IB Math AI SL — Topic 1

Financial Applications

Compound Interest & Depreciation · Amortisation · Annuities

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:________________________ Date:______________

Score:__________/120

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Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. A graphic display calculator is required throughout; support GDC answers with the values entered and the feature used.

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Key Formulae

- **Compound Interest:** $FV = PV \times \left(1 + \frac{r}{100k}\right)^{nk}$, where $r\%$ is the annual rate, k is the number of compounding periods per year, and n is the number of years.
- **Depreciation (reducing balance):** $V = V_0 \times \left(1 - \frac{d}{100}\right)^{n}$, where $d\%$ is the annual depreciation rate and n is the number of years.
- **Loan Repayment (amortisation payment):** $PMT = PV \times \frac{i(1+i)^n}{(1+i)^n - 1}$, where i is the interest rate per period and n is the total number of payment periods.
- **Total Interest Paid (loan):** Total interest $=$ (Total payments) $-$ Principal $= PMT \times N - PV$.
- **Future Value of Annuity (regular savings):** $FV = PMT \times \frac{(1+i)^n - 1}{i}$, where payments are made at the end of each period.
- **Real Value (inflation adjustment):** Real value $= \frac{\text{Nominal value}}{(1 + \text{inflation rate})^n}$.
- **Effective Annual Rate:** $r_{\text{eff}} = \left(1 + \frac{r}{100k}\right)^{k} - 1$, used to compare offers with different compounding frequencies.

GDC Tips

- **TVM Solver (Finance app):** Press `Apps` → `Finance` → `TVM Solver`. Enter all known values (N, I%, PV, PMT, FV, P/Y, C/Y), place cursor on the unknown, then press `ALPHA` → `ENTER` (Solve). Always set **P/Y** (payments per year) and **C/Y** (compounding periods per year) separately — they need not be equal.
- **Sign convention:** Money *paid out* is **negative**; money *received* is **positive**. For a loan: enter $PV > 0$ (cash received), $PMT < 0$ (repayments made), $FV = 0$ (loan cleared). For savings: enter $PMT < 0$ (deposits made), $FV > 0$ (target received).
- **Finding time (N) — integer bracketing:** After the solver returns a non-integer N, evaluate FV (or balance) at $\lfloor N \rfloor$ and $\lceil N \rceil$. Choose the whole number of periods that *meets or exceeds* the target, and state that value with its checked result.
- **Loan balance after k payments:** Re-enter the TVM Solver with the *same* I%, PV, PMT, P/Y, C/Y but set $N = k$, then solve for FV. The outstanding balance equals $|FV|$.
- **Comparing two offers:** Compute FV (or PMT, or total paid) for each offer independently in the TVM Solver, record both unrounded values, then write a comparison sentence citing those values before giving a recommendation.
- **Depreciation threshold search:** Enter $I\% = -d$ (negative rate), $PMT = 0$, solve for N. Because the context requires *complete* periods, always round **up** to the next whole year and verify by substituting back.

Common Mistakes to Avoid

1. **Compounding-frequency mismatch.** Students set P/Y $= 12$ for monthly payments but leave C/Y $= 1$ (annual compounding), or vice versa. Always check that C/Y matches the stated compounding frequency, which may differ from the payment frequency. Record both P/Y and C/Y explicitly in your parameter table.

2. **Wrong sign convention in the TVM Solver.** Entering both PV and PMT as positive (or both negative) causes the solver to return a nonsensical or zero result. Remember: cash *out* of your pocket is negative; cash *into* your pocket is positive. For a loan, PV is positive (you receive the money) and PMT is negative (you repay it).

3. **Rounding time (N) in the wrong direction.** After finding a non-integer N, students round *down* when the context requires the target to be *reached or exceeded* (e.g. a savings goal, or a value falling *below* a threshold). Always verify both $\lfloor N \rfloor$ and $\lceil N \rceil$ against the condition before choosing.

4. **Computing total interest without subtracting the principal.** A common error is to report $PMT \times N$ as the interest. Total interest is $(PMT \times N) - PV$ for a loan, or $FV - (PMT \times N)$ for a savings annuity. Always subtract the principal (or total deposits) explicitly.

5. **Premature rounding in chained calculations.** Rounding an intermediate result (e.g. a monthly payment, or a balance after k periods) and using that rounded value in the next part can cause a final answer to differ from the expected 3 s.f. value. Store all intermediate values in GDC memory and use the full unrounded figure until the final step.

6. **Confusing nominal and real values under inflation.** Students apply compound interest to find a future nominal amount, then compare it directly with today's price without adjusting for inflation, or they divide by the wrong power. To find real purchasing power, divide the future nominal value by $(1 + \text{inflation rate})^n$; do not add the inflation rate to the investment rate.

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Section A — Easy

EASY

1. Amara invests \$4 000 in a savings account that pays a nominal annual interest rate of 3% compounded annually for 5 years.
Find the value of the investment at the end of 5 years. **[3 marks]**

parameter	value
N	5
$I\%$	3
PV	$-4\,000$
PMT	0
FV	?
$P/Y = C/Y$	1

2. A car is purchased for \$18 000. Its value depreciates at a rate of 12% per year.
Find the value of the car after 4 years. Give your answer to the nearest dollar. **[3 marks]**

3. Priya takes out a loan of \$10 000 at a nominal annual interest rate of 6% compounded monthly. She repays the loan in equal monthly instalments over 3 years.
Find the monthly repayment amount. Give your answer correct to two decimal places. **[3 marks]**

4. Leon deposits \$200 at the end of each month into a savings account that pays a nominal annual interest rate of 4.8% compounded monthly. He makes deposits for 2 years.
Find the total value of the account at the end of 2 years. Give your answer correct to two decimal places. **[3 marks]**

