IB Math AI SL — Topic 3

Geometry of 3D Shapes

3D Coordinate Geometry · Volume & Surface Area

Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:_____ Date:_____

Score:_____/120

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Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. A graphic display calculator is required throughout; support GDC answers with the values entered and the feature used.

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Key Formulae

- Distance in 3D: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
- Midpoint in 3D: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$
- Volume of cylinder, cone, sphere: $V_{\text{cyl}} = \pi r^2 h$; $V_{\text{cone}} = \frac{1}{3}\pi r^2 h$; $V_{\text{sphere}} = \frac{4}{3}\pi r^3$
- Surface area of cylinder and sphere: $A_{\text{cyl}} = 2\pi rh + 2\pi r^2$; $A_{\text{sphere}} = 4\pi r^2$
- Curved surface area of cone: $A_{\text{cone}} = \pi rl$, where slant height $l = \sqrt{r^2 + h^2}$
- Volume of cuboid and pyramid: $V_{\text{cuboid}} = lwh$; $V_{\text{pyramid}} = \frac{1}{3} \times \text{base area} \times h$
- Unit conversion: 1 litre $= 1000$ cm^3; 1 m$^3 = 10^6$ cm^3

GDC Tips

- **Solving for an unknown dimension** (e.g. find r given V): store the volume equation as $f(r) =$ target, then use Graph $\rightarrow$ G-Solv $\rightarrow$ ROOT after entering $f(r) -$ target $= 0$, or use Equation $\rightarrow$ Solver.
- **Evaluating composite-solid volumes:** compute each piece separately using Run-Matrix memory variables (e.g. store $\pi r^2 h$ as A, cone volume as B, then A+B).
- **Slant height before cone surface area:** calculate $l = \sqrt{r^2 + h^2}$ in Run-Matrix first and store it; then enter $\pi r l$ to avoid retyping and rounding errors.
- **Angle in a solid:** use Run-Matrix to find the space diagonal with nested $\sqrt{\ }$, store it, then apply $\tan^{-1}$ (or $\sin^{-1}$ / $\cos^{-1}$) in the same session so no intermediate rounding occurs.
- **"How many fit" questions:** compute the ratio of volumes, display the unrounded decimal, then apply Int( ) (integer part / floor) from CATALOG — do not round up.
- **Unit conversion check:** if data are in cm but the answer requires litres, convert before pressing EXE by multiplying by $\frac{1}{1000}$; keep this as a single chain calculation to avoid premature rounding.

Common Mistakes to Avoid

1. **Using vertical height instead of slant height in cone surface area.** The formula $A = \pi r l$ requires the slant height $l = \sqrt{r^2 + h^2}$, not the vertical height h. Always compute l first as a separate step and write it down before substituting.
2. **Forgetting to convert units before calculating volume or capacity.** If dimensions are given in cm but the answer is required in litres or m^3, students who substitute raw cm values and then forget to divide by 1000 (for litres) or 10^6 (for m^3) will score zero on the final accuracy mark even if the method is correct.
3. **Rounding up instead of flooring "how many fit" answers.** When dividing a total volume by a unit volume to find how many objects fit inside, the answer must be rounded down (floor) because a fractional object does not fit. Rounding to the nearest integer or rounding up gives a physically impossible answer.
4. **Premature rounding in chained calculations.** Storing a rounded intermediate value (e.g. a slant height rounded to 3 s.f.) and using it in the next step introduces an error that compounds. Always carry the full unrounded value in GDC memory and round only the final answer.
5. **Omitting one face of a composite solid's surface area.** When two solids are joined (e.g. a cone placed on top of a cylinder), the circular face where they meet is interior and must be excluded from the total surface area. Students frequently add all faces of both solids without subtracting the two shared circles.
6. **Confusing the 3D distance formula with the 2D formula.** In three dimensions the distance formula includes a $(z_2 - z_1)^2$ term. Omitting the z-component gives the distance projected onto the horizontal plane, not the true straight-line distance, which is always longer than or equal to the 2D projection.

Section A — Easy EASY (10 questions × 3 marks = 30 marks)

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1. EASY Calculator [3 marks]

A storage room has corners at coordinates $A(0, 0, 0)$ and $B(6, 4, 3)$, where distances are in metres. Find the distance AB.

2. EASY Calculator [3 marks]

A rectangular box (cuboid) has length 8 cm, width 5 cm, and height 3 cm. Find the volume of the box.

Solid	Length (cm)	Width (cm)	Height (cm)
Cuboid	8	5	3

3. EASY Calculator [3 marks]

A cylindrical water tank has radius 2.5 m and height 4 m. Find the volume of the tank. Give your answer in m^3, correct to 3 significant figures.

4. **EASY** **Calculator** **[3 marks]**

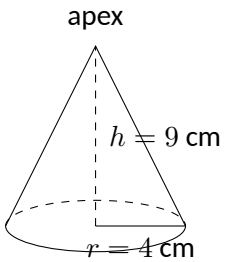
Two drones are positioned at coordinates $P(1, 3, 5)$ and $Q(7, 3, 5)$, where distances are in metres. Write down the midpoint M of PQ.

5. **EASY** **Calculator** **[3 marks]**

A solid sphere has radius 6 cm. Find the surface area of the sphere. Give your answer correct to 3 significant figures.

6. **EASY** **Calculator** **[3 marks]**

A cone has a vertical height of 9 cm and a base radius of 4 cm. Find the slant height of the cone.



(diagram not to scale)

7. **EASY** **Calculator** **[3 marks]**

A hemisphere has a radius of 5 cm. Find the total surface area of the hemisphere, including its flat circular base.

Give your answer correct to 3 significant figures.

8. EASY Calculator **[3 marks]**

Two support cables are anchored at points $A(0, 0, 0)$ and $B(3, 4, 0)$ on the ground, and both connect to a point $C(3, 4, 12)$ directly above B, where distances are in metres. Find the length of cable AC.

9. EASY Calculator **[3 marks]**

A square-based pyramid has a base side length of 10 cm and a vertical height of 12 cm. Find the volume of the pyramid.

