IB Math AI SL — Topic 1

Number Toolkit

Standard Form & Indices · Logarithms · Bounds & Percentage Error · GDC Equations

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:______________________________ Date:_______________

Score:__________/120

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Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. A graphic display calculator is required throughout; support GDC answers with the values entered and the feature used.

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Key Formulae

- **Standard form:** $a \times 10^k$, where $1 \le a < 10$ and $k \in \mathbb{Z}$
- **Laws of indices:** $a^m \times a^n = a^{m+n}$; $\quad a^m \div a^n = a^{m-n}$; $\quad (a^m)^n = a^{mn}$; $\quad a^0 = 1$; $\quad a^{-n} = \dfrac{1}{a^n}$
- **Logarithm definition:** $\log_{10} x = y \iff 10^y = x$
- **Supplied scale formula (e.g. sound level):** $L = 10\log_{10}\left(\dfrac{I}{I_0}\right)$, where I_0 is a reference intensity
- **Upper & lower bounds:** for a value v measured to the nearest unit u, $\quad v - \frac{u}{2} \le$ true value $< v + \frac{u}{2}$
- **Bounds of a quotient:** $\left(\dfrac{a}{b}\right)_{\text{max}} = \dfrac{a_{\text{max}}}{b_{\text{min}}}$, $\quad \left(\dfrac{a}{b}\right)_{\text{min}} = \dfrac{a_{\text{min}}}{b_{\text{max}}}$
- **Percentage error:** $\%$ error $= \dfrac{|v_A - v_E|}{|v_E|} \times 100\%$, where v_E is the *exact* value

GDC Tips

- **Evaluating powers and standard form:** press `[2nd]` `[EE]` (or $\times 10^x$) to enter $a \times 10^k$ directly; confirm the display shows the correct exponent before recording your answer.
- **Solving equations / finding intersections:** enter both sides as Y_1 and Y_2 in `[Y=]`, then use `[2nd]` `[CALC]` $\rightarrow$ `5: intersect`; move the cursor near each root and press `ENTER` three times — repeat for every root.
- **Evaluating $\log_{10}$:** press `[LOG]` then type the argument and close the bracket; to invert (find x given L) evaluate `10^(value)`.
- **Storing unrounded intermediates:** after a calculation press `[STO→]` `[A]` (or any letter) to save the full-precision value; recall it with `[ALPHA]` `[A]` in subsequent steps to avoid premature rounding.
- **Checking bounds on a quotient:** store a_{max}, a_{min}, b_{max}, b_{min} in variables A–D, then compute `A/D` (upper bound) and `C/B` (lower bound) directly, reading each to 3 s.f. only at the final step.
- **Polynomial roots:** for a polynomial equation set equal to zero, enter the left-hand side as Y_1 and use `[2nd]` `[CALC]` $\rightarrow$ `2: zero`; bracket each root separately with left-bound and right-bound prompts.

Common Mistakes to Avoid

1. **Wrong denominator in percentage error.** Students divide by the *approximate* value v_A instead of the *exact* value v_E. Always place the exact (theoretical or given) value in the denominator: $\dfrac{|v_A - v_E|}{|v_E|} \times 100\%$.
2. **Standard form with a outside $[1, 10)$.** Writing 12.3×10^4 instead of 1.23×10^5 violates the definition. After any arithmetic on standard-form numbers, re-check that $1 \leq a < 10$ and adjust k accordingly.
3. **Pairing bounds incorrectly for a quotient.** To maximise a quotient, pair the *upper* bound of the numerator with the *lower* bound of the denominator — not the same-direction bounds. Swapping this pairing gives an answer that is actually the minimum.
4. **Premature rounding in chained calculations.** Rounding an intermediate result (e.g. a logarithm or a bound) and then using that rounded value in the next step accumulates error. Store the full GDC value in a variable and round only the *final* answer to the required accuracy.
5. **Wrong tail / direction when inverting a logarithmic scale.** When solving $L = 10\log_{10}(I/I_0)$ for I, students sometimes apply $\log_{10}$ a second time instead of raising 10 to the power. Always isolate the $\log_{10}$ term first, then exponentiate: $I = I_0 \times 10^{L/10}$.
6. **Bounds certainty judgement without naming the worst case.** A conclusion such as “the value certainly exceeds the limit” is unsupported unless the student explicitly states *which* bound (upper or lower) represents the worst case and compares *that* bound to the threshold. An R1 mark requires the comparison to be written before the conclusion is stated.

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Section A — Easy

EASY

1. The mass of a grain of sand is approximately 0.0000647 grams.
Write this number in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$. **[3 marks]**

2. The distance from Earth to the Sun is approximately 1.496×10^8 km.
Write this distance in full, as an ordinary number. **[1 marks]**
A spacecraft travels at 4.3×10^4 km per hour. Calculate how many hours it would take to travel from Earth to the Sun. Give your answer in standard form. **[2 marks]**

3. Simplify the following expression, giving your answer as a single power of 5.

$$\frac{5^6 \times 5^3}{5^4}$$

[3 marks]

4. The loudness of a sound, L decibels (dB), is given by the formula

$$L = 10\,\log_{10}\left(\frac{I}{I_0}\right)$$

where I is the sound intensity in watts per square metre ($\mathrm{W\,m^{-2}}$) and $I_0 = 1 \times 10^{-12}$ $\mathrm{W\,m^{-2}}$.
A classroom has a sound intensity of $I = 1 \times 10^{-5}$ $\mathrm{W\,m^{-2}}$.
Calculate the loudness of the sound in the classroom. **[3 marks]**

5. A farmer measures the length of a field as 148 m, rounded to the nearest metre.
Write down the upper bound and the lower bound of the length of the field. **[2 marks]**
Hence, write down the upper bound for the perimeter of a square field with this measured side length. **[1 marks]**

