IB Math AI SL — Topic 3
Trigonometry

Right-Angled Trigonometry · Sine & Cosine Rule · Elevation & Depression · Bearings

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Difficulty	EASY	MEDIUM	HARD
Questions	10	10	10
Total Marks	30	40	50

Name:_____ Date:_____

Score:_____/120

Instructions: Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. A graphic display calculator is required throughout; support GDC answers with the values entered and the feature used.

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Key Formulae

- Pythagoras: $c^2 = a^2 + b^2$ (right-angled triangle, c is the hypotenuse)
- Right-angled trig: $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$, $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$, $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$
- Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ (use when given a side–angle pair)
- Cosine rule (side): $a^2 = b^2 + c^2 - 2bc\cos A$ (use when given two sides and the included angle, or all three sides)
- Cosine rule (angle): $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
- Area of a triangle: Area $= \dfrac{1}{2}ab\sin C$ (two sides and the included angle)
- Bearing: measured clockwise from north, always written as a three-figure number (e.g. $065°$); the included angle between two legs is derived from the north lines at each point.

GDC Tips

- **Evaluating trig ratios:** Ensure the GDC is in DEGREE mode before every calculation (Mode → Degree); entering an angle in radians when degrees are required is the single most common GDC error in this topic.
- **Inverse trig (find an angle):** Use 2nd → $\sin^{-1}$/$\cos^{-1}$/$\tan^{-1}$; store the unrounded result with STO→ before rounding.
- **Cosine rule for an angle:** Enter $\cos^{-1}\left(\dfrac{b^2+c^2-a^2}{2bc}\right)$ as a single expression on the home screen; wrap the numerator and denominator each in parentheses to avoid order-of-operations errors.
- **Area formula:** Compute $\frac{1}{2} \times a \times b \times \sin(C)$ directly; confirm the included angle C is the one between sides a and b, not opposite one of them.
- **Bearings — included angle:** Sketch both north lines on your diagram, then compute the interior angle algebraically (e.g. co-interior/alternate angles) before keying anything into the GDC; entering the bearing directly into the cosine rule is a common trap.
- **Chained calculations:** Use ANS or store intermediate sides/angles to at least 6 significant figures; never round to 3 s.f. mid-calculation or the final answer may be outside the accepted range.

Common Mistakes to Avoid

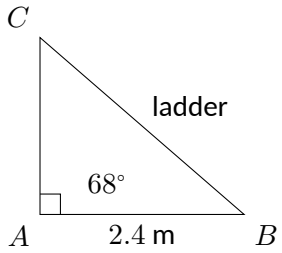
1. **Forgetting to switch to degree mode.** The GDC default after a reset is often radians. Always verify Mode → Degree at the start of every exam question; a radian-mode answer for a 38° angle looks plausible but loses every accuracy mark.
2. **Using bearings directly as triangle angles.** A bearing of 130° from P and 065° from Q does not mean the included angle in the triangle is $130^\circ - 65^\circ = 65^\circ$. Draw both north lines, apply co-interior or alternate-angle rules, and derive the true interior angle before applying the cosine rule.
3. **Premature rounding in chained triangle problems.** Rounding an intermediate side to 3 s.f. before using it in the next sine-rule or cosine-rule step can shift the final answer outside the mark-scheme tolerance. Carry all intermediate values unrounded (store in GDC memory) and only round the final answer.
4. **Misidentifying the included angle in the area formula.** Area $= \frac{1}{2}ab\sin C$ requires C to be the angle between sides a and b. Using the angle opposite one of those sides gives a completely wrong area with no method mark available.
5. **Choosing the wrong rule for the given information.** The sine rule needs a matched side–angle pair; the cosine rule is needed when two sides and the included angle (or all three sides) are given but no opposite-pair exists. Stating which rule is used and why earns the first method mark and prevents rule-substitution errors.
6. **Missing the hidden right angle in due-east/due-north journeys.** When one leg is due east and another is due north, the angle between them is exactly 90° and Pythagoras applies directly. Students who default to the cosine rule with $\cos 90^\circ$ still get the right answer, but students who use an incorrect included angle (e.g. $180^\circ - 90^\circ$) lose all accuracy marks.

Section A — Easy EASY (10 questions × 3 marks = 30 marks)

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1. EASY Calculator **[3 marks]**

A ladder leans against a vertical wall. The foot of the ladder is 2.4 m from the base of the wall, and the ladder makes an angle of 68° with the horizontal ground. Find the length of the ladder.

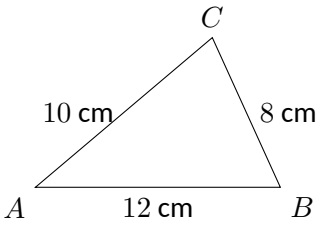


2. EASY Calculator **[3 marks]**

In triangle PQR, $PQ = 9.5$ cm, $QR = 7.2$ cm, and angle $PQR = 54^\circ$. Find the area of triangle PQR.

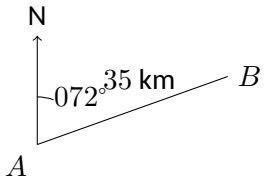
3. **EASY** **Calculator** **[3 marks]**

In triangle ABC, $AB = 12$ cm, $BC = 8$ cm, and $AC = 10$ cm. Find the size of angle BAC. Diagram not to scale.



4. **EASY** **Calculator** **[3 marks]**

A ship travels from port A on a bearing of $072°$ for a distance of 35 km to reach port B. Find how far east of A the port B is.



5. **EASY** **Calculator** **[3 marks]**

In triangle DEF, angle $D = 43°$, angle $E = 76°$, and $DE = 14.0$ m. Find the length of EF. Diagram not to scale.

6. **EASY** **Calculator** **[3 marks]**

A vertical flagpole casts a horizontal shadow of length 8.3 m. The angle of elevation of the top of the flagpole from the tip of the shadow is $38°$. Find the height of the flagpole.

7. **EASY** **Calculator** **[3 marks]**

In triangle XYZ, $XY = 7.0$ cm, $XZ = 9.5$ cm, and angle $YXZ = 62°$. Find the length of YZ. Diagram not to scale.

8. **EASY** **Calculator** **[3 marks]**

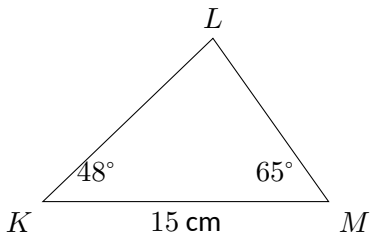
A rectangular swimming pool is 25 m long and 12 m wide. Find the length of the diagonal of the pool.

9. **EASY** **Calculator** **[3 marks]**

An observer stands at point T on horizontal ground. The angle of elevation of the top of a building at point R is $52°$. The horizontal distance from the observer to the base of the building is 18 m. Find the height of the building.

10. **EASY** **Calculator** **[3 marks]**

In triangle KLM, $KM = 15$ cm, angle $K = 48°$, and angle $M = 65°$. Find the length of KL. Diagram not to scale.

