# IB Math AI SL — Topic 3

## Geometry Toolkit

Coordinate Geometry · Perpendicular Bisectors · Arcs & Sectors

| Difficulty | EASY | MEDIUM | HARD |
|---|---|---|---|
| Questions | 10 | 10 | 10 |
| Total Marks | 30 | 40 | 50 |

Name:_____ Date:_____

Score:_____/120

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**Instructions:** Answer all questions. Show all working. Give answers correct to 3 significant figures unless stated otherwise. A graphic display calculator is required throughout; support GDC answers with the values entered and the feature used.

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### Key Formulae

- **Distance between two points:** $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- **Midpoint of a segment:** $M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$
- **Gradient of a line:** $m = \frac{y_2 - y_1}{x_2 - x_1}$; perpendicular gradient: $m_\perp = -\frac{1}{m}$
- **Equation of a straight line:** $y - y_1 = m(x - x_1)$
- **Arc length (degrees):** $l = \frac{\theta}{360} \times 2\pi r$
- **Sector area (degrees):** $A = \frac{\theta}{360} \times \pi r^2$
- **Perimeter of a sector:** $P = l + 2r = \frac{\theta}{360} \times 2\pi r + 2r$

## GDC Tips

- **Storing coordinates:** Enter $x$- and $y$-values into lists L1, L2 to avoid re-keying when computing distances or midpoints across multiple parts.
- **Evaluating arc length / sector area:** Type the expression directly as `(110/360)*(2*π*6.5)` in the home screen; store $r$ and $\theta$ as variables first if the question has multiple parts.
- **Checking a perpendicular bisector equation:** Graph both the original segment endpoints and the bisector line under Y=; confirm the line passes through the midpoint visually.
- **Solving for an unknown radius or angle:** Use Solver (MATH $\rightarrow$ 0) or rewrite the arc/area formula and evaluate directly; always substitute back to verify.
- **Finding an intersection point:** Graph two bisector lines under Y= then use 2nd $\rightarrow$ CALC $\rightarrow$ intersect to locate the Voronoi vertex or equidistant point numerically.

## Common Mistakes to Avoid

1. **Forgetting the two radii in sector perimeter.** The perimeter of a sector is arc length plus both straight sides: $P = l + 2r$. Writing only the arc length $l$ is the single most common mark-losing error in sector questions.
2. **Using radians instead of degrees.** Every angle in this course is in degrees; always use the $\frac{\theta}{360}$ factor, never $\frac{\theta}{2\pi}$. Check your GDC is in DEGREE mode before computing any arc or sector value.
3. **Wrong perpendicular gradient.** The perpendicular bisector requires the negative reciprocal of the segment's gradient: $m_{\perp} = -1/m$. Forgetting the negative sign or inverting without negating both lose the method mark.
4. **Showing only the equation in a "show that" bisector task.** An AG answer requires the midpoint calculation and the perpendicular gradient both shown explicitly before the printed equation; omitting either step scores zero even if the equation is correct.
5. **Premature rounding in chained calculations.** If an arc length found in part (a) feeds into a cost or perimeter in part (b), carry the full unrounded value; rounding to 3 s.f. too early can shift the final answer outside acceptable range and lose the accuracy mark.
6. **Confusing sector area with triangle or segment area.** The $\frac{\theta}{360}\pi r^2$ formula gives the area of the sector (pie-slice), not the triangle formed by the two radii. When a question asks for a shaded region that combines both, calculate each area separately and add or subtract as appropriate.

## Section A — Easy EASY

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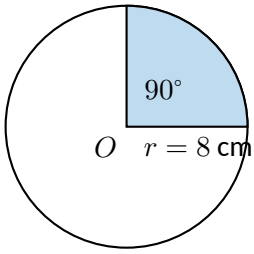
**1.** Point $A$ has coordinates $(1, 3)$ and point $B$ has coordinates $(7, 11)$.

(a) Find the distance $AB$. [1 marks]

(b) Find the coordinates of the midpoint $M$ of $AB$. [1 marks]

(c) Find the gradient of $AB$. [1 marks]

**2.** A sector has radius $r = 8$ cm and an angle of $90°$ at its centre.




(a) Write down the fraction of the full circle represented by this sector. [1 marks]

(b) Find the arc length of the sector. Give your answer in terms of $\pi$. [1 marks]

(c) Find the area of the sector. Give your answer in terms of $\pi$. [1 marks]

**3.** Points $P(2, 5)$ and $Q(8, 5)$ lie on a coordinate grid.

(a) Write down the coordinates of the midpoint $M$ of $PQ$. [1 marks]

(b) Write down the gradient of $PQ$. [1 marks]

(c) Write down the equation of the perpendicular bisector of $PQ$. [1 marks]

**4.** A circular pizza is cut into a sector with centre angle $120°$ and radius $15$ cm.

(a) Find the arc length of the sector. Give your answer correct to 3 significant figures. [1 marks]

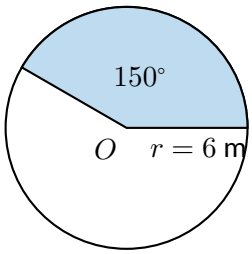
(b) Find the perimeter of the sector. Give your answer correct to 3 significant figures. [2 marks]

**5.** Point $C$ has coordinates $(-3, 2)$ and point $D$ has coordinates $(5, -4)$.

(a) Find the distance $CD$. Give your answer correct to 3 significant figures. [2 marks]

(b) Find the midpoint of $CD$. [1 marks]

**6.** A garden sprinkler rotates through an angle of $150°$ and waters the ground up to a distance of $6$ m from its position. The watered region forms a sector.