6. A student estimates the area of a circular pond by rounding the radius to 1 significant figure before calculating.
The actual radius of the pond is $r = 6.3$ m. The student uses $r = 6$ m in their calculation.
The area of a circle is $A = \pi r^2$.
Calculate the percentage error in the student's estimate of the area. Give your answer correct to 2 decimal places. **[3 marks]**

7. The table shows the results of measuring the wingspan, w cm, of five butterflies.

Butterfly	1	2	3	4	5
Wingspan, w (cm)	4.2	3.8	5.1	4.6	3.9

Each wingspan is measured to the nearest 0.1 cm.
Write down the lower bound of the wingspan of butterfly 3. **[1 marks]**
Write down the upper bound of the total wingspan of all five butterflies. **[2 marks]**

8. Use a GDC to solve the equation

$$3x^2 - 5x - 7 = 0.$$

Give both solutions correct to 3 significant figures. **[3 marks]**

9. Evaluate $5^0 + 4^{-2} + 2^3$.
Give your answer as an exact fraction. **[3 marks]**

10. The approximate population of a city is given as $2\,400\,000$.
Write this number in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$. **[1 marks]**
A second city has a population of 8.1×10^5. Calculate the combined population of the two cities, giving your answer in standard form. **[2 marks]**

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Section B — Medium

MEDIUM

11. The distance from Earth to the star Proxima Centauri is approximately 4.02×10^{13} km. The distance from Earth to the star Sirius is approximately 8.13×10^{13} km.

(a) Write down the combined distance from Earth to Proxima Centauri and from Earth to Sirius, giving your answer in standard form. **[2 marks]**

(b) The speed of light is 9.461×10^{12} km per year. Find how many times greater the distance to Sirius is compared to the distance light travels in one year, giving your answer to 3 significant figures. **[2 marks]**

12. A biologist measures the mass of a bacterium as 3.8×10^{-12} g, rounded to 2 significant figures.

(a) Write down the upper and lower bounds of this measurement. **[2 marks]**

(b) The biologist claims the mass is less than 3.85×10^{-12} g. Determine whether this claim is certainly true. Justify your answer. **[2 marks]**

13. A scientist uses the formula for sound intensity level:

$$L = 10\log_{10}\left(\frac{I}{I_0}\right)$$

where L is the sound level in decibels (dB), I is the intensity in W m^{-2}, and $I_0 = 1 \times 10^{-12}$ W m^{-2} is the reference intensity.
A vacuum cleaner produces a sound level of 75 dB. Find the intensity I of the vacuum cleaner, giving your answer in standard form to 3 significant figures. **[4 marks]**

14. A rectangular field is measured as 84 m long and 37 m wide, each measurement rounded to the nearest metre.

(a) Write down the upper bound of the length and the lower bound of the width. **[2 marks]**

(b) Find the upper bound of the area of the field, giving your answer in m^2. **[2 marks]**

15. A storage tank contains 2.56×10^4 litres of water. A pump removes water at a rate of 3.20×10^2 litres per hour.

(a) Find the number of hours it takes to empty the tank completely, giving your answer as an exact value. **[2 marks]**

(b) Write your answer to part (a) to 2 significant figures. **[2 marks]**

quantity	value
initial volume (litres)	2.56×10^4
pump rate (litres/hour)	3.20×10^2

16. A student estimates the area of a circular pond by rounding the radius to 1 significant figure. The actual radius of the pond is 6.4 m.

(a) Write down the radius rounded to 1 significant figure. **[1 marks]**

(b) Find the percentage error in the area when the rounded radius is used instead of the actual radius. Give your answer to 3 significant figures. **[3 marks]**

17. Solve the following equation, giving all solutions to 3 significant figures.

$$2x^3 - 5x^2 - 3x + 4 = 0$$

[4 marks]

18. The table shows the population of four cities and their areas.

city	population	area (km^2)
Alderton	$1\,240\,000$	312
Breva	$875\,000$	204
Calston	$2\,050\,000$	487
Dorwich	$640\,000$	98

(a) Write the population of Calston in standard form. **[1 marks]**

(b) Find the population density (people per km^2) of Dorwich, giving your answer to 3 significant figures. **[2 marks]**

(c) Find the percentage error if a student approximates Breva's population as 9×10^5. Give your answer to 3 significant figures. **[1 marks]**

19. The pH of a solution is given by the formula

$$\text{pH} = -\log_{10}[\text{H}^+]$$

where $[\text{H}^+]$ is the hydrogen ion concentration in mol L^{-1}.

(a) A fruit juice has $[\text{H}^+] = 2.5 \times 10^{-4}$ mol L^{-1}. Find the pH of the fruit juice, giving your answer to 3 significant figures. **[2 marks]**

(b) A cleaning solution has a pH of 11.3. Find its hydrogen ion concentration, giving your answer in standard form to 3 significant figures. **[2 marks]**

20. Find the values of x that satisfy the equation

$$3^x = 5x + 2$$

giving your answers to 3 significant figures. **[4 marks]**

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Section C — Hard

HARD

21. The speed of light in a vacuum is $c = 2.998 \times 10^8$ m s^{-1}. A radio telescope detects a signal from a galaxy whose distance from Earth is measured as $d = 1.54 \times 10^{23}$ m.

(a) Show that the time, T seconds, for the signal to travel this distance satisfies $T \approx 5.14 \times 10^{14}$ s. **[2 marks]**

(b) The distance d is measured to the nearest 10^{21} m. Find the upper and lower bounds of T, giving your answers in standard form to three significant figures. **[2 marks]**

(c) Hence determine, with justification, whether the signal travel time certainly exceeds 5.10×10^{14} s. **[1 marks]**

22. The pH of a liquid is given by the formula

$$\text{pH} = -\log_{10}[\text{H}^+]$$

where $[\text{H}^+]$ is the hydrogen ion concentration in mol L^{-1}.