5. Sofia invests \$6 500 in an account that pays a nominal annual interest rate of 2.4% compounded monthly for 3 years.
Find the value of the investment at the end of 3 years. Give your answer correct to two decimal places. **[3 marks]**

parameter	value
N	36
$I\%$	2.4
PV	$-6\,500$
PMT	0
FV	?
$P/Y = C/Y$	12

6. A laptop costs \$1 200 when new. Its value depreciates at a rate of 20% per year.
Write down the value of the laptop after 1 year.
Find the value of the laptop after 6 years. Give your answer to the nearest dollar. **[3 marks]**

7. Carlos borrows \$5 000 at a nominal annual interest rate of 9% compounded monthly. He repays the loan in equal monthly instalments of \$130.
Find the total amount Carlos pays over the full term of the loan. Give your answer correct to two decimal places.
You may use the following information: the loan is fully repaid after 44 monthly payments. **[3 marks]**

8. Nina saves \$500 at the end of each year in an account paying 3% interest per year compounded annually. She saves for 6 years.
Find the total amount in the account at the end of 6 years. Give your answer correct to two decimal places. **[3 marks]**

9. James invests \$8 000 at a nominal annual interest rate of 5% compounded annually.
Find the number of complete years it takes for the investment to exceed \$10 000. **[3 marks]**

10. A bank offers a savings account with a nominal annual interest rate of 6% compounded quarterly. Elena deposits $3 000 into this account and leaves it for 4 years.

Find the value of Elena's account at the end of 4 years. Give your answer correct to two decimal places. **[3 marks]**

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Section B — Medium

MEDIUM

11. Priya invests \$8 500 in a savings account that pays a nominal annual interest rate of 3.6%, compounded monthly. She makes no further deposits or withdrawals.

(a) Find the value of the investment after 5 years. **[2 marks]**

(b) Find the number of complete years it takes for the investment to exceed \$11 000. **[2 marks]**

12. A car is purchased for $24 000. Its value depreciates at a fixed annual rate of 14%.

(a) Find the value of the car after 4 years. **[2 marks]**

(b) Determine the number of complete years it takes for the car's value to fall below $8 000. Justify your answer. **[2 marks]**

13. Lucas takes out a loan of $15 000 to renovate his apartment. The loan charges a nominal annual interest rate of 6%, compounded monthly. Lucas repays the loan in equal monthly payments over 4 years.
The table below shows the TVM parameters.

Parameter	Value
N	48
$I\%$	6
PV	$15\,000$
PMT	?
FV	0
$P/Y = C/Y$	12

(a) Find the monthly payment Lucas makes. **[2 marks]**

(b) Find the total interest Lucas pays over the life of the loan. **[2 marks]**

14. Elena opens a retirement savings account. She deposits \$350 at the end of every month into an account earning a nominal annual interest rate of 4.8%, compounded monthly. She makes deposits for 20 years.

(a) Find the total value of Elena's account at the end of 20 years. **[2 marks]**

(b) Find the total amount Elena deposited over the 20 years. **[1 marks]**

(c) Find the total interest earned. **[1 marks]**

15. A factory machine costs \$62 000 when new. After 6 years the machine is sold for \$21 500.

(a) Show that the annual depreciation rate is approximately 18.1%. **[2 marks]**

(b) Using this rate, find the value of the machine after a further 3 years. **[2 marks]**

16. Marco wants to save \$20 000 for a deposit on a house. He deposits a fixed amount at the end of each month into an account paying a nominal annual interest rate of 3.6%, compounded monthly.

(a) Find the monthly deposit Marco must make if he wants to reach his goal in exactly 4 years. **[2 marks]**

(b) Find the total amount Marco deposits over the 4 years. **[1 marks]**

(c) Find the total interest earned on the account. **[1 marks]**

17. Two banks offer savings accounts for a single deposit of \$10 000 over 3 years.

	Bank A	Bank B
Nominal annual rate	4.2%	4.0%
Compounding	quarterly	monthly

(a) Find the future value of the investment with Bank A after 3 years. **[1 marks]**

(b) Find the future value of the investment with Bank B after 3 years. **[1 marks]**

(c) Determine which bank gives the greater return. Justify your answer. **[2 marks]**

18. Sophia takes a mortgage of \$180 000 at a nominal annual interest rate of 5.4%, compounded monthly. She repays the mortgage in equal monthly instalments over 25 years.

(a) Find the monthly repayment. **[2 marks]**

(b) Find the total amount Sophia pays over the 25 years. **[1 marks]**

(c) Find the total interest paid. **[1 marks]**

19. David invests \$5 000 in an account earning a nominal annual interest rate of r%, compounded quarterly. After 8 years the investment is worth \$7 390.

(a) Find the value of r. **[2 marks]**

(b) Using this rate, find how many complete years it would take for the original investment of \$5 000 to double in value. **[2 marks]**

20. A retirement fund currently has a balance of \$250 000. The fund earns a nominal annual interest rate of 4.2%, compounded monthly. A retiree withdraws \$1 500 at the end of every month.

(a) Find the number of complete months the fund will last. **[2 marks]**

(b) Find the total amount withdrawn from the fund. **[1 marks]**

(c) Find the total interest earned by the fund during this period. **[1 marks]**

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Section C — Hard

HARD

21. Amara deposits $18 000 into a savings account that pays interest at a nominal annual rate of $r\%$, compounded monthly. After exactly 6 years the account contains $24 350.

(a) Show that the annual nominal interest rate is approximately 5.08%. **[2 marks]**

(b) Amara instead considers a second account offering 5.1% nominal annual interest, compounded quarterly. Determine which account gives the greater value after 6 years. Justify your answer with a numerical comparison. **[3 marks]**

22. A car is purchased for \$32 000. Its value depreciates at a fixed annual rate of $d\%$. After 3 years the car is worth \$21 500.

(a) Find the annual depreciation rate d. **[2 marks]**

(b) Find the number of complete years it takes for the car's value to fall below \$10 000 for the first time. **[3 marks]**

23. Priya takes out a home loan of \$210 000 at a nominal annual interest rate of 4.8%, compounded monthly. She repays the loan in equal monthly instalments over 25 years.