Solid	Base side length (cm)	Vertical height (cm)
Square pyramid	10	12

10. EASY Calculator **[3 marks]**

A cylindrical tin of paint has radius 7 cm and height 15 cm. Find the total surface area of the closed tin. Give your answer correct to 3 significant figures.

Section B — Medium MEDIUM (10 questions × 4 marks = 40 marks)

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11. MEDIUM Calculator [4 marks]

A storage warehouse has a roof in the shape of a triangular prism. The cross-section of the roof is an isosceles triangle with base $AB = 18$ m and slant edges of length 15 m. The length of the prism (depth of the warehouse) is 30 m.

Dimension	Symbol	Value
Base of triangle	AB	18 m
Slant edge	l	15 m
Prism depth	d	30 m

(a) Find the vertical height of the triangular cross-section. [2]

(b) Find the total volume of the roof. [2]

12. MEDIUM Calculator [4 marks]

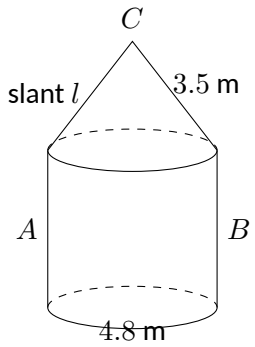
A composite trophy consists of a solid cylinder with a solid hemisphere placed on top, sharing the same circular base. The cylinder has radius $r = 3.2$ cm and height $h = 8.5$ cm. The hemisphere has the same radius.

(a) Write down the radius of the hemisphere. [1]

(b) Find the total volume of the trophy. [3]

13. **MEDIUM** **Calculator** **[4 marks]**

A grain silo consists of a cylinder of radius 2.4 m and height 6 m, with a cone of the same base radius on top. The total height of the silo (cylinder plus cone) is 9.5 m. (Diagram not to scale.)



(a) Find the vertical height of the cone. [1]

(b) Find the slant height of the cone. [1]

(c) Find the total surface area of the outside of the silo (base excluded). [2]

14. **MEDIUM** **Calculator** **[4 marks]**

A rectangular fish tank has internal dimensions 80 cm long, 35 cm wide, and 40 cm tall. Water fills the tank to 75% of its height.

(a) Find the volume of water in the tank in cm^3. [2]

(b) Convert your answer to litres. [1]

(c) A spherical decoration of radius 4 cm is fully submerged in the tank. Determine by how many centimetres the water level rises. [1]

15. MEDIUM Calculator **[4 marks]**

Two drones, P and Q, are tracked using 3D coordinates measured in metres. Drone P is at $(4, -1, 6)$ and drone Q is at $(10, 5, 2)$.

(a) Find the straight-line distance between the two drones. [2]

(b) Find the coordinates of the midpoint M between P and Q. [2]

16. MEDIUM Calculator **[4 marks]**

A solid square-based pyramid has a base side length of 10 cm and a vertical height of 12 cm.

Dimension	Symbol	Value
Base side length	s	10 cm
Vertical height	h	12 cm

(a) Find the slant height of a triangular face of the pyramid. [2]

(b) Find the total surface area of the pyramid, including the base. [2]

17. MEDIUM Calculator **[4 marks]**

A metal sphere of radius 5 cm is melted down and recast into solid cones. Each cone has base radius 2 cm and vertical height 5 cm.

(a) Show that the volume of the sphere is $\frac{500\pi}{3}$ cm^3. [1]

(b) Find the volume of one cone. [1]

(c) Find the maximum number of complete cones that can be made. [2]

18. MEDIUM Calculator **[4 marks]**

A room has the shape of a cuboid with length 7.5 m, width 4.2 m, and height 2.8 m. A cable is to be run from one bottom corner A to the diagonally opposite top corner B.

(a) Find the length of the diagonal of the base of the room. [2]

(b) Find the angle that the cable AB makes with the floor of the room. [2]

19. MEDIUM Calculator **[4 marks]**

A cylindrical water tank has an internal radius of 0.75 m and an internal height of 1.6 m. A builder needs to fill the tank using 12-litre buckets.

(a) Find the volume of the tank in m^3. [2]

(b) Convert the volume to litres. ($1\ \text{m}^3 = 1000$ litres) [1]

(c) Find the minimum number of complete buckets needed to fill the tank. [1]

20. MEDIUM Calculator **[4 marks]**

A garden ornament consists of a solid hemisphere of radius 6 cm sitting on top of a solid cuboid base of length 18 cm, width 18 cm, and height 4 cm.

Component	Dimension	Value
Hemisphere	radius	6 cm
Cuboid	length $\times$ width $\times$ height	$18 \times 18 \times 4$ cm

(a) Find the total volume of the ornament. [2]

(b) The ornament is to be painted on all exposed surfaces. Find the total surface area to be painted. [2]

Section C — Hard HARD (10 questions × 5 marks = 50 marks)

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21. HARD Calculator [5 marks]

A storage silo is composed of a cylinder with a hemisphere placed on top. The cylinder has a radius of 3.2 m and a height of 8.5 m. The hemisphere has the same radius as the cylinder.

Component	Radius (m)	Height/depth (m)
Cylinder	3.2	8.5
Hemisphere	3.2	3.2

(a) Show that the total volume of the silo is approximately 342 m^3. [2]

(b) Grain is poured into the silo. Each cubic metre of grain has a mass of 720 kg. Find the maximum mass of grain the silo can hold, giving your answer to the nearest tonne. [3]

22. HARD Calculator [5 marks]

A glass ornament is made by melting down a solid sphere of radius 6 cm and recasting the molten glass into identical small solid cones. Each cone has a base radius of 1.5 cm and a vertical height of 4 cm.

(a) Find the volume of the original sphere. [1]

(b) Find the number of complete cones that can be made. Justify your answer. [4]

23. HARD Calculator **[5 marks]**

A grain container is in the shape of a square-based pyramid. The square base has side length s metres and the vertical height of the pyramid is 6.0 m. The volume of the container is 96 m^3.