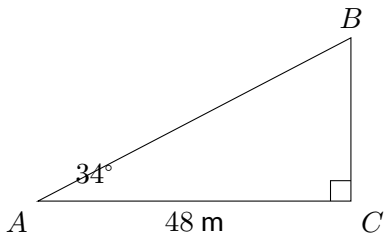


Section B — Medium MEDIUM (10 questions × 4 marks = 40 marks)

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11. MEDIUM Calculator [4 marks]

A surveyor stands at point A and observes the top of a vertical tower BC, where C is the base of the tower on horizontal ground. The angle of elevation from A to the top of the tower B is $34°$. The horizontal distance $AC = 48$ m. Diagram not to scale.

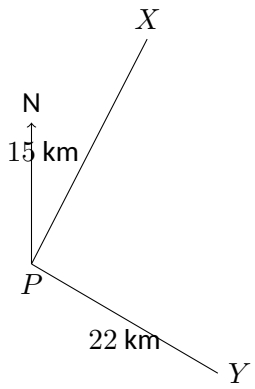


(a) Find the height BC of the tower. [2]

(b) Find the straight-line distance AB from the surveyor to the top of the tower. [2]

12. MEDIUM Calculator **[4 marks]**

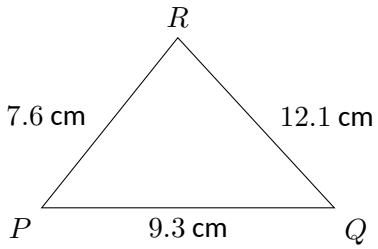
Two ships leave a port P at the same time. Ship X sails on a bearing of 072° for 15 km to reach point X. Ship Y sails on a bearing of 138° for 22 km to reach point Y. Diagram not to scale.



Find the distance XY between the two ships.

13. MEDIUM Calculator **[4 marks]**

In triangle PQR, $PQ = 9.3$ cm, $QR = 12.1$ cm and $PR = 7.6$ cm. Diagram not to scale.



(a) Find the size of angle $\widehat{P}$, the angle at vertex P. [2]

(b) Find the area of triangle PQR. [2]

14. MEDIUM Calculator **[4 marks]**

A hiker walks from point A due east for 6.4 km to point B, then due north for 4.9 km to point C.

(a) Write down the size of angle ABC. [1]

(b) Find the straight-line distance AC. [1]

(c) Find the bearing of C from A, giving your answer as a three-figure bearing. [2]

15. MEDIUM Calculator **[4 marks]**

In triangle DEF, angle $D = 47°$, angle $E = 63°$ and $DE = 14.5$ cm.

(a) Find the length EF. [2]

(b) Find the area of triangle DEF. [2]

16. MEDIUM Calculator **[4 marks]**

From a lighthouse L, a boat B is observed on a bearing of $205°$ at a distance of 3.8 km. A second boat C is observed from L on a bearing of $310°$ at a distance of 5.2 km. Find the distance BC between the two boats.

17. MEDIUM Calculator [4 marks]

A triangular plot of land has vertices J, K and L. The side $JK = 38$ m, $KL = 45$ m and the angle $\widehat{JKL} = 74°$.

(a) Find the area of the triangular plot. [2]

(b) Fencing costs \$12.50 per metre. Find the total cost of fencing the entire perimeter of the plot. [2]

18. MEDIUM Calculator [4 marks]

An observer at point A measures the angle of elevation to the top T of a vertical cliff as $28°$. The observer walks 40 m directly towards the base B of the cliff along horizontal ground to a new point A', and measures the angle of elevation to T as $51°$. Find the height TB of the cliff.

19. MEDIUM Calculator [4 marks]

In triangle GHI, $GH = 8.7$ cm, angle $\widehat{GHI} = 55°$ and angle $\widehat{HGI} = 42°$.

(a) Find the length HI. [2]

(b) Find the length GI. [2]

20. MEDIUM Calculator **[4 marks]**

A triangular sail on a yacht has sides of length 5.2 m, 6.8 m and 4.5 m.

(a) Find the largest angle in the sail. [2]

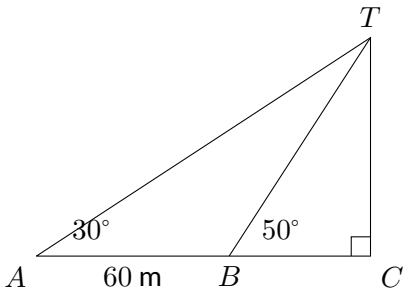
(b) The sail material costs \$85 per square metre. Find the total cost of the material needed to make the sail, giving your answer to the nearest dollar. [2]

Section C — Hard HARD (10 questions × 5 marks = 50 marks)

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21. HARD Calculator **[5 marks]**

A surveyor stands at point A and observes the top of a vertical tower TC at an angle of elevation of $30°$. She then walks 60 m directly towards the base C of the tower to point B, where the angle of elevation of T is $50°$. The base C and both observation points A and B lie on horizontal ground. Diagram not to scale.



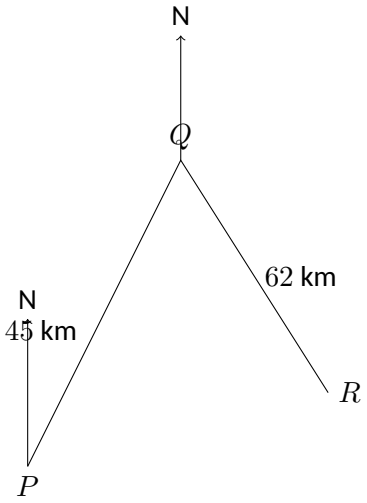
(a) Show that $BT = \dfrac{60 \sin 30°}{\sin 20°}$, where BT is the straight-line distance from B to the top of the tower T. [2]

(b) Find the height TC of the tower. [2]

(c) The surveyor claims the tower is taller than 65 m. Determine whether her claim is correct. Justify your answer. [1]

22. HARD Calculator **[5 marks]**

A ship leaves port P and sails on a bearing of 072° for 45 km to reach point Q. It then changes course and sails on a bearing of 155° for 62 km to reach point R. Diagram not to scale.



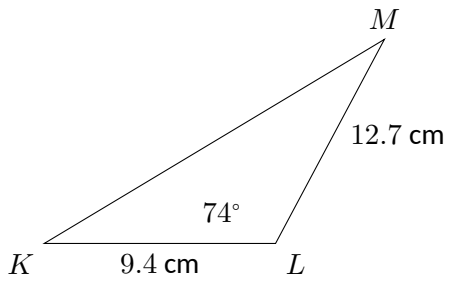
(a) Show that the angle $PQR = 97^\circ$. **[2]**

(b) Find the direct distance PR. **[2]**

(c) Find the bearing of R from P, giving your answer as a three-figure bearing. **[1]**

23. HARD Calculator **[5 marks]**

Triangle KLM has $KL = 9.4$ cm, $LM = 12.7$ cm and $\angle KLM = 74^\circ$. Diagram not to scale.