(a) Find the monthly repayment amount. **[2 marks]**

(b) Find the total interest paid over the full term of the loan. **[1 marks]**

(c) After exactly 10 years of repayments, Priya makes one additional lump-sum payment of \$15 000 directly off the principal. Find the number of further monthly payments required to clear the remaining balance, assuming the interest rate and monthly payment amount remain unchanged. **[2 marks]**

24. Leon saves for retirement by depositing a fixed amount at the end of each month into an account paying 3.6% nominal annual interest, compounded monthly. He wishes to accumulate \$500 000 in exactly 30 years.

(a) Find the required monthly deposit. **[2 marks]**

(b) Leon retires and moves the \$500 000 into a new account paying 2.4% nominal annual interest, compounded monthly. He withdraws \$2 800 at the end of each month. Determine how many complete months the fund will last before it is exhausted. **[3 marks]**

25. Two investment offers are available to Sofia.
Offer A: Invest $12 000 at 6.0% nominal annual interest, compounded monthly, for 8 years.
Offer B: Invest $12 000 at 6.2% nominal annual interest, compounded annually, for 8 years.
The table below summarises the TVM parameters for Offer A.

Parameter	Value
N	96
$I\%$	6.0
PV	$-12\,000$
PMT	0
FV	?
$P/Y = C/Y$	12

(a) Find the future value of Offer A after 8 years. **[2 marks]**

(b) Determine which offer Sofia should choose. Justify your answer with a numerical comparison. **[3 marks]**

26. A factory machine is bought for \$95 000. Its value depreciates at 12% per year. Running costs increase the effective annual loss, so the total cost of ownership in year k is modelled by the depreciation in value during that year.

(a) Show that the value of the machine at the end of year n is $95\,000 \times (0.88)^n$ dollars. **[1 marks]**

(b) Find the year in which the annual depreciation first falls below \$4 000. **[3 marks]**

(c) The company decides to sell the machine when its value first falls below \$20 000. Find the number of complete years the machine is kept. **[1 marks]**

27. Yusuf borrows \$45 000 at a nominal annual rate of 7.2%, compounded monthly. He repays \$900 per month.

(a) Find the number of complete monthly payments required to repay the loan in full. **[2 marks]**

(b) Find the total amount Yusuf pays over the life of the loan. **[1 marks]**

(c) Find the total interest paid. **[1 marks]**

(d) Yusuf's friend suggests he instead pays \$1 100 per month. Find how many months sooner the loan would be repaid. **[1 marks]**

28. Mei invests a single sum of \$P into an account paying 4.2% nominal annual interest, compounded monthly. Simultaneously, she begins withdrawing \$350 per month from a separate account that currently holds \$60 000 and earns no interest.

(a) Find the number of complete months before the separate \$60 000 account is exhausted. **[1 marks]**

(b) Mei wants her investment account to reach exactly \$80 000 at the moment the \$60 000 account runs out. Find the value of P. **[4 marks]**

29. A pension fund currently holds \$750 000. The fund earns interest at a nominal annual rate of 3.0%, compounded monthly. A retiree wishes to make equal monthly withdrawals for exactly 20 years, after which the fund should be exactly zero.

(a) Find the maximum monthly withdrawal the retiree can make. **[2 marks]**

(b) After 5 years of withdrawals, the interest rate changes to 2.4% nominal annual, compounded monthly. Using the outstanding balance at that point, find the new monthly withdrawal required to exhaust the fund in the remaining 15 years. **[3 marks]**

30. Carlos is saving to buy a boat costing \$55 000. He deposits \$600 per month into an account paying $r\%$ nominal annual interest, compounded monthly. The table below shows the TVM parameters.

Parameter	Value
N	?
$I\%$	r
PV	0
PMT	-600
FV	55 000
$P/Y = C/Y$	12

(a) If $r = 4.8$, find the number of complete months Carlos must save. **[2 marks]**

(b) Carlos would prefer to reach his target in no more than 6 years (72 months). Find the minimum nominal annual interest rate r (to two decimal places) that achieves this. **[3 marks]**

Worked Solutions

Q1 — Compound interest, annual compounding [EASY]

Set up the TVM solver with the parameters shown in the table, entering PV as negative because money is paid into the account (money out is negative). **M1**

GDC: Finance $\rightarrow$ TVM Solver, enter $N = 5$, $I\% = 3$, $PV = -4000$, $PMT = 0$, $P/Y = C/Y = 1$, solve for FV.
Unrounded value: $FV = 4637.084\ldots$ **A1**

The value of the investment at the end of 5 years is $\$$**4 637.08** (to 2 d.p.). **A1**

Warning: Ensure $P/Y = C/Y = 1$ for annual compounding; setting $C/Y = 12$ would give a different (incorrect) answer.

Q2 — Depreciation to a future value [EASY]

Apply the depreciation formula $V = V_0 \times (1 - r)^n$, where $V_0 = 18\,000$, $r = 0.12$, and $n = 4$, because the value decreases by 12% each year. **M1**

$V = 18\,000 \times (0.88)^4$
Unrounded value: $V = 18\,000 \times 0.59969\ldots = 10\,794.5\ldots$ **A1**

The value of the car after 4 years is $\$$**10 795** (to the nearest dollar). **A1**

Warning: Use the multiplier $(1 - 0.12) = 0.88$, not $(1 + 0.12)$; depreciating assets decrease in value.

Q3 — Amortisation, monthly repayment [EASY]

Set up the TVM solver for a loan of \$10 000 repaid over $3 \times 12 = 36$ monthly periods at a nominal rate of 6% compounded monthly, entering PV as positive (money received) and solving for PMT. **M1**

GDC: Finance $\rightarrow$ TVM Solver, enter $N = 36$, $I\% = 6$, $PV = 10\,000$, $FV = 0$, $P/Y = C/Y = 12$, solve for PMT.
Unrounded value: $PMT = -304.2194\ldots$ **A1**

The monthly repayment is $\$$**304.22** (to 2 d.p.). **A1**

Warning: Set $P/Y = C/Y = 12$ to match the monthly compounding; using $C/Y = 1$ gives an incorrect payment amount.

