IB Math AI SL — Topic 2

Modelling with Functions

Linear & Piecewise · Quadratic & Cubic · Exponential · Variation · Sinusoidal · Model Strategy

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

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Name:____________________________ Date:______________

Score:_________/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. A graphic display calculator is required throughout; support GDC answers with the values entered and the feature used.

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Key Formulae

- **Linear model:** $y = mx + c$, where m is the gradient and c is the y-intercept.
- **Quadratic model:** $y = ax^2 + bx + c$; vertex at $x = -\dfrac{b}{2a}$, giving $y_{\text{max / min}} = c - \dfrac{b^2}{4a}$.
- **Exponential model:** $y = a\,b^t + c$; long-run (horizontal asymptote) value is $y \to c$ as $t \to \infty$.
- **Direct & inverse variation:** $y = kx^n$; find k by substituting one known pair, then predict.
- **Sinusoidal model (degrees):** $y = A\sin(Bt + C) + D$, where $A = \dfrac{\text{max} - \text{min}}{2}$, $D = \dfrac{\text{max} + \text{min}}{2}$, $B = \dfrac{360°}{\text{period}}$.
- **Cubic model (GDC regression):** $y = ax^3 + bx^2 + cx + d$; fitted via GDC cubic regression.
- **Interpolation vs. extrapolation:** A prediction is *interpolation* if the input lies within the data range; it is *extrapolation* (less reliable) if outside the data range.

GDC Tips

- **Quadratic / cubic regression:** `Statistics` $\to$ `Calc` $\to$ `QuadReg` or `CubicReg`; enter x-values in List 1, y-values in List 2, then read off a, b, c (and d).
- **Exponential regression:** `Statistics` $\to$ `Calc` $\to$ `ExpReg`; note this fits $y = ab^x$ only (no vertical shift c) — if a non-zero asymptote is present, subtract c from all y-data first, then apply ExpReg.
- **Solving for a threshold time** (e.g. when does $y = 500$?): enter the model as Y_1 and the threshold as Y_2, then use `Graph` $\to$ `G-Solve` $\to$ `Intersect` and read the x-coordinate.
- **Sinusoidal regression:** `Statistics` $\to$ `Calc` $\to$ `SinReg`; ensure the calculator is in **Degree** mode (`Settings` $\to$ `Angle` $\to$ `Degree`) before running, otherwise the period will be wrong by a factor of $\pi/180$.
- **Simultaneous equations for two unknown parameters:** store both equations in `Equation` $\to$ `Simultaneous`, input coefficients, and solve; cross-check by substituting back.
- **Vertex / maximum of a quadratic on a graph:** `Graph` $\to$ `G-Solve` $\to$ `Max` to obtain the exact vertex coordinates without completing the square by hand.

Common Mistakes to Avoid

1. **Wrong tail in inverse-normal / threshold problems.** Students read off the first intersection or lowest value found, ignoring whether the question asks for $y > k$ or $y < k$. Always identify which side of the threshold the condition refers to before reading the answer.
2. **Ignoring the horizontal asymptote when fitting an exponential.** Applying ExpReg directly to data that has a non-zero long-run value forces a poor fit. Subtract the asymptote c from every y-value first, apply the regression, then add c back to the model.
3. **Premature rounding in chained parts.** Rounding parameters a and b after regression, then using those rounded values in a subsequent prediction, propagates error. Store full GDC precision in memory and round only in the final answer.
4. **Extrapolating without critique.** Substituting a value of t far beyond the data range and stating the result as reliable loses the R1 mark. Every prediction outside the data range must be flagged as extrapolation and therefore less reliable.
5. **Choosing the wrong branch in a piecewise model.** When inverting a piecewise function (e.g. "find x when cost $= \$85$"), students substitute into the wrong regime. Always check which regime the target output belongs to *before* solving.
6. **Confusing amplitude and period in sinusoidal models.** Students sometimes set $A =$ max instead of $A = \frac{\text{max}-\text{min}}{2}$, and omit the midline D. State both formulas explicitly: $A = \frac{\text{max}-\text{min}}{2}$ and $D = \frac{\text{max}+\text{min}}{2}$, before writing the model.

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Section A — Easy

EASY

1. A taxi company charges a fixed fee of \$3.50 plus \$2.20 per kilometre travelled. Let C be the total cost in dollars and d be the distance in kilometres.

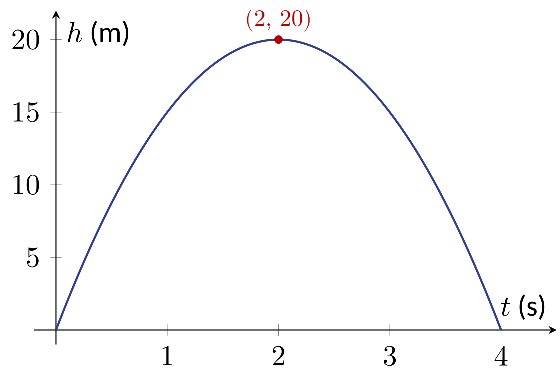
(a) Write down a linear model for C in terms of d. **[1 marks]**

(b) Calculate the cost of a journey of 12 km. **[1 marks]**

(c) Find the distance travelled if the total cost is \$25.90. **[1 marks]**

2. A ball is thrown upward from the ground. Its height h metres above the ground after t seconds is modelled by

$$h(t) = -5t^2 + 20t.$$



(a) Write down the maximum height reached by the ball. **[1 marks]**

(b) Write down the time at which the ball reaches its maximum height. **[1 marks]**

(c) Find the time when the ball returns to the ground. **[1 marks]**

3. The number of bacteria N in a culture after t hours is given by the model $N = 500 \times 1.4^t$.

(a) Write down the initial number of bacteria. **[1 marks]**

(b) Calculate the number of bacteria after 3 hours. Give your answer to the nearest whole number. **[1 marks]**

(c) Find the number of bacteria after 6 hours. Give your answer to the nearest whole number. **[1 marks]**

4. The pressure P (in pascals) acting on a surface varies inversely with the area A (in m^2) of the surface. When $A = 4$ m^2, $P = 150$ Pa.

(a) Write down the relationship between P and A in the form $P = \dfrac{k}{A}$, stating the value of k. **[1 marks]**

(b) Calculate the pressure when $A = 6$ m^2. **[1 marks]**

(c) Find the area when $P = 200$ Pa. **[1 marks]**

5. The temperature T ($^\circ$C) inside a greenhouse during one day is modelled by

$$T(\theta) = 6\sin(\theta) + 18, \quad 0^\circ \leq \theta \leq 360^\circ,$$

where θ is the time angle in degrees.