(a) Find the area of triangle KLM. **[2]**

(b) Find the length KM. **[2]**

(c) Hence find $\angle MKL$, giving your answer to the nearest degree. [1]

24. **HARD** **Calculator** **[5 marks]**

A triangular plot of land ABC has $AB = 54$ m, $BC = 38$ m and $AC = 47$ m. A farmer wants to build a straight fence from vertex A to the point D on BC such that $\angle ADB = 90^\circ$.

(a) Find $\angle ABC$ using the cosine rule. [2]

(b) Hence find the length AD. [2]

(c) The fence costs \$23.50 per metre. Find the total cost of the fence, giving your answer to the nearest dollar. [1]

25. **HARD** **Calculator** **[5 marks]**

From a point X on horizontal ground, the angle of elevation to the top T of a vertical mast TF is 28°. Point X is due west of F. Point Y is due north of F on the same horizontal ground, and the angle of elevation of T from Y is 41°.

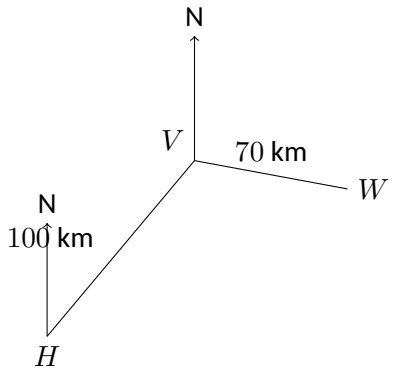
(a) Write down the sizes of the angles $\angle TFX$ and $\angle TFY$. [1]

(b) Show that $XF = h \cot 28^\circ$ and $YF = h \cot 41^\circ$, where $h = TF$. [1]

(c) Given that $XY = 95$ m, find the height h of the mast. [3]

26. **HARD** **Calculator** **[5 marks]**

A rescue helicopter flies from base H on a bearing of 040° for 100 km to reach a vessel at point V. A second vessel is at point W, which is 70 km from V on a bearing of 100° from V.



(a) Show that $\angle HVW = 120^\circ$. [2]

(b) Find the distance HW. [2]

(c) Find the bearing of W from H, giving your answer as a three-figure bearing. [1]

27. **HARD** **Calculator** **[5 marks]**

In triangle PQR, the area is 84 cm^2, $PQ = x$ cm, $QR = (x + 5)$ cm and $\angle PQR = 38^\circ$.

(a) Show that $x^2 + 5x - \dfrac{168}{\sin 38^\circ} = 0$. [2]

(b) Find the value of x. [2]

(c) Hence find the length PR. [1]

28. HARD Calculator [5 marks]

Two rangers observe a fire at point F from positions A and B on horizontal ground. Ranger A is due south of ranger B, and $AB = 6.4$ km. From A, the bearing of F is $037°$. From B, the bearing of F is $112°$.

(a) Show that $\angle FAB = 37°$ and find $\angle FBA$. [2]

(b) Hence find $\angle AFB$ and use the sine rule to find AF. [2]

(c) The ranger at A can drive at 80 km/h directly towards F over rough terrain. Determine whether she can reach F within 15 minutes. Justify your answer. [1]

29. HARD Calculator [5 marks]

A field is in the shape of quadrilateral $ABCD$ with diagonal $AC = 52$ m. In triangle ABC: $AB = 48$ m and $BC = 36$ m. In triangle ACD: $CD = 60$ m and $\angle CAD = 35°$.

(a) Find $\angle BAC$ using the cosine rule in triangle ABC. [2]

(b) Find the area of triangle ACD. [2]

(c) The total area of the field is to be seeded at a cost of \$4.50 per m^2. Find the total seeding cost to the nearest dollar. [1]

30. **HARD** **Calculator** **[5 marks]**

A communications tower stands vertically at point C on a hillside. The hillside makes an angle of 12° with the horizontal. An observer at point A, 200 m down the slope from C (measured along the slope), looks up the slope. The angle of elevation of the top T of the tower from A, measured from the horizontal, is 29°.

(a) Find the angle $\angle TAC$ inside triangle TAC, giving a reason. [1]

(b) Show that $\angle ACT = 102^\circ$, giving a reason for each angle in your working. [2]

(c) Hence find the height TC of the tower. [2]

Mark Schemes

Q1 — Ladder against a wall using right-angled trigonometry EASY

The horizontal distance from the wall is the side adjacent to the 68° angle, and the ladder is the hypotenuse, so cosine connects the two sides directly.

$$\cos 68^\circ = \frac{2.4}{\text{ladder length}} \Rightarrow \text{ladder length} = \frac{2.4}{\cos 68^\circ}$$

M1

GDC: calculate $2.4 \div \cos(68^\circ)$ in degree mode.

$$= \frac{2.4}{0.37461\ldots} = 6.4067\ldots \approx 6.41 \text{ m}$$

A1

Warning: Ensure the GDC is set to degree mode before evaluating $\cos 68^\circ$*; radian mode gives a wrong answer.*

Answer: ladder ≈ 6.41 m

Q2 — Area of a triangle using (1/2)ab sin C EASY

Two sides and the included angle are known, so the area formula Area $= \frac{1}{2}ab\sin C$ applies directly with $a = PQ = 9.5$ cm, $b = QR = 7.2$ cm, and $C = 54^\circ$.

$$\text{Area} = \tfrac{1}{2} \times 9.5 \times 7.2 \times \sin 54^\circ$$

M1

GDC: calculate $0.5 \times 9.5 \times 7.2 \times \sin(54^\circ)$ in degree mode.

$$= \tfrac{1}{2} \times 9.5 \times 7.2 \times 0.80902\ldots = 27.668\ldots \approx 27.7 \text{ cm}^2$$

A1

Warning: The angle used must be the included angle between the two given sides; here $\angle PQR$ *lies between* PQ *and* QR*, so it is correct.*

Answer: Area ≈ 27.7 cm^2

Q3 — Finding an angle using the cosine rule EASY

All three sides are known and an angle is required, so the cosine rule rearranged for an angle is the appropriate choice: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$, where side $a = BC = 8$ cm is opposite angle A, and $b = AC = 10$ cm, $c = AB = 12$ cm.

$$\cos(\angle BAC) = \frac{10^2 + 12^2 - 8^2}{2 \times 10 \times 12} = \frac{100 + 144 - 64}{240} = \frac{180}{240} = 0.75$$

M1

GDC: calculate $\cos^{-1}(0.75)$ in degree mode.

$$\angle BAC = \cos^{-1}(0.75) = 41.409\ldots^\circ \approx 41.4^\circ$$

A1

Warning: Identify the side opposite the required angle carefully; $a = BC$ is opposite $\angle BAC$, not AB or AC.