Q4 — Annuity, regular monthly savings [EASY]

Set up the TVM solver for regular end-of-month deposits of \$200 over $2 \times 12 = 24$ months at a nominal rate of 4.8% compounded monthly, with $PV = 0$ since the account starts empty. **M1**

GDC: Finance $\rightarrow$ TVM Solver, enter $N = 24$, $I\% = 4.8$, $PV = 0$, $PMT = -200$, $P/Y = C/Y = 12$, solve for FV.

Unrounded value: $FV = 5\,024.9523\ldots$ **A1**

The total value of the account at the end of 2 years is $\$$**5 024.95** (to 2 d.p.). **A1**

Warning: Ensure PMT is entered as negative (money paid in each month), and $P/Y = C/Y = 12$ to match the monthly compounding frequency.

Q5 — Compound interest, monthly compounding [EASY]

Set up the TVM solver with the parameters shown in the table; $N = 3 \times 12 = 36$ monthly periods, because the nominal rate is compounded monthly and the investment lasts 3 years. **M1**

GDC: Finance → TVM Solver, enter $N = 36$, $I\% = 2.4$, $PV = -6500$, $PMT = 0$, $P/Y = C/Y = 12$, solve for FV.
Unrounded value: $FV = 6\,978.7618\ldots$ **A1**

The value of the investment at the end of 3 years is $\$$**6 978.76** (to 2 d.p.). **A1**

Warning: The number of periods N must be 36 (months), not 3 (years); entering $N = 3$ with $C/Y = 12$ gives an incorrect future value.

Q6 — Depreciation, two parts [EASY]

After 1 year the laptop loses 20% of its value, so multiply by $(1 - 0.20) = 0.80$. **A1**

Value after 1 year $= 1\,200 \times 0.80 = \$$**960**.

Apply the depreciation formula for 6 years: $V = 1\,200 \times (0.80)^6$. **M1**

Unrounded value: $V = 1\,200 \times 0.26214\ldots = 314.572\ldots$
The value after 6 years is $\$$**315** (to the nearest dollar). **A1**

Warning: Use the multiplier 0.80 (not 0.20) raised to the power of the number of years.

Q7 — Amortisation, total amount paid [EASY]

The total amount paid is found by multiplying the fixed monthly payment by the total number of payments, because each payment is equal. **M1**

Total paid $= 130 \times 44$ **A1**

Unrounded value: $130 \times 44 = 5\,720.00$
The total amount Carlos pays is $\$$**5 720.00**. **A1**

Warning: The total amount paid is not the same as the interest paid; total interest $= \$5\,720.00 - \$5\,000.00 = \$720.00$.

Q8 — Annuity, annual savings [EASY]

Set up the TVM solver for regular end-of-year deposits of \$500 over 6 years at 3% per year compounded annually, with $PV = 0$ since the account starts empty. **M1**

GDC: Finance → TVM Solver, enter $N = 6$, $I\% = 3$, $PV = 0$, $PMT = -500$, $P/Y = C/Y = 1$, solve for FV.
Unrounded value: $FV = 3\,229.6355\ldots$ **A1**

The total amount in the account at the end of 6 years is $\$$**3 229.64** (to 2 d.p.). **A1**

Warning: Set $P/Y = C/Y = 1$ for annual compounding; the total saved without interest would be only $\$3\,000$, confirming interest has been earned.

Q9 — Compound interest, time to exceed a threshold [EASY]

Set up the TVM solver to find the number of years N for \$8 000 to grow to \$10 000 at 5% compounded annually. **M1**

GDC: Finance → TVM Solver, enter $I\% = 5$, $PV = -8000$, $PMT = 0$, $FV = 10\,000$, $P/Y = C/Y = 1$, solve for N.
Unrounded value: $N = 4.573\ldots$ years **A1**

Since N must be a whole number of complete years and $4.573\ldots > 4$, the investment first exceeds \$10 000 after **5** complete years. **A1**

Warning: Round up to the next whole year because the investment must exceed \$10 000; after 4 complete years the value is only $8\,000 \times (1.05)^4 = \$9\,724.05$, which does not yet exceed the target.

Q10 — Compound interest, quarterly compounding [EASY]

Set up the TVM solver with $N = 4 \times 4 = 16$ quarterly periods, because the nominal rate is compounded quarterly and the investment lasts 4 years. **M1**

GDC: Finance → TVM Solver, enter $N = 16$, $I\% = 6$, $PV = -3000$, $PMT = 0$, $P/Y = C/Y = 4$, solve for FV.
Unrounded value: $FV = 3\,806.0706\ldots$ **A1**

The value of Elena's account at the end of 4 years is $\$$**3 806.07** (to 2 d.p.). **A1**

Warning: Set $P/Y = C/Y = 4$ for quarterly compounding and $N = 16$ periods; using $N = 4$ with $C/Y = 1$ would ignore the quarterly compounding and give a lower, incorrect answer.

Q11 — Compound interest: future value and threshold time [MEDIUM]

(a) To find the future value, we apply compound interest with monthly compounding. The monthly rate is $3.6\% \div 12 = 0.3\%$, and the total number of periods is $5 \times 12 = 60$.
GDC: Finance → TVM Solver: $N = 60,\ I\% = 3.6,\ PV = -8500,\ PMT = 0,\ FV = ?,\ P/Y = C/Y = 12.$ **M1**

$FV = 10\,113.774\ldots \approx \$10\,113.77$ **A1**

(b) We need the smallest whole number of years n such that the future value exceeds \$11 000. Using $N = 12n$ periods in the TVM Solver:

GDC: Trial with $N = 12 \times 8 = 96$: $FV = 10\,989.57\ldots < 11\,000$; with $N = 12 \times 9 = 108$: $FV = 11\,390.14\ldots > 11\,000$. **M1**

The investment first exceeds \$11 000 after **9 complete years**. **A1**

Warning: The trap is using annual compounding ($C/Y = 1$) instead of monthly ($C/Y = 12$). The compounding frequency must match the stated "compounded monthly" condition throughout.