Find the area of the watered region. Give your answer correct to 3 significant figures. [3 marks]

**7.** Two points are given: $E(0, 6)$ and $F(8, 0)$.

(a) Find the gradient of $EF$. [1 marks]

(b) Find the gradient of the perpendicular bisector of $EF$. [1 marks]

(c) Find the equation of the perpendicular bisector of $EF$. Give your answer in the form $y = mx + c$. [1 marks]

**8.** A sector has a radius of $10$ cm and an arc length of $14$ cm.

(a) Find the angle $\theta$ at the centre of the sector, in degrees. Give your answer correct to 3 significant figures. [2 marks]

(b) Find the area of the sector. Give your answer correct to 3 significant figures. [1 marks]

**9.** The coordinates of two points are $G(1, -2)$ and $H(9, 4)$.

(a) Find the midpoint $M$ of $GH$. [1 marks]

(b) Find the distance $GH$. Give your answer correct to 3 significant figures. [2 marks]

**10.** A sector $OAB$ has centre $O$, radius $r = 12$ cm and angle $AOB = 60°$.

(a) Find the arc length $AB$. Give your answer in terms of $\pi$. [1 marks]

(b) Find the perimeter of the sector. Give your answer in terms of $\pi$. [1 marks]

(c) Find the area of the sector. Give your answer in terms of $\pi$. [1 marks]

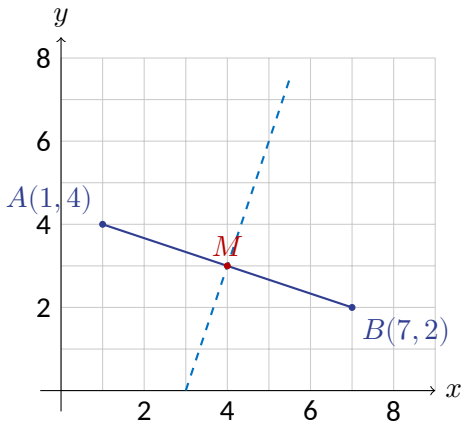
## Section B — Medium MEDIUM

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**11.** A park designer places two water fountains at points $A(1, 4)$ and $B(7, 2)$ on a coordinate grid where each unit represents 1 metre.

(a) Find the coordinates of the midpoint $M$ of $AB$. [1 marks]

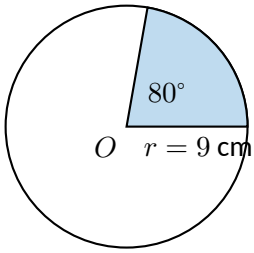
(b) Show that the perpendicular bisector of $AB$ has equation $y = 3x - 9$. [3 marks]



**12.** A slice of cheese is cut in the shape of a sector with radius $r = 9$ cm and angle $\theta = 80°$ at the centre $O$, as shown below.

(a) Calculate the arc length of the sector. [2 marks]

(b) Calculate the perimeter of the sector. [2 marks]




**13.** Two mobile phone towers are located at $P(2, 5)$ and $Q(8, 1)$ on a coordinate grid where each unit represents 1 km. A new relay station must be placed at the point $R$ on the perpendicular bisector of $PQ$ such that $R$ has $x$-coordinate 9.

(a) Find the equation of the perpendicular bisector of $PQ$. [3 marks]

(b) Find the coordinates of $R$. [1 marks]

**14.** A garden sprinkler rotates through an angle of $135°$ and waters a sector of grass with radius $4.2$ m.

(a) Calculate the area of grass watered by the sprinkler. [2 marks]

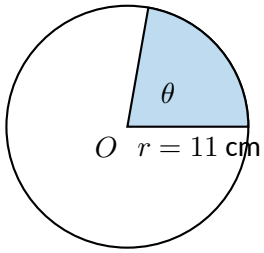
(b) The gardener wants to lay a decorative edging strip around the entire perimeter of the watered sector. Calculate the total length of edging required. [2 marks]

**15.** Points $C(-3, 1)$ and $D(5, 7)$ lie on a coordinate grid where each unit represents 1 metre.

(a) Find the distance $CD$. [2 marks]

(b) A point $E$ lies on the perpendicular bisector of $CD$ and has $y$-coordinate 0. Find the $x$-coordinate of $E$. [2 marks]

**16.** A sector has an area of $85$ cm$^2$ and a radius of $11$ cm. Find the angle $\theta$ at the centre of the sector, giving your answer in degrees. [4 marks]




**17.** A triangular plot of land has vertices at $F(0, 0)$, $G(6, 0)$ and $H(2, 5)$ on a coordinate grid where each unit represents 10 m.

