate (litres/hour)	3.20×10^2

(a) Find the number of hours it takes to empty the tank completely, giving your answer as an exact value. [2]

(b) Write your answer to part (a) to 2 significant figures. [2]

16. MEDIUM [4 marks]

A student estimates the area of a circular pond by rounding the radius to 1 significant figure. The actual radius of the pond is 6.4 m.

(a) Write down the radius rounded to 1 significant figure. [1]

(b) Find the percentage error in the area when the rounded radius is used instead of the actual radius. Give your answer to 3 significant figures. [3]

17. MEDIUM [4 marks]

Solve the following equation, giving all solutions to 3 significant figures.

$$2x^3 - 5x^2 - 3x + 4 = 0$$

18. MEDIUM [4 marks]

The table shows the population of four cities and their areas.

city	population	area (km^2)
Alderton	$1\,240\,000$	312
Breva	$875\,000$	204
Calston	$2\,050\,000$	487
Dorwich	$640\,000$	98

(a) Write the population of Calston in standard form. [1]

(b) Find the population density (people per km^2) of Dorwich, giving your answer to 3 significant figures. [2]

(c) Find the percentage error if a student approximates Breva's population as 9×10^5. Give your answer to 3 significant figures. [1]

19. MEDIUM **[4 marks]**

The pH of a solution is given by the formula

$$\text{pH} = -\log_{10}[\text{H}^+]$$

where $[\text{H}^+]$ is the hydrogen ion concentration in mol L^{-1}.

(a) A fruit juice has $[\text{H}^+] = 2.5 \times 10^{-4}$ mol L^{-1}. Find the pH of the fruit juice, giving your answer to 3 significant figures. [2]

(b) A cleaning solution has a pH of 11.3. Find its hydrogen ion concentration, giving your answer in standard form to 3 significant figures. [2]

20. MEDIUM **[4 marks]**

Find the values of x that satisfy the equation

$$3^x = 5x + 2$$

giving your answers to 3 significant figures.

Section C — Hard HARD (10 questions × 5 marks = 50 marks)

Scan me for help

21. HARD **[5 marks]**

The speed of light in a vacuum is $c = 2.998 \times 10^8$ m s^{-1}. A radio telescope detects a signal from a galaxy whose distance from Earth is measured as $d = 1.54 \times 10^{23}$ m.

(a) Show that the time, T seconds, for the signal to travel this distance satisfies $T \approx 5.14 \times 10^{14}$ s. [2]

(b) The distance d is measured to the nearest 10^{21} m. Find the upper and lower bounds of T, giving your answers in standard form to three significant figures. [2]

(c) Hence determine, with justification, whether the signal travel time certainly exceeds 5.10×10^{14} s. [1]

22. HARD **[5 marks]**

The pH of a liquid is given by the formula

$$\text{pH} = -\log_{10}[\text{H}^+]$$

where $[\text{H}^+]$ is the hydrogen ion concentration in mol L^{-1}.

(a) A fruit juice has $[\text{H}^+] = 3.16 \times 10^{-4}$ mol L^{-1}. Find its pH, correct to two decimal places. [1]

(b) A second liquid has pH $= 8.72$. Find its hydrogen ion concentration, giving your answer in standard form to three significant figures. [2]

(c) Find the ratio of the hydrogen ion concentration of the fruit juice to that of the second liquid, giving your answer as a power of ten to two decimal places. [2]

23. HARD **[5 marks]**

A scientist records the mass m grams of a radioactive sample at time t days. The model is

$$m = 250 \times k^t$$

where k is a positive constant. The table shows two recorded values.

time, t (days)	0	12
mass, m (g)	250	159

(a) Show that $k = 0.963$ correct to three significant figures. [2]

(b) Find the time at which the mass first falls below 100 g, giving your answer to the nearest day. [2]

(c) Explain why the model is not appropriate for very large values of t. [1]

24. HARD **[5 marks]**

The sound intensity level L in decibels is given by

$$L = 10 \log_{10}\left(\frac{I}{I_0}\right)$$

where I is the sound intensity in W m^{-2} and $I_0 = 1.00 \times 10^{-12}$ W m^{-2}. A loudspeaker produces a sound level of 94 dB at a distance of 1 m. A second loudspeaker of identical type is placed alongside the first. The combined intensity is exactly double.

(a) Find the intensity I_1 of one loudspeaker at 1 m, giving your answer in standard form to three significant figures. [2]

(b) Find the combined sound level of the two loudspeakers, correct to one decimal place. [2]

(c) State the increase in sound level caused by doubling the number of identical sources. [1]

25. HARD **[5 marks]**

A surveyor measures the length of a rectangular field as $l = 84$ m and the width as $w = 37$ m, both to the nearest metre.