Q12 — Depreciation: future value and threshold time [MEDIUM]

(a) Using the depreciation formula $V = 24000 \times (1 - 0.14)^4 = 24000 \times (0.86)^4$.
$(0.86)^4 = 0.54700736$, so $V = 24000 \times 0.54700736 = 13\,128.176\ldots \approx \$13\,128.18$ **M1 A1**

(b) We need the first complete year n for which $24000 \times (0.86)^n < 8000$.

Testing integer values: at $n = 11$: $24000 \times (0.86)^{11} = 24000 \times 0.20961\ldots = 5030.71\ldots$; at $n = 10$: $24000 \times (0.86)^{10} = 24000 \times 0.24364\ldots = 5847.37\ldots$; at $n = 8$: $24000 \times (0.86)^8 = 24000 \times 0.32927\ldots = 7902.2\ldots < 8000$; at $n = 7$: $24000 \times (0.86)^7 = 24000 \times 0.38288\ldots = 9189.2\ldots > 8000$. **M1**

Since the value exceeds \$8 000 at year 7 but falls below \$8 000 at year 8, it takes **8 complete years**. **A1**

Warning: Always check both n and $n - 1$ when finding a threshold crossing; accepting $n = 7$ without verification is the common error.

Q13 — Amortisation: monthly payment and total interest [MEDIUM]

(a) We set up the TVM Solver to find the monthly payment that reduces the loan to zero over 48 months. Money received is positive, so $PV = +15\,000$ and the payment found will be negative (money out).

GDC: Finance → TVM Solver: $N = 48,\ I\% = 6,\ PV = 15000,\ PMT = ?,\ FV = 0,\ P/Y = C/Y = 12$. **M1**

$PMT = -352.028\ldots$, so the monthly payment is $\$352.03$. **A1**

(b) Total amount paid $= 48 \times 352.028\ldots = 16\,897.37\ldots$

Total interest $= 16\,897.37\ldots - 15\,000 = 1897.37\ldots \approx \$1\,897.37$ **M1 A1**

Warning: Total interest is always computed as (total payments) − (original principal), not as a direct TVM output. Using the rounded monthly payment of \$352.03 to find total paid introduces rounding error; always use the unrounded value.

Q14 — Annuity: future value, deposits, and interest [MEDIUM]

(a) Elena makes $20 \times 12 = 240$ monthly payments. We find the future value of regular end-of-month deposits.

GDC: Finance → TVM Solver: $N = 240,\ I\% = 4.8,\ PV = 0,\ PMT = -350,\ FV = ?,\ P/Y = C/Y = 12$. **M1**

$FV = 138\,024.36\ldots \approx \$138\,024.36$ **A1**

(b) Total deposited $= 240 \times 350 = \$84\,000$ **A1**

(c) Total interest earned $= 138\,024.36\ldots - 84\,000 = 54\,024.36\ldots \approx \$54\,024.36$ **A1**
Warning: Set $PV = 0$ (no lump sum at the start) to model regular deposits only. Entering $PV \neq 0$ inflates the result incorrectly.

Q15 — Depreciation: find rate and apply further [MEDIUM]

(a) Let r be the annual depreciation rate. Then $62000 \times (1-r)^6 = 21500$.
$(1-r)^6 = \frac{21500}{62000} = 0.34677\ldots$
$1 - r = (0.34677\ldots)^{1/6} = 0.81895\ldots$ **M1**
$r = 1 - 0.81895\ldots = 0.18104\ldots \approx 18.1\%$ **AG**
Warning: The unrounded value $r = 0.18104\ldots$ must be shown before concluding 18.1%; writing only the rounded value does not satisfy the "show that" requirement.
(b) Starting from the value after 6 years ($21 500), apply the rate for 3 further years using $r = 0.18104\ldots$ (unrounded).
$V = 21500 \times (1 - 0.18104\ldots)^3 = 21500 \times (0.81895\ldots)^3$ **M1**
$= 21500 \times 0.54937\ldots = 11\,811.5\ldots \approx \$11\,811.50$ **A1**
Warning: Use the unrounded rate from part (a); using 18.1% (rounded) gives $\$11\,817$, which is slightly different and loses the accuracy mark.

Q16 — Annuity: find monthly deposit, total deposits, interest [MEDIUM]

(a) We need the monthly payment that grows to $20 000 in $4 \times 12 = 48$ months. Deposits are end-of-month, so $PV = 0$ and $FV = 20000$ (positive, money in).
GDC: Finance → TVM Solver: $N = 48,\ I\% = 3.6,\ PV = 0,\ PMT = ?,\ FV = 20000,\ P/Y = C/Y = 12.$ **M1**
$PMT = -390.947\ldots$, so Marco must deposit $\$390.95$ per month. **A1**
(b) Total deposited $= 48 \times 390.947\ldots = 18\,765.47\ldots \approx \$18\,765.47$ **A1**
(c) Interest earned $= 20\,000 - 18\,765.47\ldots = 1234.52\ldots \approx \$1\,234.53$ **A1**
Warning: Use the unrounded PMT when computing total deposits; using the rounded $390.95 gives a slightly different total, which then changes the interest calculation.

Q17 — Two-offer comparison: different compounding frequencies [MEDIUM]

(a) Bank A: quarterly compounding, $N = 3 \times 4 = 12$ periods.
GDC: Finance → TVM Solver: $N = 12,\ I\% = 4.2,\ PV = -10000,\ PMT = 0,\ FV = ?,\ P/Y = C/Y = 4.$
$FV_A = 13\,339.72\ldots \approx \$13\,339.72$ **A1**
(b) Bank B: monthly compounding, $N = 3 \times 12 = 36$ periods.
GDC: Finance → TVM Solver: $N = 36,\ I\% = 4.0,\ PV = -10000,\ PMT = 0,\ FV = ?,\ P/Y = C/Y = 12.$
$FV_B = 11\,272.72\ldots \approx \$11\,272.72$ **A1**
(c) Since $\$13\,339.72 > \$11\,272.72$, Bank A gives the greater return. **R1**

Bank A is the better option, yielding approximately \$2 067 more after 3 years. **A1**
Warning: The trap is mixing up the compounding frequency: Bank A has $C/Y = 4$ (quarterly) and Bank B has $C/Y = 12$ (monthly). Using $C/Y = 12$ for both, or $C/Y = 1$ for both, is a common error that changes both answers.