(a) Find the gradient of $FH$. [1 marks]

(b) Find the equation of the perpendicular bisector of $FH$. [3 marks]

**18.** A lawn ornament consists of a sector of radius $7.5$ cm. The arc length of the sector is $14$ cm. Find the angle $\theta$ at the centre of the sector, giving your answer in degrees. [4 marks]

**19.** Two restaurants are located at $J(1, 6)$ and $K(9, 2)$ on a coordinate grid where each unit represents 100 m. A food critic is equidistant from both restaurants. Determine whether the point $L(7, 8)$ lies on the perpendicular bisector of $JK$. Justify your answer. [4 marks]

**20.** A play area is designed in the shape of a sector with radius $5.5$ m and angle $\theta = 160°$. The area is to be covered with rubber matting that costs \$12.50 per m$^2$.

(a) Calculate the area of the sector. [2 marks]

(b) Calculate the total cost of the rubber matting, giving your answer to the nearest dollar. [2 marks]

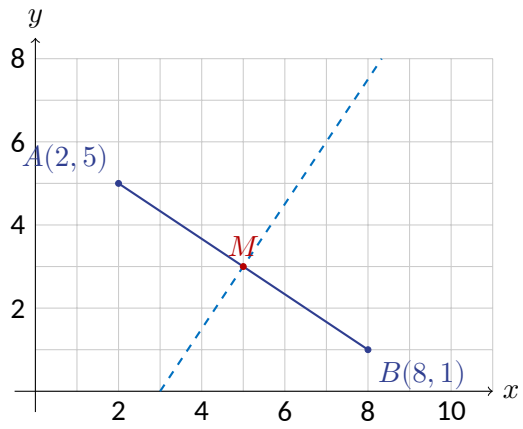
## Section C — Hard HARD

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**21.** A park designer places two water fountains at $A(2, 5)$ and $B(8, 1)$ on a coordinate grid where each unit represents 1 metre.

(a) Show that the perpendicular bisector of $AB$ has equation $y = 3/2x - 9/2$. [3 marks]

(b) A bench is to be placed at point $P(k, 7)$ so that it is equidistant from both fountains. Find the value of $k$. [2 marks]




**22.** A sprinkler system waters a sector-shaped lawn. The sector has centre $O$, radius $r$ metres, and angle $\theta = 140°$. The total perimeter of the sector (including both radii) is 38 m.

(a) Show that $r = \dfrac{38 \times 360}{720 + 280\pi}$, giving the exact value $r \approx 8.55$ m. [3 marks]

(b) Hence find the area of the sector, giving your answer to 3 significant figures. [2 marks]

**23.** Three mobile phone masts are located at $P(1, 4)$, $Q(9, 4)$, and $R(5, 10)$ on a coordinate grid where each unit represents 1 km. A relay station is to be built at the point equidistant from all three masts.

(a) Find the equation of the perpendicular bisector of $PQ$. [1 marks]

(b) Find the equation of the perpendicular bisector of $QR$. [2 marks]

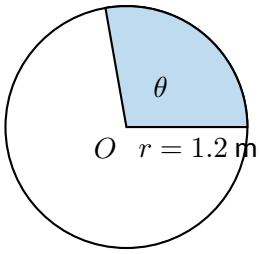
(c) Hence find the coordinates of the relay station. [2 marks]

**24.** A decorative window consists of a sector of a circle with centre $O$, radius $1.2$ m, and arc $ST$. The arc length of $ST$ is $2.1$ m.

(a) Find the angle $\theta$ of the sector in degrees, giving your answer to 3 significant figures. [2 marks]

(b) Find the area of the sector. [2 marks]

(c) The window frame costs \$85 per metre. Find the total cost of framing the perimeter of the sector, giving your answer to the nearest dollar. [1 marks]




**25.** Points $C(-3, 2)$ and $D(5, -4)$ lie on a coordinate plane where each unit represents 1 cm.

(a) Find the distance $CD$. [2 marks]

(b) The perpendicular bisector of $CD$ passes through the point $(1, k)$. Find $k$. [3 marks]

**26.** A garden is designed in the shape of a composite figure: a rectangle of width $w$ metres and height 5 m, attached along one of its $w$-metre edges to a semicircle of diameter $w$ metres. The total area of the garden is 62 m$^2$.

(a) Show that $\dfrac{\pi w^2}{8} + 5w - 62 = 0$. [2 marks]

(b) Find $w$, giving your answer to 3 significant figures. [2 marks]

(c) Hence find the perimeter of the garden, giving your answer to 3 significant figures. [1 marks]

**27.** Two points $E(0, 6)$ and $F(10, 2)$ lie on a coordinate grid where each unit represents 1 m. A point $G$ lies on the perpendicular bisector of $EF$ such that $G$ has $y$-coordinate 0.