(a) Find the value of s. [2]

(b) Hence find the slant height of the pyramid, from the midpoint of a base edge to the apex. [2]

(c) A coat of sealant is applied to all four triangular faces only. One tin of sealant covers 18 m^2. Find the minimum number of tins required. Justify your answer. [1]

24. HARD Calculator **[5 marks]**

A water tank is in the shape of a cylinder with internal radius r cm and internal height 40 cm. The tank holds exactly 60 litres when full.

(a) Show that $r = \sqrt{\dfrac{60\,000}{40\pi}}$, giving an unrounded value before stating that $r \approx 21.9$ cm. [2]

(b) Water is pumped into the empty tank at a constant rate of 2.5 litres per minute. Find the time, in minutes, for the tank to be 75% full. [3]

25. HARD Calculator **[5 marks]**

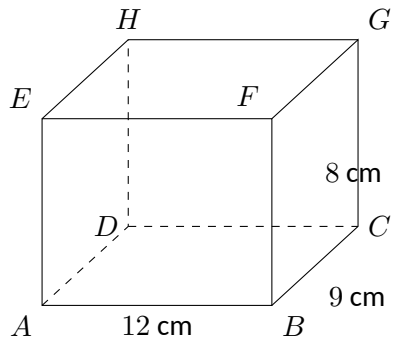
A drone flies from point $A(2, 5, 10)$ to point $B(14, 1, 4)$ and then to point $C(8, 9, 4)$, where coordinates are in metres relative to a fixed origin, with the z-axis pointing vertically upward.

(a) Find the total distance the drone travels from A to B to C. [3]

(b) Find the angle $\widehat{ABC}$, the angle at B in triangle ABC. [2]

26. HARD Calculator **[5 marks]**

A rectangular box (cuboid) has dimensions 12 cm $\times$ 9 cm $\times$ 8 cm (length $\times$ width $\times$ height). Diagram not to scale.



(a) Find the length of the space diagonal AG. [2]

(b) Find the angle that the space diagonal AG makes with the base of the cuboid. [3]

27. HARD Calculator **[5 marks]**

A cylindrical paint tin has an internal radius of 7.5 cm. Paint is sold in tins that are filled to a height of 18 cm. A manufacturer wants to design a new conical tin that holds the same volume of paint, with the same base radius of 7.5 cm.

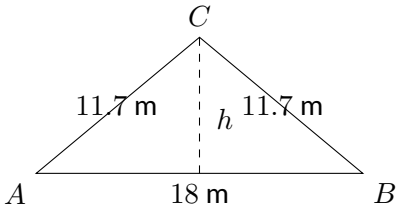
(a) Find the volume of paint in the cylindrical tin, giving your answer in litres. [2]

(b) Find the vertical height of the new conical tin. [2]

(c) The conical tin is to be made from a flat sheet of metal. The curved surface of the cone requires $\pi r l$ cm^2 of metal, where l is the slant height. Determine whether a sheet of metal of area $2\,000$ cm^2 is sufficient for the curved surface only, justifying your answer. [1]

28. HARD Calculator **[5 marks]**

A sports hall has a roof in the shape of a triangular prism. The cross-section of the roof is an isosceles triangle with base 18 m and two equal sides of length 11.7 m. The length (depth) of the hall is 30 m.



(a) Show that the vertical height h of the roof's cross-section is approximately 7.48 m (to 3 significant figures). [2]

(b) Hence find the volume of the triangular prism forming the roof. [1]

(c) The four rectangular faces of the prism (two sides and two ends, not including the triangular faces) are to be clad in aluminium panels. Find the total area to be clad. [2]

29. **HARD** **Calculator** **[5 marks]**

A spherical water balloon has a radius of r cm. When the balloon is punctured it collapses completely and all the water pours into an empty cylindrical bucket. The bucket has an internal radius of 12 cm. After the water settles, the depth of water in the bucket is 9 cm.

(a) Write down an equation relating the volume of the sphere to the volume of water in the cylinder. [1]

(b) Find the radius of the balloon, giving your answer to 3 significant figures. [2]

(c) The balloon was advertised as having a diameter of more than 20 cm. Determine whether this claim is justified. [2]

30. **HARD** **Calculator** **[5 marks]**

Two checkpoints P and Q are positioned in a three-dimensional training course. Their coordinates are $P(3, 7, 2)$ and $Q(9, 1, 14)$, where units are in metres.

(a) Find the coordinates of the midpoint M of PQ. [1]

(b) Find the distance PQ. [2]

(c) A third checkpoint R is placed so that R lies on the line segment PQ, at a point that is $\frac{1}{3}$ of the way from P to Q. Find the coordinates of R. [2]

Mark Schemes

Q1 — Distance between two 3D coordinates EASY

We apply the 3D distance formula because A and B are given as coordinates in three-dimensional space.

$$AB = \sqrt{(6-0)^2 + (4-0)^2 + (3-0)^2}$$

M1

$$AB = \sqrt{36 + 16 + 9} = \sqrt{61} = 7.81\ldots \approx 7.81 \text{ m}$$

A1

Warning: Do not forget to include all three coordinate differences; a common error is to use only two dimensions.

Answer: $AB = \sqrt{61} \approx 7.81$ m

Q2 — Volume of a cuboid EASY

The volume of a cuboid is found by multiplying its three dimensions together; this is a direct application of the formula $V = l \times w \times h$.

$$V = 8 \times 5 \times 3$$

M1

$$V = 120 \text{ cm}^3$$

A1

Warning: Always include the correct units (cm^3) in your answer; leaving units off costs a mark.

Answer: $V = 120$ cm^3

Q3 — Volume of a cylinder EASY

We use the formula $V = \pi r^2 h$ for the volume of a cylinder, substituting $r = 2.5$ m and $h = 4$ m.

$$V = \pi \times (2.5)^2 \times 4$$

M1

GDC: Type $\pi \times 2.5^2 \times 4$ directly; the display reads $78.539\ldots$

$$V = 78.539\ldots \approx 78.5 \text{ m}^3$$

A1

Warning: Use the unrounded value $78.539\ldots$ before rounding; writing 78.5 in the working line without showing the unrounded value first would not earn the method mark.