(a) Write down the upper bound of l and the lower bound of w. [1]

(b) Find the upper bound and lower bound of the area $A = lw$, giving your answers to four significant figures. [2]

(c) The surveyor claims the area exceeds $3\,000$ m^2. Determine, with justification, whether this claim is certainly correct. [2]

26. HARD **[5 marks]**

A student uses the approximation $\sqrt{2} \approx 1.4$ to estimate the value of

$$Q = \frac{200}{\sqrt{2}}.$$

(a) Write down the student's estimate of Q. [1]

(b) Calculate the percentage error in the student's estimate, correct to two significant figures. [2]

(c) A second quantity is $P = \dfrac{200}{\sqrt{2} - 0.01}$. Using the exact value of $\sqrt{2}$, find P correct to five significant figures. Hence find the percentage error if the student uses their estimate of Q as an approximation for P as well, correct to two significant figures. [2]

27. HARD **[5 marks]**

The number of bacteria N in a culture at time t hours satisfies

$$N = a \times 10^{bt}$$

where a and b are constants with $a > 0$ and $b > 0$. At $t = 0$ there are 500 bacteria, and at $t = 6$ there are $32\,000$ bacteria.

(a) Write down the value of a. [1]

(b) Show that $b = \dfrac{\log_{10}(64)}{6}$, and hence find b correct to three significant figures. [2]

(c) Find the time at which N first exceeds $1\,000\,000$, giving your answer in hours correct to two decimal places. [2]

28. HARD **[5 marks]**

A spherical droplet of oil has radius r cm, measured to the nearest 0.1 cm, with $r = 2.3$ cm.

(a) State the lower bound of r. [1]

(b) The volume of the droplet is $V = \dfrac{4}{3}\pi r^3$ cm^3. Find the upper bound and lower bound of V, giving your answers correct to four significant figures. [2]

(c) The droplet is melted and recast into a cube of side length s cm. Using the upper bound of V, find the upper bound of s, correct to three significant figures. [2]

29. HARD **[5 marks]**

The magnitude of an earthquake is given by the formula

$$M = \frac{2}{3}\log_{10}\left(\frac{E}{E_0}\right)$$

where E is the energy released in joules and $E_0 = 1.00 \times 10^{4.4}$ J.

(a) An earthquake releases energy $E_1 = 2.00 \times 10^{14}$ J. Find its magnitude M_1, correct to two decimal places. [2]

(b) A second earthquake has magnitude $M_2 = 7.20$. Find the energy E_2 it releases, giving your answer in standard form to three significant figures. [2]

(c) Find the ratio $E_2 : E_1$, correct to the nearest whole number. [1]

30. HARD **[5 marks]**

A polynomial equation is given by

$$f(x) = 2x^3 - 5x^2 - 4x + 2.$$

(a) Using a GDC, find all three roots of $f(x) = 0$, correct to three significant figures. [2]

(b) Show that one root lies between $x = 3$ and $x = 4$ by evaluating $f(3)$ and $f(4)$. [2]

(c) The equation $f(x) = k$ has exactly two real solutions. Using your GDC, find the two possible values of k, correct to three significant figures. [1]

Mark Schemes

Q1 — Standard Form of a Small Decimal EASY

Moving the decimal point 5 places to the right gives 6.47, so the power of 10 is -5. **M1**

$$0.0000647 = 6.47 \times 10^{-5}$$

$a = 6.47$ **A1**

$k = -5$ **A1**

Warning: Both a and k must be stated; writing 64.7×10^{-6} does not satisfy $1 \leq a < 10$ and earns no credit for a.

Q2 — Dividing Numbers in Standard Form EASY

Distance as an ordinary number: $1.496 \times 10^8 = 149\,600\,000$ km **A1**

Time calculation: $t = \dfrac{1.496 \times 10^8}{4.3 \times 10^4}$ **M1**

GDC: type $(1.496\text{E}8) \div (4.3\text{E}4)$, read display. Unrounded value: $3480.232\ldots$

$$t \approx 3.48 \times 10^3 \text{ hours}$$

A1

Warning: The answer must be expressed in standard form with $1 \leq a < 10$; writing 34.8×10^2 loses the accuracy mark.

Q3 — Laws of Indices EASY

Apply the multiplication law first, then the division law:

$$\frac{5^6 \times 5^3}{5^4} = \frac{5^{6+3}}{5^4} = \frac{5^9}{5^4}$$

M1

$$= 5^{9-4} = 5^5$$

A1 A1

Warning: Do not multiply the indices in the numerator ($5^{6\times3}$); the bases are the same so indices are added, not multiplied.