Q18 — Mortgage: repayment, total paid, total interest [MEDIUM]

(a) Sophia makes $25 \times 12 = 300$ monthly repayments. We find the payment that amortises \$180 000.
GDC: Finance → TVM Solver: $N = 300,\ I\% = 5.4,\ PV = 180000,\ PMT = ?,\ FV = 0,\ P/Y = C/Y = 12.$ **M1**
$PMT = -1094.823\ldots$, so the monthly repayment is $\$1\,094.82$. **A1**
(b) Total paid $= 300 \times 1094.823\ldots = 328\,447.07\ldots \approx \$328\,447.07$ **A1**
(c) Total interest $= 328\,447.07\ldots - 180\,000 = 148\,447.07\ldots \approx \$148\,447.07$ **A1**
Warning: Total interest is (total payments) − (original loan), not a direct GDC output. Use the unrounded monthly payment when computing total paid to avoid cumulative rounding error.

Q19 — Compound interest: find the rate, then threshold time [MEDIUM]

(a) We need to find r such that \$5 000 grows to \$7 390 in 8 years with quarterly compounding ($N = 32$ quarters).
GDC: Finance → TVM Solver: $N = 32,\ I\% = ?,\ PV = -5000,\ PMT = 0,\ FV = 7390,\ P/Y = C/Y = 4.$ **M1**
$I\% = 2.29887\ldots \approx 2.30\%$ (nominal annual rate). **A1**
(b) We find the number of complete years for the investment to double to \$10 000. Using $I\% = 2.29887\ldots$ (unrounded) and testing:
GDC: TVM Solver with $PV = -5000,\ FV = 10000$: at $N = 121$ quarters (≈ 30.25 years): $FV = 9996.5\ldots < 10000$; at $N = 122$ quarters (≈ 30.5 years): $FV = 10\,053.4\ldots > 10000$. **M1**
Since $N = 122$ quarters corresponds to 30 complete years (with 2 extra quarters), the investment doubles during year 31. The number of complete years required is **31 years**. **A1**
Warning: The answer must be expressed in complete years, not quarters. Convert N (quarters) to years by dividing by 4 and rounding up to the next whole year; checking both n and $n - 1$ is essential.

Q20 — Annuity drawdown: how long, total withdrawn, interest [MEDIUM]

(a) We find the number of months until the fund is exhausted. Set $FV = 0$ and solve for N.
GDC: Finance → TVM Solver: $N = ?,\ I\% = 4.2,\ PV = 250000,\ PMT = -1500,\ FV = 0,\ P/Y = C/Y = 12.$ **M1**
$N = 249.67\ldots$ months. Since the fund must last complete months, we check: at month 249 the balance is positive; at month 250 the fund is exhausted. The fund lasts **249 complete months**. **A1**
(b) Total withdrawn $= 249 \times 1500 = \$373\,500$ **A1**
(c) Total interest earned $= 373\,500 - 250\,000 = \$123\,500$ **A1**

Warning: The number of complete months is 249 (not 250), because the fund does not survive the full 250th month. Using 250 overstates both total withdrawals and interest.

Q21 — Reverse rate find then account comparison [HARD]

(a) To find the nominal annual rate we set up the TVM solver with the known future value and solve for $I\%$; money paid in is negative.

GDC: Finance → TVM Solver: $N = 72$, $PV = -18000$, $PMT = 0$, $FV = 24350$, $P/Y = C/Y = 12$; solve for $I\%$.
GDC returns $I\% = 5.0796\ldots \approx 5.08\%$ **AG**
The unrounded value $5.0796\ldots\%$ must be shown before stating the printed target. (no marks for AG line)
Warning: N must be 72 (months), not 6 (years) — the compounding-frequency mismatch trap costs the mark if $N = 6$ is used.

(b) Compute the value under Account 2 at the same $N = 6$ years with quarterly compounding ($P/Y = C/Y = 4$, $N = 24$).
GDC: Finance → TVM Solver: $N = 24$, $I\% = 5.1$, $PV = -18000$, $PMT = 0$, $FV = ?$, $P/Y = C/Y = 4$. **M1**
$FV = 24\,373.07\ldots \approx \$24\,373.07$ **A1**
Since $\$24\,373.07 > \$24\,350.00$, Account 2 gives the greater value after 6 years. **R1**
Warning: Account 1 already rounds to $\$24\,350$ — use the printed target value $\$24\,350$ (or rework with 5.08% exactly) as the comparison benchmark; do not re-derive Account 1 with $r = 5.0796\%$ or rounding errors may flip the decision.

Q22 — Depreciation rate reversal then integer threshold [HARD]

(a) The depreciation model gives $32000\,(1 - d/100)^3 = 21500$; solve for d.
GDC: Finance → TVM Solver: $N = 3$, $PV = 32000$, $PMT = 0$, $FV = -21500$, $P/Y = C/Y = 1$; solve for $I\%$. **M1**
GDC returns $I\% = 12.777\ldots$, so the annual depreciation rate is $d = 12.78\%$ (3 s.f.). **A1**
Warning: The TVM sign convention requires PV and FV to have opposite signs; entering both positive returns an error or wrong value.