Answer: $\angle BAC \approx 41.4^\circ$

Q4 — Eastward component of a bearing journey EASY

The bearing of 072° is measured clockwise from north, so the eastward component of the journey is found using sine of the bearing angle measured from north.

$$\text{Distance east} = 35 \times \sin 72^\circ$$

M1

GDC: calculate $35 \times \sin(72^\circ)$ in degree mode.

$$= 35 \times 0.95106\ldots = 33.287\ldots \approx 33.3 \text{ km}$$

A1

Warning: The eastward component uses sin *of the bearing angle measured from north (i.e.* $\sin 72^\circ$*), not* $\cos 72^\circ$*; confusing the two components is a common error.*

Answer: 33.3 km east of A

Q5 — Finding a side using the sine rule EASY

Two angles and a side are known, so the sine rule applies. First find angle F: $\angle F = 180^\circ - 43^\circ - 76^\circ = 61^\circ$. Side EF is opposite angle $D = 43^\circ$, and $DE = 14.0$ m is opposite angle $F = 61^\circ$.

$$\frac{EF}{\sin 43^\circ} = \frac{14.0}{\sin 61^\circ}$$

M1

GDC: calculate $14.0 \times \sin(43°) \div \sin(61°)$ in degree mode.

$$EF = \frac{14.0 \times \sin 43°}{\sin 61°} = \frac{14.0 \times 0.68200\ldots}{0.87462\ldots} = 10.921\ldots \approx 10.9 \text{ m}$$

A1

Warning: Always find the third angle of the triangle before applying the sine rule; using an incorrect pairing of side and opposite angle will give a wrong answer.

Answer: $EF \approx 10.9$ m

Q6 — Height of a flagpole using angle of elevation EASY

The shadow length is the horizontal side adjacent to the angle of elevation, and the height of the flagpole is the opposite side, so tangent connects them directly.

$$\tan 38° = \frac{\text{height}}{8.3} \Rightarrow \text{height} = 8.3 \times \tan 38°$$

M1

GDC: calculate $8.3 \times \tan(38°)$ in degree mode.

$$= 8.3 \times 0.78129\ldots = 6.4847\ldots \approx 6.48 \text{ m}$$

A1

Warning: The angle of elevation is measured from the horizontal to the line of sight upward; using cos or sin instead of tan here would be incorrect since opposite and adjacent are the relevant sides.

Answer: height ≈ 6.48 m

Q7 — Finding a side using the cosine rule EASY

Two sides and the included angle are known, so the cosine rule gives the third side: $YZ^2 = XY^2 + XZ^2 - 2 \cdot XY \cdot XZ \cdot \cos(\angle YXZ)$.

$$YZ^2 = 7.0^2 + 9.5^2 - 2 \times 7.0 \times 9.5 \times \cos 62°$$

M1

GDC: calculate $\sqrt{7^2 + 9.5^2 - 2 \times 7 \times 9.5 \times \cos(62°)}$ in degree mode.

$$= \sqrt{49 + 90.25 - 133 \times 0.46947\ldots} = \sqrt{139.25 - 62.440\ldots} = \sqrt{76.809\ldots} = 8.7647\ldots \approx 8.76 \text{ cm}$$

A1

Warning: Carry the full unrounded value of $\cos 62°$ into the calculation; rounding $\cos 62°$ prematurely will introduce error in the final answer.

Answer: $YZ \approx 8.76$ cm

Q8 — Diagonal of a rectangle using Pythagoras EASY

The diagonal of a rectangle forms the hypotenuse of a right-angled triangle with the length and width as the two shorter sides, so Pythagoras' theorem applies directly.

$$d = \sqrt{25^2 + 12^2}$$

M1

GDC: calculate $\sqrt{625 + 144}$ in standard mode.

$$= \sqrt{769} = 27.730\ldots \approx 27.7 \text{ m}$$

A1

Warning: Remember to square both dimensions before adding them; a common error is to add $25 + 12$ first and then square the sum.

Answer: diagonal ≈ 27.7 m

Q9 — Height of a building using angle of elevation EASY

The horizontal distance is adjacent to the angle of elevation, and the height of the building is the opposite side, so tangent is the appropriate trigonometric ratio.

$$\tan 52^\circ = \frac{\text{height}}{18} \Rightarrow \text{height} = 18 \times \tan 52^\circ$$

M1

GDC: calculate $18 \times \tan(52^\circ)$ in degree mode.

$$= 18 \times 1.27994\ldots = 23.038\ldots \approx 23.0 \text{ m}$$

A1

Warning: The angle of elevation is always measured upward from the horizontal; ensure the GDC is in degree mode so that $\tan 52^\circ$ is evaluated correctly.

Answer: height ≈ 23.0 m

Q10 — Finding a side using the sine rule with two angles given EASY

Two angles and a side are given, so the sine rule applies. First find angle L: $\angle L = 180^\circ - 48^\circ - 65^\circ = 67^\circ$. Side KL is opposite angle $M = 65^\circ$, and $KM = 15$ cm is opposite angle $L = 67^\circ$.

$$\frac{KL}{\sin 65^\circ} = \frac{15}{\sin 67^\circ}$$

M1

GDC: calculate $15 \times \sin(65^\circ) \div \sin(67^\circ)$ in degree mode.

$$KL = \frac{15 \times \sin 65^\circ}{\sin 67^\circ} = \frac{15 \times 0.90631\ldots}{0.92050\ldots} = 14.768\ldots \approx 14.8 \text{ cm}$$

A1

Warning: Always compute the third angle of the triangle first and use the correct side-angle opposite pairs in the sine rule; mixing up which angle is opposite which side is the most common error here.

Answer: $KL \approx 14.8$ cm

Q11 — Angle of elevation in a right triangle MEDIUM

(a) Since BC is opposite the angle of elevation and AC is the adjacent side, we use tan to find the opposite side.

$$BC = AC \times \tan(34^\circ) = 48 \times \tan(34^\circ) = 48 \times 0.67451\ldots = 32.376\ldots \approx 32.4 \text{ m}$$

M1 A1

(b) With the right angle at C, AB is the hypotenuse; use cos to find the hypotenuse from the adjacent side and angle.

$$AB = \frac{AC}{\cos(34^\circ)} = \frac{48}{0.82904\ldots} = 57.90\ldots \approx 57.9 \text{ m}$$

M1 A1

Warning: Using $\sin(34^\circ)$ *for part (b) instead of* $\cos(34^\circ)$ *is a common error; the angle is at* A *and* AC *is the adjacent side, so cosine is required.*

Answer: (a) $BC \approx 32.4$ m (b) $AB \approx 57.9$ m

Q12 — Bearings and the cosine rule MEDIUM

To apply the cosine rule we need the included angle $\widehat{P}$ at the port between the two directions. The bearing of X is 072° and the bearing of Y is 138°, so the angle between PX and PY measured from north is $138^\circ - 72^\circ = 66^\circ$. **M1**

Warning: Always subtract the smaller bearing from the larger to find the included angle; do not use 72° *or* 138° *alone in the cosine rule.*

We have two sides and the included angle, so apply the cosine rule:

$$XY^2 = PX^2 + PY^2 - 2 \cdot PX \cdot PY \cdot \cos(\widehat{P}) = 15^2 + 22^2 - 2(15)(22)\cos(66^\circ)$$