Q4 — Logarithm Formula for Sound Level EASY

Substitute $I = 1 \times 10^{-5}$ and $I_0 = 1 \times 10^{-12}$ into the given formula.

$$L = 10 \log_{10}\left(\frac{1 \times 10^{-5}}{1 \times 10^{-12}}\right)$$

M1

Evaluate the ratio inside the logarithm: $\dfrac{1 \times 10^{-5}}{1 \times 10^{-12}} = 1 \times 10^{7}$ **A1**

GDC: type $10 \times \log(1\text{E}7)$, read display.

$$L = 10 \times \log_{10}(10^{7}) = 10 \times 7 = 70 \text{ dB}$$

A1

Warning: Use $\log_{10}$ *(common logarithm), not* ln *(natural logarithm); on most GDCs the LOG key gives base 10.*

Q5 — Upper and Lower Bounds of a Length EASY

A measurement of 148 m rounded to the nearest metre has a precision of ± 0.5 m.
Lower bound $= 148 - 0.5 = 147.5$ m; Upper bound $= 148 + 0.5 = 148.5$ m **A1 A1**

For the perimeter of a square, use the upper bound of the side length to obtain the largest possible perimeter.

$$\text{Upper bound of perimeter} = 4 \times 148.5 = 594 \text{ m}$$

A1

Warning: The upper bound is 148.5 *m, not* 149 *m; adding 1 to the rounded value gives an incorrect bound.*

Q6 — Percentage Error in Area Estimate EASY

Calculate the exact (actual) area using $r = 6.3$ m.

$$A_{\text{exact}} = \pi \times (6.3)^2 = 124.689\ldots \text{ m}^2$$

M1

Calculate the approximate area using the student's rounded value $r = 6$ m.

$$A_{\text{approx}} = \pi \times (6)^2 = 113.097\ldots \text{ m}^2$$

Apply the percentage error formula, dividing by the exact value.

$$\%\text{ error} = \frac{|A_{\text{approx}} - A_{\text{exact}}|}{A_{\text{exact}}} \times 100 = \frac{|113.097\ldots - 124.689\ldots|}{124.689\ldots} \times 100$$

M1

Unrounded: $9.2960\ldots\%$

$$\%\text{ error} \approx 9.30\%$$

A1

Warning: Always divide by the exact value (A_{exact}), not the approximate value; dividing by A_{approx} gives a different and incorrect percentage error.

Q7 — Upper and Lower Bounds from a Table EASY

Each wingspan is measured to the nearest 0.1 cm, so the precision is ± 0.05 cm.

Lower bound of butterfly 3: the measured value is 5.1 cm, so the lower bound is $5.1 - 0.05 = 5.05$ cm. **A1**

Upper bound of total wingspan: take the upper bound of each individual measurement and add them.

$$\text{Upper bounds: } 4.25 + 3.85 + 5.15 + 4.65 + 3.95$$

M1

$$= 21.85 \text{ cm}$$

A1

Warning: Adding the upper bounds of each wingspan (not just adding 0.05 once to the sum of measured values) is required; the error accumulates across all five measurements.

Q8 — Solving a Quadratic with a GDC EASY

State the equation to be solved and use the GDC to find both roots.

$$3x^2 - 5x - 7 = 0$$

M1

GDC: Equation Solver (or Graph → find zeros): enter $3x^2 - 5x - 7$, find both x-intercepts.

Unrounded values (quadratic formula $x = \dfrac{5 \pm \sqrt{109}}{6}$): $x = 2.5733\ldots$ and $x = -0.9067\ldots$

$$x = 2.57 \quad \text{or} \quad x = -0.907 \quad \text{(both to 3 s.f.)}$$

A1 A1

Warning: A quadratic equation has two solutions; writing only one solution loses an accuracy mark. Back-substitution confirms these roots: $3(2.57)^2 - 5(2.57) - 7 \approx 0$ and $3(-0.907)^2 - 5(-0.907) - 7 \approx 0$.

Q9 — Evaluating Negative and Zero Indices EASY

Evaluate each term separately using the index laws $a^0 = 1$ and $a^{-n} = \dfrac{1}{a^n}$.

$$5^0 = 1, \qquad 4^{-2} = \frac{1}{4^2} = \frac{1}{16}, \qquad 2^3 = 8$$

M1

Add the three terms, converting to a common denominator of 16.

$$1 + \frac{1}{16} + 8 = 9 + \frac{1}{16} = \frac{144}{16} + \frac{1}{16}$$

M1

$$= \frac{145}{16}$$

A1

Warning: $5^0 = 1$, *not* 0; *any nonzero number raised to the power zero equals exactly 1.*

Q10 — Adding Numbers in Standard Form EASY

Write $2\,400\,000$ in standard form by positioning the decimal point after the first significant digit.

$$2\,400\,000 = 2.4 \times 10^6$$

A1

To add the two populations, convert both to ordinary numbers (or align powers on the GDC).

$$2.4 \times 10^6 + 8.1 \times 10^5 = 2\,400\,000 + 810\,000$$

M1

GDC: type $(2.4\text{E}6) + (8.1\text{E}5)$, read display in standard form.

$$= 3\,210\,000 = 3.21 \times 10^6$$

A1

Warning: Do not add $2.4 + 8.1$ *and keep* 10^6*; the powers of 10 are different, so the coefficients cannot be added directly without first aligning the exponents.*

Q11 — Standard Form Operations MEDIUM

(a) Since both powers of 10 are equal (10^{13}), the coefficients are added directly.

$$(4.02 + 8.13) \times 10^{13} = 12.15 \times 10^{13}$$

Rewriting so that $1 \leq a < 10$, the combined distance is 1.215×10^{14} km. **M1**

$$= 1.22 \times 10^{14} \text{ km (3 s.f.)}$$

A1

Warning: 12.15×10^{13} *is NOT in standard form; the coefficient must satisfy* $1 \leq a < 10$.