(b) We need the smallest integer n such that $32000\,(1 - 0.12778)^n < 10000$, i.e. $32000 \times (0.87222)^n < 10000$.
Test $n = 9$: $32000 \times (0.87222)^9 = 32000 \times 0.3113\ldots = 9962\ldots < 10000$ **M1**
Test $n = 8$: $32000 \times (0.87222)^8 = 32000 \times 0.3568\ldots = 11416\ldots > 10000$ **A1**
Since the value first falls below \$10 000 at $n = 9$ but is still above at $n = 8$, the car's value drops below \$10 000 after **9 complete years.** **A1**
Warning: Use the unrounded depreciation factor $0.87222\ldots$ from part (a) throughout; premature rounding to $d = 12.78\%$ can shift the integer boundary.

Q23 — Amortisation with mid-loan lump sum [HARD]

(a) Find the monthly payment using TVM; money borrowed is positive PV, payments are negative.
GDC: Finance → TVM Solver: $N = 300$, $I\% = 4.8$, $PV = 210000$, $PMT = ?$, $FV = 0$, $P/Y = C/Y = 12$. **M1**
$PMT = -1199.10\ldots \approx \$1\,199.10$ per month. **A1**
Warning: $N = 300$ (months over 25 years), not $N = 25$; using $N = 25$ with $C/Y = 12$ is the compounding-frequency mismatch trap.

(b) Total paid $= 300 \times 1199.10\ldots = 359\,730\ldots$; total interest $= 359\,730\ldots - 210\,000 = \$149\,730$. **A1**
(Use unrounded PMT: $300 \times 1199.099\ldots = 359729.7\ldots$; interest $= \$149\,729.70$ to 2 d.p.)

(c) Find the outstanding balance after 120 payments.
GDC: Finance → Balance after payment 120: $N = 120$, $I\% = 4.8$, $PV = 210000$, $PMT = -1199.10$, $P/Y = C/Y = 12$; compute balance. **M1**
Outstanding balance $= \$163\,431.58\ldots$. After the lump sum: new $PV = 163\,431.58 - 15\,000 = \$148\,431.58$.
GDC: $PV = 148431.58$, $PMT = -1199.10$, $I\% = 4.8$, $FV = 0$, $P/Y = C/Y = 12$; solve for N.
$N = 152.7\ldots$, so **153 further monthly payments** are required. **A1ft**
Warning: Always round the number of payments *up* (ceiling), not to nearest integer, since a partial period still requires a full payment.

Q24 — Annuity savings goal then drawdown duration [HARD]

(a) Find the monthly deposit to accumulate \$500 000.
GDC: Finance → TVM Solver: $N = 360$, $I\% = 3.6$, $PV = 0$, $PMT = ?$, $FV = 500000$, $P/Y = C/Y = 12$. **M1**
$PMT = -817.07\ldots \approx \$817.07$ per month. **A1**
Warning: $PV = 0$ (not -500000); FV is the target, not the deposit; mixing these is a common sign-convention error.

(b) Find how many complete months \$500 000 lasts with monthly withdrawals of \$2 800 at 2.4% compounded monthly.
GDC: Finance → TVM Solver: $N = ?$, $I\% = 2.4$, $PV = 500000$, $PMT = -2800$, $FV = 0$, $P/Y = C/Y = 12$. **M1**
$N = 227.4\ldots$; since the fund is exhausted partway through month 228, it lasts for **227 complete months**. **A1**
Verify: after 227 payments the balance is still positive; after 228 payments the balance would be negative. **A1**
Warning: Round *down* to the last complete month (floor), not up — the fund cannot support a full 228th payment.

Q25 — Two-offer compound interest comparison [HARD]

(a) Use the TVM parameters in the given table to find the future value of Offer A.
GDC: Finance → TVM Solver: $N = 96$, $I\% = 6.0$, $PV = -12000$, $PMT = 0$, $FV = ?$, $P/Y = C/Y = 12$. **M1**
$FV = 19\,380.04\ldots \approx \$19\,380.04$. **A1**
Warning: The table already shows $N = 96$; do not enter $N = 8$ — the compounding-frequency mismatch ($P/Y = C/Y = 12$) means N must be in months.

(b) Compute the future value of Offer B with annual compounding.
GDC: Finance → TVM Solver: $N = 8$, $I\% = 6.2$, $PV = -12000$, $PMT = 0$, $FV = ?$, $P/Y = C/Y = 1$. **M1**
$FV = 19\,400.21\ldots \approx \$19\,400.21$. **A1**
Since $\$19\,400.21 > \$19\,380.04$, Offer B gives the larger return; Sofia should choose **Offer B**. **R1**
Warning: Although Offer A compounds more frequently (monthly vs annually), its lower rate means it does not overtake Offer B — do not assume more frequent compounding always wins.

Q26 — Depreciation model AG then annual-drop and threshold [HARD]

(a) At the end of each year the machine retains $(1 - 0.12) = 0.88$ of its value. Starting from \$95 000, after n years the value is

$$V_n = 95000 \times (0.88)^n \text{ dollars.}$$

AG
No marks awarded for the printed target itself; the reasoning above earns the method mark.

(b) The depreciation *during* year k is $V_{k-1} - V_k = 95000(0.88)^{k-1} - 95000(0.88)^k = 95000(0.88)^{k-1}(0.12)$.
We need the smallest integer k such that $95000 \times 0.12 \times (0.88)^{k-1} < 4000$. **M1**
Test $k = 17$: $11400 \times (0.88)^{16} = 11400 \times 0.1399\ldots = 1595\ldots < 4000$.
Test $k = 12$: $11400 \times (0.88)^{11} = 11400 \times 0.2715\ldots = 3095\ldots < 4000$.
Test $k = 11$: $11400 \times (0.88)^{10} = 11400 \times 0.3085\ldots = 3517\ldots < 4000$.
Test $k = 10$: $11400 \times (0.88)^{9} = 11400 \times 0.3506\ldots = 3997\ldots < 4000$.
Test $k = 9$: $11400 \times (0.88)^{8} = 11400 \times 0.3985\ldots = 4543\ldots > 4000$. **A1**
The annual depreciation first falls below \$4 000 in year $k =$ **10**. **A1**
Warning: The depreciation *during* year k uses $(0.88)^{k-1}$, not $(0.88)^k$ — an off-by-one error shifts the answer.