Answer: $V \approx 78.5$ m^3

Q4 — Midpoint of a 3D segment EASY

The midpoint formula in 3D averages the corresponding coordinates of the two endpoints.

$$M = \left(\frac{1+7}{2}, \frac{3+3}{2}, \frac{5+5}{2}\right)$$

M1

$$M = (4, 3, 5)$$

A1

Warning: All three coordinates must be averaged, even if some are identical; failing to state the z-coordinate loses a mark.

Answer: $M = (4, 3, 5)$

Q5 — Surface area of a sphere EASY

We use the formula $SA = 4\pi r^2$ for the surface area of a sphere, with $r = 6$ cm.

$$SA = 4 \times \pi \times 6^2 = 4\pi \times 36$$

M1

GDC: Type $4 \times \pi \times 36$; the display reads $452.389\ldots$

$$SA = 452.389\ldots \approx 452 \text{ cm}^2$$

A1

Warning: The surface area formula is $4\pi r^2$, not $\frac{4}{3}\pi r^3$ (which is volume); confusing these two formulae is a very common error.

Answer: $SA \approx 452$ cm^2

Q6 — Slant height of a cone via Pythagoras EASY

The slant height l, vertical height h, and base radius r form a right triangle, so we apply Pythagoras' theorem: $l^2 = r^2 + h^2$.

$$l^2 = 4^2 + 9^2 = 16 + 81 = 97$$

M1

$$l = \sqrt{97} = 9.849\ldots \approx 9.85 \text{ cm}$$

A1

Warning: The slant height is not the vertical height; always identify which measurement is required before applying the formula.

Answer: $l = \sqrt{97} \approx 9.85$ cm

Q7 — Total surface area of a hemisphere EASY

A hemisphere's total surface area consists of the curved surface ($2\pi r^2$) plus the flat circular base (πr^2), giving $SA = 3\pi r^2$ in total, with $r = 5$ cm.

$$SA = 2\pi(5)^2 + \pi(5)^2 = 3\pi \times 25$$

M1

GDC: Type $3 \times \pi \times 25$; the display reads $235.619\ldots$

$$SA = 235.619\ldots \approx 236 \text{ cm}^2$$

A1

Warning: The question says "including its flat circular base"; forgetting the base and using only $2\pi r^2$ is a direct trap.

Answer: $SA \approx 236$ cm^2

Q8 — Distance between 3D coordinates in context EASY

We apply the 3D distance formula between points $A(0, 0, 0)$ and $C(3, 4, 12)$ to find the length of cable AC.

$$AC = \sqrt{(3-0)^2 + (4-0)^2 + (12-0)^2} = \sqrt{9 + 16 + 144} = \sqrt{169}$$

M1

$$AC = 13 \text{ m}$$

A1

Warning: All three coordinate differences must be squared and summed; omitting the vertical component (12 m) gives the incorrect ground distance $AB = 5$ m.

Answer: $AC = 13$ m

Q9 — Volume of a square-based pyramid EASY

We use the formula $V = \frac{1}{3} \times$ base area $\times\, h$, where the base is a square with side 10 cm and the vertical height is 12 cm.

$$V = \frac{1}{3} \times 10^2 \times 12 = \frac{1}{3} \times 100 \times 12$$

M1

$$V = \frac{1200}{3} = 400 \text{ cm}^3$$

A1

Warning: Do not forget the factor of $\frac{1}{3}$ in the pyramid volume formula; using $V =$ base area $\times\, h$ without this factor gives an answer three times too large.

Answer: $V = 400$ cm^3

Q10 — Total surface area of a closed cylinder EASY

The total surface area of a closed cylinder is $SA = 2\pi r^2 + 2\pi rh$, consisting of two circular ends and the curved surface, with $r = 7$ cm and $h = 15$ cm.

$$SA = 2\pi(7)^2 + 2\pi(7)(15) = 2\pi(49) + 2\pi(105)$$

M1

GDC: Type $2 \times \pi \times 49 + 2 \times \pi \times 7 \times 15$; the display reads $967.610\ldots$

$$SA = 967.610\ldots \approx 968 \text{ cm}^2$$

A1

Warning: A closed tin has two circular ends; using only one circular end (πr^2 instead of $2\pi r^2$) underestimates the surface area. Also, always evaluate $2\pi(49) + 2\pi(105)$ as a single unrounded GDC sum rather than trusting a remembered figure — re-typing intermediate results by hand is a common source of transcription error.

Answer: $SA \approx 968$ cm^2

Q11 — Triangular prism roof volume MEDIUM

(a) The cross-section is isosceles, so the vertical height is found by applying Pythagoras to the right triangle formed by the slant edge, half the base, and the height.

$$h = \sqrt{15^2 - 9^2} = \sqrt{225 - 81} = \sqrt{144} = 12 \text{ m}$$

M1 A1

(b) The volume of a prism equals the cross-sectional area multiplied by the depth; the triangular cross-

section has base 18 m and height 12 m.

$$V = \tfrac{1}{2} \times 18 \times 12 \times 30 = 3240 \text{ m}^3$$

M1 A1

Warning: A common trap is to use the slant length 15 m as the height of the triangle instead of computing the perpendicular height 12 m first. The slant is the hypotenuse, not the height.

Q12 — Cylinder plus hemisphere trophy volume MEDIUM

(a) The hemisphere sits on the same circular base as the cylinder, so the radius of the hemisphere equals the cylinder radius.

$$r = 3.2 \text{ cm}$$

A1

(b) The total volume is the sum of the cylinder volume and the hemisphere volume. The hemisphere volume is half of a full sphere.