(b) Divide the two standard-form quantities.

$$\frac{8.13 \times 10^{13}}{9.461 \times 10^{12}} = \frac{8.13}{9.461} \times 10^{13-12} = 0.8593\ldots \times 10^{1}$$

M1

Unrounded value: $8.5934\ldots$, so Sirius is 8.59 times further than one light-year (3 s.f.). **A1**

Q12 — Bounds of a Measured Value and Certainty Judgement MEDIUM

(a) The mass is 3.8×10^{-12} g rounded to 2 significant figures; half a unit in the last significant place is 0.05×10^{-12}.

Lower bound: 3.75×10^{-12} g **A1**

Upper bound: 3.85×10^{-12} g **A1**

(b) By the bounds convention (Key Formulae, page 1), the true value satisfies $v - u/2 \leq$ true value $< v + u/2$ — the upper bound is *strictly excluded*. So the mass can never actually equal 3.85×10^{-12} g; it is always strictly less than this value. **R1**

The claim "the mass is less than 3.85×10^{-12} g" is therefore **certainly true**, because the upper bound itself is never attained. **A1**

Warning: It is tempting to think the mass "could equal" the upper bound and conclude the claim is unresolved — but the bounds formula uses a strict $<$ at the upper end precisely so that the upper bound is excluded. Always re-check which inequality (strict or non-strict) applies at each end before making a certainty judgement.

Q13 — Logarithm Formula Inversion for Intensity MEDIUM

Substitute $L = 75$ and $I_0 = 1 \times 10^{-12}$ into the given formula and solve for I.

$$75 = 10 \log_{10}\left(\frac{I}{1 \times 10^{-12}}\right)$$

M1

Divide both sides by 10: $\log_{10}\left(\dfrac{I}{1\times10^{-12}}\right)=7.5$ **M1**

Exponentiate both sides (raise 10 to each side):

$$\frac{I}{1\times10^{-12}}=10^{7.5}=31\,622\,776\ldots$$

$$I=10^{7.5}\times10^{-12}=10^{-4.5}=3.16227\ldots\times10^{-5}$$

A1

GDC: evaluate $10^{7.5}\times10^{-12}$ on the home screen.

$$I=3.16\times10^{-5}\ \text{W m}^{-2}\ \text{(3 s.f.)}$$

A1

Warning: Dividing by 10 before inverting the logarithm is essential; jumping straight to $I=10^{75}\times10^{-12}$ is a common error that costs both accuracy marks.

Q14 — Bounds of Measurements and Upper Bound of Area MEDIUM

(a) Each measurement is rounded to the nearest metre, so bounds are ±0.5 m.
Upper bound of length: $84+0.5=84.5$ m **A1**

Lower bound of width: $37-0.5=36.5$ m **A1**

(b) To obtain the largest possible area, multiply the upper bound of the length by the upper bound of the width.

$$\text{Upper bound of area}=84.5\times37.5$$

M1

$$=3168.75\ \text{m}^2$$

A1

Warning: The upper bound of the area uses upper bound × upper bound, NOT upper length × lower width. Pairing incorrectly is the common trap here.

Q15 — Standard Form Division and Rounding MEDIUM

(a) Divide the total volume by the pump rate.

$$\text{time}=\frac{2.56\times10^{4}}{3.20\times10^{2}}$$

M1

$$= \frac{2.56}{3.20} \times 10^{4-2} = 0.8 \times 10^2 = 80 \text{ hours}$$

A1

(b) 80 written to 2 significant figures is 80 (both digits are already significant; the answer is exact). **A1**

Writing 80 in standard form as 8.0×10^1 confirms 2 significant figures are intended. **A1**

Warning: Students must recognise that 80 is already exact here; no further rounding occurs.

Q16 — Percentage Error in Area from Rounded Radius MEDIUM

(a) Rounding 6.4 m to 1 significant figure gives $r_{\text{approx}} = 6$ m. **A1**

(b) Calculate the exact area and the approximate area, then apply the percentage error formula with the exact value in the denominator.

Exact area: $A_E = \pi \times (6.4)^2 = \pi \times 40.96 = 128.679\ldots$ m^2 **M1**

Approximate area: $A_A = \pi \times (6)^2 = \pi \times 36 = 113.097\ldots$ m^2

$$\% \text{ error} = \frac{|A_A - A_E|}{|A_E|} \times 100 = \frac{|113.097\ldots - 128.679\ldots|}{128.679\ldots} \times 100$$

M1

$$= \frac{15.582\ldots}{128.679\ldots} \times 100 = 12.1\% \text{ (3 s.f.)}$$

A1

Warning: Percentage error always divides by the exact value ($128.679\ldots$), never by the approximate value.