(a) A fruit juice has $[\text{H}^+] = 3.16 \times 10^{-4}$ mol L^{-1}. Find its pH, correct to two decimal places. **[1 marks]**

(b) A second liquid has pH $= 8.72$. Find its hydrogen ion concentration, giving your answer in standard form to three significant figures. **[2 marks]**

(c) Find the ratio of the hydrogen ion concentration of the fruit juice to that of the second liquid, giving your answer as a power of ten to two decimal places. **[2 marks]**

23. A scientist records the mass m grams of a radioactive sample at time t days. The model is

$$m = 250 \times k^t$$

where k is a positive constant. The table shows two recorded values.

time, t (days)	0	12
mass, m (g)	250	183

(a) Show that $k = 0.963$ correct to three significant figures. **[2 marks]**

(b) Find the time at which the mass first falls below 100 g, giving your answer to the nearest day. **[2 marks]**

(c) Explain why the model is not appropriate for very large values of t. **[1 marks]**

24. The sound intensity level L in decibels is given by

$$L = 10\log_{10}\left(\frac{I}{I_0}\right)$$

where I is the sound intensity in $\mathrm{W\,m^{-2}}$ and $I_0 = 1.00 \times 10^{-12}$ $\mathrm{W\,m^{-2}}$.
A loudspeaker produces a sound level of 94 dB at a distance of 1 m. A second loudspeaker of identical type is placed alongside the first. The combined intensity is exactly double.

(a) Find the intensity I_1 of one loudspeaker at 1 m, giving your answer in standard form to three significant figures. **[2 marks]**

(b) Find the combined sound level of the two loudspeakers, correct to one decimal place. **[2 marks]**

(c) State the increase in sound level caused by doubling the number of identical sources. **[1 marks]**

25. A surveyor measures the length of a rectangular field as $l = 84$ m and the width as $w = 37$ m, both to the nearest metre.

(a) Write down the upper bound of l and the lower bound of w. **[1 marks]**

(b) Find the upper bound and lower bound of the area $A = lw$, giving your answers to four significant figures. **[2 marks]**

(c) The surveyor claims the area exceeds $3\,000\ \text{m}^2$. Determine, with justification, whether this claim is certainly correct. **[2 marks]**

26. A student uses the approximation $\sqrt{2} \approx 1.4$ to estimate the value of

$$Q = \frac{200}{\sqrt{2}}.$$

(a) Write down the student's estimate of Q. **[1 marks]**

(b) Calculate the percentage error in the student's estimate, correct to two significant figures. **[2 marks]**

(c) A second quantity is $P = \dfrac{200}{\sqrt{2} - 0.01}$. Using the exact value of $\sqrt{2}$, find P correct to five significant figures. Hence find the percentage error if the student uses their estimate of Q as an approximation for P as well, correct to two significant figures. **[2 marks]**

27. The number of bacteria N in a culture at time t hours satisfies

$$N = a \times 10^{bt}$$

where a and b are constants with $a > 0$ and $b > 0$. At $t = 0$ there are 500 bacteria, and at $t = 6$ there are $32\,000$ bacteria.

(a) Write down the value of a. **[1 marks]**

(b) Show that $b = \dfrac{\log_{10}(64)}{6}$, and hence find b correct to three significant figures. **[2 marks]**

(c) Find the time at which N first exceeds $1\,000\,000$, giving your answer in hours correct to two decimal places. **[2 marks]**

28. A spherical droplet of oil has radius r cm, measured to the nearest 0.1 cm, with $r = 2.3$ cm.

(a) State the lower bound of r. **[1 marks]**

(b) The volume of the droplet is $V = \dfrac{4}{3}\pi r^3$ cm^3. Find the upper bound and lower bound of V, giving your answers correct to four significant figures. **[2 marks]**

(c) The droplet is melted and recast into a cube of side length s cm. Using the upper bound of V, find the upper bound of s, correct to three significant figures. **[2 marks]**

29. The magnitude of an earthquake is given by the formula

$$M = \frac{2}{3}\log_{10}\left(\frac{E}{E_0}\right)$$

where E is the energy released in joules and $E_0 = 1.00 \times 10^{4.4}$ J.

(a) An earthquake releases energy $E_1 = 2.00 \times 10^{14}$ J. Find its magnitude M_1, correct to two decimal places. **[2 marks]**

(b) A second earthquake has magnitude $M_2 = 7.20$. Find the energy E_2 it releases, giving your answer in standard form to three significant figures. **[2 marks]**

(c) Find the ratio $E_2 : E_1$, correct to the nearest whole number. **[1 marks]**

30. A polynomial equation is given by

$$f(x) = 2x^3 - 5x^2 - 4x + 3.$$

(a) Using a GDC, find all three roots of $f(x) = 0$, correct to three significant figures. **[2 marks]**

(b) Show that one root lies between $x = 3$ and $x = 4$ by evaluating $f(3)$ and $f(4)$. **[2 marks]**

(c) The equation $f(x) = k$ has exactly two real solutions. Using your GDC, find the two possible values of k, correct to three significant figures. **[1 marks]**

Worked Solutions

Q1 — Standard form of a small decimal [EASY]

We write 0.0000647 in standard form by identifying how many places the decimal point must move to place the first significant digit immediately after it.
Moving the decimal point 5 places to the right gives 6.47, so the power of 10 is -5. **M1**

$$0.0000647 = 6.47 \times 10^{-5}$$

A1 ($a = 6.47$) **A1** ($k = -5$)

Warning: Both a and k must be stated; writing 64.7×10^{-6} does not satisfy $1 \le a < 10$ and earns no credit for a.

Q2 — Dividing numbers in standard form [EASY]

Distance as an ordinary number:
$1.496 \times 10^8 = 149\,600\,000$ km **A1**
Time calculation:
Dividing the two standard-form numbers gives the number of hours required.

$$t = \frac{1.496 \times 10^8}{4.3 \times 10^4}$$

M1

GDC: type $(1.496\text{E}8) \div (4.3\text{E}4)$, read display.
Unrounded value: $3480.232\ldots$

$$t \approx 3.48 \times 10^3 \text{ hours}$$

A1

Warning: The answer must be expressed in standard form with $1 \le a < 10$; writing 34.8×10^2 loses the accuracy mark.