Cylinder: $V_{\text{cyl}} = \pi r^2 h = \pi(3.2)^2(8.5) = \pi \times 10.24 \times 8.5 = 87.04\pi$ **M1**

Hemisphere: $V_{\text{hemi}} = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(3.2)^3 = \frac{2}{3}\pi(32.768) = 21.8453\pi$

$$V_{\text{total}} = \pi(87.04 + 21.8453\ldots) = 108.885\ldots \times \pi = 342.073\ldots \approx 342 \text{ cm}^3$$

M1 A1

Warning: Take care with r^2h: $3.2^2 \times 8.5 = 10.24 \times 8.5 = 87.04$ — always evaluate this directly on the GDC rather than by hand, since a slipped multiplication here is a common arithmetic error. Also, the hemisphere volume is $\frac{2}{3}\pi r^3$, NOT $\frac{4}{3}\pi r^3$; using the full sphere formula doubles the hemisphere's contribution.

Q13 — Composite silo surface area MEDIUM

(a) The vertical height of the cone equals the total silo height minus the cylinder height.

$$h_{\text{cone}} = 9.5 - 6 = 3.5 \text{ m}$$

A1

(b) The slant height is found by Pythagoras using the cone's vertical height and base radius.

$$l = \sqrt{r^2 + h^2} = \sqrt{(2.4)^2 + (3.5)^2} = \sqrt{5.76 + 12.25} = \sqrt{18.01} = 4.2436\ldots \approx 4.24 \text{ m}$$

M1 A1

(c) The outside surface area (base excluded) consists of the curved surface of the cylinder and the curved surface of the cone.

$$SA_{\text{cyl}} = 2\pi rh = 2\pi(2.4)(6) = 28.8\pi$$

$$SA_{\text{cone}} = \pi rl = \pi(2.4)(4.2436\ldots) = 10.1847\ldots\pi$$

$$SA_{\text{total}} = (28.8 + 10.1847\ldots)\pi = 38.9847\ldots\pi = 122.476\ldots \approx 122\ \text{m}^2$$

M1 A1

Warning: Use the slant height l (not the vertical height) in the cone curved surface area formula πrl. Using vertical height 3.5 m instead gives an incorrect answer.

Q14 — Fish tank water volume and level rise MEDIUM

(a) The volume of water equals 75% of the full cuboid volume.

$$V_{\text{water}} = 0.75 \times 80 \times 35 \times 40 = 0.75 \times 112\,000 = 84\,000\ \text{cm}^3$$

M1 A1

(b) Converting cm^3 to litres using 1 litre $= 1000$ cm^3.

$$84\,000 \div 1000 = 84\ \text{litres}$$

A1

(c) The rise in water level equals the volume of the sphere divided by the base area of the tank. Volume of sphere: $V_{\text{sphere}} = \frac{4}{3}\pi(4)^3 = \frac{256\pi}{3} = 268.082\ldots$ cm^3.

$$\Delta h = \frac{V_{\text{sphere}}}{L \times W} = \frac{268.082\ldots}{80 \times 35} = \frac{268.082\ldots}{2800} = 0.09574\ldots \approx 0.0957\ \text{cm}$$

A1

Warning: The unit conversion trap here is central — the volume must remain in cm^3 when dividing by the base area in cm^2 to get a rise in cm. Do not convert to litres before finding the rise.

Q15 — 3D distance and midpoint between drones MEDIUM

(a) The straight-line distance between $P(4, -1, 6)$ and $Q(10, 5, 2)$ uses the 3D distance formula.

$$PQ = \sqrt{(10-4)^2 + (5-(-1))^2 + (2-6)^2} = \sqrt{36 + 36 + 16} = \sqrt{88} = 9.3808\ldots \approx 9.38\ \text{m}$$

M1 A1

(b) The midpoint M is found by averaging corresponding coordinates.

$$M = \left(\frac{4+10}{2}, \frac{-1+5}{2}, \frac{6+2}{2}\right) = (7, 2, 4)$$

M1 A1

Warning: When computing the y-coordinate difference, $5 - (-1) = 6$, not 4. A sign error here is the built-in trap; always subtract carefully with negative coordinates.

Q16 — Square pyramid slant height and surface area MEDIUM

(a) The slant height of a triangular face is found using Pythagoras with the vertical height and the apothem (half the base side length).

$$l = \sqrt{h^2 + \left(\frac{s}{2}\right)^2} = \sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13 \text{ cm}$$

M1 A1

(b) The total surface area equals the square base plus four congruent triangular faces.

$$SA_{\text{base}} = s^2 = 10^2 = 100 \text{ cm}^2$$

$$SA_{\text{faces}} = 4 \times \tfrac{1}{2} \times s \times l = 4 \times \tfrac{1}{2} \times 10 \times 13 = 260 \text{ cm}^2$$

$$SA_{\text{total}} = 100 + 260 = 360 \text{ cm}^2$$

M1 A1

Warning: The slant height is the height of a triangular face (from midpoint of base edge to apex), NOT the distance from a base corner to the apex. The apothem is $\frac{s}{2} = 5$ cm, not $s = 10$ cm.

Q17 — Sphere melted into cones (integer count) MEDIUM

(a) Substituting $r = 5$ cm into the sphere volume formula to verify the printed result.

$$V_{\text{sphere}} = \tfrac{4}{3}\pi r^3 = \tfrac{4}{3}\pi(5)^3 = \tfrac{4}{3}\pi \times 125 = \frac{500\pi}{3} \text{ cm}^3$$

AG

(b) The volume of one cone with base radius 2 cm and height 5 cm.

$$V_{\text{cone}} = \tfrac{1}{3}\pi r^2 h = \tfrac{1}{3}\pi(2)^2(5) = \frac{20\pi}{3} \text{ cm}^3$$

A1

(c) Dividing the total available volume by the volume of one cone gives the maximum number.

$$n = \frac{500\pi/3}{20\pi/3} = \frac{500\pi}{3} \times \frac{3}{20\pi} = \frac{500}{20} = 25$$

M1 A1

Since $n = 25$ exactly, 25 complete cones can be made with no material remaining.

Warning: Only complete cones count — if the division had not been exact, the answer must be floored to the nearest whole number below, not rounded up. Always check whether a partial cone can count.