Q17 — Solving a Cubic Equation by GDC MEDIUM

Enter the cubic equation into the GDC as $y_1 = 2x^3 - 5x^2 - 3x + 4$ and find the x-intercepts.
GDC: Graph the function, then use Zeros (or Roots) feature to find each x-intercept, OR use Polynomial Solver: Algebra $\rightarrow$ Polynomial $\rightarrow$ degree 3, coefficients $2, -5, -3, 4$. **M1**

$x = -1$ is an exact root: $2(-1)^3 - 5(-1)^2 - 3(-1) + 4 = -2 - 5 + 3 + 4 = 0$. Factoring: $2x^3 - 5x^2 - 3x + 4 = (x+1)(2x^2 - 7x + 4)$. **A1**

The remaining roots solve $2x^2 - 7x + 4 = 0$: $x = \dfrac{7 \pm \sqrt{17}}{4}$. Unrounded: $x = 2.7807\ldots$ and $x = 0.7192\ldots$ **A1**

Rounded to 3 significant figures:

$$x = -1.00, \quad x = 0.719, \quad x = 2.78$$

A1

Warning: All three roots must be stated for full marks. Confirm the number of roots by examining the graph over a wide x-range; back-substitution into the original cubic confirms all three values.

Q18 — Standard Form Population Density and Percentage Error MEDIUM

(a) The population of Calston is $2\,050\,000 = 2.05 \times 10^6$. **A1**

(b) Population density of Dorwich = population ÷ area.

$$\text{density} = \frac{640\,000}{98} = 6530.612\ldots \text{ people per km}^2$$

M1

$$= 6530 \text{ people per km}^2 \text{ (3 s.f.)}$$

A1

(c) The exact population of Breva is $875\,000$; the approximation is $9 \times 10^5 = 900\,000$.

$$\%\text{ error} = \frac{|900\,000 - 875\,000|}{875\,000} \times 100 = \frac{25\,000}{875\,000} \times 100 = 2.86\% \text{ (3 s.f.)}$$

A1

Warning: Percentage error divides by the exact value ($875\,000$), not by the approximation ($900\,000$).

Q19 — Logarithm pH Formula Evaluation and Inversion MEDIUM

(a) Substitute $[\text{H}^+] = 2.5 \times 10^{-4}$ into the given formula.

$$\text{pH} = -\log_{10}(2.5 \times 10^{-4})$$

M1

GDC: evaluate $-\log_{10}(2.5 \times 10^{-4})$ on the home screen. Unrounded: $\text{pH} = 3.60206\ldots$

$$\text{pH} = 3.60 \text{ (3 s.f.)}$$

A1

(b) Set $\text{pH} = 11.3$ and solve for $[\text{H}^+]$.

$$11.3 = -\log_{10}[\text{H}^+] \implies \log_{10}[\text{H}^+] = -11.3$$

M1

$$[\text{H}^+] = 10^{-11.3} = 5.01187\ldots \times 10^{-12}$$

GDC: evaluate $10^{-11.3}$ on the home screen.

$$[\mathrm{H}^+] = 5.01 \times 10^{-12} \text{ mol L}^{-1} \text{ (3 s.f.)}$$

A1

Warning: The negative sign in the formula means $\log_{10}[H^+] = -pH$*, not* $+pH$*. A sign error when inverting produces a wildly wrong answer.*

Q20 — Solving an Exponential-Linear Equation by GDC MEDIUM

Rewrite the equation as $3^x - 5x - 2 = 0$ and find the zeros, OR graph $y_1 = 3^x$ and $y_2 = 5x + 2$ and find intersection points.
GDC: Enter $y_1 = 3^x$ and $y_2 = 5x + 2$. Use Intersection feature on a suitable window such as $-2 \leq x \leq 4$.
M1

Two intersection points are found. Scanning $3^x - 5x - 2$ for all sign changes and refining with the GDC/solver: unrounded x-coordinates $x = -0.24764\ldots$ and $x = 2.40322\ldots$ **A1**

Rounded to 3 significant figures:

$$x = -0.248 \quad \text{and} \quad x = 2.40$$

A1 A1

Warning: Students who use only a small window may miss a root, or may misread the intersection point. Back-substitute both roots into $3^x = 5x + 2$ *to confirm: e.g.* $3^{2.40} \approx 14.0$ *and* $5(2.40) + 2 = 14.0$.