Q3 — Laws of indices [EASY]

Apply the multiplication law first: add the indices in the numerator, then apply the division law by subtracting the index in the denominator.

$$\frac{5^6 \times 5^3}{5^4} = \frac{5^{6+3}}{5^4} = \frac{5^9}{5^4}$$

M1

$$= 5^{9-4} = 5^5$$

A1 A1

Warning: Do not multiply the indices in the numerator ($5^{6\times3}$); the bases are the same so indices are added, not multiplied.

Q4 — Logarithm formula for sound level [EASY]

Substitute $I = 1 \times 10^{-5}$ and $I_0 = 1 \times 10^{-12}$ into the given formula.

$$L = 10\log_{10}\left(\frac{1\times10^{-5}}{1\times10^{-12}}\right)$$

M1

Evaluate the ratio inside the logarithm:

$$\frac{1\times10^{-5}}{1\times10^{-12}} = 1\times10^{7}$$

A1

GDC: type $10 \times \log(1\text{E}7)$, read display.

$$L = 10\times\log_{10}(10^7) = 10\times7 = 70\text{ dB}$$

A1

Warning: Use $\log_{10}$ (common logarithm), not ln (natural logarithm); on most GDCs the `log` key gives base 10.

Q5 — Upper and lower bounds of a length [EASY]

A measurement of 148 m rounded to the nearest metre has a precision of ±0.5 m.
Lower bound $= 148 - 0.5 = 147.5$ m; Upper bound $= 148 + 0.5 = 148.5$ m. **A1 A1**
For the perimeter of a square, use the upper bound of the side length to obtain the largest possible perimeter.

$$\text{Upper bound of perimeter} = 4\times148.5 = 594\text{ m}$$

A1

Warning: The upper bound is 148.5 m, not 149 m; adding 1 to the rounded value gives an incorrect bound.

Q6 — Percentage error in area estimate [EASY]

Calculate the exact (actual) area using $r = 6.3$ m.

$$A_{\text{exact}} = \pi\times(6.3)^2 = 124.689\ldots\text{ m}^2$$

M1

Calculate the approximate area using the student's rounded value $r = 6$ m.

$$A_{\text{approx}} = \pi \times (6)^2 = 113.097\ldots \text{ m}^2$$

Apply the percentage error formula, dividing by the exact value.

$$\%\text{ error} = \frac{|A_{\text{approx}} - A_{\text{exact}}|}{A_{\text{exact}}} \times 100 = \frac{|113.097\ldots - 124.689\ldots|}{124.689\ldots} \times 100$$

M1

GDC: evaluate the expression above.
Unrounded: $9.2960\ldots\%$

$$\%\text{ error} \approx 9.30\%$$

A1

Warning: Always divide by the exact value (A_{exact}), not the approximate value; dividing by A_{approx} gives a different and incorrect percentage error.

Q7 — Upper and lower bounds from a table [EASY]

Each wingspan is measured to the nearest 0.1 cm, so the precision is ± 0.05 cm.
Lower bound of butterfly 3:
The measured value is 5.1 cm, so the lower bound is $5.1 - 0.05 = 5.05$ cm. **A1**
Upper bound of total wingspan:
To find the largest possible total, take the upper bound of each individual measurement and add them.

$$\text{Upper bounds: } 4.25 + 3.85 + 5.15 + 4.65 + 3.95$$

M1

$$= 21.85 \text{ cm}$$

A1

Warning: Adding the upper bounds of each wingspan (not just adding 0.05 once to the sum of measured values) is required; the error accumulates across all five measurements.

Q8 — Solving a quadratic with a GDC [EASY]

State the equation to be solved and use the GDC to find both roots.

$$3x^2 - 5x - 7 = 0$$

M1

GDC: Equation Solver (or Graph $\rightarrow$ find zeros): enter $3x^2 - 5x - 7$, find both x-intercepts.
Unrounded values: $x = 2.7967\ldots$ and $x = -0.7967\ldots$

$$x = 2.80 \quad \text{or} \quad x = -0.797 \quad \text{(both to 3 s.f.)}$$

A1 A1

Warning: A quadratic equation has two solutions; writing only one solution loses an accuracy mark.

Q9 — Evaluating negative and zero indices [EASY]

Evaluate each term separately using the index laws $a^0 = 1$ and $a^{-n} = \dfrac{1}{a^n}$.

$$5^0 = 1, \qquad 4^{-2} = \frac{1}{4^2} = \frac{1}{16}, \qquad 2^3 = 8$$

M1

Add the three terms, converting to a common denominator of 16.

$$1 + \frac{1}{16} + 8 = 9 + \frac{1}{16} = \frac{144}{16} + \frac{1}{16}$$

M1

$$= \frac{145}{16}$$

A1

Warning: $5^0 = 1$, not 0; any nonzero number raised to the power zero equals exactly 1.

Q10 — Adding numbers in standard form [EASY]

Write $2\,400\,000$ in standard form by positioning the decimal point after the first significant digit.

$$2\,400\,000 = 2.4 \times 10^6$$

A1

To add the two populations, convert both to ordinary numbers (or align powers on the GDC).

$$2.4 \times 10^6 + 8.1 \times 10^5 = 2\,400\,000 + 810\,000$$

M1

GDC: type $(2.4\text{E}6) + (8.1\text{E}5)$, read display in standard form.

$$= 3\,210\,000 = 3.21 \times 10^6$$

A1

Warning: Do not add $2.4 + 8.1$ and keep 10^6; the powers of 10 are different, so the coefficients cannot be added directly without first aligning the exponents.

Q11 — Standard form operations, two stars [MEDIUM]

(a) To find the combined distance, add the two values in standard form. Since both powers of 10 are equal (10^{13}), the coefficients are added directly.

$$(4.02 + 8.13) \times 10^{13} = 12.15 \times 10^{13}$$

Rewriting so that $1 \leq a < 10$, the combined distance is 1.215×10^{14} km. **M1**

$$= 1.22 \times 10^{14} \text{ km (3 s.f.)}$$

A1

Warning: The answer 12.15×10^{13} is NOT in standard form; the coefficient must satisfy $1 \leq a < 10$.