Q18 — Cuboid space diagonal angle with floor MEDIUM

(a) The diagonal of the base is found using Pythagoras in two dimensions with the length and width of the room.

$$d_{\text{base}} = \sqrt{7.5^2 + 4.2^2} = \sqrt{56.25 + 17.64} = \sqrt{73.89} = 8.5961\ldots \approx 8.60 \text{ m}$$

M1 A1

(b) The cable AB forms the hypotenuse of a right triangle whose base is the floor diagonal and whose vertical side is the room height. The angle θ with the floor is found using trigonometry.

$$\tan\theta = \frac{h}{d_{\text{base}}} = \frac{2.8}{8.5961\ldots} = 0.32573\ldots$$

$$\theta = \arctan(0.32573\ldots) = 18.047\ldots^\circ \approx 18.0^\circ$$

M1 A1

Warning: Carry the unrounded value of $d_{base} = 8.5961\ldots$ m into part (b). Using the rounded value 8.60 m prematurely gives $\theta = 18.043\ldots^\circ$, which still rounds correctly here, but in general premature rounding can cost a mark.

Q19 — Cylindrical tank capacity and bucket count MEDIUM

(a) The volume of a cylinder is $V = \pi r^2 h$ with $r = 0.75$ m and $h = 1.6$ m.

$$V = \pi(0.75)^2(1.6) = \pi(0.5625)(1.6) = 0.9\pi = 2.82743\ldots \approx 2.83 \text{ m}^3$$

M1 A1

(b) Converting from m^3 to litres using the given conversion $1\ \text{m}^3 = 1000$ litres.

$$V = 2.82743\ldots \times 1000 = 2827.43\ldots \approx 2830 \text{ litres}$$

A1

(c) Each bucket holds 12 litres; the number of complete buckets is found by dividing and rounding up (ceiling), since partial buckets must still be filled to deliver the required volume.

$$n = \left\lceil \frac{2827.43\ldots}{12} \right\rceil = \lceil 235.619\ldots \rceil = 236 \text{ buckets}$$

A1

Warning: This is a "minimum buckets needed" question, so the answer is rounded up, not down. Flooring to 235 buckets would leave the tank slightly underfilled. Always read whether the context demands a floor or ceiling.

Q20 — Hemisphere on cuboid: volume and painted surface area MEDIUM

(a) The total volume is the sum of the hemisphere and the cuboid.

$$V_{\text{hemi}} = \tfrac{2}{3}\pi r^3 = \tfrac{2}{3}\pi(6)^3 = \tfrac{2}{3}\pi(216) = 144\pi = 452.389\ldots \text{ cm}^3$$

$$V_{\text{cuboid}} = 18 \times 18 \times 4 = 1296 \text{ cm}^3$$

$$V_{\text{total}} = 452.389\ldots + 1296 = 1748.389\ldots \approx 1750 \text{ cm}^3$$

M1 A1

(b) Exposed surfaces: the curved surface of the hemisphere, the four vertical sides of the cuboid, and the top face of the cuboid minus the circular base of the hemisphere (since that face is hidden at the join). The base of the cuboid is not painted.

$$SA_{\text{hemi}} = 2\pi r^2 = 2\pi(36) = 72\pi = 226.194\ldots \text{ cm}^2$$

$$SA_{\text{4 sides}} = 2(18)(4) + 2(18)(4) = 144 + 144 = 288 \text{ cm}^2$$

$$SA_{\text{top}} = 18^2 - \pi r^2 = 324 - 36\pi = 324 - 113.097\ldots = 210.902\ldots \text{ cm}^2$$

$$SA_{\text{total}} = 226.194\ldots + 288 + 210.902\ldots = 725.097\ldots \approx 725 \text{ cm}^2$$

M1 A1

Warning: The circular patch where the hemisphere meets the cuboid top is not a painted surface — it is the internal join. Forgetting to subtract $\pi r^2 = 36\pi$ cm^2 from the cuboid top is the built-in trap and overcounts the painted area.

Q21 — Composite solid volume and mass HARD

(a) The total volume is the sum of the cylinder and the hemisphere; we compute each component separately and add them before rounding, so that a single unrounded total can be shown.

$$V_{\text{cyl}} = \pi r^2 h = \pi(3.2)^2(8.5) = \pi \times 10.24 \times 8.5 = 87.04\pi \approx 273.44\ldots \text{ m}^3$$

$$V_{\text{hemi}} = \tfrac{2}{3}\pi r^3 = \tfrac{2}{3}\pi(3.2)^3 = \tfrac{2}{3}\pi(32.768) = 21.8453\pi \approx 68.63\ldots \text{ m}^3$$

Combining before rounding (premature rounding trap avoided): **M1**

$$V_{\text{total}} = 273.44\ldots + 68.63\ldots = 342.07\ldots \text{ m}^3$$

GDC: Enter $\pi(3.2)^2(8.5) + \tfrac{2}{3}\pi(3.2)^3$ directly $= 342.073\ldots \approx 342$ m^3 **AG**

Warning: $3.2^2 \times 8.5 = 10.24 \times 8.5 = 87.04$ — always recompute r^2h directly on the GDC rather than by hand, since a slipped multiplication here is a common arithmetic slip that changes the final total.

(b) Using the total volume of $342.073\ldots$ m^3 (not the rounded 342) to avoid an AG-firewall error:

Mass $= 342.073\ldots \times 720 = 246\,292.8\ldots$ kg **M1**

Converting to tonnes: $246\,292.8\ldots \div 1000 = 246.292\ldots$ tonnes **A1**

≈ 246 tonnes (to the nearest tonne) **A1**

Warning: Always carry the full unrounded GDC value for the volume forward into the mass calculation, not a rounded intermediate figure.

Q22 — Melt and recast: sphere into cones HARD

(a) The volume of the sphere of radius 6 cm is found directly:

$$V_{\text{sphere}} = \tfrac{4}{3}\pi r^3 = \tfrac{4}{3}\pi(6)^3 = 288\pi = 904.778\ldots \approx 905 \text{ cm}^3$$

A1

(b) First find the volume of one cone; then divide the sphere's volume by the cone's volume; finally apply the integer floor rule.