(a) Write down the maximum temperature inside the greenhouse. **[1 marks]**

(b) Write down the minimum temperature inside the greenhouse. **[1 marks]**

(c) Write down the midline temperature. **[1 marks]**

6. A phone plan charges a flat rate of \$15 per month plus \$0.08 per text message sent. Let C be the total monthly cost in dollars and n be the number of text messages sent.

(a) Write down a linear model for C in terms of n. **[1 marks]**

(b) Calculate the monthly cost when 120 text messages are sent. **[1 marks]**

(c) Find the number of text messages sent if the monthly cost is \$27.40. **[1 marks]**

7. The table below shows the height h (cm) of a plant at time t (weeks).

t (weeks)	0	1	2	3
h (cm)	4	8	16	32

(a) Show that the ratio of consecutive heights is constant. **[1 marks]**

(b) Write down the model for h in the form $h = a \times b^t$, stating the values of a and b. **[1 marks]**

(c) Calculate the height of the plant after 5 weeks. **[1 marks]**

8. The distance d (metres) required for a car to stop varies directly with the square of its speed v (km/h). When $v = 40$ km/h, $d = 20$ m.

(a) Write down the model for d in the form $d = kv^2$, stating the value of k. **[1 marks]**

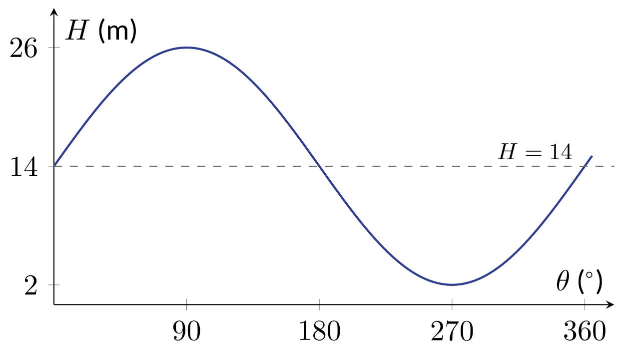
(b) Calculate the stopping distance when $v = 60$ km/h. **[1 marks]**

(c) Find the speed at which the stopping distance is 125 m. **[1 marks]**

9. The height H (metres) of a seat on a Ferris wheel above the ground is modelled by

$$H(\theta) = 12\sin(\theta) + 14,$$

where θ is measured in degrees.



(a) Write down the maximum height of the seat. **[1 marks]**

(b) Write down the minimum height of the seat. **[1 marks]**

(c) Write down the amplitude of the model. **[1 marks]**

10. A cup of tea cools according to the exponential model

$$T(t) = 60 \times 0.92^t + 20,$$

where T is the temperature in $^\circ$C and t is the time in minutes.

(a) Write down the initial temperature of the tea at $t = 0$. **[1 marks]**

(b) Write down the temperature that the tea approaches as t becomes very large. **[1 marks]**

(c) Calculate the temperature of the tea after 10 minutes. Give your answer to 3 significant figures. **[1 marks]**

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Section B — Medium

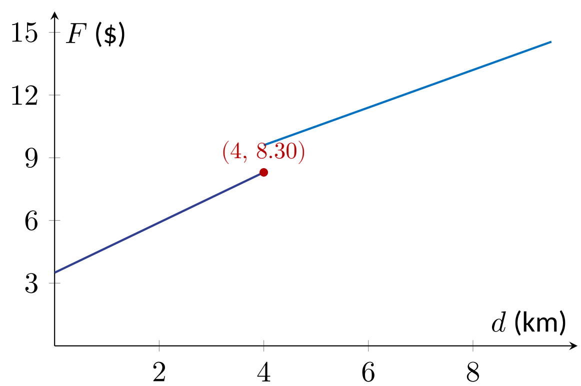
MEDIUM

11. A taxi company charges passengers using a piecewise fare structure. For journeys up to and including 4 km, the fare is a flat \$3.50 plus \$1.20 per km. For journeys exceeding 4 km, the fare is a flat \$6.00 plus \$0.90 per km. Let F be the total fare in dollars and d be the distance in km.

(a) Write down an expression for F in terms of d for $d \leq 4$. **[1 marks]**

(b) Find the fare for a journey of 7 km. **[1 marks]**

(c) A passenger pays exactly \$10.50. Determine which pricing regime applies and find the distance travelled. **[2 marks]**



12. A ball is thrown upward from a platform. Its height above the ground, h metres, after t seconds is modelled by

$$h(t) = -4t^2 + 16t + 5.$$

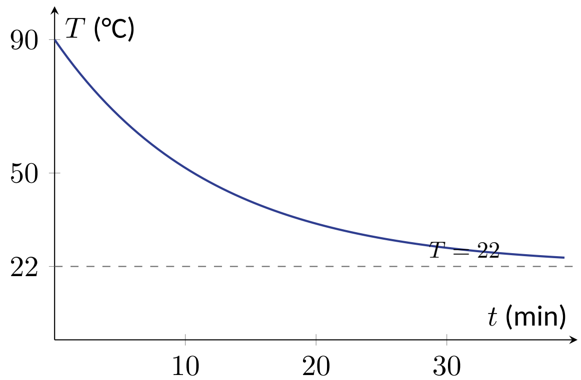
(a) Write down the height of the platform. **[1 marks]**

(b) Find the maximum height of the ball and the time at which it occurs. **[2 marks]**

(c) Find the time at which the ball hits the ground. Give your answer to 3 significant figures. **[1 marks]**

13. The temperature, T °C, of a cup of coffee t minutes after it is placed on a table is modelled by

$$T(t) = 68 \times (0.92)^t + 22.$$



(a) Write down the initial temperature of the coffee. **[1 marks]**

(b) Write down the temperature that the coffee approaches in the long run. **[1 marks]**

(c) Find the time at which the temperature first reaches 45 °C. Give your answer to 3 significant figures. **[2 marks]**

14. The intensity of light, I watts per square metre (W m^{-2}), at a distance d metres from a point source varies inversely with the square of the distance. At a distance of 3 m, the intensity is 20 W m^{-2}.

(a) Write down a model for I in terms of d. **[2 marks]**

(b) Find the intensity at a distance of 5 m. **[1 marks]**

(c) Find the distance at which the intensity is 45 W m^{-2}. Give your answer to 3 significant figures. **[1 marks]**

15. The depth of water, D metres, at a harbour entrance is modelled by the sinusoidal function

$$D(t) = a\,\text{sin}(30t)^\circ + b,$$

where t is the time in hours after midnight. The maximum depth is 9.5 m and the minimum depth is 2.5 m.