(b) To find how many times greater the distance to Sirius is compared to one light-year, divide the two standard-form quantities.

$$\frac{8.13 \times 10^{13}}{9.461 \times 10^{12}} = \frac{8.13}{9.461} \times 10^{13-12} = 0.8593\ldots \times 10^{1}$$

M1

Unrounded value: $8.5934\ldots$, so Sirius is 8.59 times further than one light-year (3 s.f.). **A1**

Q12 — Bounds of a measured value and certainty judgement [MEDIUM]

(a) The mass is given as 3.8×10^{-12} g rounded to 2 significant figures. The last significant figure is in the first decimal place of the coefficient (the tenths place), so half a unit in that place is 0.05×10^{-12}.

Lower bound: 3.75×10^{-12} g **A1**

Upper bound: 3.85×10^{-12} g **A1**

(b) The upper bound of the measurement is exactly 3.85×10^{-12} g, meaning the mass could equal this value. Since the claim is that the mass is *strictly less than* 3.85×10^{-12} g, this cannot be guaranteed. **R1**

The claim is NOT certainly true, because the mass could be as large as 3.85×10^{-12} g. **A1**

Warning: The upper bound is not included in the possible values (the measurement rounds to 3.8), but the claim uses strict inequality "$<$"; the boundary case means the claim cannot be certified.

Q13 — Logarithm formula inversion for intensity [MEDIUM]

Substitute $L = 75$ and $I_0 = 1 \times 10^{-12}$ into the given formula and solve for I.

$$75 = 10 \log_{10}\left(\frac{I}{1 \times 10^{-12}}\right)$$

M1

Divide both sides by 10:

$$\log_{10}\left(\frac{I}{1 \times 10^{-12}}\right) = 7.5$$

M1

Exponentiate both sides (raise 10 to each side):

$$\frac{I}{1 \times 10^{-12}} = 10^{7.5} = 31\,622\,776\ldots$$

$$I = 10^{7.5} \times 10^{-12} = 10^{-4.5} = 3.16227\ldots \times 10^{-5}$$

A1

GDC: evaluate $10^{7.5} \times 10^{-12}$ on the home screen.

$$I = 3.16 \times 10^{-5} \text{ W m}^{-2} \text{ (3 s.f.)}$$

A1

Warning: Dividing by 10 *before* inverting the logarithm is essential; jumping straight to $I = 10^{75} \times 10^{-12}$ is a common error that costs both accuracy marks.

Q14 — Bounds of measurements and upper bound of area [MEDIUM]

(a) Each measurement is rounded to the nearest metre, so bounds are ± 0.5 m.
Upper bound of length: $84 + 0.5 = 84.5$ m **A1**
Lower bound of width: $37 - 0.5 = 36.5$ m **A1**
(b) To obtain the largest possible area, multiply the upper bound of the length by the upper bound of the width (both as large as possible gives the largest product).

$$\text{Upper bound of area} = 84.5 \times 37.5$$

M1

$$= 3168.75 \text{ m}^2$$

A1

Warning: The upper bound of the area uses upper bound $\times$ upper bound, NOT upper length $\times$ lower width. Pairing incorrectly is the common trap here.

Q15 — Standard form division and rounding [MEDIUM]

(a) To find the time to empty the tank, divide the total volume by the pump rate.

$$\text{time} = \frac{2.56 \times 10^4}{3.20 \times 10^2}$$

M1

$$= \frac{2.56}{3.20} \times 10^{4-2} = 0.8 \times 10^2 = 80 \text{ hours}$$

A1

(b) 80 written to 2 significant figures is 80 (the two significant figures are already present; the answer is exact). **A1**
Alternatively, writing 80 to 2 s.f. requires confirming both digits are significant: 8.0×10^1. **A1**
Warning: Students must recognise that 80 is already exact here; no rounding occurs. Writing 80 in standard form as 8.0×10^1 confirms 2 significant figures are intended.

Q16 — Percentage error in area from rounded radius [MEDIUM]

(a) Rounding 6.4 m to 1 significant figure gives $r_{\text{approx}} = 6$ m. **A1**
(b) Calculate the exact area and the approximate area, then apply the percentage error formula with the exact value in the denominator.
Exact area: $A_E = \pi \times (6.4)^2 = \pi \times 40.96 = 128.679\ldots$ m^2 **M1**
Approximate area: $A_A = \pi \times (6)^2 = \pi \times 36 = 113.097\ldots$ m^2

$$\%\text{ error} = \frac{|A_A - A_E|}{|A_E|} \times 100 = \frac{|113.097\ldots - 128.679\ldots|}{128.679\ldots} \times 100$$

M1

$$= \frac{15.582\ldots}{128.679\ldots} \times 100 = 12.1\% \text{ (3 s.f.)}$$

A1
Warning: Percentage error always divides by the *exact* value ($128.679\ldots$), never by the approximate value. Using the approximate area in the denominator is the trap.

Q17 — Solving a cubic equation by GDC [MEDIUM]

Enter the cubic equation into the GDC as $y_1 = 2x^3 - 5x^2 - 3x + 4$ and find the x-intercepts (zeros/roots).
GDC: Graph the function $y = 2x^3 - 5x^2 - 3x + 4$, then use Zeros (or Roots) feature to find each x-intercept, OR use Polynomial Solver: Algebra $\to$ Polynomial $\to$ degree 3, coefficients $2, -5, -3, 4$. **M1**
Three roots found (unrounded): $x = -0.80178\ldots,\ x = 0.67315\ldots,\ x = 3.62862\ldots$ **A1**
Rounded to 3 significant figures: **A1**

$$x = -0.802, \quad x = 0.673, \quad x = 3.63$$

A1
Warning: All three roots must be stated for full marks. Students who find only one or two roots by graphing a restricted window miss the remaining solutions. Confirm the number of roots by examining the graph over a wide x-range.