Volume of one cone: **M1**

$$V_{\text{cone}} = \tfrac{1}{3}\pi r^2 h = \tfrac{1}{3}\pi(1.5)^2(4) = \tfrac{1}{3}\pi(9) = 3\pi = 9.4247\ldots \text{ cm}^3$$

Number of cones (unrounded):

$$n = \frac{288\pi}{3\pi} = \frac{288}{3} = 96 \text{ (exact)}$$

M1

Since $n = 96$ exactly, all the glass is used with no remainder. **A1**

The number of complete cones that can be made is 96. **A1**

Warning: The answer here is exact; had the division given 95.7 the answer would be 95 not 96 — rounding up is physically impossible as there is insufficient glass for another complete cone. Always floor non-integer results for "how many complete" questions.

Q23 — Pyramid: reverse volume & surface area HARD

(a) The volume of a square-based pyramid is $V = \tfrac{1}{3}s^2h$. Setting this equal to 96 m^3 and solving for s: **M1**

$$\tfrac{1}{3}s^2(6.0) = 96 \implies 2s^2 = 96 \implies s^2 = 48 \implies s = \sqrt{48} = 6.928\ldots \approx 6.93 \text{ m}$$

A1

(b) The slant height l runs from the midpoint of a base edge to the apex. The distance from the centre of the base to the midpoint of a base edge is $\frac{s}{2}$. Using Pythagoras with the vertical height: **M1**

$$l = \sqrt{h^2 + \left(\frac{s}{2}\right)^2} = \sqrt{(6.0)^2 + \left(\frac{6.928\ldots}{2}\right)^2} = \sqrt{36 + 12.000\ldots} = \sqrt{48.000\ldots} = 6.928\ldots \approx 6.93 \text{ m}$$

A1

Warning: Use the unrounded value of s from part (a), not 6.93, to avoid a premature-rounding error in

the slant height.

(c) Total area of four triangular faces (lateral surface area):

$$LSA = 4 \times \tfrac{1}{2} \times s \times l = 4 \times \tfrac{1}{2} \times 6.928\ldots \times 6.928\ldots = 2 \times 48.000\ldots = 96.0\ldots \text{ m}^2$$

Number of tins $= 96.0\ldots \div 18 = 5.33\ldots$ tins.

Since $5.33\ldots > 5$, five tins are insufficient; the minimum number of tins required is 6. **R1**

Warning: Rounding 5.33 down to 5 tins leaves part of the surface uncoated — always ceiling (round up) for "how many tins" questions.

Q24 — Cylinder reverse dimension and fill-time HARD

(a) The volume of a cylinder is $V = \pi r^2 h$. Converting 60 litres to cm^3: $60 \times 1000 = 60\,000$ cm^3. **M1**

Setting $\pi r^2(40) = 60\,000$:

$$r^2 = \frac{60\,000}{40\pi} \implies r = \sqrt{\frac{60\,000}{40\pi}}$$

AG

GDC: Compute $\sqrt{60000 \div (40\pi)} = 21.8509\ldots \approx 21.9$ cm **AG**

Warning: The unit conversion from litres to cm^3 ($\times 1000$) is essential — omitting it gives $r \approx 2.19$ cm, which is unreasonably small.

(b) 75% of the tank volume $= 0.75 \times 60 = 45$ litres. **M1**

$$\text{Time} = \frac{45 \text{ litres}}{2.5 \text{ litres per minute}} = 18 \text{ minutes (exact).}$$ **A1**

The time for the tank to reach 75% full is 18 minutes. **A1**

Warning: The radius found in part (a) is not needed here — working in litres throughout avoids any unit mixing in this part.

Q25 — 3D coordinates: total path distance and angle HARD

(a) Distance AB: **M1**

$$AB = \sqrt{(14-2)^2 + (1-5)^2 + (4-10)^2} = \sqrt{144 + 16 + 36} = \sqrt{196} = 14 \text{ m}$$

Distance BC:

$$BC = \sqrt{(8-14)^2 + (9-1)^2 + (4-4)^2} = \sqrt{36 + 64 + 0} = \sqrt{100} = 10 \text{ m}$$

Total distance $= 14 + 10 = 24$ m. **A1**

(b) Distance AC:

$$AC = \sqrt{(8-2)^2 + (9-5)^2 + (4-10)^2} = \sqrt{36 + 16 + 36} = \sqrt{88} = 9.3808\ldots \text{ m}$$

Using the cosine rule at B (sides $BA = 14$, $BC = 10$, $AC = \sqrt{88}$): **M1**

$$\cos(\widehat{B}) = \frac{14^2 + 10^2 - 88}{2(14)(10)} = \frac{196 + 100 - 88}{280} = \frac{208}{280} = 0.74286\ldots$$

$$\widehat{B} = \arccos(0.74286\ldots) = 42.02\ldots^\circ \approx 42.0^\circ$$

A1

Warning: Using rounded values of AB, BC or AC before applying the cosine rule may affect the angle — carry unrounded values throughout. Full precision: $\arccos(208/280) = 42.0246\ldots^\circ$, *which rounds to* 42.0°.

Q26 — Cuboid: space diagonal and angle with base HARD

(a) The space diagonal of a cuboid with dimensions $l \times w \times h$ is $AG = \sqrt{l^2 + w^2 + h^2}$. **M1**

$$AG = \sqrt{12^2 + 9^2 + 8^2} = \sqrt{144 + 81 + 64} = \sqrt{289} = 17 \text{ cm}$$

A1

(b) First find the diagonal of the base $d = \sqrt{12^2 + 9^2} = \sqrt{144 + 81} = \sqrt{225} = 15$ cm. The space diagonal $AG = 17$ cm rises from the end of the base diagonal to the top corner; the angle θ with the base satisfies: **M1**

$$\tan\theta = \frac{h}{d} = \frac{8}{15} = 0.53\overline{3}$$

M1

$$\text{GDC: } \theta = \arctan\left(\frac{8}{15}\right) = 28.072\ldots^\circ \approx 28.1^\circ$$

A1

Warning: Use the base diagonal (15 cm), not a single base edge, as the horizontal component — confusing the base edge with the base diagonal is the built-in trap here.