M1

GDC: evaluate $225 + 484 - 660\cos(66^\circ) = 709 - 660 \times 0.40674\ldots = 709 - 268.45\ldots = 440.55\ldots$

$$XY = \sqrt{440.55\ldots} = 20.989\ldots \approx 21.0 \text{ km}$$

A2

Answer: $XY \approx 21.0$ km

Q13 — Cosine rule for angle then area MEDIUM

(a) We are given all three sides and must find angle $\widehat{P}$, so the cosine rule in rearranged form is required. Side QR (opposite $\widehat{P}$) $= 12.1$ cm.

$$\cos(\widehat{P}) = \frac{PQ^2 + PR^2 - QR^2}{2 \cdot PQ \cdot PR} = \frac{9.3^2 + 7.6^2 - 12.1^2}{2(9.3)(7.6)}$$

M1

$$\text{GDC: } = \frac{86.49 + 57.76 - 146.41}{141.36} = \frac{-2.16}{141.36} = -0.015279\ldots$$

$$\widehat{P} = \arccos(-0.015279\ldots) = 90.876\ldots^\circ \approx 90.9^\circ$$

A1

(b) The area formula using two sides and the included angle applies here.

$$\text{Area} = \tfrac{1}{2} \times PQ \times PR \times \sin(\widehat{P}) = \tfrac{1}{2} \times 9.3 \times 7.6 \times \sin(90.876\ldots^\circ)$$

M1

Warning: Use the unrounded value of $\widehat{P}$ stored in the GDC; using 90.9° introduces a premature-rounding error.

$$\text{Area} = \tfrac{1}{2} \times 9.3 \times 7.6 \times 0.99988\ldots = 35.345\ldots \approx 35.3\ \text{cm}^2$$

A1

Answer: (a) $\widehat{P} \approx 90.9^\circ$ (b) Area ≈ 35.3 cm^2

Q14 — Hidden right angle and bearing MEDIUM

(a) Walking due east then turning due north creates a right angle at B; the angle ABC is exactly 90°. **A1**

(b) AC is the hypotenuse of the right-angled triangle; by Pythagoras:

$$AC = \sqrt{6.4^2 + 4.9^2} = \sqrt{40.96 + 24.01} = \sqrt{64.97} = 8.0605\ldots \approx 8.06\ \text{km}$$

A1

(c) The bearing of C from A is measured clockwise from north at A. The angle that AC makes east of north satisfies:

$$\tan(\theta) = \frac{AB}{BC} = \frac{6.4}{4.9}$$

M1

Warning: The bearing is measured from north (clockwise), not from east; the ratio must use the eastward leg over the northward leg.

GDC: $\theta = \arctan\left(\frac{6.4}{4.9}\right) = 52.5614\ldots^\circ$, which rounds to 053° as a three-figure bearing (nearest degree).

$$\text{Bearing of } C \text{ from } A = 053^\circ$$

A1

Answer: (a) 90° (b) $AC \approx 8.06$ km (c) 053°

Q15 — Sine rule for side then area MEDIUM

(a) The third angle is $F = 180^\circ - 47^\circ - 63^\circ = 70^\circ$. Side EF is opposite angle $D = 47^\circ$; side DE is opposite angle $F = 70^\circ$, so the sine rule applies. **M1**

$$\frac{EF}{\sin D} = \frac{DE}{\sin F} \Rightarrow EF = \frac{14.5 \times \sin(47^\circ)}{\sin(70^\circ)}$$

GDC: $EF = \frac{14.5 \times 0.73135\ldots}{0.93969\ldots} = \frac{10.6046\ldots}{0.93969\ldots} = 11.285\ldots \approx 11.3$ cm **A1**

(b) Two sides and the included angle are now available: $DE = 14.5$ cm, $EF \approx 11.285\ldots$ cm, included angle $E = 63^\circ$. **M1**

Warning: Use the unrounded value of EF from part (a) stored in the GDC to avoid a premature-rounding penalty.

$$\text{Area} = \tfrac{1}{2} \times DE \times EF \times \sin E = \tfrac{1}{2} \times 14.5 \times 11.285\ldots \times \sin(63^\circ)$$
$$= \tfrac{1}{2} \times 14.5 \times 11.285\ldots \times 0.89101\ldots = 72.900\ldots \approx 72.9 \text{ cm}^2$$

A1

Answer: (a) $EF \approx 11.3$ cm (b) Area ≈ 72.9 cm^2

Q16 — Bearings and the cosine rule via included angle MEDIUM

The bearing of B from L is 205° and the bearing of C from L is 310°. The angle $\widehat{BLC}$ between the two directions is $310^\circ - 205^\circ = 105^\circ$. **M1**

Warning: Bearings greater than 180° are measured past south; always compute the included angle as the simple difference of the two bearings when they are on the same side of the reference point.

We have two sides $LB = 3.8$ km, $LC = 5.2$ km and included angle 105°, so the cosine rule gives: **M1**

$$BC^2 = 3.8^2 + 5.2^2 - 2(3.8)(5.2)\cos(105^\circ)$$

GDC: $= 14.44 + 27.04 - 39.52 \times (-0.25882\ldots) = 41.48 + 10.228\ldots = 51.708\ldots$

$$BC = \sqrt{51.708\ldots} = 7.1909\ldots \approx 7.19 \text{ km}$$

A2

Answer: $BC \approx 7.19$ km

Q17 — Area and perimeter of triangle MEDIUM

(a) We have two sides and the included angle, so apply the area formula directly. **M1**

$$\text{Area} = \tfrac{1}{2} \times JK \times KL \times \sin(\widehat{JKL}) = \tfrac{1}{2} \times 38 \times 45 \times \sin(74^\circ)$$
$$= \tfrac{1}{2} \times 38 \times 45 \times 0.96126\ldots = 821.878\ldots \approx 822 \text{ m}^2$$

A1

(b) First find the third side JL using the cosine rule: $JL^2 = 38^2 + 45^2 - 2(38)(45)\cos(74^\circ)$. **M1**

GDC: $= 1444 + 2025 - 3420 \times 0.27564\ldots = 3469 - 942.69\ldots = 2526.31\ldots$

$$JL = \sqrt{2526.31\ldots} = 50.262\ldots \text{ m}$$

Warning: The cost requires the full perimeter; a common trap is to use only two sides and omit JL.

Perimeter $= 38 + 45 + 50.262\ldots = 133.262\ldots$ m

$$\text{Total cost} = 133.262\ldots \times 12.50 = \$1665.77\ldots \approx \$1665.78$$

A1

Answer: (a) Area ≈ 822 m^2 (b) Cost $\approx \$1665.78$

Q18 — Two elevation angles and the height of a cliff MEDIUM

Let h = height TB and d = horizontal distance $A'B$. From point A (which is $d+40$ m from B): $\tan(28^\circ) = \dfrac{h}{d+40}$, so $h = (d+40)\tan(28^\circ)$. From point A': $\tan(51^\circ) = \dfrac{h}{d}$, so $h = d\tan(51^\circ)$. **M1**

Setting equal: $(d+40)\tan(28^\circ) = d\tan(51^\circ)$ **M1**

$d\tan(28^\circ) + 40\tan(28^\circ) = d\tan(51^\circ)$ $40\tan(28^\circ) = d(\tan(51^\circ) - \tan(28^\circ))$

Warning: Do not round d before substituting back to find h; premature rounding costs the final accuracy mark.