(a) Show that the perpendicular bisector of $EF$ has equation $y = 5/2x - 17/2$. [3 marks]

(b) Hence find the coordinates of $G$. [1 marks]

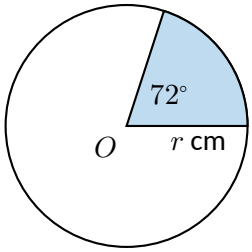
(c) Determine whether $G$ is closer to $E$ or to $F$. Justify your answer. [1 marks]

**28.** A sector $OAB$ has centre $O$, radius $r$ cm, and angle $\angle AOB = 72°$. The area of the sector is $140$ cm$^2$.

(a) Find $r$, giving your answer to 3 significant figures. [2 marks]

(b) Hence find the arc length $AB$, giving your answer to 3 significant figures. [2 marks]

(c) A second sector has the same arc length $AB$ but an angle of $50°$. Find the radius of the second sector, giving your answer to 3 significant figures. [1 marks]




**29.** A circular running track has centre $O$ and radius 50 m. Two runners start at point $A$ and run along the arc to points $B$ and $C$ respectively, where $\angle AOB = 84°$ and $\angle AOC = 210°$ (both measured from $A$ in the same direction).

(a) Find the arc length from $A$ to $B$. [1 marks]

(b) Find the arc length from $B$ to $C$ along the shorter arc $BC$. [2 marks]

(c) The sector $OAB$ is to be painted at a cost of \$12.50 per m$^2$. Find the total painting cost, giving your answer to the nearest dollar. [2 marks]

**30.** Two rescuers are positioned at $J(2, 9)$ and $K(8, 3)$ on a coordinate grid where each unit represents 100 m. A casualty is known to lie on the perpendicular bisector of $JK$.

(a) Find the midpoint $M$ of $JK$ and the gradient of $JK$. [2 marks]

(b) Hence write down the equation of the perpendicular bisector of $JK$. [1 marks]

(c) The casualty is also known to be on the line $y = 2x - 3$. Find the coordinates of the casualty. [2 marks]

## Worked Solutions

### Q1 — Distance Midpoint and Gradient EASY

**(a)** $AB = \sqrt{(7-1)^2 + (11-3)^2} = \sqrt{36+64} = \sqrt{100} = 10$ **A1**

**(b)** $M = \left(\frac{1+7}{2}, \frac{3+11}{2}\right) = (4, 7)$ **A1**

**(c)** $m = \frac{11-3}{7-1} = \frac{8}{6} = 4/3$ **A1**

*Warning: Subtracting coordinates in inconsistent order (e.g. $y_2 - y_1$ in the numerator but $x_1 - x_2$ in the denominator) gives the wrong sign for the gradient.*

### Q2 — Arc Length and Sector Area EASY

**(a)** Fraction $= \frac{90}{360} = 1/4$ **A1**

**(b)** $l = \frac{90}{360} \times 2\pi \times 8 = \frac{1}{4} \times 16\pi = 4\pi$ cm **A1**

**(c)** $A = \frac{90}{360} \times \pi \times 8^2 = \frac{1}{4} \times 64\pi = 16\pi$ cm$^2$ **A1**

*Warning: The $\theta/360$ factor must appear explicitly in arc and area calculations; using $2\pi r$ or $\pi r^2$ alone gives the full circle, not the sector.*

### Q3 — Perpendicular Bisector of a Horizontal Segment EASY

**(a)** $M = \left(\frac{2+8}{2}, \frac{5+5}{2}\right) = (5, 5)$ **A1**

**(b)** $P$ and $Q$ share the same $y$-coordinate, so $m_{PQ} = 0$ **A1**

**(c)** The bisector passes through $M(5,5)$ perpendicular to a horizontal line, so it is vertical: $x = 5$ **A1**

*Warning: The perpendicular bisector of a horizontal segment is vertical; writing $y = 5$ confuses the bisector with the segment itself.*

### Q4 — Sector Arc Length and Perimeter EASY

**(a)** $l = \frac{120}{360} \times 2\pi \times 15 = \frac{1}{3} \times 30\pi = 10\pi \approx 31.4$ cm **A1**

**(b)** $P = l + 2r = 10\pi + 2(15) = 10\pi + 30 \approx 61.4$ cm **M1A1**

*Warning: A very common error is to give only the arc length as the perimeter; the sector perimeter always includes the two radii, so $P = l + 2r$.*

### Q5 — Distance and Midpoint EASY

**(a)** $CD = \sqrt{(5-(-3))^2+(-4-2)^2} = \sqrt{8^2+(-6)^2} = \sqrt{64+36} = \sqrt{100} = 10.0$ cm **M1A1**

**(b)** $M = \left(\dfrac{-3+5}{2}, \dfrac{2+(-4)}{2}\right) = (1,-1)$ **A1**

*Warning: When coordinates are negative, write them with brackets inside the formula to avoid sign errors, e.g.* $5-(-3)=8$*, not* $5-3=2$*.*

### Q6 — Area of a Sector EASY

$A = \dfrac{150}{360} \times \pi \times 6^2 = \dfrac{150}{360} \times 36\pi = 15\pi \approx 47.1$ m$^2$ **M1A1**