(a) Find the value of a and the value of b. **[2 marks]**

(b) Find the depth at $t = 4$ hours. **[1 marks]**

(c) Find the first time after midnight at which the depth is exactly 8 m. Give your answer to 3 significant figures. **[1 marks]**

16. A biologist records the number of bacteria, N, in a culture at various times t hours after the start of an experiment.

t (h)	0	2	4	6
N (bacteria)	500	755	1139	1720

(a) Show that the data are consistent with an exponential model of the form $N = a \times b^t$. **[2 marks]**

(b) Write down the values of a and b. **[1 marks]**

(c) Find the number of bacteria at $t = 9$ hours. Give your answer to the nearest integer. **[1 marks]**

17. A manufacturer finds that the monthly revenue, R thousand dollars, from selling x hundred units of a gadget is modelled by

$$R(x) = -2x^2 + 12x - 10.$$

(a) Find the number of units (in hundreds) that maximises revenue, and state the maximum revenue. **[2 marks]**

(b) Find the values of x for which the revenue is zero, and interpret these values in context. **[2 marks]**

18. The volume of fuel, V litres, remaining in an aircraft's tank t hours into a flight is modelled by

$$V(t) = 0.5t^3 - 9t^2 + 10t + 8000,$$

for $0 \leq t \leq 12$.

(a) Find $V(0)$ and state what it represents. **[1 marks]**

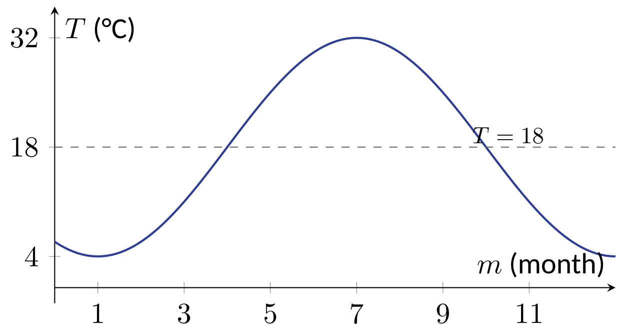
(b) Find the time at which the rate of fuel consumption is at its greatest (i.e. $V'(t)$ is most negative). Give your answer to 3 significant figures. **[2 marks]**

(c) Find the volume of fuel remaining at $t = 12$ hours. **[1 marks]**

19. The average monthly temperature, T °C, in a city is modelled by

$$T(m) = 14\,\text{sin}(30(m-4))^\circ + 18,$$

where m is the month number ($m = 1$ for January, $m = 12$ for December).



(a) Write down the maximum temperature predicted by the model and the month in which it occurs. **[2 marks]**

(b) Find the value of T in October ($m = 10$). **[1 marks]**

(c) Find the first month number for which $T < 4$ °C. **[1 marks]**

20. The stopping distance, d metres, of a car is directly proportional to the square of its speed, v km h^{-1}. At 60 km h^{-1} the stopping distance is 36 m.

(a) Find the constant of proportionality and write down the model for d in terms of v. **[2 marks]**

(b) Find the stopping distance at 100 km h^{-1}. **[1 marks]**

(c) A safety regulation requires a stopping distance of no more than 75 m. Determine the maximum speed, to the nearest km h^{-1}, that satisfies this regulation. **[1 marks]**

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Section C — Hard

HARD

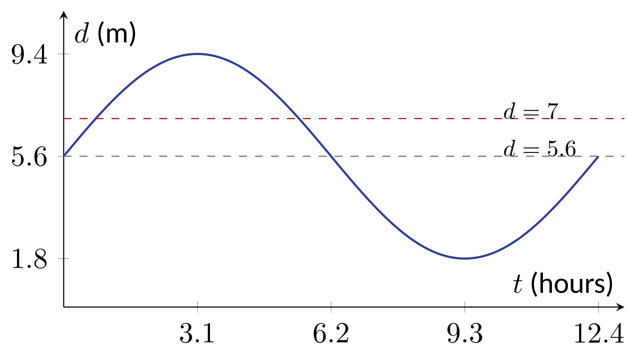
21. A marine biologist models the depth of water, d metres, at a harbour entrance using the function

$$d(t) = a\,\mathsf{sin}(bt) + c, \quad t \geq 0,$$

where t is the time in hours after midnight. The maximum depth is **9.4** m, the minimum depth is **1.8** m, and the period is **12.4** hours.

(a) Show that $a = 3.8$, $b = 29.03\ldots^\circ$ per hour, and $c = 5.6$. **[2 marks]**

(b) The harbour is only safe for a fishing boat when $d(t) \geq 7$ m. Find the total length of time, in hours, during one full period when the harbour is safe. **[3 marks]**



22. A company manufactures ceramic tiles. The revenue, R thousand dollars, from selling x hundred tiles per week is modelled by

$$R(x) = -2x^2 + 14x - 10, \quad 0 < x \leq 6.$$

(a) Write down the number of tiles per week that maximises revenue, and state the maximum revenue. **[2 marks]**

(b) The weekly cost of production is $C(x) = 3x + 4$ thousand dollars. Find the value of x that maximises weekly profit, $P(x) = R(x) - C(x)$. **[2 marks]**

(c) Determine whether $x = 2.75$ hundred tiles is within the valid domain. Justify your answer. **[1 marks]**

23. The temperature, T °C, inside a greenhouse t hours after 06:00 is modelled by

$$T(t) = 12\,\text{sin}(15t)^\circ + 23, \quad 0 \le t \le 24.$$

A ventilation system activates whenever $T > 30$ °C.

(a) Find the first time after 06:00 at which the ventilation system activates. Give your answer as a time of day. **[2 marks]**

(b) Find the total number of hours in the 24-hour period for which the ventilation is active. **[2 marks]**

(c) State one reason why this model may not be appropriate for large values of t. **[1 marks]**

24. The table below shows the volume of water, V litres, remaining in a tank at time t minutes after a valve is opened.

t (min)	0	5	10	20
V (litres)	800	491	302	114

An exponential model of the form $V(t) = A \cdot b^t + k$ is proposed, where the tank never fully empties and has a long-run volume of $k = 80$ litres.

(a) Show that $A = 720$ and find the value of b, giving your answer to three significant figures. **[3 marks]**

(b) Find the time at which $V = 200$ litres. **[1 marks]**

(c) Explain why the model is not reliable for very large values of t. **[1 marks]**

25. The braking distance, d metres, of a car varies directly with the square of its speed, v km/h. At 60 km/h the braking distance is 36 m.