GDC: $d = \dfrac{40\tan(28^\circ)}{\tan(51^\circ) - \tan(28^\circ)} = \dfrac{40 \times 0.53171\ldots}{1.23490\ldots - 0.53171\ldots} = \dfrac{21.2685\ldots}{0.70319\ldots} = 30.2457\ldots$ m

$$h = d\tan(51^\circ) = 30.2457\ldots \times 1.23490\ldots = 37.350\ldots \approx 37.4 \text{ m}$$

A2

Answer: $h \approx 37.4$ m

Q19 — Sine rule with two sides MEDIUM

(a) First find angle $\hat{I}$: $\hat{I} = 180° - 55° - 42° = 83°$. Side HI is opposite angle $G = 42°$; side GH is opposite angle $I = 83°$; the sine rule links them. **M1**

$$\frac{HI}{\sin G} = \frac{GH}{\sin I} \Rightarrow HI = \frac{8.7 \times \sin(42°)}{\sin(83°)}$$

GDC: $HI = \dfrac{8.7 \times 0.66913\ldots}{0.99255\ldots} = \dfrac{5.82146\ldots}{0.99255\ldots} = 5.8653\ldots \approx 5.87$ cm **A1**

(b) Side GI is opposite angle $H = 55°$; use the same sine rule ratio. **M1**

$$GI = \frac{8.7 \times \sin(55°)}{\sin(83°)} = \frac{8.7 \times 0.81915\ldots}{0.99255\ldots} = \frac{7.12665\ldots}{0.99255\ldots} = 7.1800\ldots \approx 7.18 \text{ cm}$$

A1

Warning: Use the unrounded sine-rule ratio $\dfrac{GH}{\sin I}$ stored in the GDC rather than recomputing from rounded intermediate values.

Answer: (a) $HI \approx 5.87$ cm (b) $GI \approx 7.18$ cm

Q20 — Cosine rule for largest angle and area of triangle MEDIUM

(a) The largest angle is opposite the longest side. The longest side is 6.8 m. Let the sides be $a = 5.2$, $b = 4.5$, $c = 6.8$ (side c is opposite angle C). **M1**

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{5.2^2 + 4.5^2 - 6.8^2}{2(5.2)(4.5)} = \frac{27.04 + 20.25 - 46.24}{46.8} = \frac{1.05}{46.8} = 0.022436\ldots$$

GDC: $C = \arccos(0.022436\ldots) = 88.714\ldots° \approx 88.7°$ **A1**

(b) Use the area formula with two sides and the included angle C. **M1**

Warning: Use the unrounded angle C stored in the GDC; also note that the cost requires the area in m^2, which matches the given side units (metres).

$$\text{Area} = \tfrac{1}{2} \times 5.2 \times 4.5 \times \sin(88.714\ldots°) = \tfrac{1}{2} \times 5.2 \times 4.5 \times 0.99974\ldots = 11.696\ldots \text{ m}^2$$

$$\text{Cost} = 11.696\ldots \times 85 = 994.21\ldots \approx \$994$$

A1

Answer: (a) $C \approx 88.7°$ (b) Cost $\approx \$994$

Q21 — Two-observation tower height via sine rule then right triangle HARD

(a) We need the angle at T in triangle ABT to apply the sine rule. The angle of elevation from A is $30°$, so $\angle TAB = 30°$. The angle of elevation from B is $50°$, so $\angle TBC = 50°$, which means $\angle TBA = 180° - 50° = 130°$ (angles on a straight line at B). The angle at T is therefore $\angle ATB = 180° - 30° - 130° = 20°$. **M1**

Applying the sine rule in triangle ABT with $AB = 60$ m: side BT is opposite $\angle TAB = 30°$, and side AB is opposite $\angle ATB = 20°$.

$$\frac{BT}{\sin 30°} = \frac{AB}{\sin 20°} \Rightarrow BT = \frac{60 \sin 30°}{\sin 20°}$$

AG

Warning: The angle $\angle TBA$ inside triangle ABT is the supplement of the elevation angle $50°$ at B, not $50°$ itself. Using $50°$ directly gives the wrong angle at T and breaks the AG. Also note carefully that this sine-rule relation gives BT, the slant distance from B up to T — not the horizontal distance BC.

(b) GDC: $BT = \dfrac{60 \sin 30°}{\sin 20°} = 87.714\ldots$ m. **M1**

In right triangle TBC (right angle at C), BT is the hypotenuse and the angle at B is $50°$, so the height is the opposite side:

$$TC = BT \times \sin 50° = 87.714\ldots \times \sin 50° = 87.714\ldots \times 0.76604\ldots = 67.19\ldots \approx 67.2 \text{ m}$$

A1

Warning: Since BT is the hypotenuse of right triangle TBC, the height uses $\sin 50°$ (opposite side), not $\tan 50°$ (which would require the horizontal leg BC, not BT).

(c) Since $TC \approx 67.2$ m > 65 m, the surveyor's claim is correct. **R1**

Answer: (a) shown (b) $TC \approx 67.2$ m (c) claim correct

Q22 — Bearings journey with cosine rule and return bearing HARD

(a) The bearing from P to Q is $072°$, so the back-bearing (direction QP from Q) is $072° + 180° = 252°$. The bearing QR is $155°$. The included angle at Q, measured from QP round to QR, is $\angle PQR = 252° - 155° = 97°$. **(M1)(M1)**

$$\angle PQR = 97°$$

AG

Warning: The back-bearing of PQ is $072° + 180° = 252°$, not $072° - 180°$. Always add $180°$ when the forward bearing is less than $180°$.

(b) Applying the cosine rule in triangle PQR with $PQ = 45$, $QR = 62$, $\angle PQR = 97°$: **M1**

GDC: $PR^2 = 45^2 + 62^2 - 2(45)(62)\cos 97° = 2025 + 3844 - 5580\cos 97° = 5869 - (-679.8\ldots) = 6548.8\ldots$

$$PR = \sqrt{6548.8\ldots} = 80.9\ldots \approx 80.9 \text{ km}$$

A1

(c) GDC: $\sin(\angle QPR) = \dfrac{62\sin 97^\circ}{80.9\ldots}$; $\angle QPR = \arcsin(0.7607\ldots) = 49.5\ldots^\circ$.

$$\text{Bearing of } R \text{ from } P = 072^\circ + 49.5\ldots^\circ = 121.5\ldots^\circ \approx 122^\circ$$

A1

Answer: (a) shown (b) $PR \approx 80.9$ km (c) 122°

Q23 — Area and cosine rule side and sine rule angle fusion HARD

(a) Using the area formula with the included angle $\angle KLM = 74^\circ$, $KL = 9.4$ cm, $LM = 12.7$ cm: **M1**

$$\text{Area} = \tfrac{1}{2}(9.4)(12.7)\sin 74^\circ = \tfrac{1}{2}(119.38)(0.9613\ldots) = 57.37\ldots \approx 57.4 \text{ cm}^2$$