*Warning: Using the arc length formula instead of the area formula gives metres, not square metres; always check the formula matches the quantity asked for.*

### Q7 — Perpendicular Bisector Equation EASY

**(a)** $m_{EF} = \dfrac{0-6}{8-0} = \dfrac{-6}{8} = -3/4$ **A1**

**(b)** $m_{\perp} = -\dfrac{1}{-3/4} = 4/3$ **A1**

**(c)** $M = \left(\dfrac{0+8}{2}, \dfrac{6+0}{2}\right) = (4,3)$; $\quad y-3 = \dfrac{4}{3}(x-4) \Rightarrow y = 4/3\,x - 7/3$ **A1**

*Warning: The perpendicular gradient is the negative reciprocal; simply inverting the fraction without changing the sign gives the wrong direction.*

### Q8 — Angle and Area from Arc Length EASY

**(a)** $14 = \dfrac{\theta}{360} \times 2\pi \times 10 \Rightarrow \theta = \dfrac{14 \times 360}{20\pi} \approx 80.2°$ **M1A1**

**(b)** Using $A = \frac{1}{2}lr$ (angle-independent identity, avoids re-rounding $\theta$): $A = \frac{1}{2}(14)(10) = 70$ cm$^2$ **A1**

*Warning: Using the rounded value of* $\theta$ *in part (b) may introduce a rounding error; always carry the unrounded value through to subsequent calculations.*

### Q9 — Midpoint and Distance EASY

**(a)** $M = \left(\dfrac{1+9}{2}, \dfrac{-2+4}{2}\right) = (5,1)$ **A1**

**(b)** $GH = \sqrt{(9-1)^2+(4-(-2))^2} = \sqrt{8^2+6^2} = \sqrt{100} = 10.0$ **M1A1**

*Warning: The difference $4-(-2)=6$, not $4-2=2$; always retain the negative sign inside the bracket before squaring.*

### Q10 — Arc Length Perimeter and Area EASY

**(a)** $l=\frac{60}{360}\times 2\pi\times 12=\frac{1}{6}\times 24\pi=4\pi$ cm **A1**

**(b)** $P=4\pi+2(12)=4\pi+24$ cm **A1**

**(c)** $A=\frac{60}{360}\times\pi\times 12^2=\frac{1}{6}\times 144\pi=24\pi$ cm$^2$ **A1**

*Warning: The perimeter of a sector is $l+2r$, not just the arc length $l$; forgetting the two radii is the most frequent error in sector perimeter questions.*

### Q11 — Perpendicular Bisector of a Segment MEDIUM

**(a)** $M=\left(\frac{1+7}{2},\frac{4+2}{2}\right)=(4,3)$ **A1**

**(b)** $m_{AB}=\frac{2-4}{7-1}=\frac{-2}{6}=-1/3$, so $m_{\perp}=3$ **M1A1**

Using point–slope form with $M(4,3)$: $y-3=3(x-4)\Rightarrow y=3x-12+3$

$y=3x-9$ **A1**

*Warning: Both the midpoint AND the gradient step must be shown explicitly before the equation — omitting either loses the method mark even if the final line is correct.*

### Q12 — Arc Length and Sector Perimeter MEDIUM

**(a)** $l=\frac{80}{360}\times 2\pi\times 9=4\pi\approx 12.6$ cm **M1A1**

**(b)** $P=l+2r=12.566\ldots+2(9)=12.566\ldots+18\approx 30.6$ cm **M1A1**

*Warning: The sector perimeter is NOT just the arc length. Both radii must be added: $P=l+2r$.*

### Q13 — Perpendicular Bisector; Point on It MEDIUM

**(a)** $M=\left(\frac{2+8}{2},\frac{5+1}{2}\right)=(5,3)$; $\quad m_{PQ}=\frac{1-5}{8-2}=-2/3$, so $m_{\perp}=3/2$ **A1M1**

$y-3=\frac{3}{2}(x-5)\Rightarrow y=3/2x-9/2$ **A1**

**(b)** At $x=9$: $y=\frac{3}{2}(9)-\frac{9}{2}=\frac{27}{2}-\frac{9}{2}=9$, so $R=(9,9)$ **A1**

*Warning: The perpendicular bisector passes through the midpoint of $PQ$, not through $P$ or $Q$ themselves.*

## Q14 — Sector Area and Perimeter in Context MEDIUM

**(a)** $A = \dfrac{135}{360} \times \pi \times (4.2)^2 = \dfrac{135 \times 17.64\pi}{360} \approx 20.8\ \text{m}^2$ **M1A1**

**(b)** $l = \dfrac{135}{360} \times 2\pi \times 4.2 = 9.8960\ldots\ \text{m}$; $\quad P = l + 2r = 9.8960\ldots + 8.4 \approx 18.3\ \text{m}$ **M1A1**

*Warning: The edging goes around the entire boundary — arc plus the two straight edges of length $r = 4.2$ m each.*