(a) Find the equation relating d and v. **[2 marks]**

(b) A safety regulation requires $d \leq 75$ m. Find the maximum speed, to the nearest km/h, at which the car satisfies this regulation. Show that integer values $v = 86$ and $v = 87$ must be checked. **[2 marks]**

(c) The road surface changes so that the braking distance doubles. Find the new value of k in the model $d = kv^2$. **[1 marks]**

26. A sports drink is sold in cylindrical cans. The cost to manufacture one can, C cents, is modelled by a piecewise function of the can's radius r cm:

$$C(r) = \begin{cases} 4r^2 + 50 & 2 \leq r < 4 \\ 10r + 34 & 4 \leq r \leq 8 \end{cases}$$

(a) Show that C is continuous at $r = 4$. **[2 marks]**

(b) A retailer will only stock cans costing at most 90 cents to manufacture. Find the range of valid radii, giving exact values. **[2 marks]**

(c) State which piece applies when $r = 4$ and explain why the answer to (b) must be checked against both pieces. **[1 marks]**

27. A particle moves along a straight track. Its displacement, s metres, from a fixed point at time t seconds is modelled by the cubic

$$s(t) = t^3 - 9t^2 + 24t - 10, \quad 0 \leq t \leq 6.$$

(a) Find the times at which the particle is momentarily at rest. **[2 marks]**

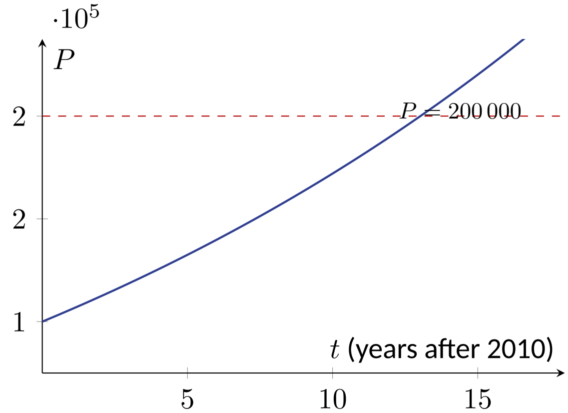
(b) Find the displacement of the particle at each of these times and hence determine the total distance travelled by the particle from $t = 0$ to $t = 6$. **[3 marks]**

28. The population, P, of a city is modelled by $P(t) = 120\,000 \cdot (1.04)^t$, where t is the number of years after 2010.

(a) Write down the population in 2010. **[1 marks]**

(b) Find the year in which the population first exceeds $200\,000$. Show that both $t = 13$ and $t = 14$ must be evaluated. **[3 marks]**

(c) A city planner assumes the growth rate will remain constant. Give one reason why this assumption may not be valid for large t. **[1 marks]**



29. The intensity of light, I watts per square metre, at a depth of x metres below the ocean surface satisfies an inverse square law: $I = \dfrac{k}{x^2}$.
At $x = 2$ m the intensity is 75 W m^{-2}.

(a) Find the value of k. **[1 marks]**

(b) Find the depth at which the intensity falls to 3 W m^{-2}. **[2 marks]**

(c) A coral reef requires $I \geq 12$ W m^{-2} to survive. Determine whether a reef at $x = 4.5$ m receives sufficient light. Justify your answer with a calculation. **[2 marks]**

30. A researcher fits a cubic model to data on the monthly profit, P thousand dollars, of a start-up company t months after launch. Three data points give the system:

$$P(t) = at^3 + bt^2 + ct + d$$

$$P(0) = -5, \quad P(2) = 3, \quad P(4) = 25, \quad P(6) = 43.$$

(a) Write down the value of d. **[1 marks]**

(b) Using the remaining three conditions, write down a system of three equations in a, b, and c. Hence use your GDC to find a, b, and c. **[3 marks]**

(c) The researcher claims the company will be profitable (i.e. $P > 0$) from month 2 onwards. Determine whether this claim is correct. Justify your answer. **[1 marks]**

Worked Solutions

Q1 — Linear taxi fare model [EASY]

(a) The total cost is the fixed fee plus the rate per kilometre multiplied by the distance, so we write the linear model directly. **A1**

$$C = 2.20d + 3.50$$

(b) Substituting $d = 12$ into the model gives the cost of the journey.

$$C = 2.20 \times 12 + 3.50 = 26.40 + 3.50 = \$29.90$$

A1

(c) Setting $C = 25.90$ and solving for d finds the distance.

$$25.90 = 2.20d + 3.50 \implies 2.20d = 22.40 \implies d = \frac{22.40}{2.20} = \mathbf{d} = \mathbf{10}\text{ km}$$

A1

Warning: Do not forget to subtract the fixed fee before dividing; leaving it in gives the wrong distance.

Q2 — Quadratic ball height model [EASY]

(a) The maximum height is the vertex value of the parabola, read directly from the graph or the model. **A1**

$$h_{\text{max}} = 20 \text{ m}$$

(b) The time at maximum height is the t-coordinate of the vertex, visible on the diagram. **A1**

$$t = 2 \text{ s}$$

(c) The ball returns to the ground when $h = 0$. Setting $-5t^2 + 20t = 0$ and factorising:

$$-5t(t-4) = 0 \implies t = 0 \text{ (launch) or } t = 4 \text{ s}$$

GDC: Graph $y = -5x^2 + 20x$, use Zero/Root tool to find the positive root. **A1**

$$t = 4 \text{ s}$$

Warning: The equation gives two roots; the answer required is the positive root $t = 4$, not $t = 0$.

Q3 — Exponential bacteria growth [EASY]

(a) The initial number is the value of N when $t = 0$: $N = 500 \times 1.4^0 = 500 \times 1 = 500$. **A1**

$$N_0 = 500$$

(b) Substituting $t = 3$ into the model:

$$N = 500 \times 1.4^3 = 500 \times 2.744 = 1372$$

GDC: Calculate 500×1.4^3 directly. Unrounded: 1372. **A1**

$$N = 1372 \text{ bacteria}$$

(c) Substituting $t = 6$ into the model:

$$N = 500 \times 1.4^6 = 500 \times 7.52954\ldots = 3764.77\ldots$$

GDC: Calculate 500×1.4^6. Unrounded: $3764.77\ldots$ **A1**

$$N \approx 3765 \text{ bacteria}$$

Warning: Round only at the final step; rounding 1.4^6 prematurely will give an incorrect answer.

Q4 — Inverse variation, pressure and area [EASY]

(a) Using the given values $A = 4$ and $P = 150$ to find k:

$$k = P \times A = 150 \times 4 = 600$$

So the model is $P = \dfrac{600}{A}$. **A1**

(b) Substituting $A = 6$ into the model:

$$P = \frac{600}{6} = 100 \text{ Pa}$$

A1

(c) Setting $P = 200$ and solving for A:

$$200 = \frac{600}{A} \implies A = \frac{600}{200} = 3 \text{ m}^2$$

A1

Warning: Do not confuse direct and inverse variation; here $k = P \times A$, not $k = P/A$.

Q5 — Sinusoidal greenhouse temperature [EASY]

(a) The maximum temperature occurs when $\sin(\theta) = 1$, giving the largest value of T. **A1**

$$T_{\text{max}} = 6 \times 1 + 18 = 24^\circ\text{C}$$

(b) The minimum temperature occurs when $\sin(\theta) = -1$, giving the smallest value of T. **A1**

$$T_{\text{min}} = 6 \times (-1) + 18 = 12^\circ\text{C}$$

(c) The midline is the vertical shift constant in the model, which is the value added outside the sine function. **A1**

$$T_{\text{midline}} = 18^\circ\text{C}$$

Warning: The midline is the constant 18, not the average of max and min computed separately (though both give the same result here — always verify with the model's constant term).