A1

(b) Applying the cosine rule to find the side opposite the known angle: **M1**

GDC: $KM^2 = 9.4^2 + 12.7^2 - 2(9.4)(12.7)\cos 74^\circ = 88.36 + 161.29 - 238.76\cos 74^\circ = 249.65 - 65.85\ldots = 183.80\ldots$

$$KM = \sqrt{183.80\ldots} = 13.55\ldots \approx 13.6 \text{ cm}$$

A1

(c) Using the sine rule: $\dfrac{\sin(\angle MKL)}{12.7} = \dfrac{\sin 74^\circ}{13.55\ldots}$; $\sin(\angle MKL) = \dfrac{12.7 \times \sin 74^\circ}{13.55\ldots} = 0.9015\ldots$ GDC: $\angle MKL = \arcsin(0.9015\ldots) = 64.4\ldots^\circ \approx 64^\circ$. **A1**

Warning: The instruction says "hence", so the sine rule must use the value of KM found in (b); starting afresh with the angle sum loses the method mark.

Answer: (a) ≈ 57.4 cm^2 (b) $KM \approx 13.6$ cm (c) $\angle MKL \approx 64^\circ$

Q24 — Cosine rule angle and right-triangle perpendicular and cost HARD

(a) Applying the cosine rule to find $\angle ABC$ from three known sides $AB = 54$, $BC = 38$, $AC = 47$: **M1**

$$\cos(\angle ABC) = \frac{AB^2 + BC^2 - AC^2}{2 \cdot AB \cdot BC} = \frac{54^2 + 38^2 - 47^2}{2(54)(38)} = \frac{2916 + 1444 - 2209}{4104} = \frac{2151}{4104} = 0.52411\ldots$$

GDC: $\angle ABC = \arccos(0.52411\ldots) = 58.39\ldots^\circ \approx 58.4^\circ$. **A1**

(b) Since $\angle ADB = 90^\circ$, triangle ABD is right-angled at D. Using right-triangle trigonometry: **M1**

$$AD = AB\sin(\angle ABC) = 54 \times \sin(58.39\ldots^\circ) = 54 \times 0.85178\ldots = 45.99\ldots \approx 46.0 \text{ m}$$

A1

Warning: Use the unrounded angle from (a) in the sine calculation. Rounding to 58.4° first gives the same

3 s.f. result here, but carry unrounded values through chained calculations to avoid a premature-rounding penalty in general.

(c) Cost $= 45.99\ldots \times 23.50 = \$1080.8\ldots \approx \$1081$. **A1**

Answer: (a) $\angle ABC \approx 58.4°$ (b) $AD \approx 46.0$ m (c) $\approx \$1081$

Q25 — 3D mast with hidden right angle and Pythagoras parameter hunt HARD

(a) Since the mast is vertical, $\angle TFX = \angle TFY = 90°$. **A1**

(b) In right triangle TFX: $\tan 28° = \dfrac{h}{XF}$, so $XF = \dfrac{h}{\tan 28°} = h\cot 28°$. Similarly $YF = h\cot 41°$. **AG**

Warning: The hidden right angle here is $\angle XFY = 90°$ because X is due west of F and Y is due north of F; these directions are perpendicular. Forgetting this gives a wrong distance formula in (c).

(c) Since $\angle XFY = 90°$, by Pythagoras: **M1**

$$XY^2 = XF^2 + YF^2 = (h\cot 28°)^2 + (h\cot 41°)^2 = h^2(\cot^2 28° + \cot^2 41°)$$

M1

GDC: $\cot 28° = \dfrac{1}{\tan 28°} = 1.8807\ldots$; $\cot 41° = \dfrac{1}{\tan 41°} = 1.1503\ldots$

$$95^2 = h^2(1.8807\ldots^2 + 1.1503\ldots^2) = h^2(3.5370\ldots + 1.3233\ldots) = h^2(4.8603\ldots)$$

$$h^2 = \frac{9025}{4.8603\ldots} = 1856.9\ldots; \quad h = \sqrt{1856.9\ldots} = 43.09\ldots \approx 43.1 \text{ m}$$

A1

Answer: (a) $90°, 90°$ (b) shown (c) $h \approx 43.1$ m

Q26 — Helicopter bearings with cosine rule distance and return bearing HARD

(a) The bearing from H to V is $040°$, so the back-bearing (direction VH from V) is $040° + 180° = 220°$. The bearing VW is $100°$. The included angle at V, measured from VH round to VW, is $\angle HVW = 220° - 100° = 120°$. **(M1)(M1)**

$$\angle HVW = 120°$$

AG

Warning: The back-bearing of HV is $040° + 180° = 220°$, not $040° - 180°$. Always add $180°$ when the forward bearing is less than $180°$.

(b) Cosine rule: $HW^2 = HV^2 + VW^2 - 2(HV)(VW)\cos(\angle HVW)$ **M1**

$$= 100^2 + 70^2 - 2(100)(70)\cos 120° = 10000 + 4900 - 14000 \times (-0.5) = 14900 + 7000 = 21900$$

$$HW = \sqrt{21900} = 147.98\ldots \approx 148 \text{ km}$$

A1

(c) GDC: sine rule: $\sin(\angle VHW) = \dfrac{VW \sin(\angle HVW)}{HW} = \dfrac{70 \sin 120^\circ}{147.98\ldots} = \dfrac{60.621\ldots}{147.98\ldots} = 0.40965\ldots$; $\angle VHW = 24.18\ldots^\circ$.

$$\text{Bearing of } W \text{ from } H = 040^\circ + 24.18\ldots^\circ = 064.18\ldots^\circ \approx 064^\circ$$

A1

Answer: (a) shown (b) $HW \approx 148$ km (c) 064°

Q27 — Reverse area to quadratic and cosine rule fusion HARD

(a) Using the area formula: Area $= \frac{1}{2} \cdot PQ \cdot QR \cdot \sin(\angle PQR)$: **M1**

$$84 = \tfrac{1}{2} \cdot x \cdot (x+5) \cdot \sin 38^\circ$$
$$168 = x(x+5)\sin 38^\circ = (x^2+5x)\sin 38^\circ$$
$$x^2 + 5x = \frac{168}{\sin 38^\circ}, \quad \text{so} \quad x^2 + 5x - \frac{168}{\sin 38^\circ} = 0$$

AG

Warning: The factor of $\frac{1}{2}$ in the area formula means the right-hand side becomes $\dfrac{168}{\sin 38^\circ}$, not $\dfrac{84}{\sin 38^\circ}$; omitting the factor of 2 gives a different (wrong) quadratic.