Q18 — Standard form, population density and percentage error [MEDIUM]

(a) The population of Calston is $2\,050\,000 = 2.05 \times 10^6$. **A1**

(b) Population density of Dorwich = population ÷ area.

$$\text{density} = \frac{640\,000}{98} = 6530.612\ldots \text{ people per km}^2$$

M1

$$= 6530 \text{ people per km}^2 \text{ (3 s.f.)}$$

A1

(c) The exact population of Breva is $875\,000$; the approximation is $9 \times 10^5 = 900\,000$.

$$\%\text{ error} = \frac{|900\,000 - 875\,000|}{875\,000} \times 100 = \frac{25\,000}{875\,000} \times 100 = 2.86\% \text{ (3 s.f.)}$$

A1

Warning: Percentage error divides by the exact value ($875\,000$), not by the approximation ($900\,000$). Reversing the denominator is the flagged trap.

Q19 — Logarithm pH formula, evaluation and inversion [MEDIUM]

(a) Substitute $[\text{H}^+] = 2.5 \times 10^{-4}$ into the given formula.

$$\text{pH} = -\log_{10}(2.5 \times 10^{-4})$$

M1

GDC: evaluate $-\log_{10}(2.5 \times 10^{-4})$ on the home screen.
Unrounded: pH $= 3.60206\ldots$

$$\text{pH} = 3.60 \text{ (3 s.f.)}$$

A1

(b) Set pH $= 11.3$ and solve for $[\text{H}^+]$.

$$11.3 = -\log_{10}[\text{H}^+] \implies \log_{10}[\text{H}^+] = -11.3$$

M1

$$[\text{H}^+] = 10^{-11.3} = 5.01187\ldots \times 10^{-12}$$

GDC: evaluate $10^{-11.3}$ on the home screen.

$$[\text{H}^+] = 5.01 \times 10^{-12} \text{ mol L}^{-1} \text{ (3 s.f.)}$$

A1

Warning: The negative sign in the formula means $\log_{10}[\text{H}^+] = -\text{pH}$, not $+\text{pH}$. A sign error when inverting produces a wildly wrong answer.

Q20 — Solving an exponential-linear equation by GDC [MEDIUM]

Rewrite the equation as $3^x - 5x - 2 = 0$ and find the zeros, OR graph $y_1 = 3^x$ and $y_2 = 5x + 2$ and find intersection points.
GDC: Enter $y_1 = 3^x$ and $y_2 = 5x + 2$. Use Intersection feature (G-Solve $\to$ Intersect or 2nd $\to$ Calc $\to$ Intersect on a suitable window such as $-2 \le x \le 4$). **M1**
Two intersection points are found. Unrounded x-coordinates: $x = -0.33186\ldots$ and $x = 2.9575\ldots$ **A1**
Rounded to 3 significant figures: **A1**

$$x = -0.332 \quad \text{and} \quad x = 2.96$$

A1
Warning: Students who use only a small window (e.g. $0 \le x \le 3$) may miss the negative solution. Both solutions must be stated. Confirm by checking the graph extends into negative x values.

Q21 — Standard form bounds and certainty judgement [HARD]

(a) To find T we divide distance by speed; the calculation is set up so that the printed value is the target (AG).

$T = \dfrac{d}{c} = \dfrac{1.54 \times 10^{23}}{2.998 \times 10^{8}}$ **M1**

$T = \dfrac{1.54}{2.998} \times 10^{23-8} = 0.51367\ldots \times 10^{15} = 5.1367\ldots \times 10^{14}$ s $\approx 5.14 \times 10^{14}$ s **AG**

Warning: The unrounded value $5.1367\ldots \times 10^{14}$ must be shown before stating the rounded target; omitting this loses the method mark.
(b) Since d is measured to the nearest 10^{21} m, its bounds are $d_{\text{max}} = 1.545 \times 10^{23}$ m and $d_{\text{min}} = 1.535 \times 10^{23}$ m. The speed c is exact, so bounds of T come from bounds of d alone. **M1**

$T_{\text{max}} = \dfrac{1.545 \times 10^{23}}{2.998 \times 10^{8}} = 5.1534\ldots \times 10^{14} \approx 5.15 \times 10^{14}$ s;

$T_{\text{min}} = \dfrac{1.535 \times 10^{23}}{2.998 \times 10^{8}} = 5.1200\ldots \times 10^{14} \approx 5.12 \times 10^{14}$ s **A1**

Warning: The upper bound of T uses the upper bound of d; swapping the pairing is the flagged trap.
(c) The lower bound $T_{\text{min}} = 5.12 \times 10^{14}$ s $> 5.10 \times 10^{14}$ s, so even in the worst case the travel time exceeds 5.10×10^{14} s. **R1**

Therefore the signal travel time certainly exceeds 5.10×10^{14} s.

Q22 — Logarithm formula inversion and ratio [HARD]

(a) Substituting directly into the formula gives the pH; no inversion needed here.

$\text{pH} = -\log_{10}(3.16 \times 10^{-4}) = -(\log_{10} 3.16 + \log_{10} 10^{-4}) = -(0.4997\ldots - 4) = 3.4997\ldots$

GDC: evaluate $-\log_{10}(3.16 \times 10^{-4})$ directly. pH $= 3.50$ (2 d.p.) **A1**
(b) To find $[\text{H}^+]$ we invert the logarithm formula: $[\text{H}^+] = 10^{-\text{pH}}$. **M1**

$[\text{H}^+] = 10^{-8.72}$; GDC: $10^{-8.72} = 1.9054\ldots \times 10^{-9}$

$[\text{H}^+] = 1.91 \times 10^{-9}$ mol L^{-1} (3 s.f.) **A1**

Warning: The answer must be expressed in standard form with $1 \le a < 10$; writing 19.1×10^{-10} loses the accuracy mark.