Q21 — Standard Form Bounds and Certainty Judgement HARD

(a) To find T we divide distance by speed; the calculation is set up so that the printed value is the target (AG).

$$T = \frac{d}{c} = \frac{1.54 \times 10^{23}}{2.998 \times 10^{8}}$$

M1

$$T = \frac{1.54}{2.998} \times 10^{23-8} = 0.51367\ldots \times 10^{15} = 5.1367\ldots \times 10^{14} \text{ s} \approx 5.14 \times 10^{14} \text{ s}$$

AG

Warning: The unrounded value $5.1367\ldots \times 10^{14}$ *must be shown before stating the rounded target; omitting this loses the method mark.*

(b) Since d is measured to the nearest 10^{21} m, its bounds are $d_{\text{max}} = 1.545 \times 10^{23}$ m and $d_{\text{min}} = 1.535 \times 10^{23}$ m. The speed c is exact, so bounds of T come from bounds of d alone.
M1

$$T_{\text{max}} = \frac{1.545 \times 10^{23}}{2.998 \times 10^{8}} = 5.1534\ldots \times 10^{14} \approx 5.15 \times 10^{14}\ \text{s};$$

$$T_{\text{min}} = \frac{1.535 \times 10^{23}}{2.998 \times 10^{8}} = 5.1200\ldots \times 10^{14} \approx 5.12 \times 10^{14}\ \text{s}$$

A1

Warning: The upper bound of T uses the upper bound of d; swapping the pairing is the flagged trap.

(c) The lower bound $T_{\text{min}} = 5.12 \times 10^{14}$ s $> 5.10 \times 10^{14}$ s, so even in the worst case the travel time exceeds 5.10×10^{14} s. **R1**

Therefore the signal travel time certainly exceeds 5.10×10^{14} s.

Q22 — Logarithm Formula Inversion and Ratio HARD

(a) Substituting directly into the formula gives the pH; no inversion needed here.

$$\text{pH} = -\log_{10}(3.16 \times 10^{-4}) = -(\log_{10} 3.16 + \log_{10} 10^{-4}) = -(0.4997\ldots - 4) = 3.4997\ldots$$

GDC: evaluate $-\log_{10}(3.16 \times 10^{-4})$ directly. pH $= 3.50$ (2 d.p.) **A1**

(b) To find $[\text{H}^+]$ we invert the logarithm formula: $[\text{H}^+] = 10^{-\text{pH}}$. **M1**

$[\text{H}^+] = 10^{-8.72}$; GDC: $10^{-8.72} = 1.9054\ldots \times 10^{-9}$

$[\text{H}^+] = 1.91 \times 10^{-9}$ mol L^{-1} (3 s.f.) **A1**

Warning: The answer must be expressed in standard form with $1 \leq a < 10$; writing 19.1×10^{-10} loses the accuracy mark.

(c) The ratio is $\dfrac{3.16 \times 10^{-4}}{1.9054\ldots \times 10^{-9}} = \dfrac{3.16}{1.9054\ldots} \times 10^{5} = 1.6586\ldots \times 10^{5}$. **M1**

Expressing as a power of ten: $\log_{10}(1.6586\ldots \times 10^{5}) = 5.2197\ldots \approx 5.22$, so ratio $= 10^{5.22}$ (2 d.p.). **A1**

Q23 — Index Equation Reversal and Domain Critique HARD

(a) At $t = 12$, $m = 159$ g; substituting into the model and solving for k provides the printed target. **M1**

$$159 = 250 \times k^{12} \implies k^{12} = \frac{159}{250} = 0.636 \implies k = 0.636^{1/12}$$

GDC: $0.636^{(1/12)} = 0.96299\ldots$, so $k = 0.963$ (3 s.f.) **AG**

Warning: The unrounded value $0.96299\ldots$ must appear before rounding to 0.963; the printed target is worth nothing without showing the route.

(b) We need the smallest integer t for which $250 \times 0.963^{t} < 100$. Set up the equation $250 \times 0.963^{t} = 100$ and solve. **M1**

GDC: Graph $\rightarrow$ Intersection of $y_1 = 250 \times 0.963^{x}$ and $y_2 = 100$; intersection at $x = 24.30\ldots$ days.

Since mass is decreasing, the mass first falls below 100 g at $t = 25$ days. **A1**

Warning: Always check the bracketing: $m(24) = 250 \times 0.963^{24} = 101.1\ldots > 100$ *and* $m(25) = 97.4\ldots < 100$, *confirming* $t = 25$ *days is correct. Rounding down to 24 is the trap.*

(c) As $t \to \infty$, $m \to 0$, which predicts the mass becomes negligibly small; however the model cannot account for atomic-scale lower limits on measurable mass, and exponential decay models lose physical validity once only a few atoms remain. **A1**

Q24 — Logarithm Formula Inversion and Combined Intensity HARD

(a) To find I_1, invert the decibel formula: $\dfrac{I_1}{I_0} = 10^{L/10}$. **M1**

$I_1 = I_0 \times 10^{94/10} = 1.00 \times 10^{-12} \times 10^{9.4} = 10^{-12+9.4} = 10^{-2.6}$
GDC: $10^{-2.6} = 2.5118\ldots \times 10^{-3}$, so $I_1 = 2.51 \times 10^{-3}$ W m^{-2} (3 s.f.) **A1**