(b) GDC: solve $x^2 + 5x - \dfrac{168}{\sin 38^\circ} = 0$. Numerically: $\dfrac{168}{\sin 38^\circ} = \dfrac{168}{0.61566\ldots} = 272.87\ldots$ **M1**

$x^2 + 5x - 272.87\ldots = 0$. GDC: quadratic formula: $x = \dfrac{-5 + \sqrt{25 + 4(272.87\ldots)}}{2} = \dfrac{-5 + \sqrt{1116.5\ldots}}{2} = \dfrac{-5 + 33.41\ldots}{2} = \dfrac{28.41\ldots}{2} = 14.20\ldots \approx 14.2$ cm. (Reject the negative root since $x > 0$.) **A1**

(c) $PQ = 14.20\ldots$ cm, $QR = 14.20\ldots + 5 = 19.20\ldots$ cm, $\angle PQR = 38^\circ$. Cosine rule: **(M1)**

$$PR^2 = 14.20\ldots^2 + 19.20\ldots^2 - 2(14.20\ldots)(19.20\ldots)\cos 38^\circ$$
$$= 201.7\ldots + 368.8\ldots - 545.7\ldots \times 0.7880\ldots = 570.5\ldots - 430.0\ldots = 140.5\ldots$$
$$PR = \sqrt{140.5\ldots} = 11.85\ldots \approx 11.9 \text{ cm}$$

A1

Answer: (a) shown (b) $x \approx 14.2$ cm (c) $PR \approx 11.9$ cm

Q28 — Two-observer fire location with sine rule and distance-time decision HARD

(a) A is due south of B, so the line AB runs north–south. The bearing of F from A is $037°$; the angle between north (direction AB) and AF is $037°$, so $\angle FAB = 37°$. **A1**

The bearing of F from B is $112°$. Since A is due south of B, the direction BA from B is due south, i.e. $180°$. The angle between BA (bearing $180°$) and BF (bearing $112°$) is $180° - 112° = 68°$, so $\angle FBA = 68°$. **A1**

Warning: The bearing of F from B is $112°$ (east of south), so the angle at B is measured from the direction BA (due south) to BF; this gives $180° - 112° = 68°$, not $112°$.

(b) $\angle AFB = 180° - 37° - 68° = 75°$. **M1**

Sine rule: $\dfrac{AF}{\sin(\angle FBA)} = \dfrac{AB}{\sin(\angle AFB)}$; $AF = \dfrac{6.4\sin 68°}{\sin 75°} = \dfrac{6.4 \times 0.92718\ldots}{0.96592\ldots} = \dfrac{5.9339\ldots}{0.96592\ldots} = 6.144\ldots \approx 6.14$ km. **A1**

(c) Distance to travel $= 6.144\ldots$ km at 80 km/h: time $= \dfrac{6.144\ldots}{80} = 0.07680\ldots$ hours $= 4.608\ldots$ min < 15 min. Since 4.6 min < 15 min, the ranger can reach F within 15 minutes. **R1**

Answer: (a) $37°$, $68°$ (b) $AF \approx 6.14$ km (c) reachable in time

Q29 — Composite quadrilateral area via cosine and sine rule HARD

(a) In triangle ABC: $AB = 48$ m, $BC = 36$ m, $AC = 52$ m (all three sides known, so the cosine rule is used to find $\angle BAC$). **M1**

$$\cos(\angle BAC) = \frac{AB^2 + AC^2 - BC^2}{2 \cdot AB \cdot AC} = \frac{48^2 + 52^2 - 36^2}{2(48)(52)}$$

$$= \frac{2304 + 2704 - 1296}{4992} = \frac{3712}{4992} = 0.74359\ldots$$

GDC: $\angle BAC = \arccos(0.74359\ldots) = 41.96\ldots° \approx 42.0°$. **A1**

(b) In triangle ACD: $AC = 52$ m, $CD = 60$ m, $\angle CAD = 35°$ (two sides and a non-included angle; since $CD > AC$, the sine rule gives a unique triangle here). **M1**

$$\frac{\sin(\angle ADC)}{AC} = \frac{\sin(\angle CAD)}{CD} \Rightarrow \sin(\angle ADC) = \frac{52\sin 35°}{60} = \frac{29.828\ldots}{60} = 0.49713\ldots$$

GDC: $\angle ADC = \arcsin(0.49713\ldots) = 29.81\ldots°$. Then $\angle ACD = 180° - 35° - 29.81\ldots° = 115.19\ldots°$.

$$\text{Area of } ACD = \tfrac{1}{2}(AC)(CD)\sin(\angle ACD) = \tfrac{1}{2}(52)(60)\sin(115.19\ldots°)$$

$$= \tfrac{1}{2}(3120)(0.90506\ldots) = 1411.6\ldots \approx 1410 \text{ m}^2$$

A1

(c) Area of $ABC = \tfrac{1}{2}(AB)(AC)\sin(\angle BAC) = \tfrac{1}{2}(48)(52)\sin(41.96\ldots°) = \tfrac{1}{2}(2496)(0.66862\ldots) = 834.46\ldots$ m^2. Total area $= 834.46\ldots + 1411.6\ldots = 2246.08\ldots$ m^2.

$$\text{Cost} = 2246.08\ldots \times 4.50 = 10107.3\ldots \approx \$10\,107$$

A1

Warning: Carry both unrounded areas into the sum before multiplying by the cost rate; rounding each area first can shift the final dollar amount.

Answer: (a) $\angle BAC \approx 42.0^\circ$ (b) Area $ACD \approx 1410$ m^2 (c) $\approx \$10\,107$

Q30 — Tower on hillside with angle geometry and sine rule height HARD

(a) The angle of elevation of T from A measured from the horizontal is 29°. The line AC lies along the slope, which is inclined at 12° above horizontal. The angle $\angle TAC$ inside triangle TAC (measured from AC to AT) equals the angle of elevation minus the slope angle: $\angle TAC = 29^\circ - 12^\circ = 17^\circ$. **A1**

(b) The mast TC is vertical, so it makes an angle of 90° with the horizontal. The slope from C descends to A at 12° below horizontal. The angle between the upward-vertical mast CT and the down-slope direction CA is $90^\circ + 12^\circ = 102^\circ$. Hence $\angle ACT = 102^\circ$. **M1**

Reason: the mast is perpendicular to the horizontal (90° from horizontal); the slope CA descends at 12° below horizontal; the angle between vertical CT and the downslope direction CA is $90^\circ + 12^\circ = 102^\circ$. **AG**

Warning: A common error is to use $90^\circ - 12^\circ = 78^\circ$; this gives the angle between the mast and the upslope direction, not the interior angle at C in triangle TAC where A is downhill from C.

(c) Angle sum in triangle TAC: $\angle ATC = 180^\circ - 17^\circ - 102^\circ = 61^\circ$. **M1**

Sine rule: $\dfrac{TC}{\sin(\angle TAC)} = \dfrac{AC}{\sin(\angle ATC)}$; $TC = \dfrac{200 \sin 17^\circ}{\sin 61^\circ} = \dfrac{200 \times 0.29237\ldots}{0.87461\ldots} = \dfrac{58.474\ldots}{0.87461\ldots} = 66.86\ldots \approx 66.9$ m. **A1**

Answer: (a) 17° (b) shown (c) $TC \approx 66.9$ m