(c) The ratio is $\dfrac{3.16 \times 10^{-4}}{1.9054\ldots \times 10^{-9}} = \dfrac{3.16}{1.9054\ldots} \times 10^5 = 1.6586\ldots \times 10^5$. **M1**

Expressing as a power of ten: $\log_{10}(1.6586\ldots \times 10^5) = 5.2197\ldots \approx 5.22$, so ratio $= 10^{5.22}$ (2 d.p.). **A1**

Q23 — Index equation reversal and domain critique [HARD]

(a) At $t = 12$, $m = 183$ g; substituting into the model and solving for k provides the printed target. **M1**

$$183 = 250 \times k^{12} \Rightarrow k^{12} = \frac{183}{250} = 0.732 \Rightarrow k = 0.732^{1/12}$$

GDC: $0.732^{(1/12)} = 0.96296\ldots$, so $k = 0.963$ (3 s.f.) **AG**

Warning: The unrounded value $0.96296\ldots$ must appear before rounding to 0.963; the printed target is worth nothing without showing the route.

(b) We need the smallest integer t for which $250 \times 0.963^t < 100$. Set up the equation $250 \times 0.963^t = 100$ and solve. **M1**

GDC: Graph $\to$ Intersection of $y_1 = 250 \times 0.963^x$ and $y_2 = 100$; intersection at $x = 24.86\ldots$ days.

Since mass is decreasing, the mass first falls below 100 g at $t = 25$ days. **A1**

Warning: Always check the bracketing: $m(24) = 250 \times 0.963^{24} = 101.6\ldots > 100$ and $m(25) = 97.9\ldots < 100$, confirming $t = 25$ days is correct. Rounding down to 24 is the trap.

(c) As $t \to \infty$, $m \to 0$, which predicts the mass becomes negligibly small; however the model cannot account for atomic-scale lower limits on measurable mass, and exponential decay models lose physical validity once only a few atoms remain. **A1**

Q24 — Logarithm formula, inversion, combined intensity [HARD]

(a) To find I_1, invert the decibel formula: $\dfrac{I_1}{I_0} = 10^{L/10}$. **M1**

$I_1 = I_0 \times 10^{94/10} = 1.00 \times 10^{-12} \times 10^{9.4} = 10^{-12+9.4} = 10^{-2.6}$

GDC: $10^{-2.6} = 2.5118\ldots \times 10^{-3}$, so $I_1 = 2.51 \times 10^{-3}$ W m^{-2} (3 s.f.) **A1**

Warning: $I_0 = 10^{-12}$ must be included; omitting it and computing $10^{9.4}$ alone is the flagged trap.

(b) Combined intensity $I_{\text{total}} = 2I_1 = 2 \times 2.5118\ldots \times 10^{-3} = 5.0237\ldots \times 10^{-3}$ W m^{-2}. **M1**

$$L_{\text{total}} = 10\log_{10}\left(\frac{5.0237\ldots \times 10^{-3}}{1.00 \times 10^{-12}}\right) = 10\log_{10}(5.0237\ldots \times 10^9) = 10 \times 9.7010\ldots = 97.010\ldots$$

$L_{\text{total}} = 97.0$ dB (1 d.p.) **A1**

(c) Increase $= 97.0 - 94.0 = 3.0$ dB. **A1**

Q25 — Bounds of a product and certainty judgement [HARD]

(a) Since l is measured to the nearest metre, its upper bound is 84.5 m; since w is measured to the nearest metre, its lower bound is 36.5 m. **A1**

(b) To find the upper bound of the area, pair the upper bound of l with the upper bound of w; for the lower bound, pair the lower bounds. **M1**

$A_{\text{max}} = 84.5 \times 37.5 = 3168.75 \approx 3169$ m^2 (4 s.f.);

$A_{\text{min}} = 83.5 \times 36.5 = 3047.75 \approx 3048$ m^2 (4 s.f.) **A1**

Warning: Pairing upper l with lower w (or vice versa) does not give a true bound — both bounds of each variable must be matched correctly.

(c) The lower bound of A is 3047.75 m^2 > 3000 m^2. **R1**

Since even the minimum possible area exceeds 3000 m^2, the surveyor's claim is certainly correct. **A1**

Q26 — Percentage error with chained approximation trap [HARD]

(a) Using $\sqrt{2} \approx 1.4$: estimate $= \dfrac{200}{1.4} = 142.857\ldots \approx 142.9$. The student's estimate of Q is **142.9** (4 s.f.) or equivalently $\dfrac{1000}{7}$. **A1**

(b) The exact value is $Q_{\text{exact}} = \dfrac{200}{\sqrt{2}} = 100\sqrt{2} = 141.421\ldots$ **M1**

$\%$ error $= \dfrac{|142.857\ldots - 141.421\ldots|}{141.421\ldots} \times 100 = \dfrac{1.4359\ldots}{141.421\ldots} \times 100 = 1.015\ldots\% \approx 1.0\%$ (2 s.f.) **A1**

Warning: The exact value $141.421\ldots$ must appear in the denominator; using the approximate value 142.9 in the denominator is the flagged percentage-error trap.

(c) $P = \dfrac{200}{\sqrt{2} - 0.01} = \dfrac{200}{1.41421\ldots - 0.01} = \dfrac{200}{1.40421\ldots} = 142.432\ldots \approx 142.43$ (5 s.f.) **M1**

$\%$ error $= \dfrac{|142.857\ldots - 142.432\ldots|}{142.432\ldots} \times 100 = \dfrac{0.4246\ldots}{142.432\ldots} \times 100 = 0.2980\ldots\% \approx 0.30\%$ (2 s.f.) **A1**

Warning: Use the exact value of P (not Q) in the denominator when computing the percentage error for part (c).