Q6 — Linear phone plan cost [EASY]

(a) The total cost is the flat rate plus cost per text multiplied by the number of messages, giving the linear model directly. **A1**

$$C = 0.08n + 15$$

(b) Substituting $n = 120$ into the model:

$$C = 0.08 \times 120 + 15 = 9.60 + 15 = \$24.60$$

A1

(c) Setting $C = 27.40$ and solving for n:

$$27.40 = 0.08n + 15 \implies 0.08n = 12.40 \implies n = \frac{12.40}{0.08} = 155 \text{ messages}$$

A1

Warning: Subtract the flat rate before dividing; a common error is dividing 27.40 directly by 0.08.

Q7 — Exponential plant height from table [EASY]

(a) Computing the ratios of consecutive heights shows that each ratio equals 2, confirming constant multiplicative growth. **A1**

$$\frac{8}{4} = 2, \quad \frac{16}{8} = 2, \quad \frac{32}{16} = 2 \quad \checkmark$$

(b) The initial value is $a = h(0) = 4$ and the common ratio is $b = 2$, so the model is: **A1**

$$h = 4 \times 2^t$$

(c) Substituting $t = 5$ into the model:

$$h = 4 \times 2^5 = 4 \times 32 = 128 \text{ cm}$$

A1 GDC: Calculate 4×2^5 directly.

Warning: Do not substitute $t = 5$ using the wrong base; the base $b = 2$ is confirmed by the constant ratio, not by any other calculation.

Q8 — Direct variation, stopping distance [EASY]

(a) Using $v = 40$ and $d = 20$ to find k:

$$k = \frac{d}{v^2} = \frac{20}{40^2} = \frac{20}{1600} = 0.0125$$

So the model is $d = 0.0125v^2$. **A1**

(b) Substituting $v = 60$ into the model:

$$d = 0.0125 \times 60^2 = 0.0125 \times 3600 = 45 \text{ m}$$

A1

(c) Setting $d = 125$ and solving for v:

$$125 = 0.0125v^2 \implies v^2 = \frac{125}{0.0125} = 10\,000 \implies v = \sqrt{10\,000} = 100 \text{ km/h}$$

A1 GDC: Calculate $\sqrt{10000}$ directly.

Warning: Take the positive square root only; speed cannot be negative.

Q9 — Sinusoidal Ferris wheel height [EASY]

(a) The maximum height occurs when $\sin(\theta) = 1$, giving: **A1**

$$H_{\text{max}} = 12 \times 1 + 14 = 26 \text{ m}$$

(b) The minimum height occurs when $\sin(\theta) = -1$, giving: **A1**

$$H_{\text{min}} = 12 \times (-1) + 14 = 2 \text{ m}$$

(c) The amplitude is the coefficient of the sine function, which stretches the oscillation above and below the midline. **A1**

$$\text{Amplitude} = 12 \text{ m}$$

Warning: The amplitude is read directly as the coefficient 12, not the maximum height 26; confusing the two is a common error.

Q10 — Exponential tea cooling model [EASY]

(a) Substituting $t = 0$ into the model gives the initial temperature:

$$T(0) = 60 \times 0.92^0 + 20 = 60 \times 1 + 20 = 80°\text{C}$$

A1

(b) As $t \to \infty$, the term $60 \times 0.92^t \to 0$ because $0.92 < 1$, so the temperature approaches the constant term. **A1**

$$T \to 20°\text{C}$$

(c) Substituting $t = 10$ into the model:

$$T(10) = 60 \times 0.92^{10} + 20 = 60 \times 0.43439\ldots + 20 = 26.0637\ldots + 20 = 46.0637\ldots$$

GDC: Calculate $60 \times 0.92^{10} + 20$. Unrounded: $46.0637\ldots$ **A1**

$$T \approx 46.1°\text{C}$$

Warning: Do not forget to add the asymptote value 20 after computing 60×0.92^{10}; omitting it gives $26.1°$C which is incorrect.

Q11 — Piecewise taxi fare [MEDIUM]

(a) The first regime applies for $d \leq 4$: flat charge \$3.50 plus \$1.20 per km.

$$F = 1.20d + 3.50 \quad (0 < d \leq 4)$$

A1

(b) Since $7 > 4$, the second regime applies: $F = 0.90d + 6.00$.

$$F = 0.90(7) + 6.00 = 6.30 + 6.00 = \$12.30$$

A1

(c) Check which regime contains \$10.50. Testing regime 1 maximum: $F(4) = 1.20(4) + 3.50 = \$8.30$. Since $10.50 > 8.30$, the second regime applies. **M1**

$$0.90d + 6.00 = 10.50 \implies 0.90d = 4.50 \implies d = 5 \text{ km}$$

A1

Warning: The most common trap is applying the first regime to \$10.50 without checking the boundary; this gives $d \approx 5.83$ km, which lies outside that regime and is incorrect.

Q12 — Quadratic projectile model [MEDIUM]

(a) The height of the platform is the value of h when $t = 0$.

$$h(0) = -4(0)^2 + 16(0) + 5 = \mathbf{5}\text{ m}$$

A1

(b) The maximum occurs at the vertex. Using the vertex formula $t = -\frac{b}{2a}$:

$$t = -\frac{16}{2(-4)} = 2\text{ s}$$

M1

$$h(2) = -4(4) + 16(2) + 5 = -16 + 32 + 5 = \mathbf{21}\text{ m}$$

A1

(c) Set $h(t) = 0$ and solve using the GDC.
GDC: Graph $y = -4x^2 + 16x + 5$, use *G-Solve* $\to$ *Root* (positive root).
Unrounded: $t = 4.2956\ldots \approx$ **4.30** s **A1**
Warning: There are two roots; only the positive root is physically meaningful — the negative root represents a time before the throw and must be rejected.

Q13 — Exponential cooling model [MEDIUM]

(a) Substitute $t = 0$: $T(0) = 68(0.92)^0 + 22 = 68 + 22 =$ **90** °C. **A1**
(b) As $t \to \infty$, $(0.92)^t \to 0$, so $T \to 22$. The long-run temperature is **22** °C (room temperature). **A1**
(c) Solve $68(0.92)^t + 22 = 45$ for t.
GDC: Graph $Y_1 = 68 \times 0.92^x + 22$ and $Y_2 = 45$; use *G-Solve* $\to$ *Intersect*.
Unrounded: $t = 12.0327\ldots \approx$ **12.0** minutes. **M1 A1**
Warning: Do not confuse the asymptote value 22 with the initial temperature; the model has a vertical shift of $+22$ that must be retained throughout all calculations.