(c) We need the smallest n such that $95000 \times (0.88)^n < 20000$.
GDC: Table or TVM: $(0.88)^n < 20000/95000 = 0.2105\ldots$; trying $n = 13$: $(0.88)^{13} = 0.2002\ldots < 0.2105$ ⍰; $n = 12$: $(0.88)^{12} = 0.2157\ldots > 0.2105$.
The machine is kept for **13 complete years**. **A1**

Q27 — Loan with fixed payment: duration, total, interest, early payoff [HARD]

(a) Find the number of complete monthly payments to clear $45 000.
GDC: Finance → TVM Solver: $N = ?$, $I\% = 7.2$, $PV = 45000$, $PMT = -900$, $FV = 0$, $P/Y = C/Y = 12$. **M1**
$N = 67.6\ldots$, so **68 complete monthly payments** are required (the 68th payment clears the remaining balance). **A1**
Warning: Always round up to the next whole payment; 67 payments leave a positive balance.

(b) Total amount paid $= 68 \times 900 = \$61\,200$. **A1**
(Note: the final payment is slightly less than $900; for IB purposes the standard approach multiplies the rounded N by PMT.)

(c) Total interest paid $= 61\,200 - 45\,000 = \$16\,200$. **A1**

(d) Repeat with $PMT = -1100$:
GDC: $N = ?$, $I\% = 7.2$, $PV = 45000$, $PMT = -1100$, $FV = 0$, $P/Y = C/Y = 12$; $N = 51.4\ldots \Rightarrow$ 52 payments.
Months saved $= 68 - 52 =$ **16** months sooner. **A1**
Warning: Both durations must be rounded up before subtracting; do not subtract the unrounded N values.

Q28 — Two-technique fusion: exhaustion duration then reverse PV [HARD]

(a) The $60 000 account has no interest and pays out $350 per month, so it lasts $60000 \div 350 = 171.4\ldots$ months.
Since only complete months are counted, the account is exhausted after **171 complete months** (a partial 172nd withdrawal would exceed the balance). **A1**
Warning: Round *down*; the account cannot fund a 172nd full withdrawal.

(b) Mei's investment account must reach $80 000 after exactly 171 months. We find the present value P (a single lump sum, $PMT = 0$).
GDC: Finance → TVM Solver: $N = 171$, $I\% = 4.2$, $PV = ?$, $PMT = 0$, $FV = 80000$, $P/Y = C/Y = 12$. **M1**
$PV = -80000 \div (1 + 0.042/12)^{171}$
$= -80000 \div (1.0035)^{171}$
$= -80000 \div 1.8198\ldots$ **M1**
$PV = -43\,961.4\ldots$
So $P = \$43\,961.44$ (to 2 d.p.). **A1**
Warning: Use $N = 171$ (from part (a)) — carrying forward the unrounded $171.4\ldots$ is incorrect since the investment horizon is the number of *complete* months the $60 000 account lasts.
The sign of PV from the GDC is negative (money out); state P as a positive deposit. **A1ft**

Q29 — Annuity drawdown then mid-term rate change [HARD]

(a) Find the monthly withdrawal that exhausts $750 000 in exactly 20 years ($N = 240$ months).
GDC: Finance → TVM Solver: $N = 240$, $I\% = 3.0$, $PV = 750000$, $PMT = ?$, $FV = 0$, $P/Y = C/Y = 12$. **M1**
$PMT = -4\,161.85\ldots \approx \$4\,161.85$ per month. **A1**
Warning: $FV = 0$ is essential — the fund must be exactly zero at the end; omitting $FV = 0$ or entering a different value gives a wrong PMT.

(b) First find the outstanding balance after 5 years (60 payments) at the original rate.
GDC: Finance → TVM Solver (balance): $N = 60$, $I\% = 3.0$, $PV = 750000$, $PMT = -4161.85$, $P/Y = C/Y = 12$; compute FV. **M1**
Outstanding balance $= \$624\,379.8\ldots \approx \$624\,379.82$.
Now find the new monthly withdrawal to exhaust this balance in the remaining $15 \times 12 = 180$ months at the new rate.
GDC: Finance → TVM Solver: $N = 180$, $I\% = 2.4$, $PV = 624379.82$, $PMT = ?$, $FV = 0$, $P/Y = C/Y = 12$. **M1**
$PMT = -4\,068.89\ldots \approx \$4\,068.89$ per month. **A1**
Warning: Use the unrounded balance $\$624\,379.8\ldots$ as the new PV; rounding to $\$624\,380$ before re-entering the solver introduces a small but penalisable error at 2 d.p.

Q30 — Annuity duration then minimum rate reversal [HARD]

(a) Find N when $r = 4.8$.
GDC: Finance → TVM Solver: $N = ?$, $I\% = 4.8$, $PV = 0$, $PMT = -600$, $FV = 55000$, $P/Y = C/Y = 12$. **M1**
$N = 74.9\ldots$; since Carlos needs the *full* target to be reached, he must save for **75 complete months**. **A1**
Warning: Round up (ceiling); after 74 months the balance is still below $55 000.

(b) We need the minimum r such that $N \leq 72$. Set $N = 72$ and solve for $I\%$.
GDC: Finance → TVM Solver: $N = 72$, $I\% = ?$, $PV = 0$, $PMT = -600$, $FV = 55000$, $P/Y = C/Y = 12$. **M1**
$I\% = 9.1488\ldots$ **A1**
Verify the boundary: at $r = 9.14\%$, $N = 72.02\ldots > 72$ (target not reached in 72 months); at $r = 9.15\%$, $N = 71.97\ldots < 72$ (target reached within 72 months). **M1**
The minimum nominal annual interest rate is $r = \mathbf{9.15}\%$ (to 2 d.p.). **A1**
Warning: Because a higher r means fewer months needed, round the rate *up* to the next second decimal place (9.15%, not 9.14%) to guarantee the target is met within 72 months — this is the integer-bracketing trap applied to a rate rather than a period.