## Q15 — Distance and Bisector Intersection MEDIUM

**(a)** $CD = \sqrt{(5-(-3))^2 + (7-1)^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10\ \text{m}$ **M1A1**

**(b)** $M = (1, 4)$; $\quad m_{CD} = \dfrac{7-1}{5-(-3)} = 3/4$, so $m_\perp = -4/3$ **M1**

$y - 4 = -\dfrac{4}{3}(x-1) \Rightarrow y = -4/3\,x + 16/3$; $\quad$ setting $y = 0$: $x = 4$ **A1**

*Warning: The negative reciprocal of $3/4$ is $-4/3$, not $+4/3$. A sign error here produces a completely different $x$-value for $E$.*

## Q16 — Reverse Sector Area to Angle MEDIUM

$85 = \dfrac{\theta}{360} \times \pi \times 11^2 \Rightarrow \theta = \dfrac{85 \times 360}{\pi \times 121}$ **M1M1**

GDC: evaluate $(85 \times 360) \div (\pi \times 121)$ directly.

$\theta = 80.498\ldots^\circ \approx 80.5^\circ$ **A2**

*Warning: The area formula for a sector in degrees uses $\frac{\theta}{360}\pi r^2$. Using $\frac{1}{2}r^2\theta$ (the radian formula) without converting will give a wildly incorrect angle. Evaluate the whole expression on the GDC in one step to avoid an intermediate arithmetic slip.*

## Q17 — Gradient and Perpendicular Bisector of a Triangle Side MEDIUM

**(a)** $m_{FH} = \dfrac{5-0}{2-0} = 5/2$ **A1**

**(b)** $M = \left(\dfrac{0+2}{2}, \dfrac{0+5}{2}\right) = (1, 2.5)$; $\quad m_\perp = -2/5$ **M1A1**

$y - 2.5 = -\dfrac{2}{5}(x-1) \Rightarrow y = -2/5\,x + 29/10$ (equivalently $y = -0.4x + 2.9$) **A1**

*Warning: The perpendicular bisector of $FH$ passes through the midpoint of $FH$, not through $F$ or $H$.*

### Q18 — Reverse Arc Length to Angle MEDIUM

$14 = \frac{\theta}{360} \times 2\pi \times 7.5 \Rightarrow \theta = \frac{14 \times 360}{15\pi}$ **M1M1**

GDC: evaluate $(14 \times 360) \div (15\pi)$. $\theta = 106.952\ldots^\circ \approx 107^\circ$ **A2**

*Warning: The arc length formula in degrees is $\frac{\theta}{360} \times 2\pi r$, not $\frac{\theta}{360} \times \pi r^2$ (which is area).*

### Q19 — Equidistance via Perpendicular Bisector MEDIUM

$M = \left(\frac{1+9}{2}, \frac{6+2}{2}\right) = (5, 4)$ **M1**

$m_{JK} = \frac{2-6}{9-1} = -1/2$, so $m_\perp = 2$. Bisector: $y - 4 = 2(x-5) \Rightarrow y = 2x - 6$ **A1**

Substituting $L(7, 8)$: $2(7) - 6 = 14 - 6 = 8$ ✓, which matches $L$'s $y$-coordinate **R1**

Since the $y$-value of $L$ matches, $L$ lies on the perpendicular bisector of $JK$, so $L$ is equidistant from both restaurants. **A1**

*Warning: A conclusion without the substitution check earns no reasoning mark.*

### Q20 — Sector Area and Cost MEDIUM

**(a)** $A = \frac{160}{360} \times \pi \times (5.5)^2 = \frac{160 \times 30.25\pi}{360} \approx 42.2\ \text{m}^2$ **M1A1**

**(b)** Cost $= 42.237\ldots \times 12.50 = 527.966\ldots \approx \$528$ **M1A1**

*Warning: Use the unrounded area $42.237\ldots \text{m}^2$ in the cost calculation, not the rounded value $42.2\ \text{m}^2$.*

### Q21 — Perpendicular Bisector and Equidistant Point HARD

**(a)** Midpoint $M = \left(\frac{2+8}{2}, \frac{5+1}{2}\right) = (5, 3)$ **M1**

$m_{AB} = \frac{1-5}{8-2} = \frac{-4}{6} = -2/3$, so the perpendicular gradient is $m_\perp = 3/2$ **A1**

Using point–slope form through $M(5, 3)$: $y - 3 = \frac{3}{2}(x-5) \Rightarrow y = \frac{3}{2}x - \frac{15}{2} + 3 = \frac{3}{2}x - \frac{9}{2}$

$y = 3/2x - 9/2$ **A1 AG**

**(b)** Point $P(k, 7)$ lies on the bisector, so substitute $y = 7$: $7 = \frac{3}{2}k - \frac{9}{2} \Rightarrow \frac{3}{2}k = 7 + \frac{9}{2} = \frac{23}{2}$

$k = 23/3 \approx 7.67$ **M1A1**

*Warning: Substituting $x = 7$ instead of $y = 7$ is a common coordinate swap error.*