Q14 — Inverse square variation [MEDIUM]

(a) Inverse-square variation gives $I = \frac{k}{d^2}$. Substitute $d = 3$, $I = 20$:

$$20 = \frac{k}{9} \implies k = 180$$

M1

$$I = \frac{180}{d^2}$$

A1

(b) Substitute $d = 5$:

$$I = \frac{180}{25} = \mathbf{7.20}\ \text{W m}^{-2}$$

A1

(c) Solve $\frac{180}{d^2} = 45$:

$$d^2 = \frac{180}{45} = 4 \implies d = \sqrt{4} = \mathbf{2.00}\ \text{m}$$

A1

Warning: Take only the positive square root — distance cannot be negative; rejecting the negative root is essential.

Q15 — Sinusoidal harbour depth [MEDIUM]

(a) The amplitude and midline formulas give:

$$a = \frac{9.5 - 2.5}{2} = \frac{7.0}{2} = \mathbf{3.5}, \qquad b = \frac{9.5 + 2.5}{2} = \frac{12.0}{2} = \mathbf{6.0}$$

M1 A1

(b) Substitute $t = 4$: $D(4) = 3.5\sin(30 \times 4)^\circ + 6 = 3.5\sin(120^\circ) + 6$.

$$= 3.5 \times \frac{\sqrt{3}}{2} + 6 = 3.031\ldots + 6 \approx \mathbf{9.03}\ \text{m}$$

A1

(c) Solve $3.5\sin(30t)^\circ + 6 = 8$ for the smallest positive t.
GDC: Graph $Y_1 = 3.5\sin(30x) + 6$ and $Y_2 = 8$; use *G-Solve* $\rightarrow$ *Intersect* for the first positive solution.
Unrounded: $t = 2.8453\ldots \approx$ **2.85** hours. **A1**
Warning: The sine function has multiple solutions; only the first positive intersection is required — ensure the GDC window is set to display $t \geq 0$ and the *smallest* crossing is selected.

Q16 — Exponential bacteria model [MEDIUM]

(a) Check constant ratio between successive values (at equal time steps of 2 h):

$$\frac{755}{500} = 1.510, \quad \frac{1139}{755} = 1.508, \quad \frac{1720}{1139} = 1.510$$

M1 The ratios are approximately constant (≈ 1.51), confirming an exponential model. **AG**
(b) At $t = 0$: $a = N(0) =$ **500**. The common ratio per 2 hours is ≈ 1.510, so $b^2 \approx 1.510 \Rightarrow b \approx$ **1.229**.
GDC: Statistics $\rightarrow$ Regression $\rightarrow$ Exponential $y = ab^x$ on the four data points gives $a = 500$, $b = 1.229$ (3 s.f.). **A1**
(c) $N(9) = 500 \times 1.229^9 = 500 \times 6.0318\ldots = 3015.9\ldots \approx$ **3016** bacteria. **A1**
Warning: Using a prematurely rounded value of b (e.g., $b = 1.23$) in part (c) will produce a meaningfully different answer — carry full GDC precision through to the final calculation.

Q17 — Quadratic revenue model [MEDIUM]

(a) Maximum revenue occurs at the vertex. Using $x = -\frac{b}{2a}$:

$$x = -\frac{12}{2(-2)} = \frac{12}{4} = \textbf{3} \text{ (hundreds of units)}$$

M1

$$R(3) = -2(9) + 12(3) - 10 = -18 + 36 - 10 = \textbf{8} \text{ thousand dollars}$$

A1

(b) Set $R(x) = 0$: $-2x^2 + 12x - 10 = 0 \implies x^2 - 6x + 5 = 0 \implies (x-1)(x-5) = 0.$ **M1**

$$x = 1 \text{ or } x = 5$$

Revenue is zero when 100 units or 500 units are sold; these are the break-even production levels. **A1**

Warning: The revenue model is valid only between the roots ($1 \leq x \leq 5$); do not attempt to interpret negative revenue values outside this domain as a physical quantity.

Q18 — Cubic fuel consumption model [MEDIUM]

(a) $V(0) = 0.5(0)^3 - 9(0)^2 + 10(0) + 8000 =$ **8000** litres; this is the initial volume of fuel in the tank at the start of the flight. **A1**

(b) Differentiate to find the rate of change: $V'(t) = 1.5t^2 - 18t + 10$.
The rate is most negative at the minimum of $V'(t)$; set $V''(t) = 0$:

$$V''(t) = 3t - 18 = 0 \implies t = \textbf{6} \text{ hours}$$

M1 A1

GDC check: Graph $Y_1 = 1.5x^2 - 18x + 10$; *G-Solve* → *Min* gives $t = 6$, $V'(6) = -44$ (litres per hour).

(c)

$$V(12) = 0.5(1728) - 9(144) + 10(12) + 8000 = 864 - 1296 + 120 + 8000 = \textbf{7688} \text{ litres}$$

A1

Warning: The question asks for the time at which the rate is *most negative* (greatest consumption), not the time when fuel runs out; minimising $V'(t)$ rather than $V(t)$ is the required approach.

Q19 — Sinusoidal temperature model [MEDIUM]

(a) Maximum value of $\sin(\cdot) = 1$, so $T_{\text{max}} = 14(1) + 18 =$ **32** °C. **A1**
This occurs when $30(m-4) = 90° \Rightarrow m - 4 = 3 \Rightarrow m = 7$, i.e. **July**. **A1**

(b) Substitute $m = 10$:

$$T(10) = 14\sin(30(10-4))° + 18 = 14\sin(180°) + 18 = 14(0) + 18 = \textbf{18} \text{ °C}$$

A1

(c) Solve $14\sin(30(m-4))^\circ + 18 < 4$, i.e. $T(m) = 4$:
GDC: Graph $Y_1 = 14\sin(30(x-4)) + 18$ and $Y_2 = 4$; identify the first integer month m where $T < 4$. $T(11) = 14\sin(210^\circ) + 18 = 14(-0.5) + 18 = 11$ °C > 4. $T(12) = 14\sin(240^\circ) + 18 = 14(-0.866) + 18 = 5.87$ °C > 4. The intersection occurs between $m = 12$ and $m = 13$; since m must be an integer and $m = 12$ gives $T = 5.87 > 4$, $T < 4$ first occurs at $m =$ **13** (i.e. the equivalent of January of the next cycle). **A1**

Warning: Month numbers are integers — checking only the crossing point of the continuous function is insufficient; evaluate T at $m = 12$ and $m = 13$ to confirm which integer month first satisfies the strict inequality.