Warning: $I_0 = 10^{-12}$ *must be included; omitting it and computing* $10^{9.4}$ *alone is the flagged trap.*

(b) Combined intensity $I_{\text{total}} = 2I_1 = 2 \times 2.5118\ldots \times 10^{-3} = 5.0237\ldots \times 10^{-3}$ W m^{-2}. **M1**

$$L_{\text{total}} = 10\log_{10}\left(\frac{5.0237\ldots \times 10^{-3}}{1.00 \times 10^{-12}}\right) = 10\log_{10}(5.0237\ldots \times 10^{9}) = 10 \times 9.7010\ldots = 97.010\ldots$$

$$L_{\text{total}} = 97.0 \text{ dB (1 d.p.)}$$

A1

(c) Increase $= 97.0 - 94.0 = 3.0$ dB. **A1**

Q25 — Bounds of a Product and Certainty Judgement HARD

(a) Since l is measured to the nearest metre, its upper bound is 84.5 m; since w is measured to the nearest metre, its lower bound is 36.5 m. **A1**

(b) To find the upper bound of the area, pair the upper bound of l with the upper bound of w; for the lower bound, pair the lower bounds. **M1**

$A_{\text{max}} = 84.5 \times 37.5 = 3168.75 \approx 3169$ m^2 (4 s.f.);
$A_{\text{min}} = 83.5 \times 36.5 = 3047.75 \approx 3048$ m^2 (4 s.f.) **A1**

Warning: Pairing upper l *with lower* w *(or vice versa) does not give a true bound — both bounds of each variable must be matched correctly.*

(c) The lower bound of A is 3047.75 m$^2 > 3000$ m^2. **R1**
Since even the minimum possible area exceeds 3000 m^2, the surveyor's claim is certainly correct. **A1**

Q26 — Percentage Error with Chained Approximation Trap HARD

(a) Using $\sqrt{2} \approx 1.4$: estimate $= \dfrac{200}{1.4} = 142.857\ldots \approx 142.9$ (4 s.f.), or equivalently $\dfrac{1000}{7}$. **A1**

(b) The exact value is $Q_{\text{exact}} = \dfrac{200}{\sqrt{2}} = 100\sqrt{2} = 141.421\ldots$ **M1**

$$\%\,\text{error} = \frac{|142.857\ldots - 141.421\ldots|}{141.421\ldots} \times 100 = \frac{1.4359\ldots}{141.421\ldots} \times 100 = 1.015\ldots\,\% \approx 1.0\%\ \text{(2 s.f.)}$$

A1

Warning: The exact value $141.421\ldots$ must appear in the denominator; using the approximate value 142.9 in the denominator is the flagged percentage-error trap.

(c) $P = \dfrac{200}{\sqrt{2} - 0.01} = \dfrac{200}{1.41421\ldots - 0.01} = \dfrac{200}{1.40421\ldots} = 142.432\ldots \approx 142.43$ (5 s.f.) **M1**

$$\%\,\text{error} = \frac{|142.857\ldots - 142.432\ldots|}{142.432\ldots} \times 100 = \frac{0.4246\ldots}{142.432\ldots} \times 100 = 0.2980\ldots\,\% \approx 0.30\%\ \text{(2 s.f.)}$$

A1

Warning: Use the exact value of P (not Q) in the denominator when computing the percentage error for part (c).

Q27 — Two-Parameter Index Model Reversal and Threshold Search HARD

(a) At $t = 0$: $N = a \times 10^0 = a = 500$. **A1**

(b) At $t = 6$: $500 \times 10^{6b} = 32\,000 \implies 10^{6b} = 64 \implies 6b = \log_{10}(64)$, so $b = \dfrac{\log_{10}(64)}{6}$. **M1**

GDC: $\log_{10}(64) = 1.80617\ldots$; $b = \dfrac{1.80617\ldots}{6} = 0.30102\ldots \approx 0.301$ (3 s.f.) **AG / A1**

Warning: The step $6b = \log_{10}(64)$ must be shown explicitly before evaluating; asserting $b = 0.301$ without the log step forfeits the method mark.

(c) Solve $500 \times 10^{0.30102\ldots \times t} = 1\,000\,000$, i.e. $10^{0.30102\ldots\, t} = 2000$. **M1**

Using the precise (unrounded) $b = 0.301030\ldots$: $t = \dfrac{\log_{10}(2000)}{0.301030\ldots} = 10.9658\ldots$
N first exceeds $1\,000\,000$ at $t = 10.97$ hours (2 d.p.) **A1**

Warning: Using the rounded $b = 0.301$ (rather than the full-precision value) introduces premature-rounding error and can shift the crossing time in the third significant figure; always solve with the stored, unrounded value of b. Back-substitution confirms: $N(10.97) \approx 1\,002\,900 > 1\,000\,000$, while $N(10.96) \approx 998\,700 < 1\,000\,000$.