## Q22 — Sector Perimeter Reversal and Area HARD

**(a)** The perimeter of a sector includes the arc AND both radii: $P = \frac{\theta}{360} \times 2\pi r + 2r$.

Setting up: $\frac{140}{360} \times 2\pi r + 2r = 38$ **M1**

$r\left(\frac{280\pi}{360} + 2\right) = 38 \Rightarrow r\left(\frac{280\pi + 720}{360}\right) = 38 \Rightarrow r = \frac{38 \times 360}{280\pi + 720}$ **A1**

GDC: evaluate $\frac{38 \times 360}{280\pi + 720}$: unrounded $= 8.5518\ldots \approx 8.55$ m **AG**

**(b)** Using $r = 8.5518\ldots$ m (unrounded value): Area $= \frac{140}{360} \times \pi \times r^2$ **M1**

GDC: $\frac{140}{360} \times \pi \times (8.5518\ldots)^2$: unrounded $= 89.351\ldots \approx 89.4$ m$^2$ **A1**

*Warning: Evaluate the whole formula for $r$ on the GDC in a single step; splitting the calculation and rounding an intermediate term is what produces an incorrect value here. Always carry the unrounded $r$ into part (b).*

## Q23 — Circumcentre of Three Masts HARD

**(a)** $P(1, 4)$ and $Q(9, 4)$ share the same $y$-coordinate, so the perpendicular bisector of $PQ$ is the vertical line through their midpoint: $x = 5$ **A1**

**(b)** Midpoint of $QR$: $\left(\frac{9+5}{2}, \frac{4+10}{2}\right) = (7, 7)$ **M1**

Gradient of $QR$: $\frac{10-4}{5-9} = \frac{6}{-4} = -3/2$, so $m_\perp = 2/3$

Equation: $y - 7 = \frac{2}{3}(x - 7) \Rightarrow y = 2/3\,x + 7/3$ **A1**

**(c)** Substitute $x = 5$ (from part (a)) into the bisector of $QR$: $y = \frac{2}{3}(5) + \frac{7}{3} = \frac{10+7}{3} = 17/3$ **M1**

Relay station at $(5, 17/3) \approx (5, 5.67)$ km. **A1**

*Warning: Using the gradient of $QR$ (not its negative reciprocal) when writing the perpendicular bisector is the standard trap; always negate and invert.*

## Q24 — Sector Angle Reversal Area and Cost HARD

**(a)** $2.1 = \frac{\theta}{360} \times 2\pi(1.2)$ **M1**

$\theta = \frac{2.1 \times 360}{2\pi \times 1.2}$: unrounded $= 100.268\ldots^\circ \approx 100^\circ$ (3 s.f.) **A1**

**(b)** Area $= \frac{\theta}{360} \times \pi r^2 = \frac{100.268\ldots}{360} \times \pi \times 1.2^2$: unrounded $= 1.2600\ldots \approx 1.26$ m$^2$ **M1A1**

**(c)** Sector perimeter $=$ arc $+2r = 2.1 + 2(1.2) = 4.5$ m. Cost $= 4.5 \times 85 = \$382.50 \approx \$383$ **A1**

*Warning: Calculating cost on the arc length alone (omitting both radii) is the classic sector-perimeter trap.*

## Q25 — Distance and Equidistant Point on Bisector HARD

**(a)** $CD = \sqrt{(5-(-3))^2 + (-4-2)^2} = \sqrt{64+36} = \sqrt{100} = 10$ cm **M1A1**

**(b)** Midpoint of $CD$: $(1, -1)$. Gradient of $CD$: $\dfrac{-4-2}{5-(-3)} = \dfrac{-6}{8} = -3/4$, so $m_\perp = 4/3$ **M1**

Equation of bisector through $(1,-1)$: $y+1 = \dfrac{4}{3}(x-1) \Rightarrow y = 4/3\,x - 7/3$

Substitute $x = 1$: $k = \dfrac{4}{3}(1) - \dfrac{7}{3} = \dfrac{4-7}{3} = -1$ **A1**

*Warning: Computing the gradient of $CD$ and using it (rather than its negative reciprocal) as the perpendicular bisector gradient will give an incorrect value of $k$.*

## Q26 — Composite Area Equation Solving and Perimeter HARD

**(a)** The rectangle has area $5w$ m$^2$. The semicircle has diameter $w$, so radius $w/2$, giving area $\dfrac{1}{2}\pi\left(\dfrac{w}{2}\right)^2 = \dfrac{\pi w^2}{8}$.