Q27 — Cylinder to cone recast; slant height and area check HARD

(a) Volume of paint in cylinder:

$$V = \pi r^2 h = \pi(7.5)^2(18) = 1012.5\pi = 3180.86\ldots \text{ cm}^3$$

M1

Converting to litres ($\div 1000$): $3180.86\ldots \div 1000 = 3.18086\ldots \approx 3.18$ litres. **A1**

Warning: 1 *litre* $= 1000$ *cm*^{3}, *not* 100 *cm*^{3} *— dividing by* 100 *is a common unit-conversion error.*

(b) Setting the cone's volume equal to the cylinder's volume (1012.5π cm^3): **M1**

$$\tfrac{1}{3}\pi(7.5)^2 H = 1012.5\pi \implies \tfrac{1}{3}(56.25)H = 1012.5 \implies H = \frac{3 \times 1012.5}{56.25} = 54 \text{ cm}$$

A1

(c) Slant height: $l = \sqrt{r^2 + H^2} = \sqrt{(7.5)^2 + 54^2} = \sqrt{56.25 + 2916} = \sqrt{2972.25} = 54.518\ldots$ cm.
Curved surface area $= \pi r l = \pi(7.5)(54.518\ldots) = 1284.6\ldots$ cm^2.
Since $1284.6\ldots < 2000$, the sheet of area 2000 cm^2 is sufficient. **R1**
Warning: Use the slant height l, not the vertical height H, in the formula $\pi r l$ — using H instead gives $1272.3\ldots$ cm^2 and still passes the check here, but the method is wrong.

Q28 — Triangular prism roof: height & cladding area HARD

(a) Drop a perpendicular from the apex C to the base AB of the isosceles triangle. The base is split into two equal halves of 9 m each. Using Pythagoras: **M1**

$$h = \sqrt{11.7^2 - 9^2} = \sqrt{136.89 - 81} = \sqrt{55.89} = 7.4760\ldots \approx 7.48 \text{ m (to 3 s.f.)}$$

AG

Warning: The slant edge 11.7 m is the hypotenuse, not the height — always compute the perpendicular height using Pythagoras rather than assuming a round-number result.

(b) Volume of triangular prism $= \frac{1}{2} \times$ base $\times\, h \times$ length: **M1**

$$V = \tfrac{1}{2}(18)(7.4760\ldots)(30) = 9 \times 7.4760\ldots \times 30 = 2021.5\ldots \approx 2020 \text{ m}^3$$

A1

(c) The four faces to be clad are: two rectangular side faces (each 11.7×30 m) and the two triangular end faces (each with area $\frac{1}{2}(18)(7.4760\ldots)$). **M1**
Rectangular side faces: $2 \times (11.7 \times 30) = 2 \times 351 = 702$ m^2.
Triangular end faces: $2 \times \frac{1}{2}(18)(7.4760\ldots) = 2 \times 67.284\ldots = 134.568\ldots$ m^2.
Total area to be clad $= 702 + 134.568\ldots = 836.568\ldots \approx 837$ m^2 **A1**
Warning: The question states four faces are clad — two rectangular sides and two triangular ends — so both face types must be included, even though only two of the four are rectangles.

Q29 — Sphere radius from equal volumes; diameter claim check HARD

(a) Setting the volume of the sphere equal to the volume of water in the cylinder (equal volumes since water is conserved): **A1**

$$\tfrac{4}{3}\pi r^3 = \pi(12)^2(9)$$

(b) Solving for r: **M1**

$$\tfrac{4}{3}\pi r^3 = 144\pi(9) = 1296\pi \implies r^3 = 1296 \times \tfrac{3}{4} = 972$$

GDC: $r = \sqrt[3]{972} = 9.9065\ldots \approx 9.91$ cm (3 s.f.) **A1**

Warning: Remember to isolate r^3 completely before taking the cube root — a common error is computing $r = \sqrt[3]{1296}$ without multiplying by $\frac{3}{4}$ first.

(c) The diameter of the balloon $= 2r = 2(9.9065\ldots) = 19.813\ldots$ cm.

Since $19.813\ldots < 20$ cm, the diameter is less than 20 cm. **R1**

Therefore the claim that the diameter exceeds 20 cm is not justified. **A1**

Warning: Compare the unrounded diameter to 20 cm, not the rounded value — using $2 \times 9.91 = 19.82$ cm still leads to the same conclusion, but always base the decision on the unrounded computation.

Q30 — 3D coordinates: midpoint & division point HARD

(a) The midpoint M of PQ where $P(3, 7, 2)$ and $Q(9, 1, 14)$:

$$M = \left(\frac{3+9}{2}, \frac{7+1}{2}, \frac{2+14}{2}\right) = (6, 4, 8)$$

A1

(b) The distance PQ: **M1**

$$PQ = \sqrt{(9-3)^2 + (1-7)^2 + (14-2)^2} = \sqrt{36+36+144} = \sqrt{216} = 6\sqrt{6} = 14.696\ldots \approx 14.7 \text{ m}$$

A1

(c) Point R is $\frac{1}{3}$ of the way from P to Q. Using the section formula: **M1**

$$R = P + \tfrac{1}{3}(Q - P) = \left(3 + \tfrac{1}{3}(6),\ 7 + \tfrac{1}{3}(-6),\ 2 + \tfrac{1}{3}(12)\right) = (3+2,\ 7-2,\ 2+4) = (5, 5, 6)$$

A1

Warning: $\frac{1}{3}$ of the way from P to Q means adding $\frac{1}{3}$ of the vector $\overrightarrow{PQ}$ to P, not computing $\frac{1}{3}$ of each coordinate of Q — the latter is a common error that ignores the starting point P.