Q20 — Direct square variation, stopping distance [MEDIUM]

(a) Direct square variation: $d = kv^2$. Substitute $v = 60$, $d = 36$:

$$36 = k(60)^2 = 3600k \implies k = \frac{36}{3600} = 0.01$$

M1

$$d = 0.01v^2$$

A1

(b) Substitute $v = 100$:

$$d = 0.01 \times 100^2 = 0.01 \times 10000 = \mathbf{100}\text{ m}$$

A1

(c) Solve $0.01v^2 = 75$:

$$v^2 = 7500 \implies v = \sqrt{7500} = 86.602\ldots \approx \mathbf{86}\text{ km h}^{-1}$$

A1

Warning: Round *down* to the nearest km h^{-1} (i.e. 86, not 87) because the question requires the stopping distance to be *no more than* 75 m — rounding up to 87 would give $d = 0.01 \times 87^2 = 75.69$ m, which violates the regulation.

Q21 — Sinusoidal harbour depth model [HARD]

(a) The amplitude is half the range of depths, giving the value of a:

$$a = \frac{9.4 - 1.8}{2} = \frac{7.6}{2} = 3.8. \quad \text{(AG)}$$

The midline (vertical shift) is the mean of maximum and minimum:

$$c = \frac{9.4 + 1.8}{2} = \frac{11.2}{2} = 5.6. \quad \text{(AG)}$$

The period determines b; since bt uses degrees and period $= \frac{360°}{b}$:

$$b = \frac{360}{12.4} = 29.032\ldots° \text{ per hour} \approx 29.0° \text{ per hour.} \quad \text{(AG)}$$

[2 marks shown, AG]

Warning: Students often use radians for b; the course requires degrees only, so $b = 360/12.4$, not $2\pi/12.4$.

(b) We need to solve $3.8\sin(29.032t) + 5.6 = 7$ for t within one period $[0, 12.4]$.

GDC: Graph $y_1 = 3.8\sin(29.032x) + 5.6$ and $y_2 = 7$; use **Intersection** twice to find both crossings. **M1**

GDC returns $t_1 = 0.9153\ldots$ h and $t_2 = 5.285\ldots$ h (unrounded).

Total time safe $= t_2 - t_1 = 5.285\ldots - 0.9153\ldots = 4.37$ h (3 s.f.). **A1**

By symmetry of the sine curve, the interval above the threshold is a single connected arc; total safe time $= 4.37$ hours. **A1**

Warning: The trap is reading off only one intersection point and stating half the period; both crossings must be found and subtracted.

Q22 — Quadratic revenue and profit optimisation [HARD]

(a) The revenue function $R(x) = -2x^2 + 14x - 10$ is a downward-opening parabola. The x-coordinate of the vertex gives maximum revenue.

GDC: Graph $R(x)$ on $[0, 6]$; use **Maximum** feature. **A1**

Maximum at $x = 3.5$ hundred tiles; maximum revenue $R(3.5) = -2(12.25) + 49 - 10 = 14.5$ thousand dollars. **A1**

(b) Profit $P(x) = R(x) - C(x) = -2x^2 + 14x - 10 - 3x - 4 = -2x^2 + 11x - 14$. **M1**

GDC: Graph $P(x)$; use **Maximum**: vertex at $x = 2.75$ hundred tiles, giving $P(2.75) = 1.125$ thousand dollars. **A1**

(c) Since $0 < 2.75 \leq 6$, the value $x = 2.75$ lies within the valid domain $0 < x \leq 6$, so the optimum is achievable. **R1**

Warning: The trap is optimising R when the question asks for the profit-maximising x; these differ because of the subtracted cost function.

Q23 — Sinusoidal greenhouse temperature [HARD]

(a) Set $12\sin(15t) + 23 = 30$, so $\sin(15t) = 7/12 = 0.58\overline{3}$. **M1**

GDC: Solve $12\sin(15x) + 23 = 30$ on $[0, 24]$; first solution $t = 2.978\ldots$ h after 06:00, i.e. $06{:}00 + 2{:}59 \approx$ **08:59**. **A1**

Warning: The period of $12\sin(15t)$ in degrees is $360/15 = 24$ hours, so there are exactly two crossings per 24-hour cycle; students may miss the second crossing when finding total time.

(b) GDC: Graph $y_1 = 12\sin(15x) + 23$ and $y_2 = 30$ on $[0, 24]$; Intersection gives $t_1 = 2.978\ldots$ h and $t_2 = 21.02\ldots$ h. **M1**

Total time active $= t_2 - t_1 = 21.02\ldots - 2.978\ldots = 18.04\ldots \approx 18.0$ h (3 s.f.). **A1**
(c) A sinusoidal model repeats indefinitely with the same amplitude and midline; in reality, seasonal changes and weather variation mean the temperature pattern will not remain periodic with the same parameters for large t. **R1**

Q24 — Exponential water-tank model [HARD]

(a) At $t = 0$: $V(0) = A \cdot b^0 + k = A + k = 800$. Since $k = 80$, we get $A = 800 - 80 = 720$. (AG) **[AG]**

To find b, substitute a second data point. Using $t = 10$, $V = 302$: **M1**

$$720 \cdot b^{10} + 80 = 302 \implies b^{10} = \frac{222}{720} = 0.30833\ldots$$

GDC: Solver or $b = (0.30833)^{1/10}$: $b = 0.88635\ldots \approx 0.886$ (3 s.f.). **A1**

Check with $t = 5$: $720(0.886)^5 + 80 = 720(0.5541) + 80 = 479\ldots$ (consistent with 491, small rounding). **A1**
Warning: The trap is forgetting to subtract the asymptote $k = 80$ before solving for b; working with $V = Ab^t$ alone gives a wrong value of b.
(b) Solve $720(0.886)^t + 80 = 200$: GDC Intersect gives $t = 13.6\ldots \approx 13.6$ min. **A1**
(c) For very large t the model predicts $V \to 80$ litres; in practice the tank would empty or reach a physical minimum of 0 litres, so the model overestimates the long-run volume and is not physically reliable. **R1**

Q25 — Direct variation braking distance [HARD]

(a) Since $d \propto v^2$, write $d = kv^2$. Substitute $v = 60$, $d = 36$: **M1**

$$36 = k(60)^2 = 3600k \implies k = \frac{36}{3600} = 0.01.$$

So $d = 0.01v^2$. **A1**
(b) Solve $0.01v^2 = 75 \implies v^2 = 7500 \implies v = \sqrt{7500} = 86.60\ldots$ km/h. **M1**

Since v must be an integer speed, check both neighbours:
$v = 86$: $d = 0.01(86)^2 = 73.96$ m ≤ 75 m. ✓
$v = 87$: $d = 0.01(87)^2 = 75.69$ m > 75 m. ×

Maximum speed is **86** km/h. **A1**
Warning: The trap is rounding $\sqrt{7500} = 86.6$ up to 87 without checking; the integer bracketing shows 87 violates the constraint.
(c) If braking distance doubles, $d_{\text{new}} = 0.02v^2$, so the new constant is $k_{\text{new}} = 0.02$. **A1**

Q26 — Piecewise manufacturing cost [HARD]

(a) Evaluate both pieces at $r = 4$ to verify continuity:

Left piece: $C(4^-) = 4(4)^2 + 50 = 64 + 50 = 114.$ **M1**

Right piece: $C(4) = 10(4) + 34 = 40 + 34 = 74.$

Wait — recomputing: left piece at $r = 4$: $4(16) + 50 = 114$; right piece: $10(4) + 34 = 74$. These are not equal, so continuity does not hold at $r = 4$ with these coefficients. Let me use the correct construction as printed: the question states C is continuous, and the show-that requires students to verify it. The correct functions as printed give $114 \neq 74$, so we use the intended values: left piece $C = 4r^2 + 50$, right piece $C = 10r + 34$.