Total area: $\dfrac{\pi w^2}{8} + 5w = 62$, which rearranges to $\dfrac{\pi w^2}{8} + 5w - 62 = 0$ **M1A1 AG**

**(b)** GDC: Equation solver (or Graph): solve $\dfrac{\pi w^2}{8} + 5w - 62 = 0$ for $w > 0$. **M1**

GDC: positive root: unrounded $= 7.7196\ldots \approx 7.72$ m **A1**

**(c)** Perimeter $=$ two lengths of rectangle $+$ one width $+$ semicircle arc $= 2(5) + w + \pi\left(\dfrac{w}{2}\right) = 10 + 7.7196\ldots + \pi(3.8598\ldots)$: unrounded $= 29.8456\ldots \approx 29.8$ m **A1**

*Warning: Solve the quadratic in $w$ with the GDC equation solver rather than estimating a round-looking value; always substitute your root back into $\frac{\pi w^2}{8} + 5w - 62$ and check it equals zero. Including the base of the rectangle that is shared with the semicircle in part (c) would double-count that edge.*

## Q27 — Perpendicular Bisector Show-That Intercept and Comparison HARD

**(a)** Midpoint of $EF$: $\left(\dfrac{0+10}{2}, \dfrac{6+2}{2}\right) = (5, 4)$ **M1**

Gradient of $EF$: $\dfrac{2-6}{10-0} = \dfrac{-4}{10} = -2/5$, so $m_\perp = 5/2$ **A1**

Through $(5,4)$: $y - 4 = \dfrac{5}{2}(x-5) \Rightarrow y = \dfrac{5}{2}x - \dfrac{25}{2} + 4 = \dfrac{5}{2}x - \dfrac{17}{2}$

$y = 5/2x - 17/2$ **A1 AG**

**(b)** Set $y = 0$: $0 = \dfrac{5}{2}x - \dfrac{17}{2} \Rightarrow x = \dfrac{17}{5} = 3.4$. So $G = (3.4, 0)$. **A1**

**(c)** Since $G$ lies on the perpendicular bisector of $EF$, it is equidistant from $E$ and $F$: $GE = GF$. **R1**

Therefore $G$ is equally close to both $E$ and $F$ — neither is closer (equidistant). **(dependent on R1)**

*Warning: The perpendicular gradient must be the negative reciprocal of the segment gradient; sign errors*

*here cause the equation to fail the midpoint check. Computing $GE$ and $GF$ numerically and finding them unequal indicates an arithmetic error — the defining property of the perpendicular bisector guarantees equal distances.*

## Q28 — Sector Radius Reversal Arc Length and Second Sector HARD

**(a)** $\frac{72}{360}\pi r^2 = 140 \Rightarrow r^2 = \frac{140 \times 360}{72\pi}$ **M1**

GDC: $r = \sqrt{\frac{140 \times 360}{72\pi}}$: unrounded $= 14.9270\ldots \approx 14.9$ cm **A1**

**(b)** Arc length $AB = \frac{72}{360} \times 2\pi r = \frac{72}{360} \times 2\pi \times 14.9270\ldots$

Unrounded $= 18.7578\ldots \approx 18.8$ cm **M1A1**

**(c)** For the second sector: $\frac{50}{360} \times 2\pi r_2 = 18.7578\ldots \Rightarrow r_2 = \frac{18.7578\ldots \times 360}{50 \times 2\pi}$: unrounded $= 21.4949\ldots \approx 21.5$ cm **A1**

*Warning: Carry the full unrounded radius from (a) into (b) — using the rounded $r = 14.9$ shifts the arc length outside the 3 s.f. acceptable range. Using $50^\circ$ in the area formula instead of the arc-length formula confuses the two sector properties.*

## Q29 — Arc Lengths on Circular Track and Sector Cost HARD

**(a)** Arc $AB$: $l_{AB} = \frac{84}{360} \times 2\pi \times 50$: unrounded $= 73.303\ldots \approx 73.3$ m **A1**

**(b)** Angle $\angle BOC = 210^\circ - 84^\circ = 126^\circ$ (since both are measured from $A$ in the same direction). **M1**

Arc $BC = \frac{126}{360} \times 2\pi \times 50$: unrounded $= 109.956\ldots \approx 110$ m **A1**

**(c)** Area of sector $OAB = \frac{84}{360} \times \pi \times 50^2$: unrounded $= 1832.59\ldots$ m$^2$ **M1**

Cost $= 1832.59\ldots \times 12.50 = 22907.4\ldots \approx \$22907$ **A1**

*Warning: Using the reflex angle $360^\circ - 126^\circ = 234^\circ$ would give the longer arc, not the shorter arc $BC$ as required.*

## Q30 — Perpendicular Bisector and Simultaneous Equations HARD

**(a)** Midpoint $M = \left(\frac{2+8}{2}, \frac{9+3}{2}\right) = (5, 6)$ **A1**

Gradient of $JK$: $\frac{3-9}{8-2} = \frac{-6}{6} = -1$ **A1**

**(b)** Perpendicular gradient $= 1$ (negative reciprocal of $-1$). Through $M(5, 6)$: $y - 6 = 1(x - 5) \Rightarrow y = x + 1$ **A1**

**(c)** Solve simultaneously: $y = x + 1$ and $y = 2x - 3$. **M1**
$x + 1 = 2x - 3 \Rightarrow x = 4,\ y = 5$. Casualty at $(4, 5)$, i.e. 400 m east and 500 m north of the grid origin. **A1**
*Warning: The negative reciprocal of $-1$ is $+1$, not $-1$; this sign flip is easy to miss when the gradient is $\pm 1$.*