Q28 — Bounds and Cube-Root Reversal HARD

(a) Since $r = 2.3$ cm is measured to the nearest 0.1 cm, the lower bound of r is 2.25 cm. **A1**

(b) The upper bound of V uses the upper bound of $r = 2.35$ cm; the lower bound of V uses the lower bound $r = 2.25$ cm. **M1**

$V_{\text{max}} = \frac{4}{3}\pi(2.35)^3 = \frac{4}{3}\pi \times 12.977\ldots = 54.357\ldots \approx 54.36$ cm^3 (4 s.f.);

$V_{\text{min}} = \frac{4}{3}\pi(2.25)^3 = \frac{4}{3}\pi \times 11.390\ldots = 47.713\ldots \approx 47.71$ cm^3 (4 s.f.) **A1**

Warning: The cube means a small change in r has a large effect on V; using the rounded value $r = 2.3$ for both bounds is the flagged trap.

(c) The cube has $s^3 = V$, so $s = V^{1/3}$. Using $V_{\text{max}} = 54.357\ldots$ cm^3: **M1**

$s_{\text{max}} = (54.357\ldots)^{1/3} = 3.7889\ldots \approx 3.79$ cm (3 s.f.) **A1**

Warning: Use the full unrounded V_{max} inside the cube root; rounding V_{max} to 4 s.f. before taking the root introduces premature-rounding error.

Q29 — Earthquake Magnitude Formula Inversion and Ratio HARD

(a) First evaluate $E_0 = 10^{4.4} = 25\,118.8\ldots$ J. Then substitute $E_1 = 2.00 \times 10^{14}$: **M1**

$$M_1 = \frac{2}{3}\log_{10}\left(\frac{2.00 \times 10^{14}}{10^{4.4}}\right) = \frac{2}{3}\log_{10}(2.00 \times 10^{9.6}) = \frac{2}{3}(9.6 + \log_{10} 2.00) = \frac{2}{3}(9.6 + 0.30103)$$

$$= \frac{2}{3}(9.90103) = 6.6006\ldots$$

$M_1 = 6.60$ (2 d.p.) **A1**

(b) Invert the formula: $\frac{E_2}{E_0} = 10^{3M_2/2}$, so $E_2 = E_0 \times 10^{3\times 7.20/2} = 10^{4.4} \times 10^{10.8} = 10^{15.2}$. **M1**

GDC: $10^{15.2} = 1.58489\ldots \times 10^{15}$, so $E_2 = 1.58 \times 10^{15}$ J (3 s.f.) **A1**

Warning: The factor $\frac{2}{3}$ in the formula means inverting gives an exponent of $\frac{3}{2}M$, not $\frac{1}{2}M$; confusing these is the flagged trap.

(c) $\frac{E_2}{E_1} = \frac{1.58489\ldots \times 10^{15}}{2.00 \times 10^{14}} = \frac{15.8489\ldots}{2.00} = 7.924\ldots \approx 8$ (nearest whole number). **A1**

Q30 — Polynomial Roots via GDC and Two-Solution Threshold HARD

(a) Enter $f(x) = 2x^3 - 5x^2 - 4x + 2$ into the GDC.
GDC: Graph → x-intercepts (zeros) of $y = 2x^3 - 5x^2 - 4x + 2$; three real roots found. **M1**

Unrounded: $x = -0.9092\ldots$, $x = 0.3608\ldots$, $x = 3.0485\ldots$
Roots: $x = -0.909$, $x = 0.361$, $x = 3.05$ (all 3 s.f.) **A1**

Warning: All three roots must be stated; quoting only two forfeits the accuracy mark.

(b) To confirm a root between 3 and 4, evaluate f at both endpoints and apply the sign-change argument. **M1**

$f(3) = 2(27) - 5(9) - 4(3) + 2 = 54 - 45 - 12 + 2 = -1$;
$f(4) = 2(64) - 5(16) - 4(4) + 2 = 128 - 80 - 16 + 2 = 34$
Since $f(3) = -1 < 0$ and $f(4) = 34 > 0$, there is a sign change, so a root lies strictly between $x = 3$ and $x = 4$ (confirmed by the GDC root $x = 3.05$). **A1**

Warning: Neither endpoint is itself a root here ($f(3) \neq 0$, $f(4) \neq 0$), so the sign-change argument is valid; always check the endpoints are not themselves roots before citing the Intermediate Value Theorem.

(c) $f(x) = k$ has exactly two real solutions when k equals a local maximum or local minimum value of f. GDC: Graph $\rightarrow$ Maximum/Minimum of $y = 2x^3 - 5x^2 - 4x + 2$; solving $f'(x) = 6x^2 - 10x - 4 = 0$ gives $x = -\frac{1}{3}$ and $x = 2$, with local max $f(-\frac{1}{3}) = \frac{73}{27} = 2.70$ (3 s.f.) and local min $f(2) = -10.0$ (3 s.f.)
The two values are $k = 2.70$ and $k = -10.0$ (3 s.f.). **A1**