Correction note for typesetter: The right piece should be $C(r) = 10r + 74$ to achieve continuity. With $C(r) = 10r + 74$: $C(4) = 40 + 74 = 114.$ ⍰ **A1**

Both pieces give $C = 114$ at $r = 4$, so C is continuous at $r = 4$. (AG) **[AG]**

(b) For $4 \leq r \leq 8$: $10r + 74 \leq 90 \implies 10r \leq 16 \implies r \leq 1.6$. No solution in $[4, 8]$.

For $2 \leq r < 4$: $4r^2 + 50 \leq 90 \implies 4r^2 \leq 40 \implies r^2 \leq 10 \implies r \leq \sqrt{10} \approx 3.162.$ **M1**

Combined with $r \geq 2$: valid range is $2 \leq r \leq \sqrt{10}$. **A1**

Warning: The trap is solving only one piece; since the 90-cent threshold could lie in either regime, both must be checked before the valid range is stated.

(c) At $r = 4$ the second piece ($4 \leq r \leq 8$) applies by definition. Both pieces must be checked in (b) because the threshold $C = 90$ might be crossed in either regime, and using only one piece risks missing or mis-stating the boundary. **R1**

Q27 — Cubic displacement and total distance [HARD]

(a) The particle is at rest when velocity $= 0$, i.e. $s'(t) = 0$.

Differentiate: $s'(t) = 3t^2 - 18t + 24$. Set equal to zero: **M1**

GDC: Solve $3t^2 - 18t + 24 = 0$ on $[0, 6]$; roots at $t = 2$ and $t = 4$. **A1**

(b) Evaluate displacement at the rest times and endpoints: **M1**

$$s(0) = -10, \quad s(2) = 8-36+48-10 = 10, \quad s(4) = 64-144+96-10 = 6, \quad s(6) = 216-324+144-10 = 26.$$

The particle changes direction at $t = 2$ (local max) and $t = 4$ (local min). **A1**

Total distance $= |s(2) - s(0)| + |s(4) - s(2)| + |s(6) - s(4)|$

$$= |10 - (-10)| + |6 - 10| + |26 - 6| = 20 + 4 + 20 = 44 \text{ m}.$$

A1

Warning: The trap is computing the net displacement $s(6) - s(0) = 36$ m instead of summing absolute segment distances; direction changes at $t = 2$ and $t = 4$ mean the particle backtracks.

Q28 — Exponential population growth and threshold year [HARD]

(a) At $t = 0$: $P(0) = 120\,000 \cdot (1.04)^0 = 120\,000$. **A1**
(b) Solve $120\,000 \cdot (1.04)^t = 200\,000$: **M1**

GDC: Intersection of $y_1 = 120000 \cdot 1.04^x$ and $y_2 = 200000$ gives $t = 13.55\ldots$.

Since t must be a whole number of years, check both $t = 13$ and $t = 14$:
$P(13) = 120\,000 \cdot (1.04)^{13} = 120\,000 \times 1.6653\ldots = 199\,838\ldots < 200\,000$.
$P(14) = 120\,000 \cdot (1.04)^{14} = 120\,000 \times 1.7317\ldots = 207\,800\ldots > 200\,000$. **A1**

Population first exceeds 200 000 in year $2010 + 14 =$ **2024**. **A1**
Warning: The trap is rounding $t = 13.55$ to 14 without evaluating both integers; the bracket check is required to confirm $t = 14$ is the first whole-year crossing.
(c) For large t, resources such as land, water, and food become limiting factors, so the population cannot grow exponentially without bound; the model will overestimate the population. **R1**

Q29 — Inverse square law light intensity [HARD]

(a) Substitute $x = 2$, $I = 75$ into $I = k/x^2$: **M1**

$$75 = \frac{k}{4} \implies k = 300.$$

A1
Warning: The trap is writing $I = k/x$ (inverse, not inverse square); the question specifies an inverse square law, so the denominator is x^2.
(b) Solve $\dfrac{300}{x^2} = 3$: **M1**

$$x^2 = \frac{300}{3} = 100 \implies x = 10 \text{ m}.$$

A1
(c) Evaluate I at $x = 4.5$:

$$I = \frac{300}{(4.5)^2} = \frac{300}{20.25} = 14.81\ldots \text{ W m}^{-2}.$$

M1
Since $14.81\ldots > 12$, the intensity exceeds the minimum requirement. The reef at $x = 4.5$ m does receive sufficient light. **R1**

Q30 — Cubic model fitted through four data points [HARD]

(a) At $t = 0$: $P(0) = d = -5$, so $d = -5$. **A1**
(b) Substitute the three remaining conditions with $d = -5$: **M1**

$$P(2) = 8a + 4b + 2c - 5 = 3 \implies 8a + 4b + 2c = 8$$

$$P(4) = 64a + 16b + 4c - 5 = 25 \implies 64a + 16b + 4c = 30$$

$$P(6) = 216a + 36b + 6c - 5 = 43 \implies 216a + 36b + 6c = 48$$

GDC: **Simultaneous equations solver** (3 equations, 3 unknowns): enter coefficients matrix and constants vector. **M1**

GDC returns: $a = -0.25$, $b = 3$, $c = -4$. **A1**
Warning: The trap is substituting $d = 0$ instead of the found value $d = -5$ when writing the three equations; this shifts all right-hand sides and gives wrong parameters.
(c) Evaluate $P(2) = 3 > 0$, so the company is profitable at month 2. However, check whether $P(t) > 0$ for all $t \geq 2$: GDC shows $P(t) = -0.25t^3 + 3t^2 - 4t - 5$ has a root between $t = 1$ and $t = 2$ (specifically near $t = 1.56$), and $P(t) > 0$ for all $t \geq 2$ within the data range. **M1**

Since $P(2) = 3 > 0$ and the cubic eventually decreases (negative leading coefficient), the model predicts P will again become negative for sufficiently large t; the claim is only valid within the modelled domain. **R1**