Q27 — Two-parameter index model, reversal and threshold search [HARD]

(a) At $t = 0$: $N = a \times 10^0 = a = 500$. **A1**

(b) At $t = 6$: $500 \times 10^{6b} = 32\,000 \Rightarrow 10^{6b} = 64 \Rightarrow 6b = \log_{10}(64)$, so $b = \dfrac{\log_{10}(64)}{6}$. **M1**

GDC: $\log_{10}(64) = 1.80617\ldots$; $b = \dfrac{1.80617\ldots}{6} = 0.30102\ldots \approx 0.301$ (3 s.f.) **AG / A1**

Warning: The step $6b = \log_{10}(64)$ must be shown explicitly before evaluating; asserting $b = 0.301$ without the log step forfeits the method mark.

(c) Solve $500 \times 10^{0.30102\ldots\times t} = 1\,000\,000$, i.e. $10^{0.30102\ldots\, t} = 2000$. **M1**

GDC: Graph $\to$ Intersection of $y_1 = 500 \times 10^{0.301t}$ and $y_2 = 1\,000\,000$; intersection at $t = 11.0038\ldots$

N first exceeds $1\,000\,000$ at $t = 11.00$ hours (2 d.p.) **A1**

Q28 — Bounds, cube-root reversal (melt and recast) [HARD]

(a) Since $r = 2.3$ cm is measured to the nearest 0.1 cm, the lower bound of r is **2.25** cm. **A1**

(b) The upper bound of V uses the upper bound of $r = 2.35$ cm; the lower bound of V uses the lower bound $r = 2.25$ cm. **M1**

$V_{\text{max}} = \frac{4}{3}\pi(2.35)^3 = \frac{4}{3}\pi \times 12.977\ldots = 54.357\ldots \approx 54.36$ cm^3 (4 s.f.);

$V_{\text{min}} = \frac{4}{3}\pi(2.25)^3 = \frac{4}{3}\pi \times 11.390\ldots = 47.713\ldots \approx 47.71$ cm^3 (4 s.f.) **A1**

Warning: The cube means a small change in r has a large effect on V; using the rounded value $r = 2.3$ for both bounds is the flagged trap.

(c) The cube has $s^3 = V$, so $s = V^{1/3}$. Using $V_{\text{max}} = 54.357\ldots$ cm^3: **M1**

$s_{\text{max}} = (54.357\ldots)^{1/3} = 3.7889\ldots \approx 3.79$ cm (3 s.f.) **A1**

Warning: Use the full unrounded V_{max} inside the cube root; rounding V_{max} to 4 s.f. before taking the root introduces premature-rounding error.

Q29 — Earthquake magnitude formula, inversion and ratio [HARD]

(a) First evaluate $E_0 = 10^{4.4} = 25118.8\ldots$ J. Then substitute $E_1 = 2.00 \times 10^{14}$: **M1**

$M_1 = \frac{2}{3}\log_{10}\left(\frac{2.00 \times 10^{14}}{10^{4.4}}\right) = \frac{2}{3}\log_{10}(2.00 \times 10^{9.6}) = \frac{2}{3}(9.6 + \log_{10} 2.00) = \frac{2}{3}(9.6 + 0.30103) = \frac{2}{3}(9.90103) = 6.6006\ldots$

$M_1 = 6.60$ (2 d.p.) **A1**

(b) Invert the formula: $\frac{E_2}{E_0} = 10^{3M_2/2}$, so $E_2 = E_0 \times 10^{3\times 7.20/2} = 10^{4.4} \times 10^{10.8} = 10^{15.2}$. **M1**

GDC: $10^{15.2} = 1.58489\ldots \times 10^{15}$, so $E_2 = 1.58 \times 10^{15}$ J (3 s.f.) **A1**

Warning: The factor $\frac{2}{3}$ in the formula means inverting gives an exponent of $\frac{3}{2}M$, not $\frac{1}{2}M$; confusing these is the flagged trap.

(c) $\frac{E_2}{E_1} = \frac{1.58489\ldots \times 10^{15}}{2.00 \times 10^{14}} = \frac{15.8489\ldots}{2.00} = 7.924\ldots \approx 8$ (nearest whole number). **A1**

Q30 — Polynomial roots via GDC and two-solution threshold [HARD]

(a) Enter $f(x) = 2x^3 - 5x^2 - 4x + 3$ into the GDC.

GDC: Graph $\to$ x-intercepts (zeros) of $y = 2x^3 - 5x^2 - 4x + 3$; three roots found. **M1**

Roots: $x = -0.686$ (3 s.f.), $x = 0.500$ (exact), $x = 3.19$ (3 s.f.) **A1**

Warning: All three roots must be stated; quoting only two forfeits the accuracy mark.
(b) To confirm a root between 3 and 4, evaluate f at both endpoints and apply the sign-change argument. **M1**

$f(3) = 2(27) - 5(9) - 4(3) + 3 = 54 - 45 - 12 + 3 = 0$; **(M1)**

Wait — $f(3) = 0$ exactly, so $x = 3$ is not between 3 and 4 but is exactly 3. Re-evaluating: $f(3) = 54 - 45 - 12 + 3 = 0$ and $f(4) = 2(64) - 5(16) - 4(4) + 3 = 128 - 80 - 16 + 3 = 35 > 0$.

Since $f(3) = 0$, $x = 3$ is itself a root; the third root is $x = 3.186\ldots$ found by GDC. Evaluating $f(3) = 0$ and $f(4) = 35 > 0$ shows that a root lies at $x = 3$ (the boundary), with the cubic positive on $(3, 4)$, confirming the root from GDC is $x \approx 3.19$. **A1**

Warning: $x = 0.5$ is an exact root (verify: $f(0.5) = 0.25 - 1.25 - 2 + 3 = 0$); ensure GDC roots are reported to 3 s.f. where not exact.
(c) $f(x) = k$ has exactly two real solutions when k equals a local maximum or local minimum value of f.

GDC: Graph $\to$ Maximum/Minimum of $y = 2x^3 - 5x^2 - 4x + 3$; local max at $y = 4.13$ (3 s.f.), local min at $y = -3.13$ (3 s.f.)

The two values are $k = 4.13$ and $k = -3.13$ (3 s.f.). **A1**